FAKULTA
STAVEBNÍ
ČVUT V PRAZE

CM01 – Concrete and Masonry Structures 1

HW7 – Flat slab (dimensions, moment)

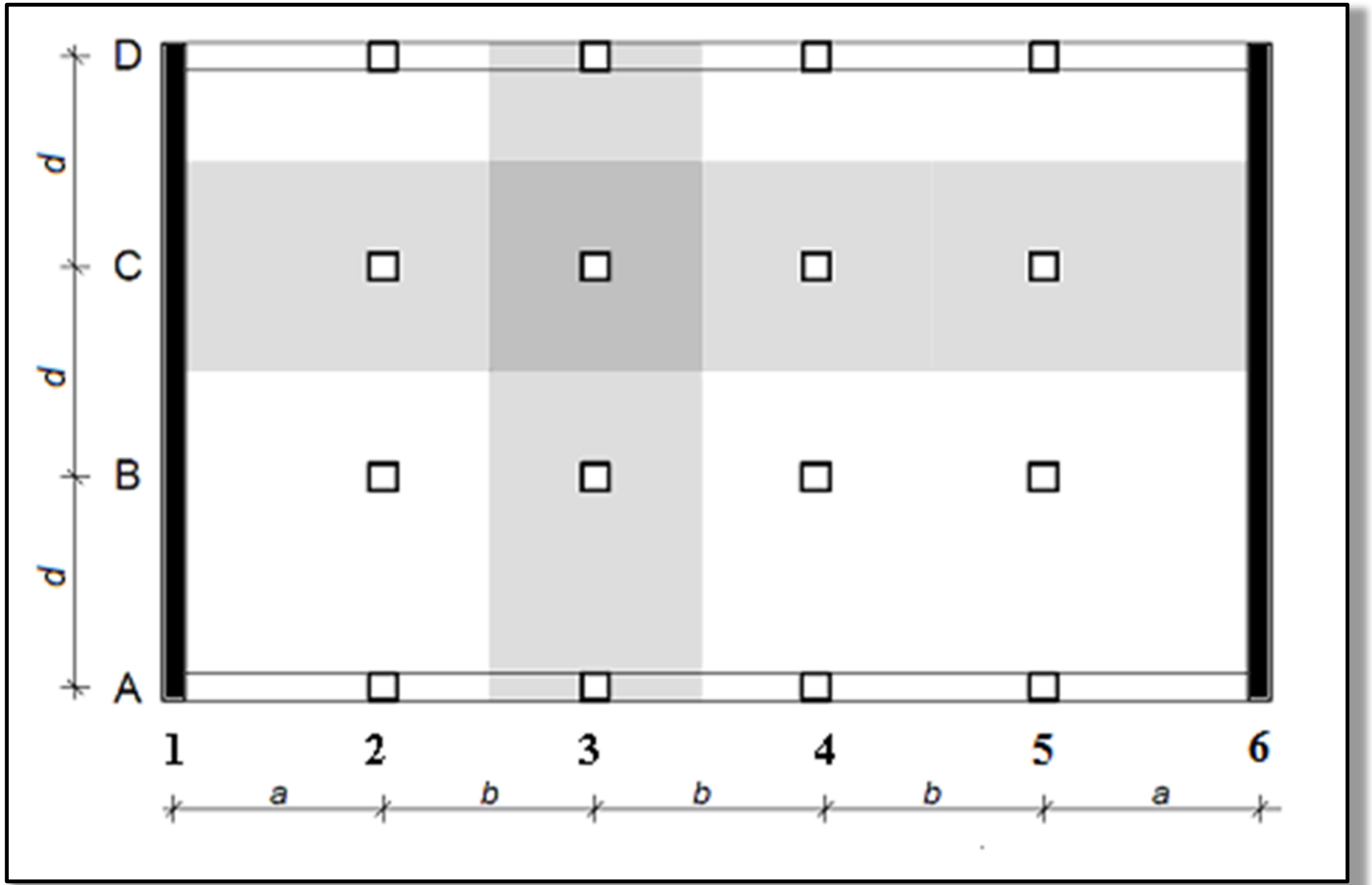
Author: Jakub Holan

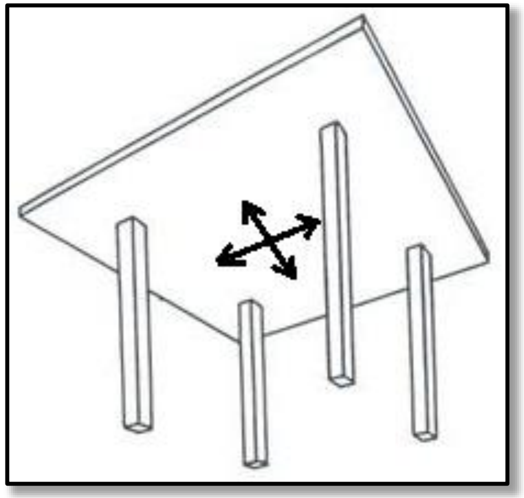
Last update: 01.11.2023 10:37

Task 3

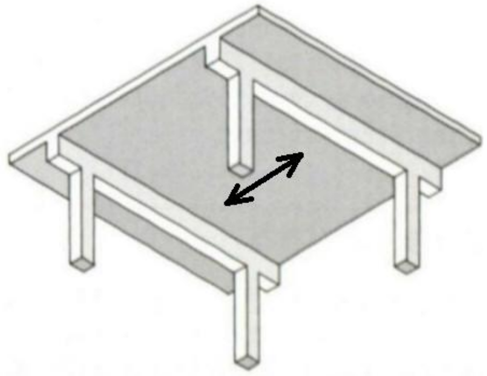
Task 3 – Flat slab

In Task 3, a two-way flat slab (slab supported by columns) will be designed.

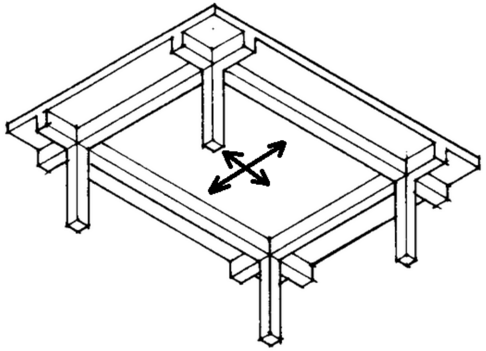




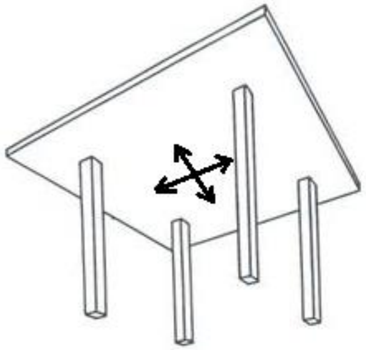
Comparison of Tasks 1 to 3



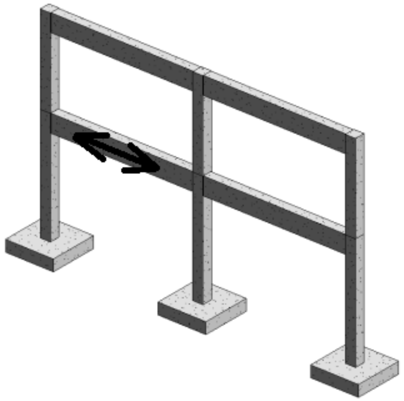
One-way slab –
Task 1



Two-way slab supported on 4 sides – **Task 2**



Two-way flat slab – **Task 3**



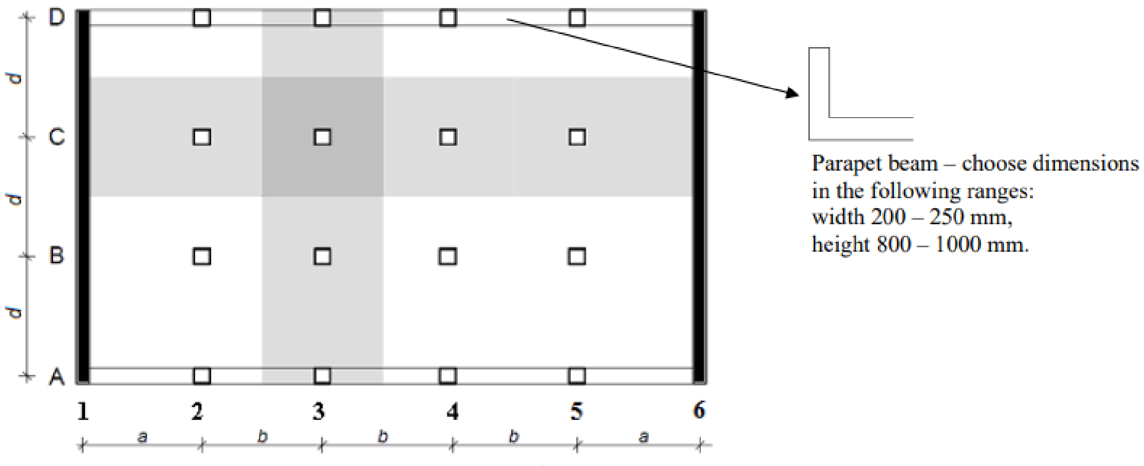
Beam (frame) –
Task 1

Task 3 – Assignment

Task 3: Two-way slab supported on columns

Two-way slab supported on columns, edge beam in axes A and D, walls in axes 1 and 6.

Scheme of the structure:



Individual parameters (parameters in **bold** you can find on teacher's website):

Geometry: ***a*, *b*, *d*** [m] – horizontal dimensions of the structure (***a*** see 1st task, ***b*** see 2nd task), ***n*** – number of floors (see 1st task)

Materials: **see 1st task**

Loads: **see 1st task**, values for typical floor (except the self-weight, which will be different)

Please work out:

1. **Design of the dimensions of the load-bearing elements** (slab, columns). Choose the thickness of the wall as 200 mm or 250 mm.
2. **Sketch of the structure.**
3. **Structural analysis of the slab**:
   - Bending moments in strips C and 3 using the *Direct design method*.
   - Draw moment curves for both strips.
   - Design slab reinforcements (rebars) for the calculated moments.
   - Design the punching reinforcement for column C3.
4. **Layout of reinforcement** (separately for upper and lower layer of reinforcement).

Task 3 – Assignment goals

1) Design of the **dimensions** of the load-bearing elements and **sketch the structure**.
2) Preliminarily **check punching**.
3) Calculate **bending moments** in lanes C and 3.
4) Design slab **bending reinforcements** (rebars) for the calculated moments.
5) Design the **punching reinforcement** for column C3.
6) Draw a **layout of the reinforcement** .

1) Design of dimensions

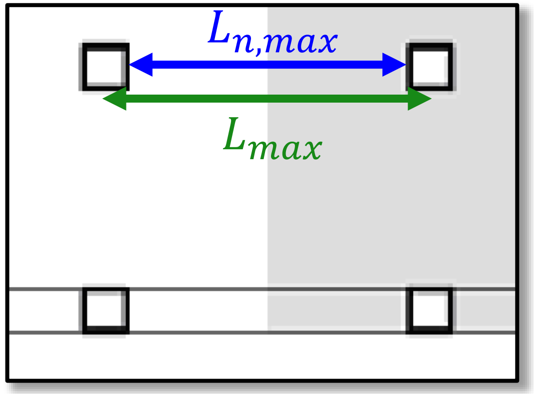
Design of dimensions

We need to design the dimensions of all structural elements, i.e.:

- thickness of the concrete slab,
- thickness of the concrete wall,
- dimensions of the perimeter beam,
- dimensions of the column.

Slab thickness

We will design the slab thickness using the **empirical relationship** and check the **span/depth** ratio (as in Task 1).



$$h_{d,1} = \frac{L_{n,max}}{30}$$

$$h_{d,2} = c + \frac{\varnothing_s}{2} + \frac{L_{max}}{\kappa_{c1}\kappa_{c2}\kappa_{c3}\lambda_{d,tab}}$$

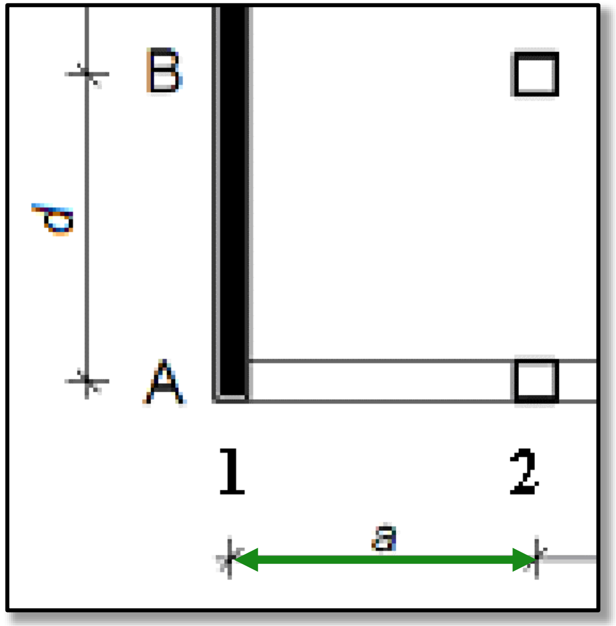
Minimal slab thickness for a slat slab is 200 mm!

ρ [%]	Concrete class								
	C 12/15	C 16/20	C 20/25	C 25/30	C 30/37	C 35/45	C 40/50	C 45/55	C 50/60
0,5	17,5	19,0	20,4	22,2	24,6	27,6	30,9	34,5	38,4
1,5	14,6	15,1	15,6	16,2	16,8	17,4	18,0	18,6	19,2

Wall thickness

For $a \leq 6\ \mathrm{m}$, design the wall as 200 mm thick.

For $a > 6\ \mathrm{m}$, design the wall as 250 mm thick.

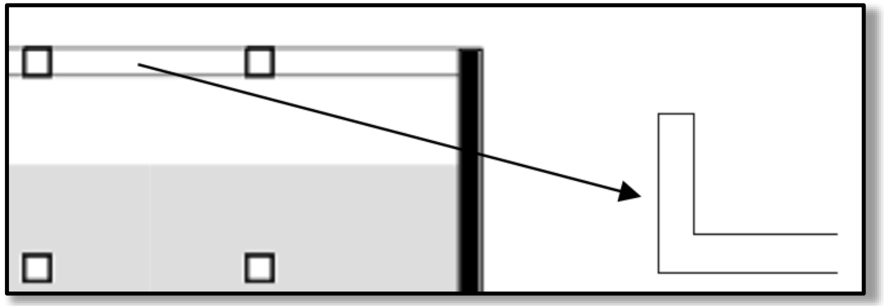


Note: This is a BIG simplification used only for this task. **Do not design walls in this way in real projects!**

Perimeter beam dimensions

For the perimeter beam, choose the dimensions anywhere in the following ranges:

- width 200 mm to 250 mm,
- height 800 mm to 1000 mm.



Both width and height should be a **multiple of 50 mm**.

Note: This is a BIG simplification used only for this task. **Do not design walls in this way in real projects!**

Column dimensions

Design the column dimensions in the same way as in Task 1 – i.e.:

1) Calculate the total area load of the slab $\boldsymbol{f_d}$ $[\mathbf{kN/m^2}]$ (apart from self-weight, all other loads are the same as in Task 1).
2) Calculate the tributing area of the column.
3) Determine the normal force at the foot of the 1st floor column (N_{Ed}).
4) Assuming the centric-load condition, we calculate the **required cross-sectional area**:

$$A_{c,req} = \frac{N_{Ed}}{0.8 f_{cd} + 0.02 \cdot 400\ \mathrm{MPa}}$$

5) Design a **square cross-section** with cross-sectional area (A_c) greater than the required cross-sectional area ($A_{c,req}$).

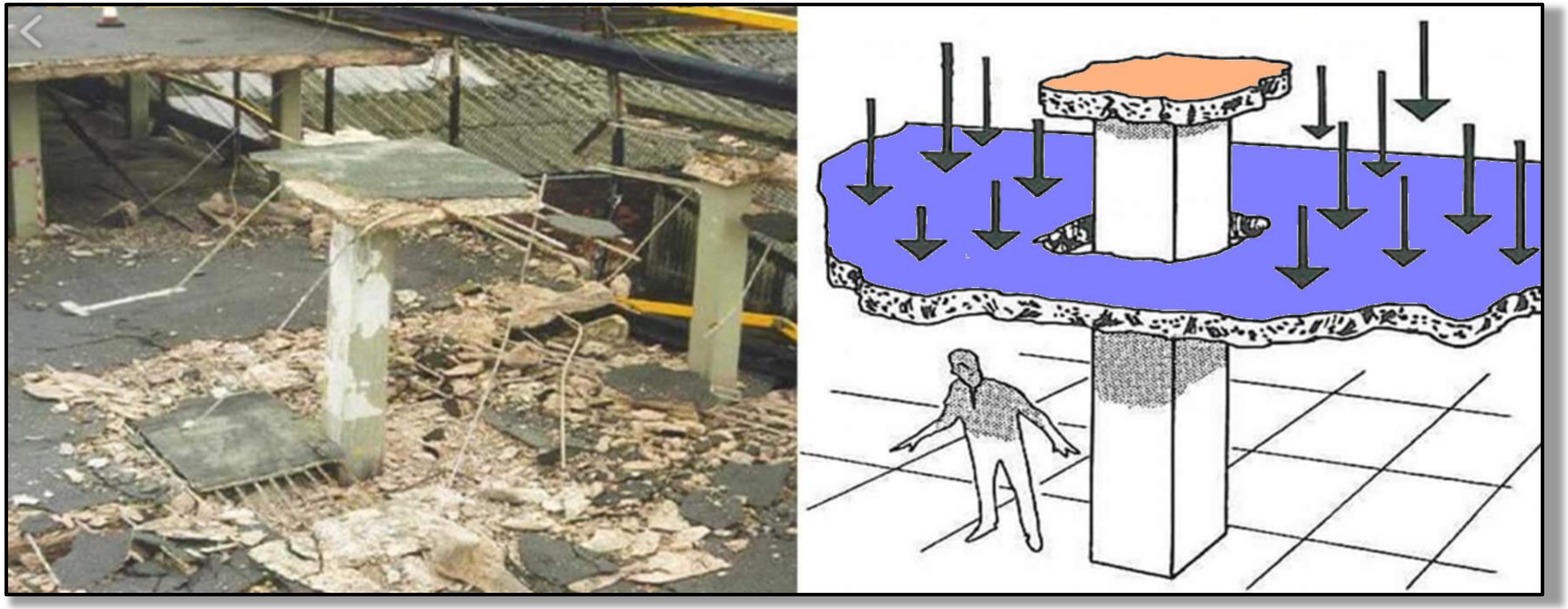
Sketch

After you design all dimensions, do a simple sketch of the plan of your structure with your dimensions (i.e., the given floor dimension and designed member dimensions).

2) Preliminary check of slab punching

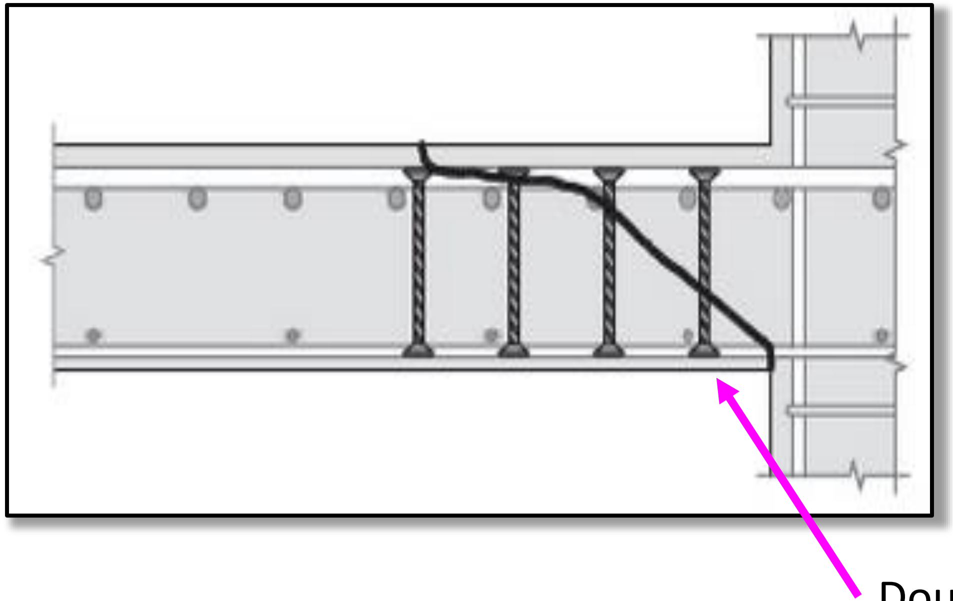
Slab punching

Flat slab have **no beams** ⟶ **load from a large area** of the slab is transferred directly to the column **through a small area** ⟶ **big concentrated stresses** occurs near the column ⟶ this can lead to **slab punching** (a shear failure of the flat slab).



Punching reinforcement

To avoid punching of the slab, punching reinforcement must be placed near the column.



Double-headed studs

Types of punching failure

There are **two types of punching failure which can occur** in the slab.

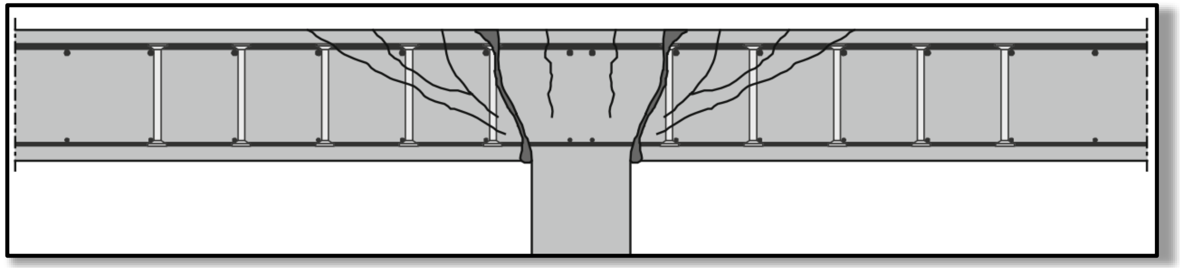
Failure of compressed concrete

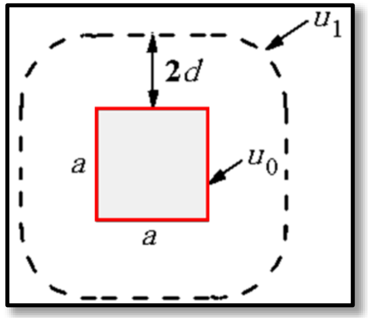
Failure of punching reinforcement

Types of punching failure

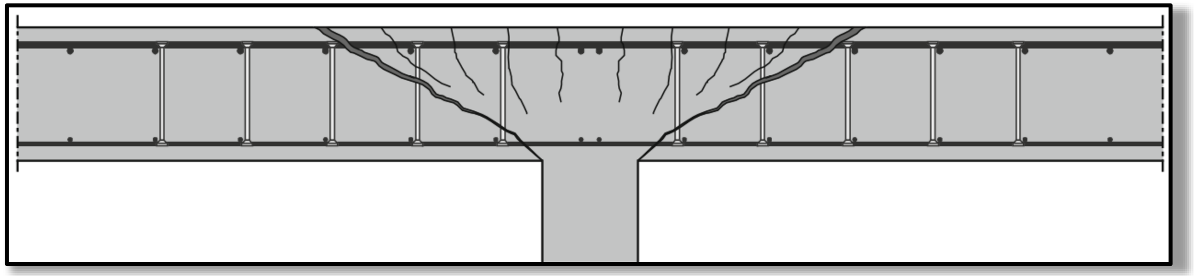
There are **two types of punching failure which can occur** in the slab.

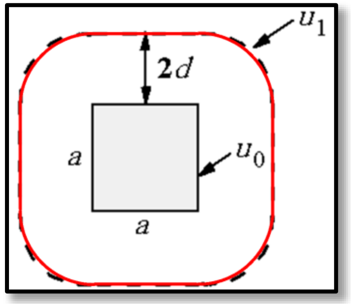
Failure of compressed concrete (at u_0 perimeter)





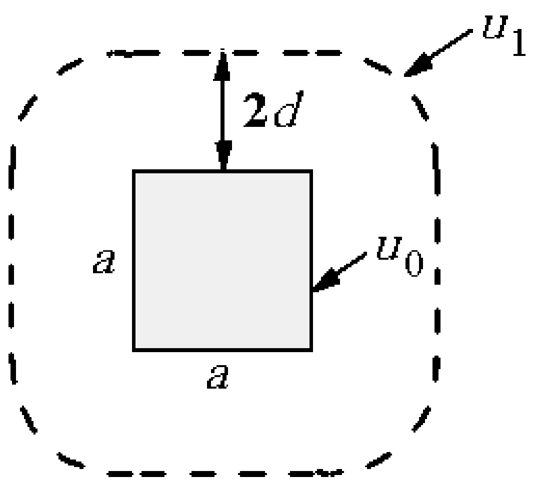
Failure of punching reinforcement (at u_1 perimeter)





Control perimeters

The control perimeters, in which the punching is assumed to occur, are at the **face of the column** (u_0) and at the distance of $\mathbf{2d}$ **from the face of the column** (u_1).



$$u_0 = 4a$$

$$u_1 = 4a + 2\pi \cdot 2d$$

Effective depth of your slab

Failure of compressed concrete

In perimeter u_0, the shear force induces high compressive stress in concrete. We must check whether the **load-bearing capacity of compressed concrete** (i.e., the "*Maximum punching shear resistance*") is sufficient.

Stress in perimeter u_0

Maximum punching shear resistance

$$v_{\mathrm{Ed},0} = \frac{\beta V_{\mathrm{Ed}}}{u_0 d} \leq v_{\mathrm{Rd,max}} = 0.4 \nu f_{\mathrm{cd}}$$

Coefficient expressing the position of the column (for inner columns β = 1.15)

Shear force equal to the normal force in the column from **<u>ONE</u>** floor (do <u>not</u> sum the forces from all floors!)

Coefficient expressing the effect of additional stresses:

$$\nu = 0,6\left(1 - \frac{f_{\mathrm{ck}}}{250}\right)$$

Coefficient expressing the effect of shear on compressive strength

Note:
Punching reinforcement does NOT help with the maximum punching shear resistance!

Failure of compressed concrete

If the condition for the *maximum punching shear resistance* is **not satisfied**, the structure will fail due to crushing of concrete, no matter how much reinforcement you provide.

Thus, you **must adjust your design** and **recalculate the condition** with the new values.

Adjusting the design

If the condition for the maximum punching shear resistance is not satisfied, you can:

- ?

$$v_{\mathrm{Ed},0} = \frac{\beta V_{\mathrm{Ed}}}{u_0 d} \leq v_{\mathrm{Rd,max}} = 0.4\nu f_{\mathrm{cd}}$$

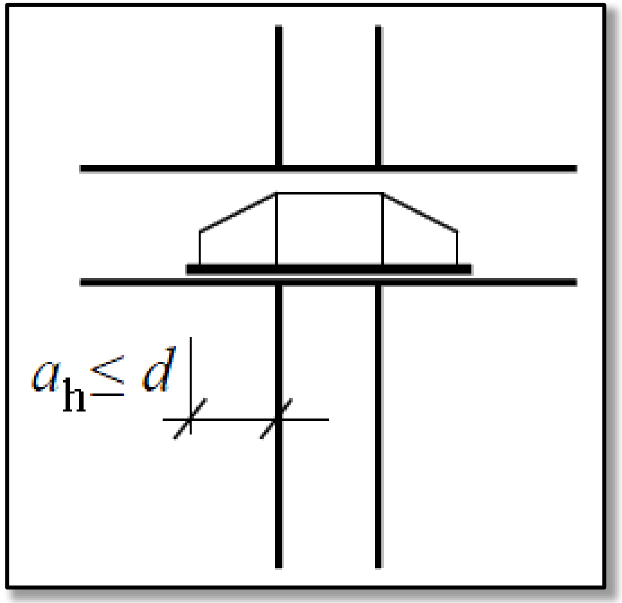
Adjusting the design

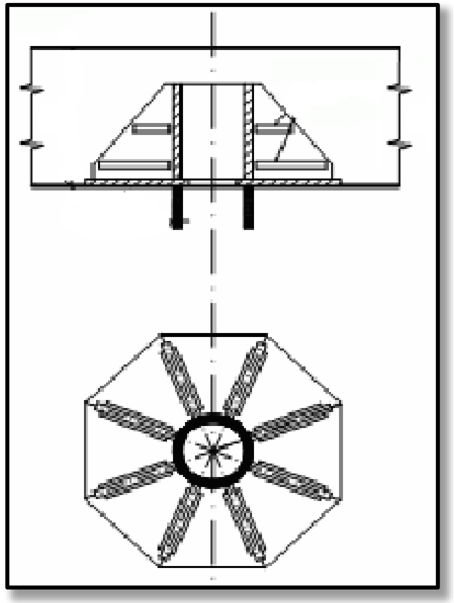
If the condition for the maximum punching shear resistance is not satisfied, you can:

- increase **depth of the slab** (not effective because the self-wight is also increased),
- increase **dimension of the column** (effective, but floor area is decreased),
- increase **concrete class** (expensive),
- design a slab with **drops or flat beams** (complicated),
- design columns with **hidden caps**.

Columns with hidden caps

In the homeworks, we will use steel flanged caps/collars (welded steel details) that are put into the slab-column joint.

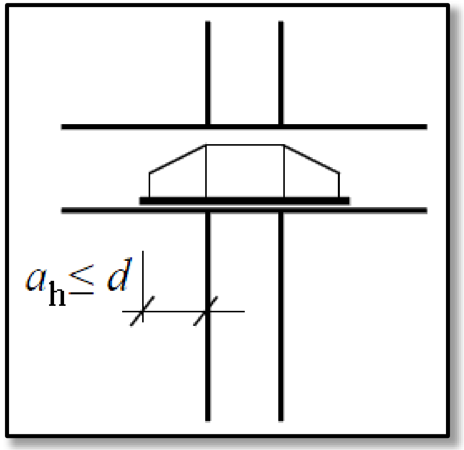




Columns with hidden caps

The **cap increases both control perimeters**:

$$u_0 = 4 \cdot (a + 2a_h)$$
$$u_1 = 4 \cdot (a + 2a_h) + 2\pi 2d$$



Thanks to the increased control perimeters, the **stresses in the control perimeters will decrease**. (Note: Only the stresses will be affected. Resistances will not be changed.)

$$v_{Ed,0} = \frac{\beta V_{Ed}}{u_0 d} \leq v_{Rd,max} = 0.4\nu f_{cd}$$

Failure of punching reinforcement

We will design and assess the **punching reinforcement at later stages** of the task.

However, at this point, **we must assess whether we will be able to anchor the punching reinforcement**.

Failure of punching reinforcement

Condition for the anchorage of the punching reinforcement in concrete:

$$v_{\mathrm{Ed},1} = \frac{\beta V_{\mathrm{Ed}}}{u_1 d} \leq k_{\max} \cdot v_{\mathrm{Rd,c}} = k_{\max} \cdot C_{\mathrm{Rd,c}} \cdot k \cdot \sqrt[3]{(100\rho_l \cdot f_{\mathrm{ck}})}$$

Reinforcement ratio of tensile reinforcement (estimate 0.005)

Effect of depth: $k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$

Reduction factor (0.12)

Coefficient of maximum resistance, see table below.

	effective depth of the slab	k_{max}
STIRRUPS	$d \leq$ 200 mm	1,45
	200 mm $\leq d \leq$ 700 mm	interpolation
	$d \geq$ 700 mm	1,70
Double-headed studs connected to a spacer bar		1,80

Failure of punching reinforcement

If the condition for the anchorage is **not satisfied**, the punching reinforcement needed to withstand the loads will not be anchored in concrete sufficiently, and therefore, the structure will fail.

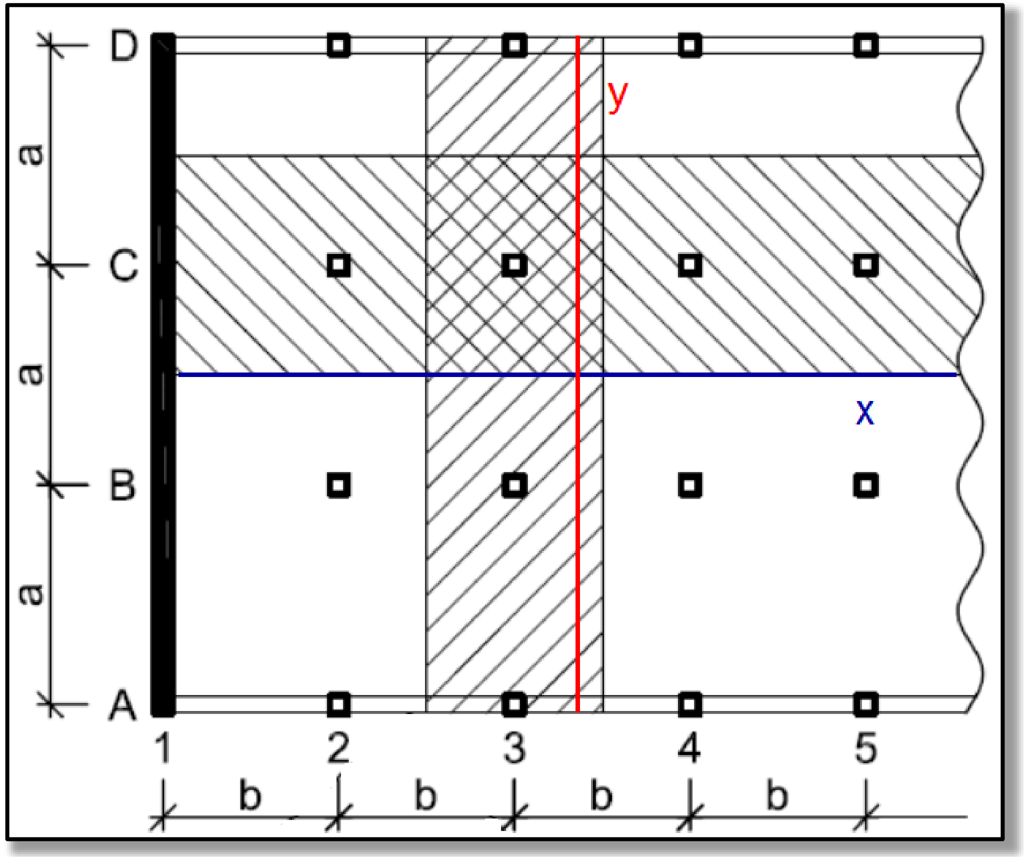
Thus, you **must adjust your design** and **recalculate the condition** with the new values.

You can **adjust the design using the same changes as presented above** for the first condition (e.g., steel caps).

3) Calculate bending moments

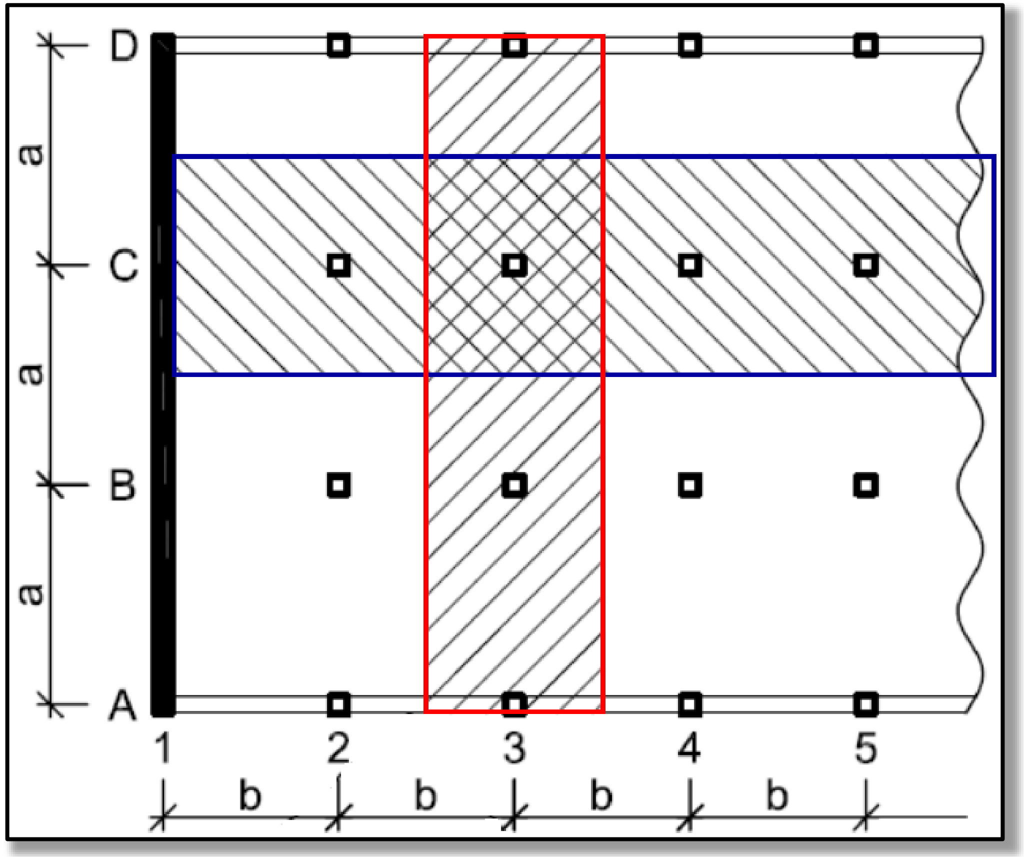
Calculation of bending moments

We must calculate the bending moments in both directions of the slab.



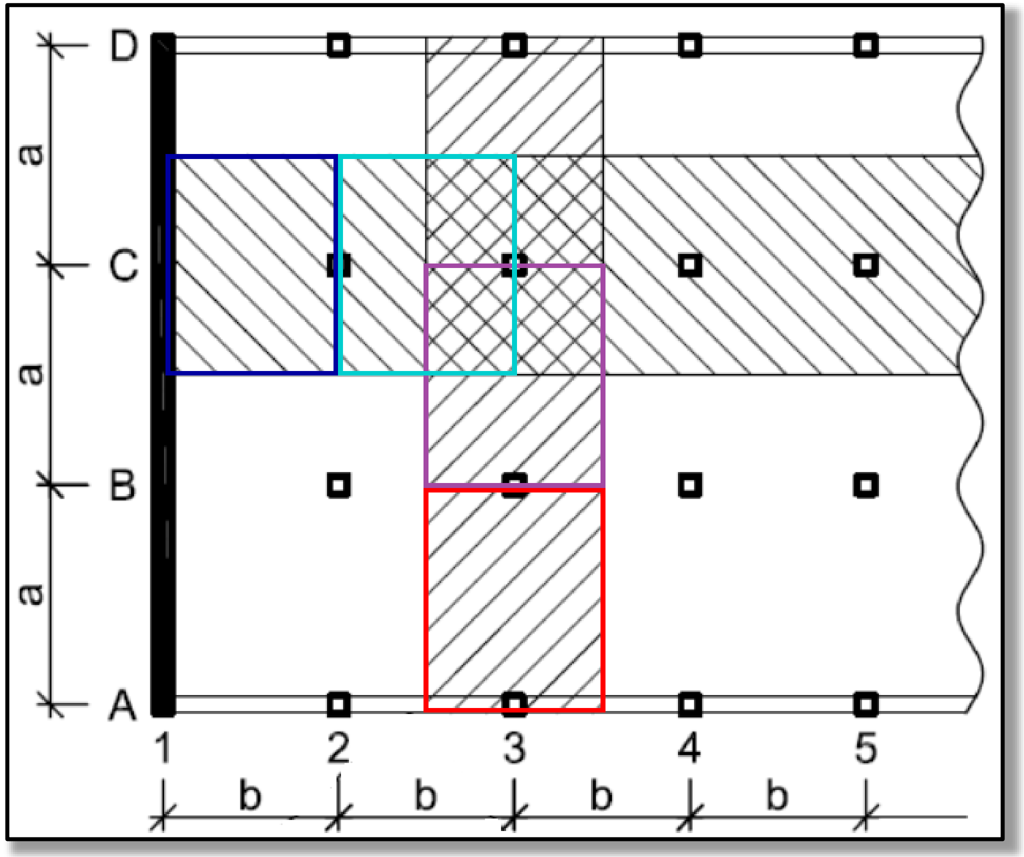
Calculation of bending moments

We will calculate the bending moments only in **lane C** and **lane 3**, and we will use the **Direct Design Method** to calculate them.



Calculation of bending moments

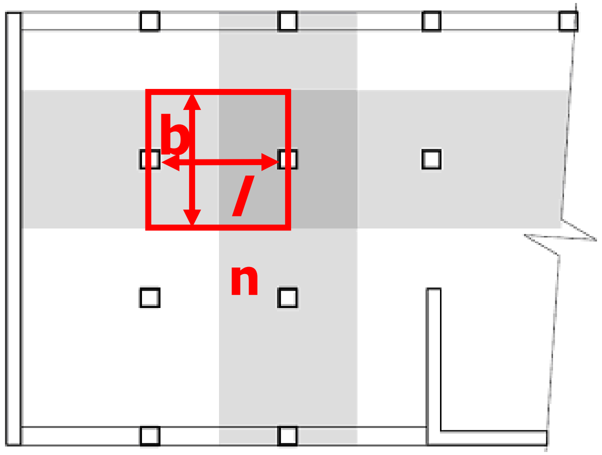
In each lane, we will calculate the moments only in the first two panels (outer panel and the adjacent inner panel).



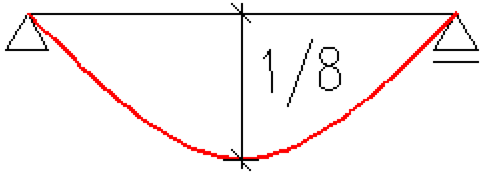
Total bending moment

In each panel, we first must calculate the **Total bending moment** (M_{tot}) as

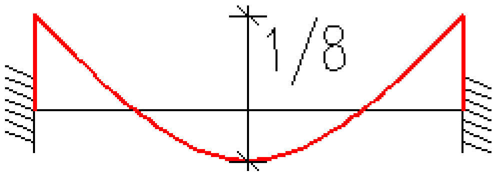
$$M_{tot} = \frac{1}{8} f_d b l_n^2$$



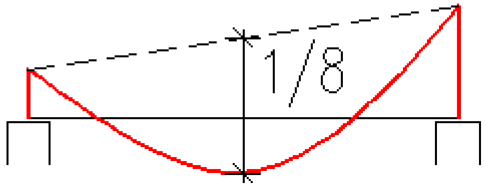
Note: The total bending moment is in kNm (per whole panel width).



Simply supported



Fixed supports



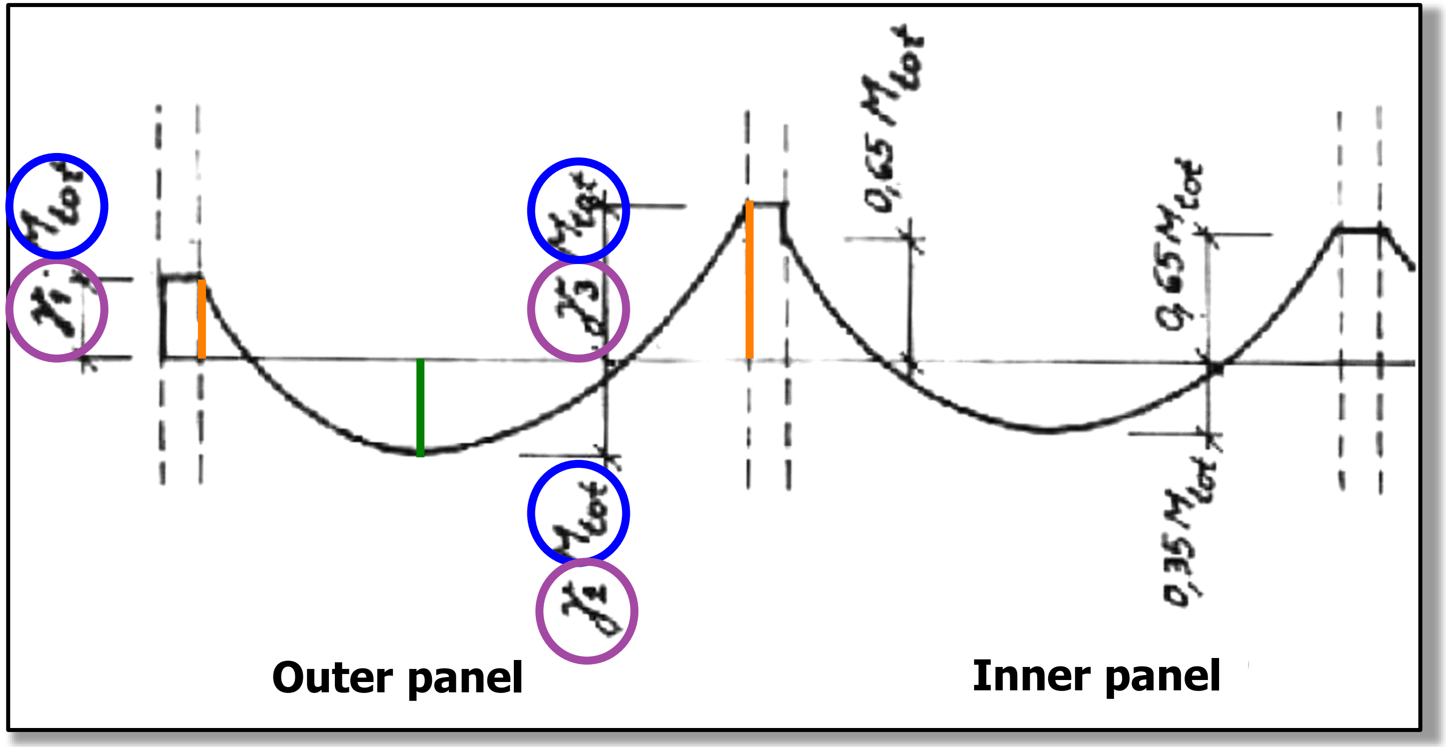
General supports

Support and mid-span moments

Using precalculated coefficients γ, we can divide the Total moment into support and mid-span moments:

$$M = \gamma_i \, M_{tot}$$

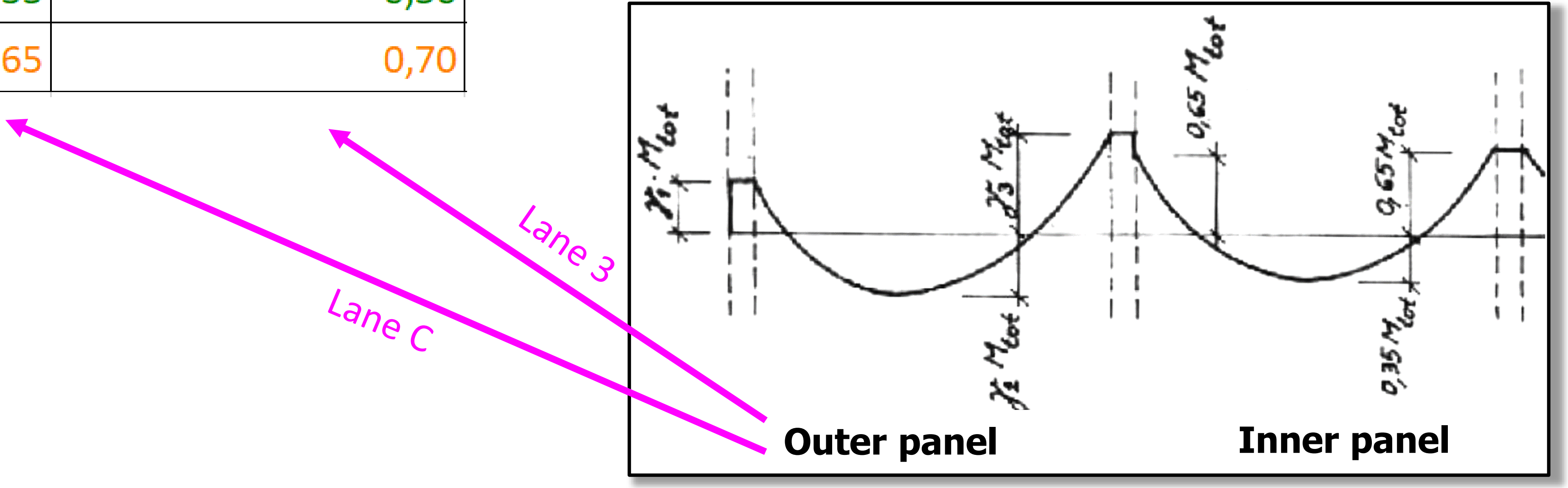
Note:
The support and mid-span moments are in kNm (per whole panel width).



Support and mid-span moments

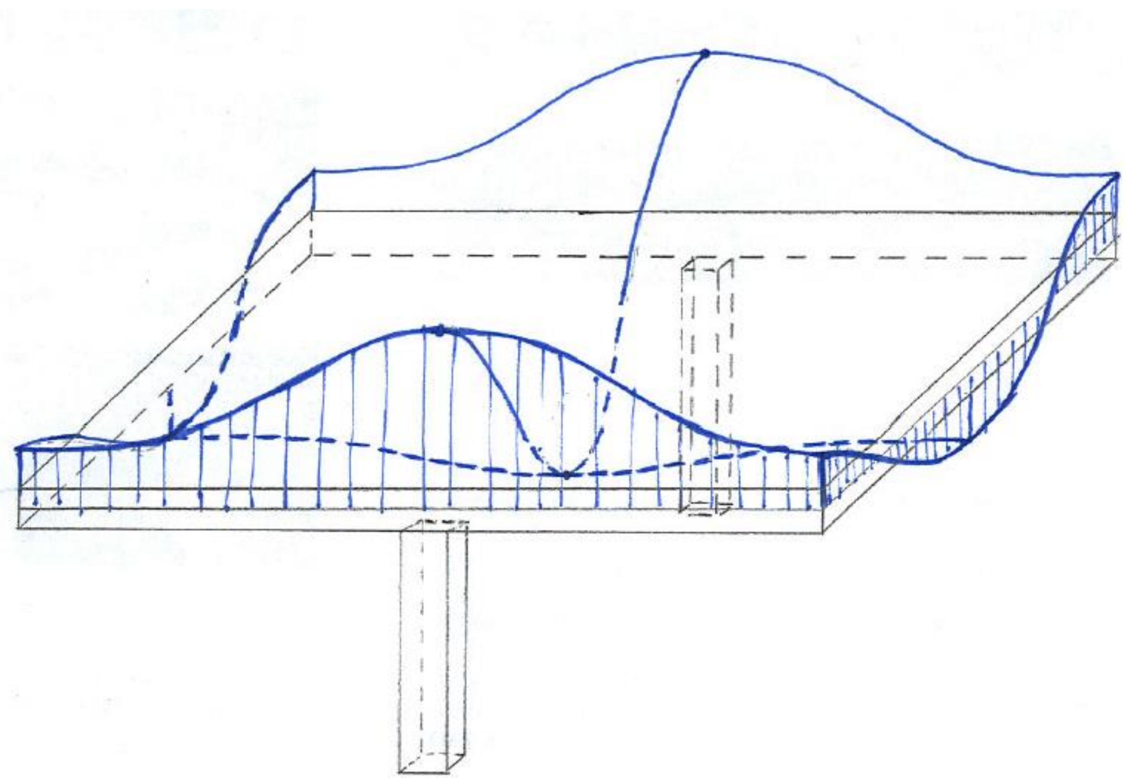
The precalculated coefficients γ are:

	Fixed support (wall or inner column)	Edge beam
γ_1	0,65	0,30
γ_2	0,35	0,50
γ_3	0,65	0,70



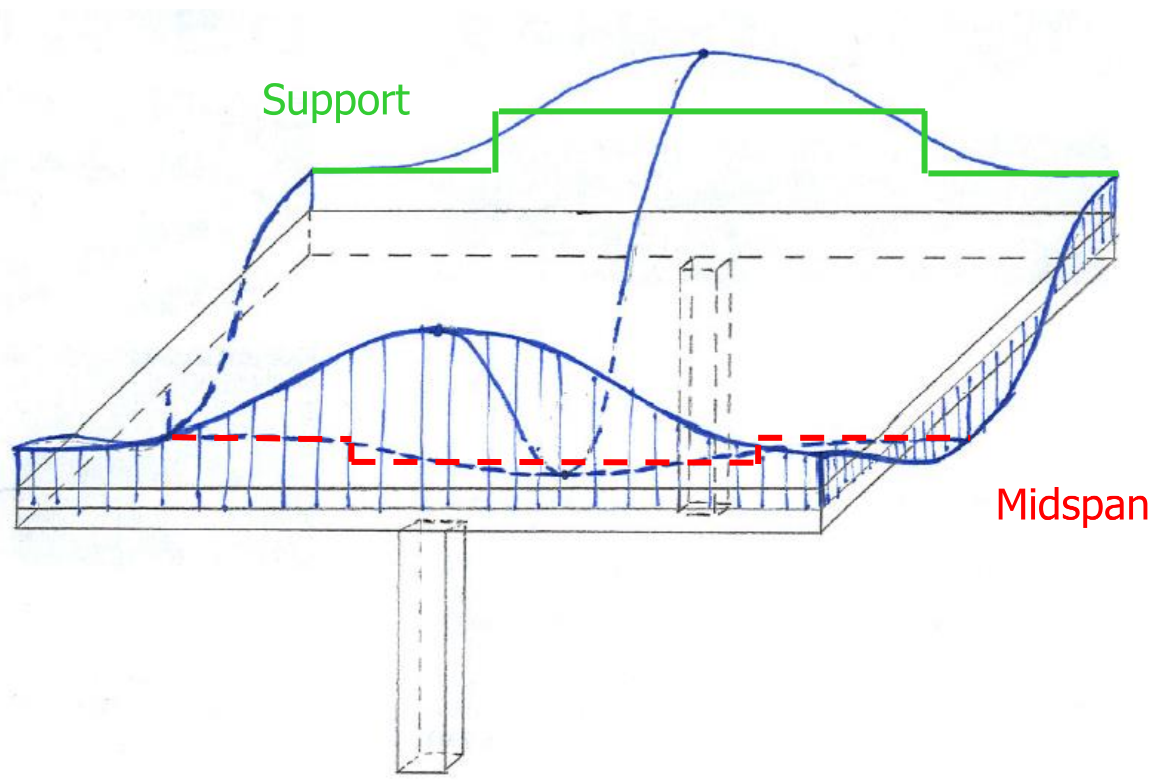
Moments in column and middle strips

The real distribution of bending moments is a 2D curve.



Moments in column and middle strips

However, from practical point of view, it is impossible to provide reinforcement exactly for these moments, and therefore, we need to calculate **„representative moments“** for particular strips of the slab.

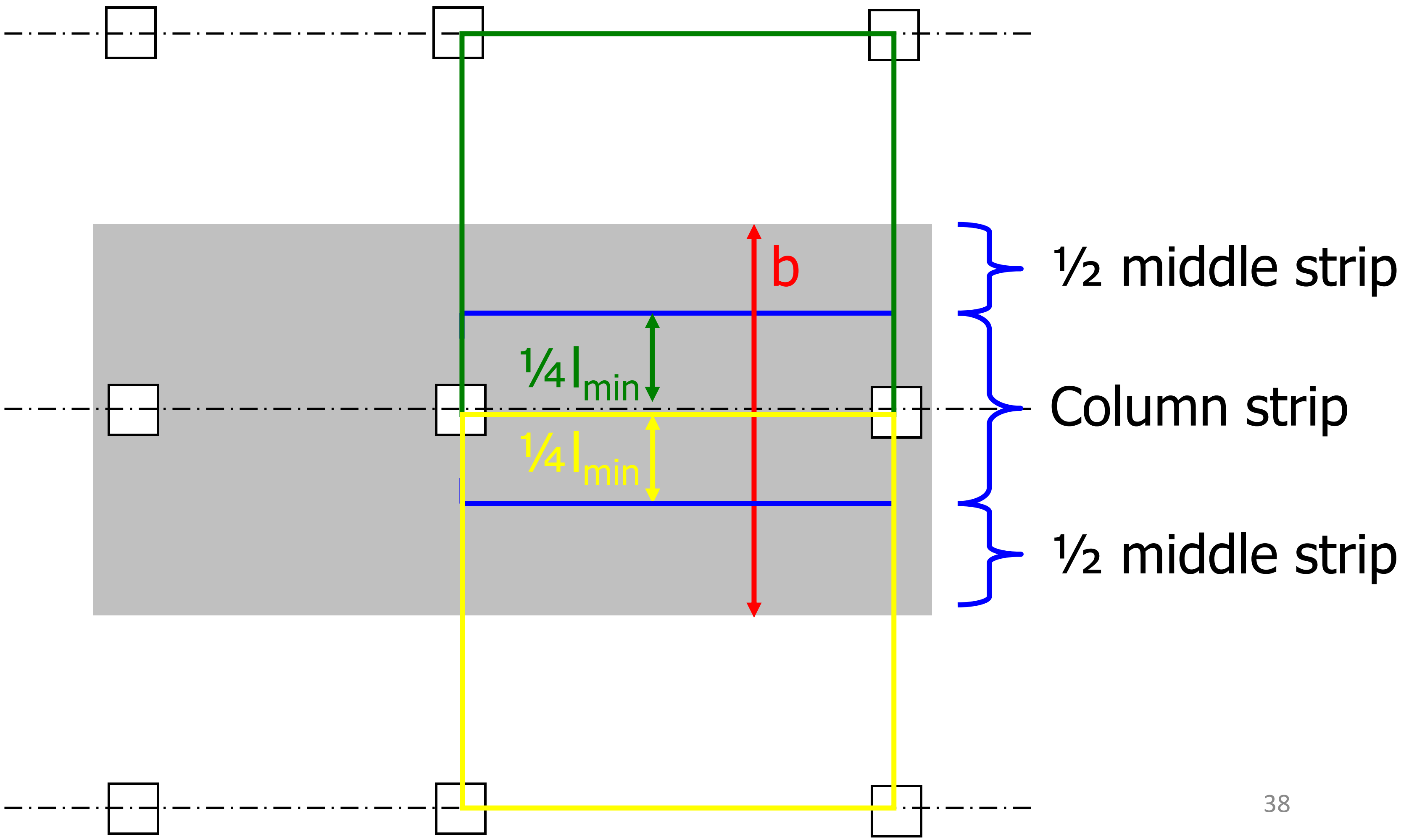


Moments in column and middle strips

We must divide the lanes into **column strips** (more loaded) and **middle strips** (less loaded).

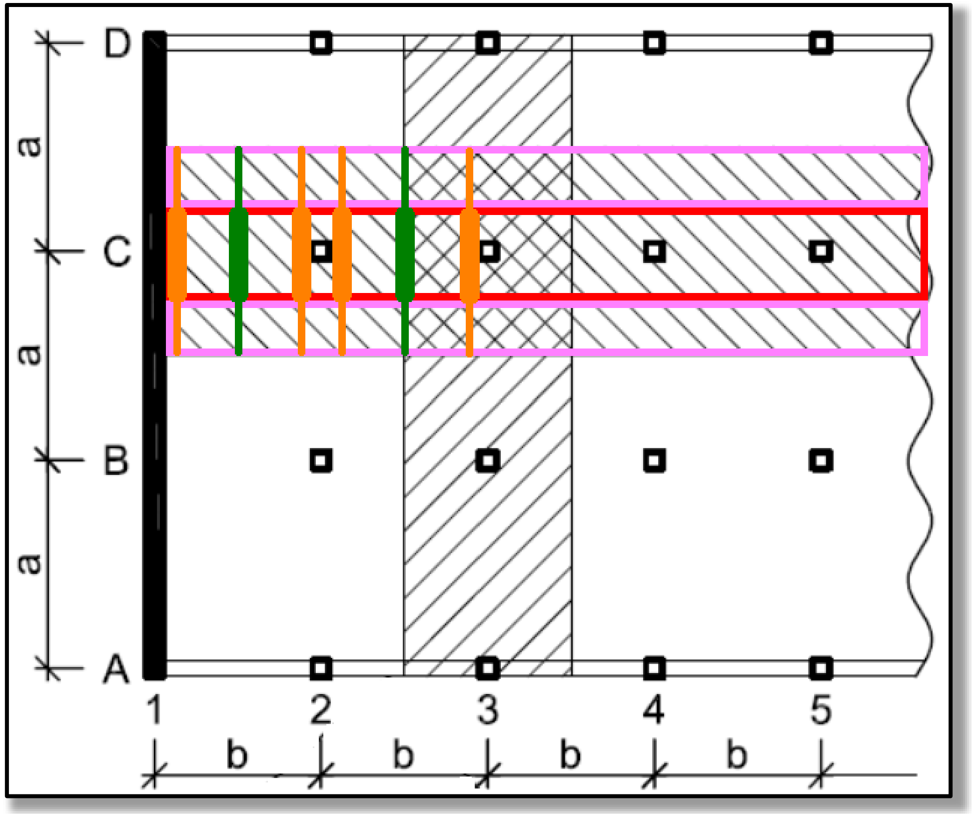
The width of the column strip is ¼ of the shorter span of the panel.

The width of the middle strip is the rest of the width of the panel.



Moments in column and middle strips

After we determine the column and middle strips, we must **split the** support moments ($\gamma_1 M_{tot}$ and $\gamma_3 M_{tot}$) and mid-span moments ($\gamma_2 M_{tot}$) **into moments in column and middle strips**.



Moments in column and middle strips

We split the support and mid-span moments into column and middle strips using precalculated coefficients ω, and the **moments in the column and middle strips** are:

$$M_{col} = \omega \times (\gamma_i M_{tot}),$$

$$M_{mid} = (1 - \omega) \times (\gamma_i M_{tot}).$$

Note: The column and middle strip moments are in kNm (per strip width).

Example:
Support moment in inner panel: $M = M_{tot} \cdot \gamma_1 = 100\ \mathrm{kNm}$ (per panel width).
Moment in column strip: $M_{col} = 100 \cdot 0.75 = 75\ \mathrm{kNm}$ (per column strip width).
Moment in column strip: $M_{mid} = 100 \cdot (1 - 0.75) = 25\ \mathrm{kNm}$ (per middle strips width).

Moments in column and middle strips

The precalculated coefficients ω are:

- in mid-span: $\omega = 0.6$,
- at inner support: $\omega = 0.75$,
- at edge beam: $\omega = \min\left(\max\left(1 - \frac{\beta_t}{10}; 0.75\right); 1\right)$,
- at wall: ω is not used.

β – refers to rigidity of the edge beam

Rigidity of the edge beam

Rigidity coefficient of edge beam is:

$$\beta_t = \frac{I_t}{2I_s}$$

Rigidity of the edge beam

The moment of inertia I_s of the slab in lane 3 is:

$$I_s = \frac{1}{12} b h_s^3$$

Width of belt 3

Depth of the slab

The torsion moment of inertia of edge beam I_t is:

$$I_t = \sum_{i=1}^{n} \left(1 - 0,63 \frac{t_i}{a_i}\right) \cdot \frac{t_i^3 a_i}{3}$$

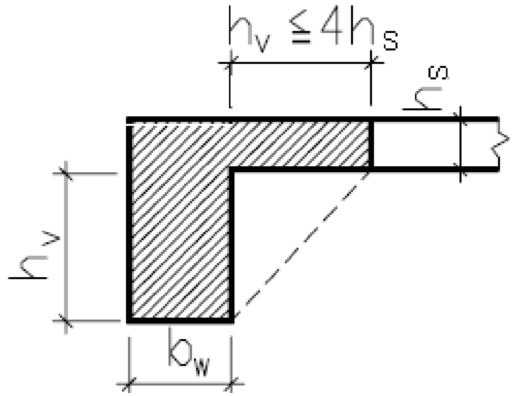
You have to sum torsion moments of all parts of the cross-section

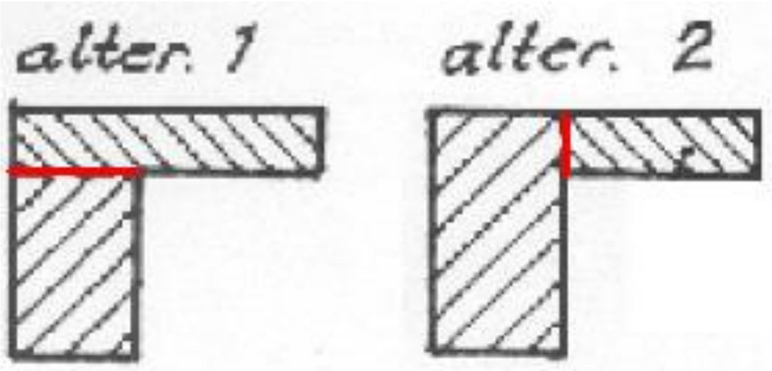
Longer side of a part of the cross-section

Shorter side of a part of the cross-section

Rigidity of the edge beam

We must divide the cross-section of the edge beam into parts (2 alternatives):

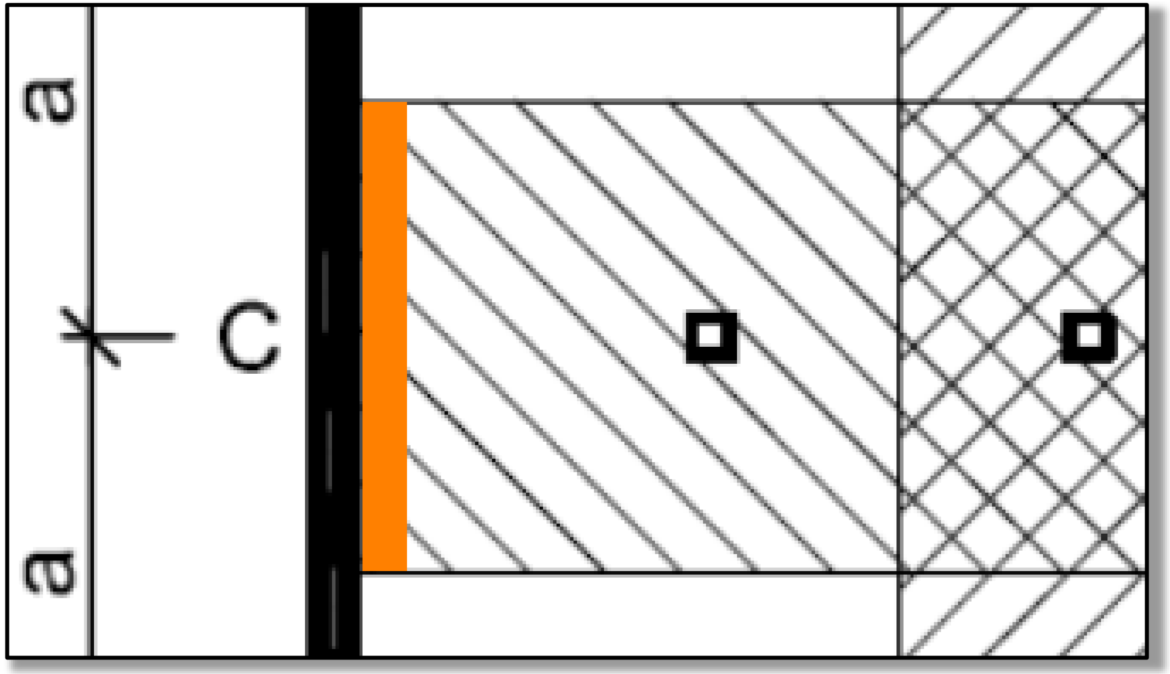




Calculate I_t for both alternatives and use the higher value to calculate β_t.

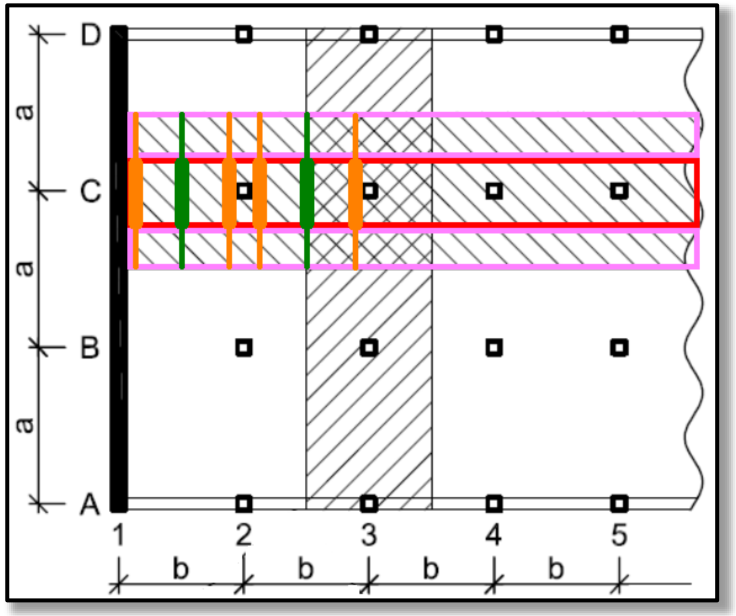
Moments near wall

For the **support at a wall**, we **don´t split** the moment into column and middle strip, and we assume that the support moment is **uniformly distributed** along the lane width.



Moments per 1 meter

Finally, we will **calculate bending moments per 1 meter width** by dividing the moments in the column and middle strips (which are in kNm per strip width) by the widths of column or middle strips.



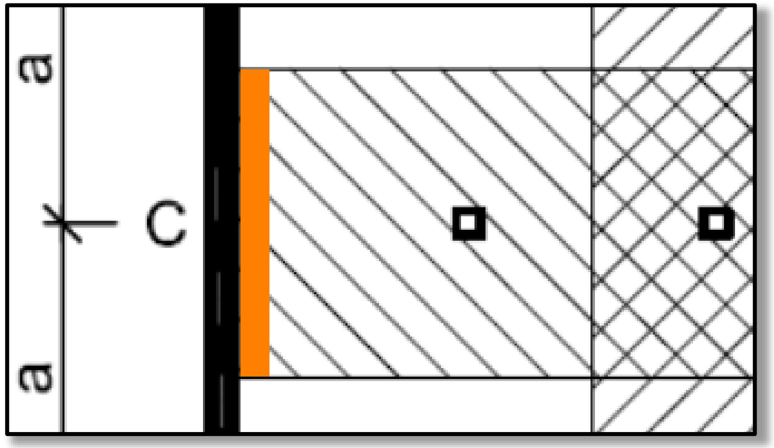
The **final moments will be in kNm/m**.

Example:
Moment in column strip: $M_{col} = 75 \text{ kNm}$ (per column strip width).
Column width: $b_{col} = 3.25 \text{ m}$.
Unit moment in column strip: $m_{col} = 75/3.25 = 23.1 \text{ kNm/m}$.

Moments per 1 meter near wall

Remember, at the **support at a wall**, we **did not split** the moment into column and middle strip, and we assumed that the support moment is **uniformly distributed** along the lane width.

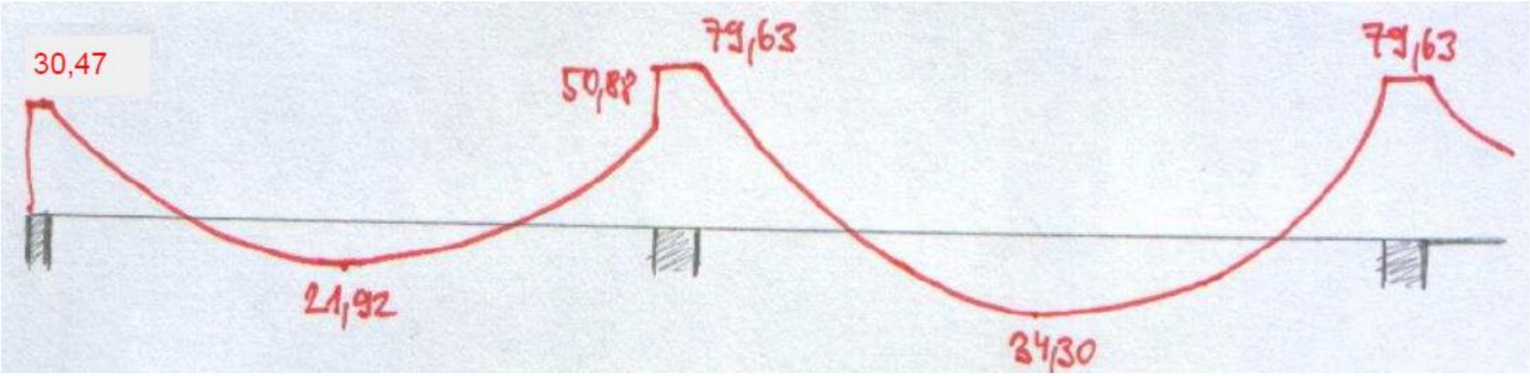
Therefore, at the wall, the bending moments per 1 meter width is calculated by dividing the moment above support (which is in kNm per panel width) by the widths of the panel.



Moment drawing

Finally, when we get all of the bending moment values, we can draw moment curves for calculated moments per 1 meter. There will be 4 curves:

- Belt C – column strip
- Belt C – middle strip
- Belt 3 – column strip
- Belt 3 – middle strip



Homework format

Calculate total moments and support/midspan moments by hand, then input them into table and calculate column/middle strip moments in the table. See example calculation.

Moments in column and middle strips							
Panel	Cross-section	Positive/negative moment M_i [kNm]	Strip	ω	Moment in column/middle strip M_j [kNm]	Width of the strip s_j [m]	Moment per 1 m of the slab m_j [kNm/m]
C_o	1 (left support)	179,76	no division	1,00	179,76	5,90	30,47
	2 (midspan)	96,80	Column	0,60	58,08	2,65	21,92
			Middle		38,72	3,25	11,91
	3 (right support)	179,76	Column	0,75	134,82	2,65	50,88
			Middle		44,94	3,25	13,83
C_{in}	1 (left support)	313,20	Column	0,75	234,90	2,95	79,63
			Middle		78,30	2,95	26,54
	2 (midspan)	168,66	Column	0,60	101,20	2,95	34,30
			Middle		67,46	2,95	22,87
	3 (right support)	313,20	Column	0,75	234,90	2,95	79,63
			Middle		78,30	2,95	26,54
3_o	1 (left support)	119,11	Column	0,99	117,44	2,95	39,81
			Middle		1,67	4,05	0,41
	2 (midspan)	198,50	Column	0,60	119,10	2,95	40,37
			Middle		79,40	4,05	19,60
	3 (right support)	277,92	Column	0,75	208,44	2,95	70,66
			Middle		69,48	4,05	17,16
3_{in}	1 (left support)	258,07	Column	0,75	193,55	2,95	65,61
			Middle		64,52	4,05	15,93
	2 (midspan)	138,96	Column	0,60	83,38	2,95	28,26
			Middle		55,58	4,05	13,72
	3 (right support)	258,07	Column	0,75	193,55	2,95	65,61
			Middle		64,52	4,05	15,93

thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.