CM01 – Concrete and Masonry Structures 1

HW6 – Slab supported on four sides

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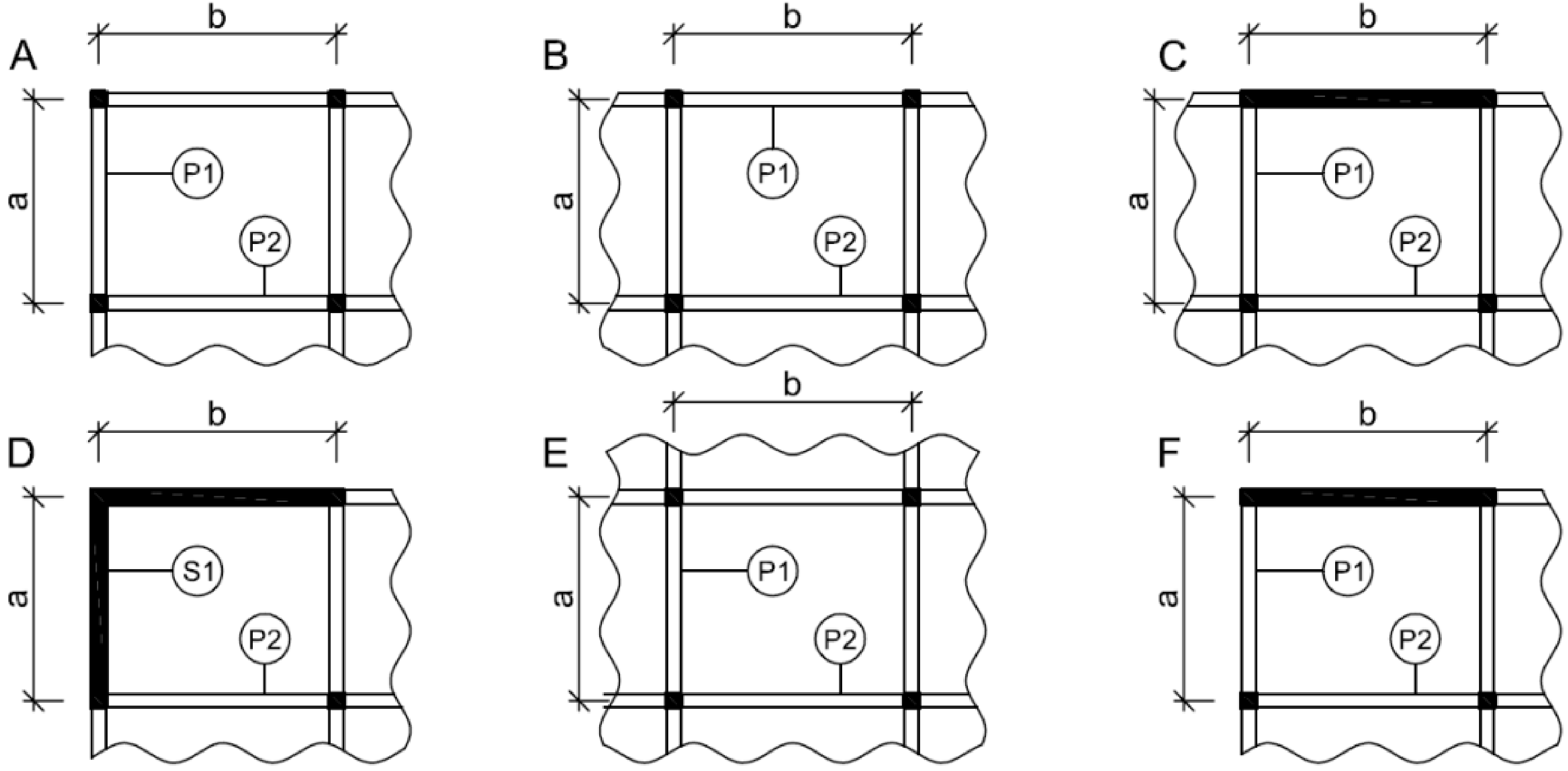
Task 2

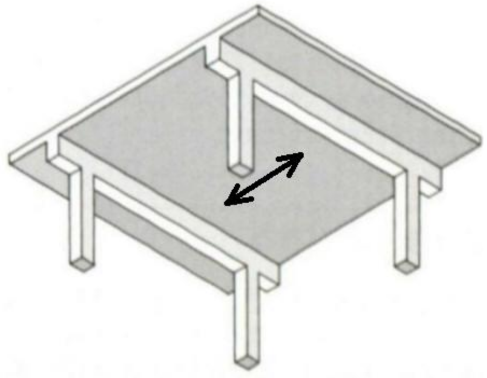
Task 2 – Slab supported on four sides

In Task 2, two-way slab supported on four sides will be designed.

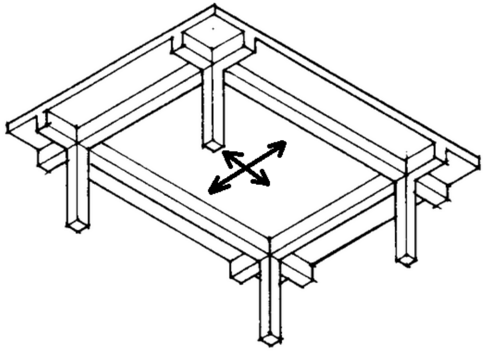
Schemes of the structures:



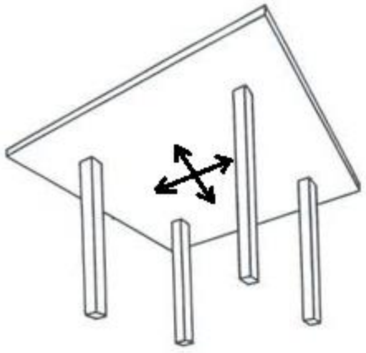
Comparison of Tasks 1 to 3



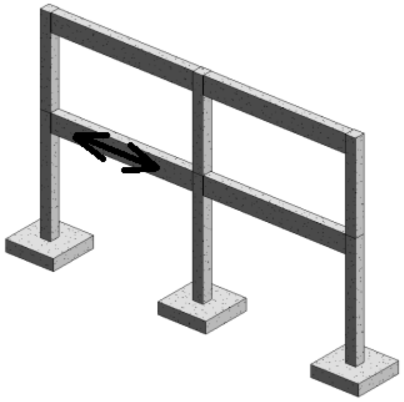
One-way slab – **Task 1**



Two-way slab supported on 4 sides – **Task 2**



Two-way flat slab – **Task 3**

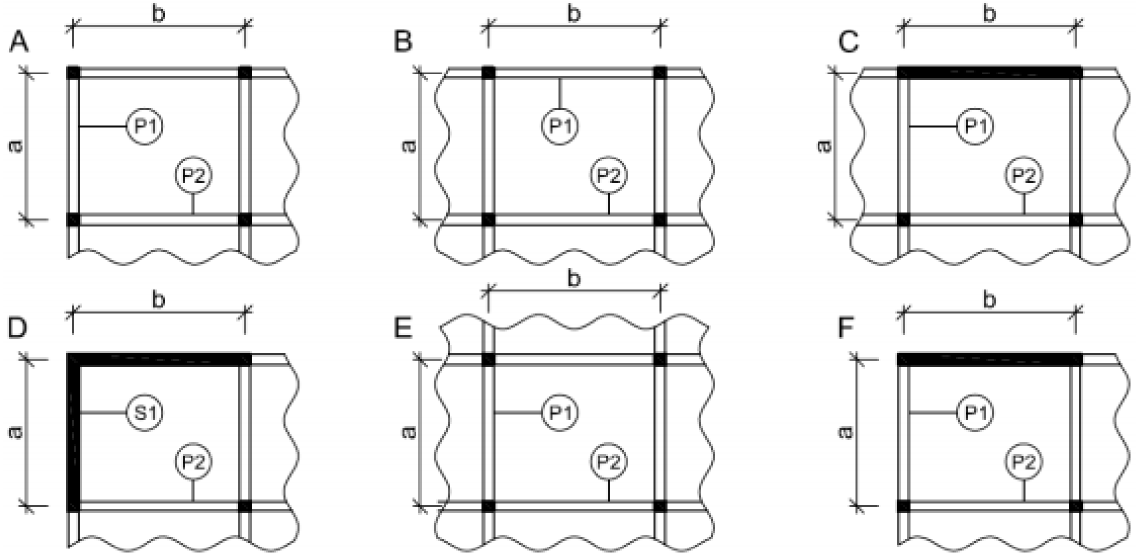


Beam (frame) – **Task 1**

Task 2 – Assignment

Consider a reinforced concrete structure of multifloor building composed of walls, columns and continuous slabs. All spans of the slab are supported by walls or rigid beams on four sides. There are no openings in the slab.

Schemes of the structures:



Individual parameters (parameters in **bold** you can find on teacher's website):

Scheme: given **scheme**, given **beam (P) or wall (S)**

Geometry: ***a***, ***b*** [m] – horizontal dimensions of the structure (***a*** see 1st task), h_s [mm] – depth of the slab

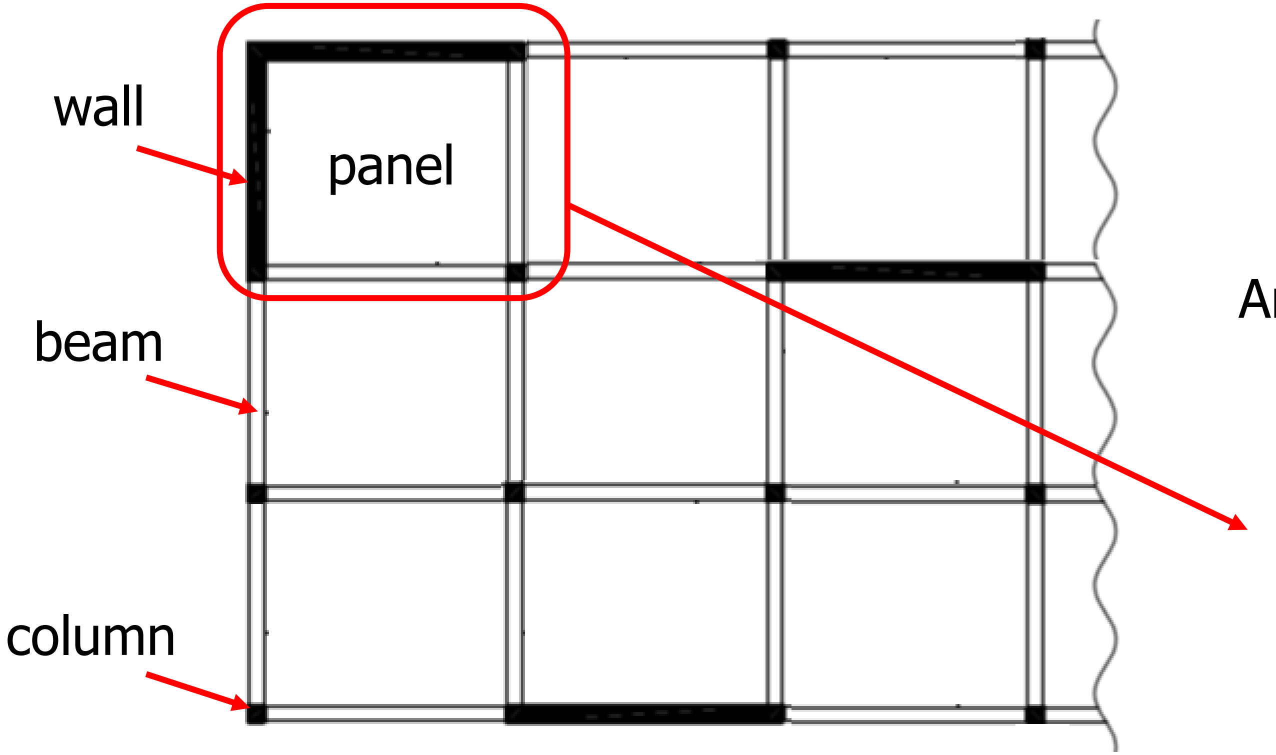
Materials: **see 1st task**

Loads: **see 1st task**, values for typical floor (except the self weight, which will be different)

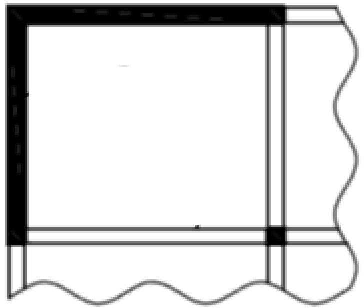
Please work out:

1. **Calculation of bending moments in the slab**:
   a) Using linear analysis (do not consider the effect of torsion moments caused by prevented lifting of the corners of the slab). Proceed from the assumption that the deflections in *x* and *y* directions are equal.
   b) Using precalculated tables based on the theory of plasticity (effect of torsion moment is included).
2. **Check of given depth of the slab** – consider bending moments from1b)
   (if the slab is not checked, just propose the adjustment, **do not** recalculate bending moments!)
3. **Calculation of loading of given beam or wall.**

Task 2 – Assignment



Analyze one slab in your homework



Task 2 – Assignment goals

<u>Our goal</u> will be to:

1) Calculate **bending moments**:
   - using calculation based on linear analysis,
   - using precalculated tables based on the theory of plasticity.
2) Preliminarily **check the slab depth***.
3) Calculate **loading of a supporting element** (beam or wall).

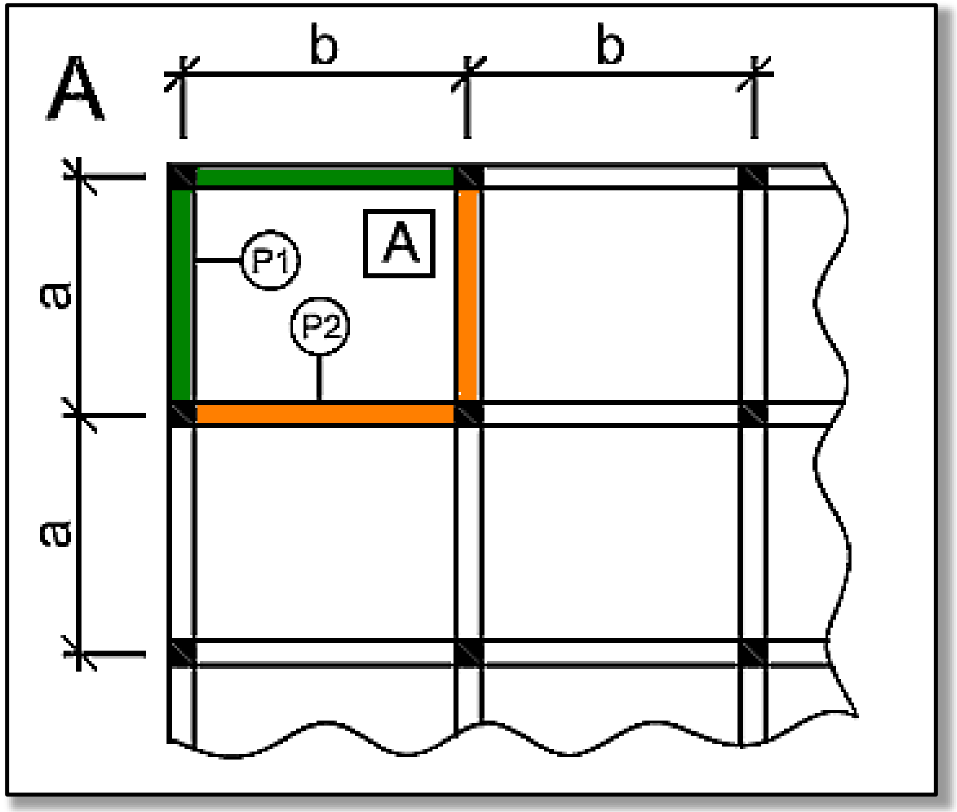
* Note: Slab depth (thickness) is already assigned, we will only check it.

Supports and static schemes

Supports

Based on the real supports of the slab, we must identify support types:

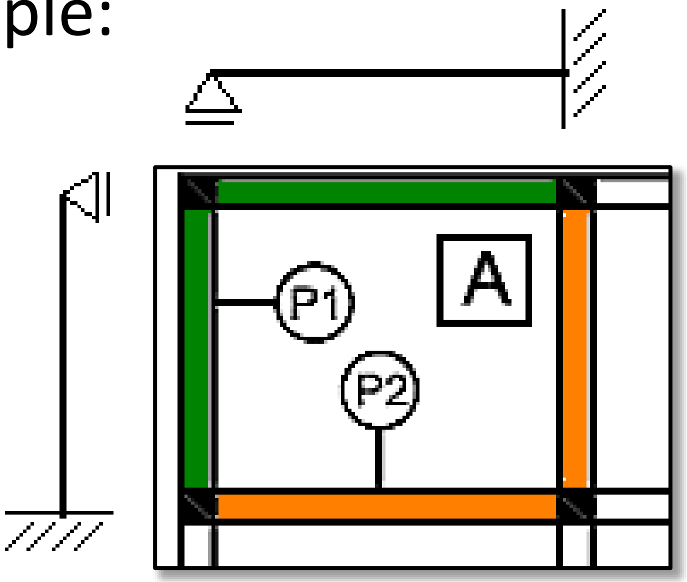
- fixed support – for wall and inner beam,
- hinged (point) support – for outer beam.



Static schemes

Based on the support types, we can determine static scheme in each direction:

For example:



Loading of the slab

Loading of the slab

Before any calculations, we must first **calculate the total area load** on the slab. The slab is loaded by:

- self-weight,
- other dead loads,
- live loads.

Do the calculation in a table!

* Note: Slab depth (thickness) is already assigned.

Bending moments

Bending moments

We will use two methods for the calculation of bending moments:

- **Strip Method** (based on elastic theory)
  - assumes ideally linear behaviour of materials,
  - always applicable,
  - results usually not close to real behaviour of reinforced-concrete structures,
  - always on the safe side.
- **Yield Line Theory** (based on plastic theory)
  - assumes ideally plastic behaviour of materials,
  - NOT always applicable (sufficient plastic hinge rotational capacity is necessary),
  - results close to real behaviour of reinforced-concrete structures,
  - NOT always on the safe side.

Bending moments

We will use two … ments:

- **Strip Method**
  - assumes ide…
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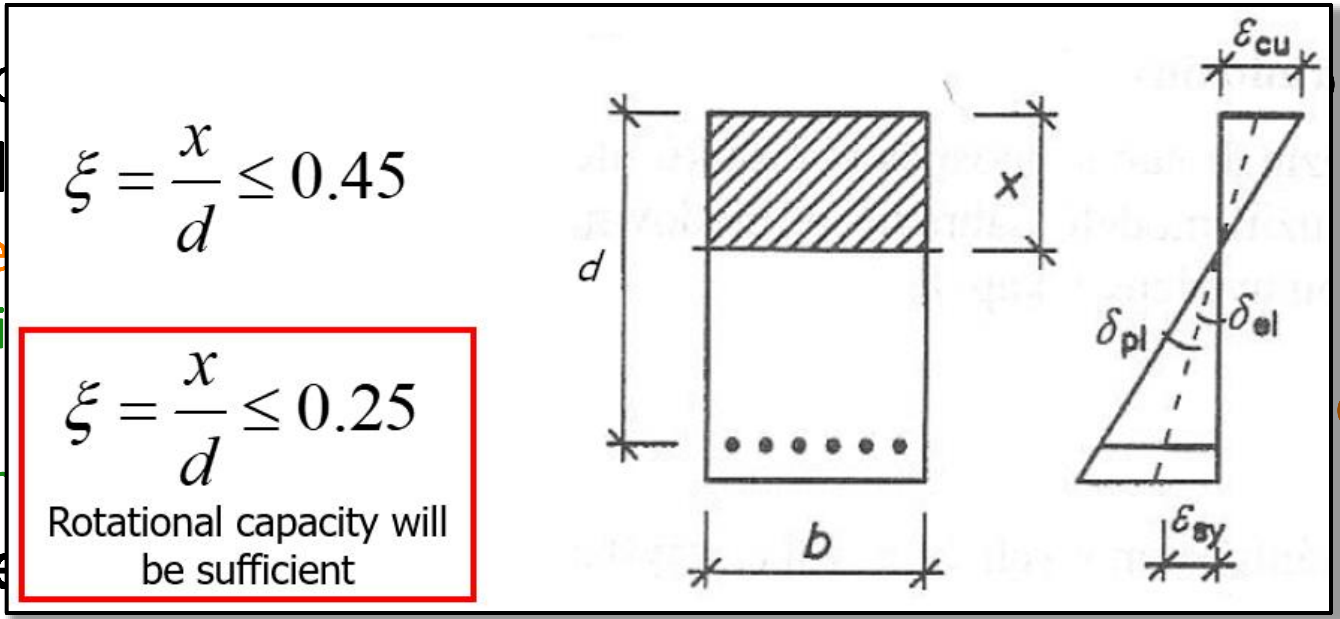
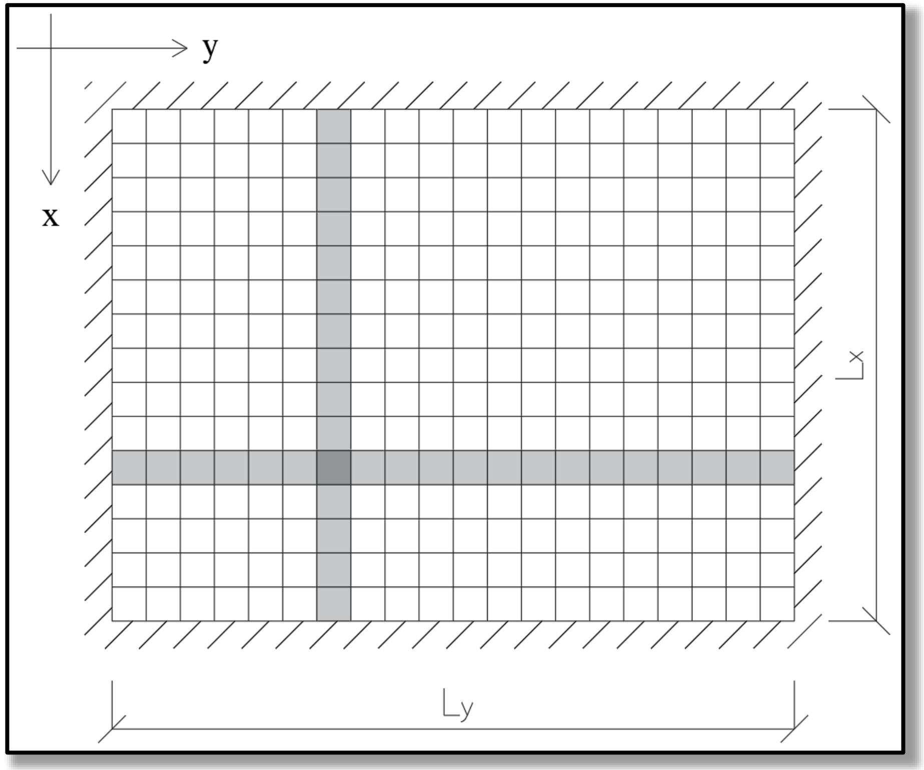


Bending moments – Strip Method

Strip Method (elastic)

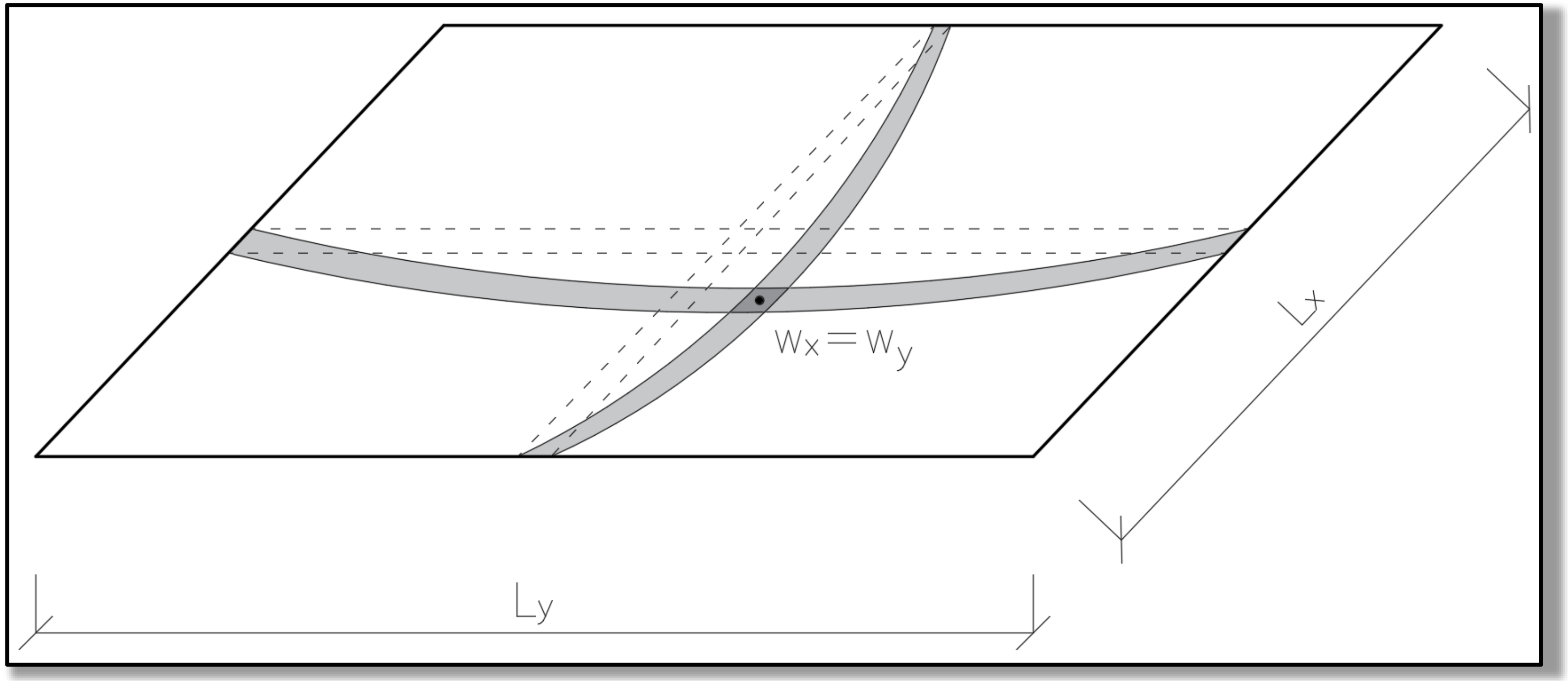
In this method, we assume that the slabs consists of **two perpendicular sets of strips**, and we calculate the **loading and moments in the two directions individually**.



Loading in each direction

First, **we must calculate the value of load in each direction** (f_x and f_y). We calculate these values using the assumption that the deflection of the slab in both directions must be the same

$$w_x\left(\frac{l_x}{2}\right) = w_y\left(\frac{l_y}{2}\right).$$



Loading in each direction

Using the equation for deflection in the middle of a beam

$$w = k\frac{fl^4}{EI},$$

we can derive:

$$w_x\left(\frac{l_x}{2}\right) = w_y\left(\frac{l_y}{2}\right)$$

$$k_x\frac{f_x l_x^4}{EI} = k_y\frac{f_y l_y^4}{EI},$$

$$f_y = f_x\frac{k_x l_x^4}{k_y l_y^4}.$$

$$k = \frac{1}{384}$$

$$k = \frac{2}{384}$$

$$k = \frac{5}{384}$$

Loading in each direction

As the sum of the loads in both directions must be equal to total area load

$$f = f_x + f_y.$$

we can derive

$$f = f_x + f_x \frac{k_x l_x^4}{k_y l_y^4},$$

$$f_x = \frac{f}{1 + \frac{k_x l_x^4}{k_y l_y^4}}.$$

Loading in each direction

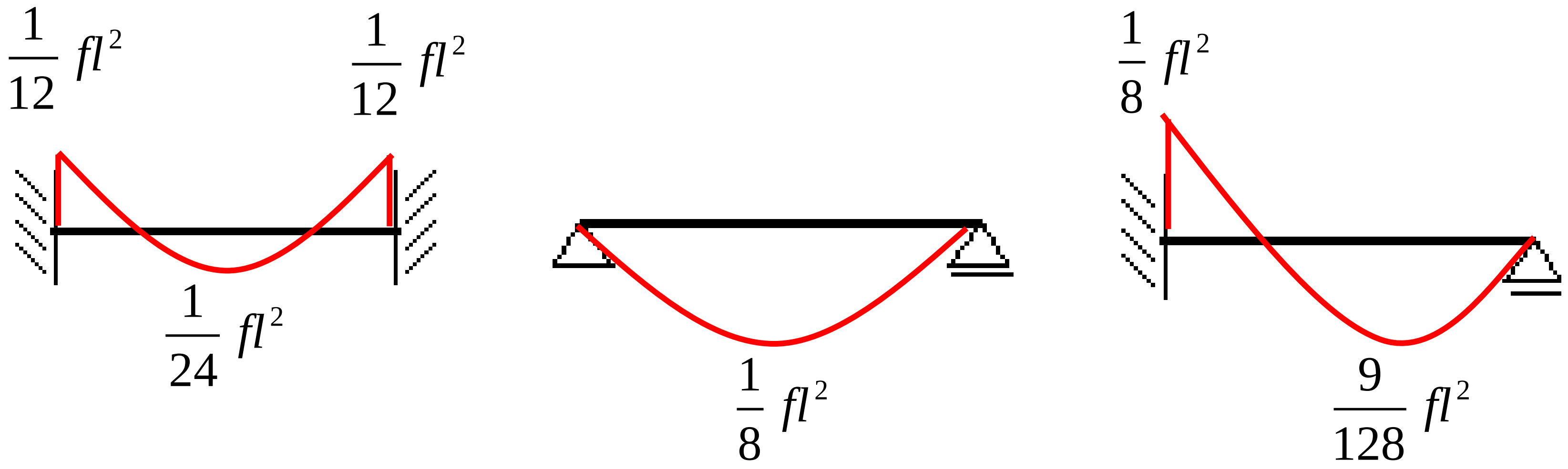
The final equations for loads in each direction are:

$$f_x = \frac{f}{1 + \frac{k_x l_x^4}{k_y l_y^4}}$$

$$f_y = f - f_x$$

Bending moments

Using the loads in individual directions (f_x and f_y), we can calculate bending moments in each direction using these values for given static scheme.



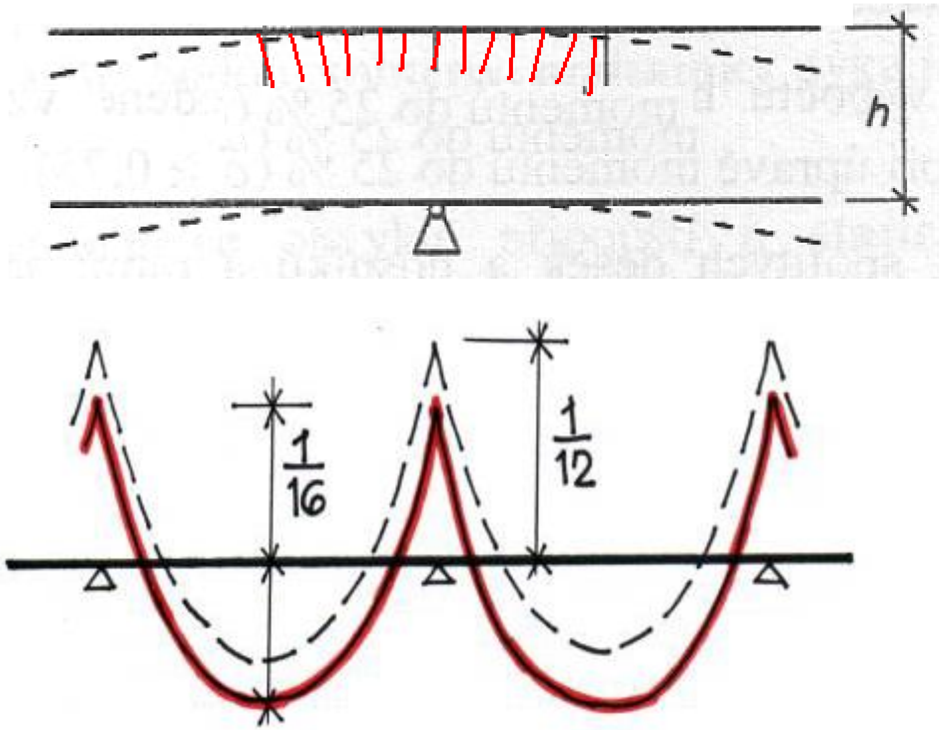
We calculate the moments on strips of 1 m width. The load is therefore:

$$f_{lin}\,[kN/m] = f_{area} \cdot 1\ \mathrm{m}$$

Bending moments – Yield Line Theory

Yield Line Theory

Yield Line Theory considers: **cracks** in the structure ⟶ changes in **bending stiffness** ⟶ **redistribution** of internal forces ⟶ real behaviour.



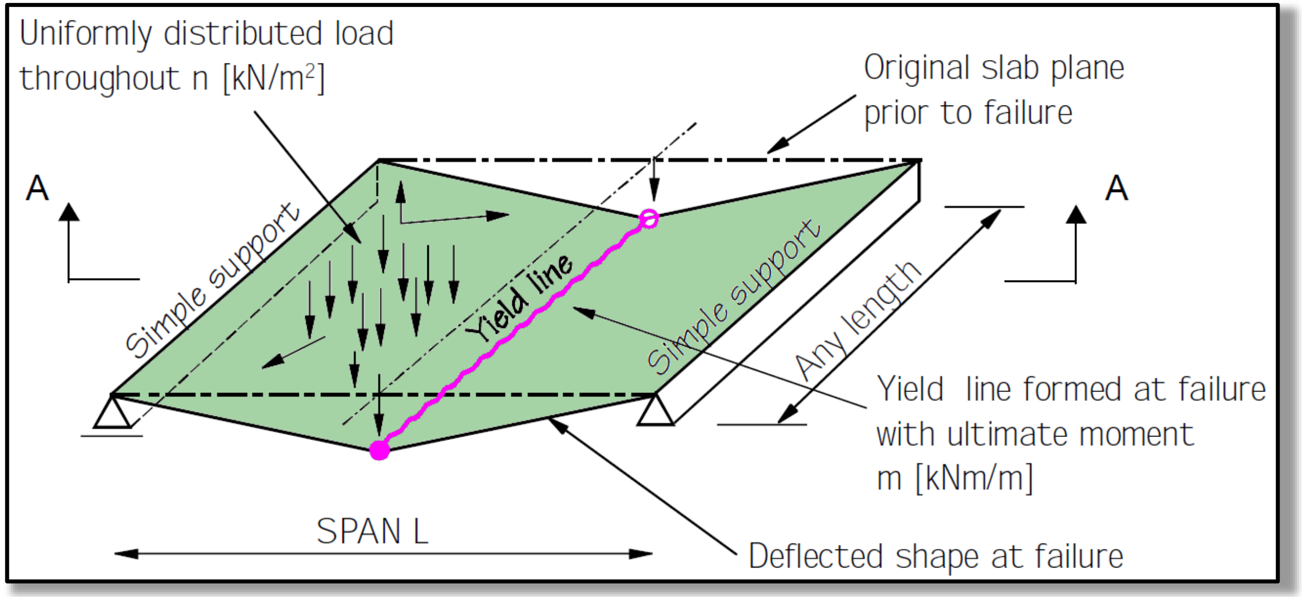
Yield Line Theory (plastic)

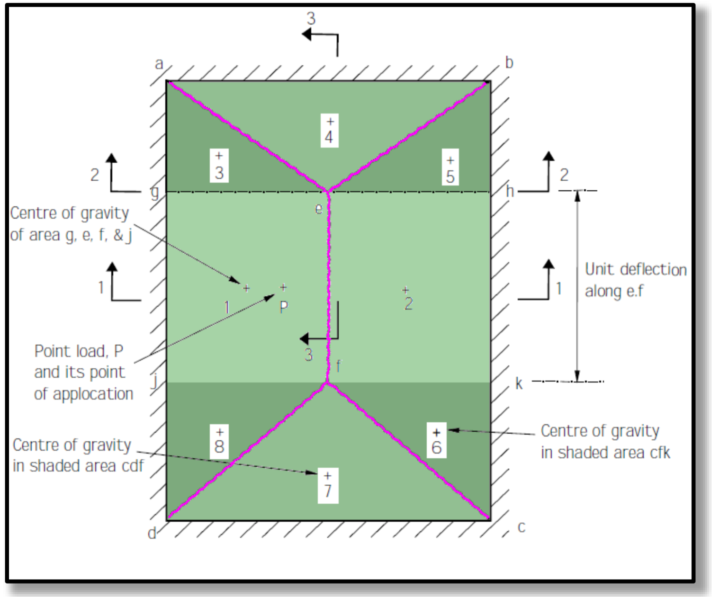
In order to use Yield Line Theory, multiple conditions must be satisfied:

- slab must have constant depth,
- supports must be rigid,
- corners must be prevented from lifting,
- adjacent panels must have same loads,
- adjacent panels must have same spans,
- reinforcement must have sufficient ductility (steel class B, C),
- rotational capacity must be sufficient ($x/d \leq 0.25$).

Yield Line Theory (plastic)

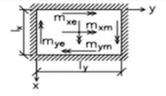
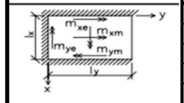
The Yield Line Theory is based on the assumption that **local parts are ideally plastic (like hinges)** and **the rest of the slab is ideally rigid**.





Yield Line Theory (plastic)

The theory is very complicated, and thus, **we use tables generated using this theory**.

Typ podepření		ly/lx										
		1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	βxe	-0.032	-0.038	-0.043	-0.047	-0.051	-0.053	-0.057	-0.058	-0.060	-0.062	-0.064
	βxm	0.024	0.028	0.032	0.035	0.038	0.040	0.042	0.044	0.045	0.047	0.048
	βye	-0.032										
	βym	0.024										
	βxe	-0.038	-0.044	-0.048	-0.052	-0.055	-0.058	-0.060	-0.062	-0.064	-0.066	-0.067
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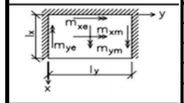
	βye	-0.038										
	βym	0.029										
	βxe	-0.038	-0.048	-0.056	-0.062	-0.068	-0.072	-0.077	-0.080	-0.083	-0.087	-0.090
	βxm	0.029	0.036	0.042	0.046	0.051	0.054	0.058	0.060	0.063	0.065	0.067
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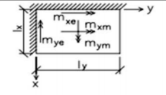	βxe	-0.047	-0.055	-0.063	-0.069	-0.074	-0.078	-0.083	-0.085	-0.088	-0.091	-0.094
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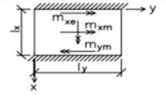
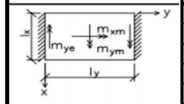
	βye	-0.047										
	βym	0.035										
	βxe	-0.046	-0.051	-0.055	-0.058	-0.061	-0.063	-0.065	-0.067	-0.068	-0.070	-0.071
	βxm	0.035	0.038	0.041	0.043	0.045	0.047	0.049	0.050	0.051	0.052	0.053
	βye	0										
	βym	0.035										
	βxe	0										
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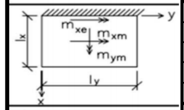
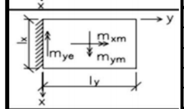
	βym	0.035										
	βxe	-0.058	-0.066	-0.072	-0.077	-0.082	-0.085	-0.090	-0.092	-0.095	-0.097	-0.100
	βxm	0.044	0.049	0.054	0.058	0.062	0.064	0.067	0.069	0.071	0.073	0.075
	βye	0										
	βym	0.044										
	βxe	0										
	βxm	0.044	0.055	0.065	0.072	0.080	0.085	0.091	0.095	0.099	0.102	0.106
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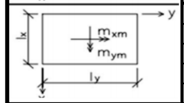
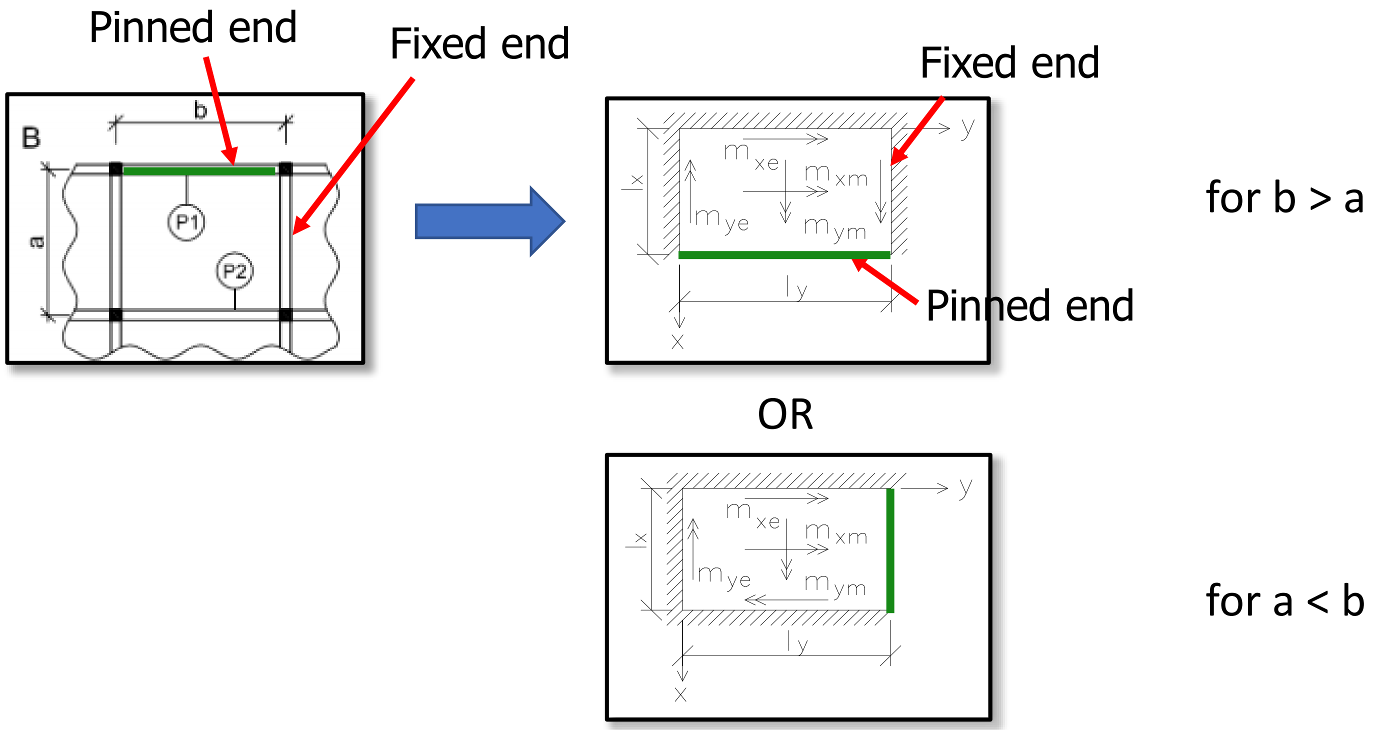
	βym	0.044										
	βxe	0										
	βxm	0.056	0.066	0.075	0.082	0.089	0.093	0.099	0.102	0.106	0.109	0.113
	βye	0										
	βym	0.056										

Yield Line Theory (plastic)

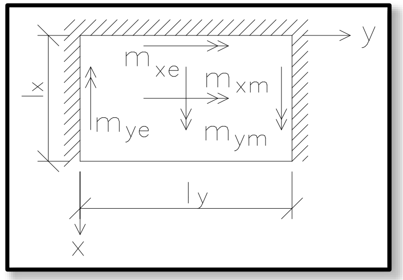
When using the tables, we first select the type of the panel based on the assigned panel.



Yield Line Theory (plastic)

When using the tables, we first select the type of the panel based on the assigned panel.

Be careful when selecting the type of the panel!



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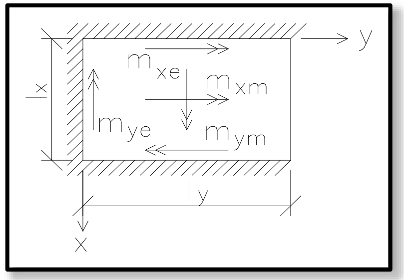
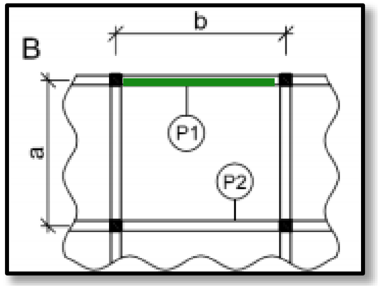


Yield Line Theory (plastic)

Then we calculate ratio of spans (l_y/l_x).

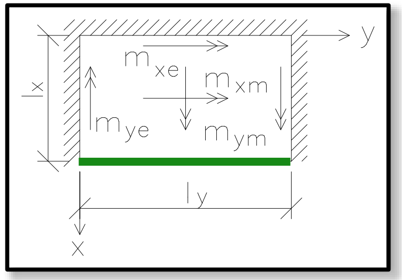
Be careful assigning a and b to l_x and l_y! For all panel types, l_x is the shorter span.

For assignment B:



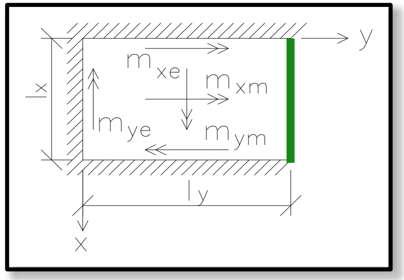
If b > a:

$$l_y = b$$
$$l_x = a$$



If b < a:

$$l_y = a$$
$$l_x = b$$



Yield Line Theory (plastic)

Using the selected type of panel and calculated ratio of spans, we lookup β_{ii} coefficients in the table. (Use linear interpolation to calculate β_{xi} coefficients.)

Typ podepření		ly/lx										
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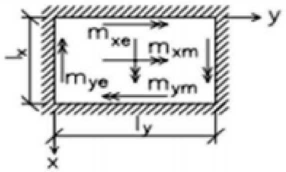
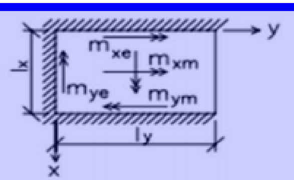
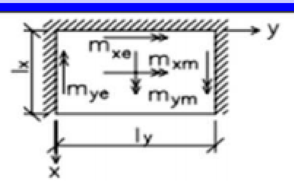
	βxe	-0.032	-0.038	-0.043	-0.047	-0.051	-0.053	-0.057	-0.058	-0.060	-0.062	-0.064
	βxm	0.024	0.028	0.032	0.035	0.038	0.040	0.042	0.044	0.045	0.047	0.048
	βye	-0.032										
	βym	0.024										
	βxe	-0.038	-0.044	-0.048	-0.052	-0.055	-0.058	-0.060	-0.062	-0.064	-0.066	-0.067
	βxm	0.029	0.033	0.036	0.039	0.041	0.043	0.045	0.047	0.048	0.049	0.051
	βye	-0.038										
	βym	0.029										
	βxe	-0.038	-0.048	-0.056	-0.062	-0.068	-0.072	-0.077	-0.080	-0.083	-0.087	-0.090
	βxm	0.029	0.036	0.042	0.046	0.051	0.054	0.058	0.060	0.063	0.065	0.067
	βye	-0.038										
	βym	0.029										

Yield Line Theory (plastic)

Calculate the bending moments using the following equations.

$$m_{\mathrm{xe}} = \beta_{\mathrm{xe}} m_0$$

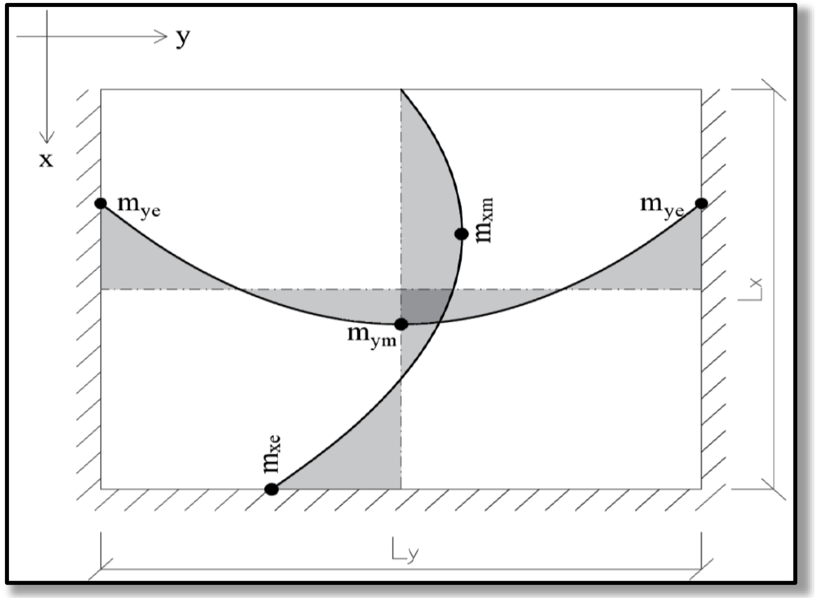
$$m_{\mathrm{xm}} = \beta_{\mathrm{xm}} m_0$$

$$m_{\mathrm{ye}} = \beta_{\mathrm{ye}} m_0$$

$$m_{\mathrm{ym}} = \beta_{\mathrm{ym}} m_0$$

$$\boxed{m_0 = f_{\mathrm{d}} \cdot l_{\mathrm{x}}^2}$$

Basic value of bending moment.



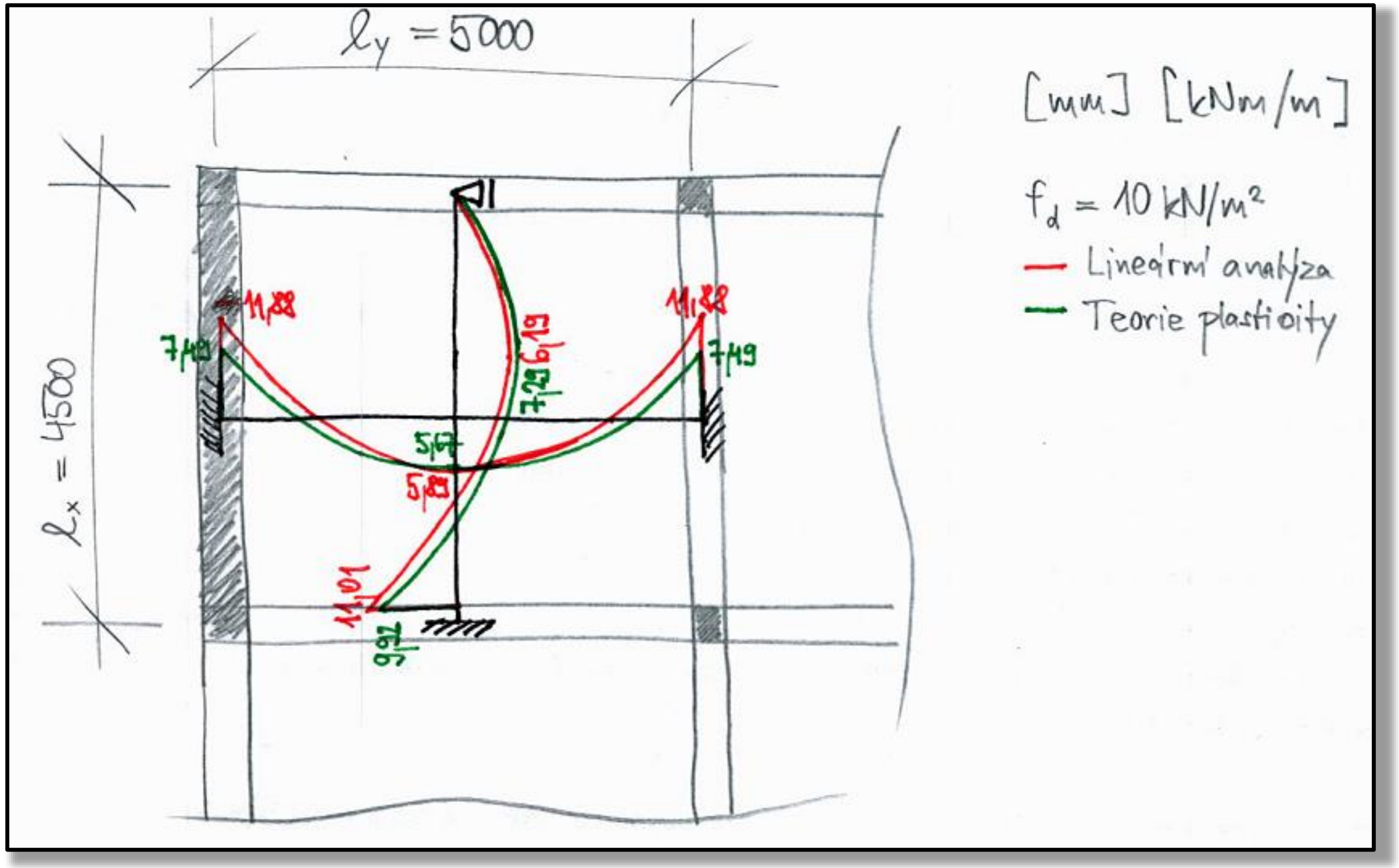
Indices:

- x, y – directions of a moments (m$_x$ is the moment in the direction of l_x)
- m – midspan moment
- e – support (edge) moment

Bending moments – schemes

Bending moments – schemes

Compare elastic (Strip Method) and plastic (Yield Line Theory) moments in one scheme.



Design of reinforcement

Design of reinforcement

In the homework, you **DO NOT HAVE TO design the reinforcement**.

Just remember, that the procedure of design of bending reinforcement for two-way slabs is almost identical to beams (see HW3).

The only difference is that you design the **reinforcement in 2 directions** and that the **width of the cross-section is taken as b = 1 m**.

Preliminarily check of the slab depth

Preliminarily check of the slab depth

Check the given value of h_s for the **biggest moment from plastic analysis** (which we will denote as $m_{Ed,max}$).

We will check the slab from the point of view of:

- estimated area of reinforcement (can the slab be reinforced?),
- estimated compressed zone (is the rotational capacity sufficient?),
- span / depth ratio (will deflections be ok?).

Check of the slab depth – area of reinforcement

First, calculate the **required cross-sectional area of reinforcement**:

$$a_{s,rqd} = \frac{m_{Ed,\max}}{0.9 d f_{yd}}$$

Calculation of effective depth – see HW3. (Estimate 10 mm rebars and take cover depth from the frame structure.)

Then, calculate the **minimal and maximal area of reinforcement** ($a_{s,min}$ and $a_{s,max}$) – see HW3.

Finally, **check the condition** for the area of reinforcement:

$$a_{s,max} \geq a_{s,req} \geq a_{s,min}$$

Check of the slab depth – compressed zone

First, estimate the **depth of the compressed zone**:

$$x = \frac{1.2a_{\mathrm{s,rqd}}f_{\mathrm{yd}}}{0.8bf_{\mathrm{cd}}}$$

Estimation of $a_{s,prov}$

The bending moment $m_{Ed,max}$ and area $a_{s,req}$ were calculated for a strip of 1 m width. Therefore, we must assume b = 1 m here.

Then, **check the condition** for the relative depth of the compressed zone:

$$\frac{x}{d} \leq 0.45.$$

Check of the slab depth – span/depth

Check the span/depth ratio (deflection control) – see HW1.

$$\lambda \leq \lambda_d$$

$$\frac{l}{d} \leq \kappa_{c1}\kappa_{c2}\kappa_{c3}\lambda_{d,tab}$$

Check of the slab depth

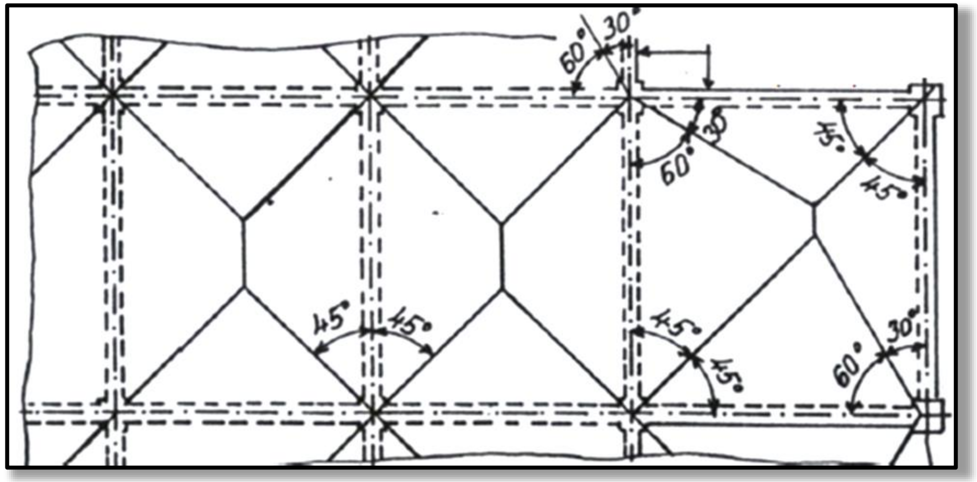
If all of the conditions are satisfied, the assigned slab depth is suitable.

If some of the conditions are not checked, propose a solution (just describe it, don't calculate anything).

Load of beam/wall

Load of beam/wall

Draw tributary areas of all the supporting elements.



The angle between **identical supports** (fixed/fixed, pinned/pinned) is **45°**.

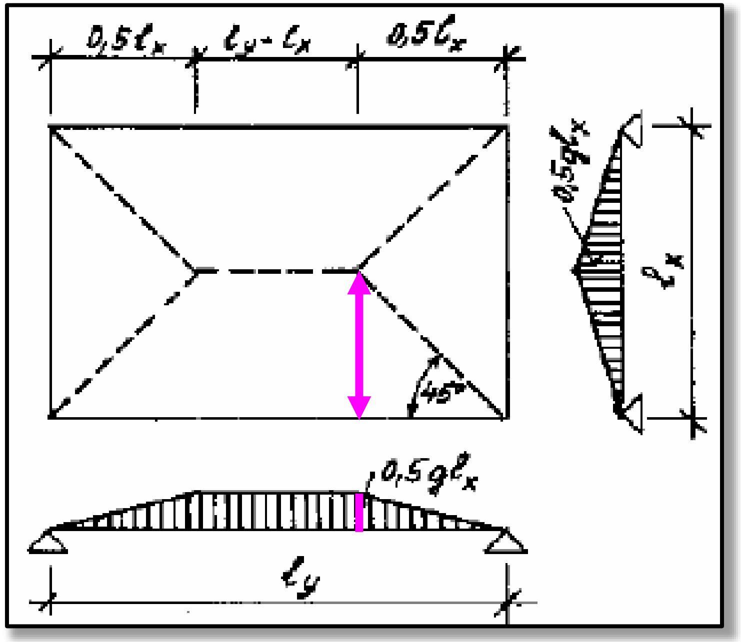
The angle between **fixed and pinned support** is 60°.

Load of beam/wall

For your given supporting element (wall or beam), **draw load diagram** and **calculate the load**.

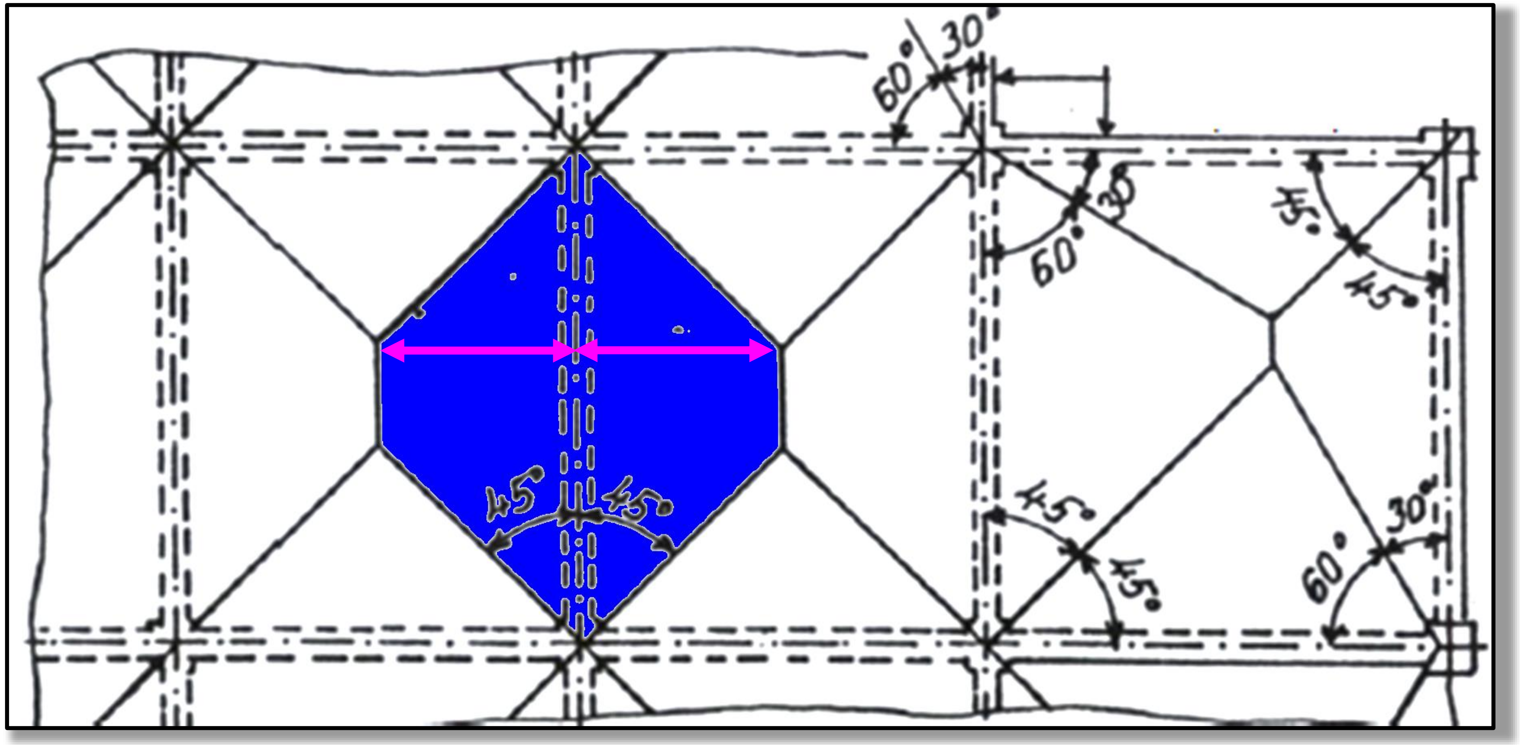
The load in each point is:

total load of the slab (f_d) × width of the tributary area



Load of beam/wall

Be aware that inner walls and beams are loaded by 2 adjacent panels!



thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.