### THIS PRESENTATION IS <u>MACHINE</u> <u>TRANSLATED</u> FROM CZECH LANGUAGE!

Use the presentation mainly for eqautions, and beware of text descriptions as they may be translated in cofusing ways.



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Concrete and reinforced concrete footings Design and assessment of footings

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Extra Theory

#### THEORY - Stresses on the feet



#### Stresses on the feet

Unlike most other concrete structures, the **compressive stresses are not as important for the footing**.

<u>The tensile stress</u> (transverse and bending) of the footing is <u>important</u>.

The tensile strength of the concrete is important for a plain concrete footing.

→For reinforced concrete footing, it is necessary to design tension/ flexural reinforcement.

As far as pressure is concerned, the <u>compressive capacity of the soil</u> is important.



Stresses on the feet



important.



Extra Theory

#### THEORY - Eccentricity of the normal force



# Eccentricity of the normal force

**The total eccentricity** is the eccentricity of the applied normal force in the footing and is determined as the ratio of the moment to the normal force applied in the footing joint

$$e=\frac{M}{N}$$
.

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The moment in the foundation joint is caused by

- torque from the upper structure,
- horizontal force on the upper edge of the footing (displacement force from the upper structure) on a shoulder equal to the height of the footing (see below). 9



# Eccentricity of the normal force

**The total eccentricity** is the eccentricity of the applied normal force in the footing and is determined as the ratio of the moment to the normal force applied in the footing joint

$$e=\frac{M}{N}.$$

The normal force in the foundation joint is caused by

- force from the upper structure,
- by the weight of the foot.



#### Eccentricity of the normal force



Normal force and bending moment from the upper structure acting at the base of the column.



#### Eccentricity of the normal force



Normal force and bending moment from the upper structure acting in the foundation joint.



#### Eccentricity of the normal force



The sliding force acting at the base of the column.



#### Eccentricity of the normal force



Effect of the displacement force at the base of the column on the internal forces in the foundation joint - development of the bending moment from the displacement force



#### Eccentricity of the normal force



All the normal forces in the foundation joint from the **upper** structure and from its own weight.



#### Eccentricity of the normal force



All bending moments in the foundation joint from the **bending** moment and from the **displacement** force.



#### Eccentricity of the normal force



All the internal forces in the foundation joint - from the normal force and bending moment, from the displacement force and from the dead weight.



#### Eccentricity of the normal force



The total eccentricity of the applied vertical force is determined from the total moment and the total vertical forces.



## Eccentricity of the normal force

The total eccentricity of the applied vertical force is determined from the total moment and the total vertical forces



Extra Theory

#### THEORY - Effective load area



If the **normal force were applied in the axis**, the **load area** would correspond to the **plan area** 





The normal force acting in the heel of the foot generally does not act in the axis of the foot, but acts at a certain eccentricity.





The effective loading area depends precisely on the eccentricity of the applied normal force.



The effective loading area depends precisely on the eccentricity of the applied normal force.

$$A_{eff} = 2(b/2 - e) \cdot b$$

**Design** and **assess a footing** loaded by a combination of vertical and horizontal forces and bending moment.



#### Tasks

- Design and assess a footing made of plain concrete.
- Design and assess the reinforced concrete footing.
- Draw a **drawing of <u>the shape of the plain concrete footing</u> (including the starter reinforcement).**
- Draw the <u>reinforcement</u> drawing of the <u>reinforced</u> concrete footing (complete, including the reinforcement statement).

#### Task

So we design <u>two different footings</u> for the specified load from the upper structure, i.e.

- one foot of plain concrete,
- one foot of reinforced concrete.

The feet **will only differ**:

- Height,
- reinforcement.

### The plan dimensions of the footing (width and length) will be the same in both cases.

### Task progress

- **1) Design of** footing **plan dimensions** and their verification (preliminary verification of stresses in the foundation joint).
- 2) Design of plain concrete footing height and footing design.
- 3) Choice of reinforced concrete footing height and footing design.
- 4) Drawing of shape and reinforcement.

#### Footprint dimensions

#### Footprint dimensions

When designing the plan dimensions, we assume that the footing design must assume that the stress in the foundation joint must be less than the specified soil bearing capacity

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \le R_d.$$

#### Effective area required

From the condition for the stress in the foundation joint

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \le R_d$$

we obtain the **relation for calculating the required effective load area** 

$$A_{eff,req} = \frac{N_{Ed} + G_{0,d}}{R_d},$$

where is the normal force from the upper structure (entered),

is the dead weight of the footing (estimated as ),

is the soil bearing capacity (specified).

#### Foot width

The plan dimensions of the footing must be selected with regard to the required effective loading area .

For simplicity, we will design a square foot with a width of .

While the design can be done analytically (see <u>instructions</u>), it is much faster to **iteratively search for a solution** - i.e., **estimate the plan dimension** appropriately and verify the **stresses in the soil**. (If the verification passes, continue. If it fails, we increase the dimension. If it passes, but the margin is large, reduce the dimension.)

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#### Foot width

Select the width of the footing so that the plan area is at least 25% larger\* than the required effective load area

 $A_p = b^2 \ge 1.25 \cdot A_{eff,req}.$ 

For the first estimation of the foot width, the relation

 $b \geq \sqrt{1.25 \cdot A_{eff,req}}.$ 

#### Select the width of the foot as a multiple of 50 mm.

\*This is not an exact mathematical relationship. It is only an "engineering estimate". We consider that the resulting effective load area (which we will calculate below) will be 20% less than the total floor area.

### Verification of foot width

Verify the design of the footing width by assessing the stresses in the foundation joint

$$\sigma = \frac{N_{Ed} + G_{0_d}}{A_{eff}} \le R_d.$$

But to determine the tension, we must first determine

- the height of the foot,
- the dead weight of the foot,
- effective load area.

#### Foot width verification - foot height

Estimate the height of the footing by assuming that the spreading angle of the load in plain concrete should be at least 60°\*,


### Foot width verification - foot dead weight

Calculate the dead weight of the footing as

 $G_{0,d} = 1.35 \cdot 25 \cdot b^2 h$ ,

where is the foot width (suggested above), is the foot height (suggested above).

## Verification of the footing width - effective loading area

The effective load area for the designed dimensions is calculated from the relation

 $A_{eff}=b(b-2e),$ 

where is the width of the footing (designed above), , where is from the upper structure (entered), is the displacement force upper structure (entered), is the height of the footing (designe the normal force from the upper structure (entered), is the de the footing force from the upper structure (calculated above).



## Verification of foot width

Once the dead load and the effective load area have been determined, the stresses in the foundation joint can be verified

$$\sigma = \frac{N_{Ed} + G_{0_d}}{A_{eff}} \le R_d.$$

If the verification **succeeds**, we can continue with the task.

If verification **fails**, the design must be modified - i.e. **increase the width of the foot \***.

## Summary of footing width design



## Foot of plain concrete Proposal

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## Foot of plain concrete

The width of the foot is already determined from the previous calculation and is not adjusted in any way.

The height of the footing is calculated more accurately from the condition for stress in the concrete\*.

We will finally assess the proposed footing made of plain concrete.

\*The previous calculation of the height of the pushed area from was only approximate and only for the purpose of verifying the design of the foot width.

#### Static diagram

For the calculations, the footing is modelled as a bent **cantilever** with an effective **length** loaded by the **load from the subgrade** (i.e. the stress applied to the footing by the subgrade).



#### Static diagram

For the calculations, the footing is modelled as a bent **cantilever** with an effective **length** loaded by the **load from the subgrade** (i.e. the stress



## Calculation procedure

For the design and design of the footing, we must **determine** 

- 1) the stress that the bedding exerts on the footing.
- 2) Load applied by the subgrade to the footing ,
- 3) moment from the load in the theoretical cantilever linkage,
- 4) design tensile strength of plain concrete,
- 5) the height of the foot ,
- 6) Load eccentricity and effective area.

# 1) Tension applied to the foot

The stress exerted by the substrate on the footing is determined from the relation\*

 $\sigma_d = \frac{N_{Ed}}{A_{eff}},$ 

where will be taken over for n of the stress in the soil.



\*The dead weight of the footing is not considered here because it is eliminated. The footing is bent upwards by the load including the self weight  $\sigma$  (previous chapter) and downwards by the self weight. The result is a load without dead load .

Foot of plain concrete

## 2) Load acting on the console

The load applied by the substrate to the footing is obtained by multiplying the stress (i.e. the area load) by the loading width (i.e. the length of the footing)



## 3) Moment in theoretical bracket insertion

The moment in the console insertion is

$$M_c = \frac{1}{2} f_d a^2,$$

where is the linear load (calculated above), is the distance from the face of the column to the edge of the footing (given by the designed geometry).



# 4) Design tensile strength of plain concrete

The design tensile strength of concrete is given by

 $f_{ctd} = rac{lpha_{ct}f_{ctk,0.05}}{\gamma_c},$ 

where is the coefficient of adverse load effects on the tensile strength of concrete\*, is the characteristic tensile strength of concrete (from the table below).

Note: is calculated from (not ).

# 4) Design tensile strength of plain concrete

The	Tabulka 3.1 – Pevnostní a deformační charakteristiky betonu																
f				-	Analytické vztahy/ vysvětlivky												
<b></b>	f <sub>ck</sub> (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90		
whe	f <sub>ск.cube</sub> (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	ł	h c
cond	f <sub>om</sub> (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{\rm cm} = f_{\rm ck} + 8$ (MPa)	abl
belo	f <sub>ctm</sub> (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5	$f_{\rm ctm} = 0.30 \times f_{\rm ck}^{(2/3)} \le C50/60$ $f_{\rm ctm} = 2.12 \cdot \ln(1 + (f_{\rm cm}/10)) > C50/60$	
	f <sub>ctk.0.05</sub> (MPa)	1,1	1,3	1,5	1,8	2	2,2	2,5	2,7	2,9	3	3,1	3,2	3,4	3,5	$f_{\text{ctk},0.05} = 0.7 \times f_{\text{ctm}}$ 5% kvantil	
Note	f <sub>сtk,0,95</sub> (MPa)	2	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6	6,3	6,6	f <sub>ctk:0,95</sub> = 1,3 × f <sub>ctm</sub> 95% kvantil	
	E <sub>cm</sub> (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{\rm cm} = 22(f_{\rm cm}/10)^{0.3}$ ( $f_{\rm cm}$ v MPa)	

## 5) Height of the footing

When calculating the height of the footing, we assume that we do not want the tension to enter at the place of the most stretched (outermost) fibres

$$\sigma_{ct} = \frac{M_c}{W} = \frac{\frac{1}{2}f_da^2}{\frac{1}{6}bh^2} \le f_{ctd}.$$

By modifying the above condition we obtain the **relation\* for the design of the footing height** 

$$h \geq \frac{a}{0.85} \sqrt{\frac{3f_d}{bf_{ctd}}}.$$

\*Add a coefficient of 0.85 to the relation, which introduces the effect of shear stress on the load <sup>52</sup>

## 6) Centricity and effective area

The effective load area is again\* determined using the relation

 $A_{eff} = b(b-2e),$ 

where is the footing width (designed in the first part of the problem), , where is the moment from the top structure (entered), is the displacement force from the top structure (entered), is the footing height (designed on the previous slide), is the normal force from the top structure (entered), .

## 6) Centricity and effective area

The effective load area is again\* determined using the relation

 $A_{eff} = b(b - 2e),$ NOTICE W After we insert the newly designed (from the condition for tension) height of the foot. (We do not use the height calculated according to anymore.) We have to recalculate the deadweight for the newly designed footing height. (We no longer use the one calculated when designing the plan dimensions or the one estimated at the beginning, i.e.).

#### Foot of plain concrete Assessment

## Assessment of a footing made of plain concrete

The proposed footing must be assessed from two perspectives.

- 1) Footing stress: the stress in the **tensile fibres of** the footing **must be less than the tensile strength of the concrete**.
- 2) Soil stress: the stress in the **foundation joint must be less than the strength of the soil**.





#### Assessment of tension in the drawn fibres of the footing

The tensile stress in the tensile fibres of the footing must be less than the tensile strength of the concrete

$$\sigma_{ct} = \frac{M_c}{W} = \frac{\frac{1}{2}f_da^2}{\frac{1}{6}bh^2} \le f_{ctd}$$

Where , where is the "most current" effective area (



#### Assessment of tension in the drawn fibres of the footing



## Stress assessment in the foundation joint

The stress in the foundation joint must be less than the strength of the soil

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \le R_d,$$

where and are the "most recently" calculated values (see slide 52).

## Stress assessment in the foundation joint

The stress in the foundation joint must be less than the strength of the soil

 $N_{Ed} + G_{0,d} \sim \mathbf{P}$ 

For the deadweight we use the most recently calculated value (slide 27). (We no longer use the one calculated when designing the plan dimensions or the one estimated at the beginning, i.e. ).

NOTICE

The effective area is the most recently calculated value (slide 52). (We no longer use any of the previous estimated .)

## Reinforced concrete footing Proposal

## Reinforced concrete footing

In the design of reinforced concrete footings:

- **1) Design the bending reinforcement** in the footing.
- 2) We'll assess the reinforced concrete footing eventually.

In our role:

- We have already determined the width of the foot from the previous calculation and do not modify it in any way.
- Choose the height of the foot as appropriate (see below).

## Reinforced concrete footing

In the design of reinforced concrete footings:

- 1) Design the bending reinforcement in the footing
- 2) We The only thing we take from the previous calculations is the width of the footing. The other values (etc.) we In our real-culate new ones.
- The width of the foot is already determined from the previous calculation and is not adjusted in any way.
- Choose the height of the foot as appropriate (see below).

## Static diagram

The foot is again modelled as a bent **cantilever**, but now **with an effective** length of

 $l_k = a + 0.15b_s,$ 

Where



#### Calculation procedure

For the design and design of the footing, we must **determine** 

- 1) the height of the foot (we choose) and the actual weight of the foot,
- 2) Load eccentricity and effective area .
- 3) stress and the load applied by the subgrade to the footing,
- 4) moment from the load in the theoretical cantilever linkage,
- 5) area of the bending reinforcement (we will design).

## 1) Height and deadweight

Select a **footing height half that of a plain concrete footing** and **round the** height down to 50 mm<sup>\*</sup>.

Determine the **dead weight of the footing** as  $G_{0,d} = 1.35 \cdot 25 \cdot b^2 h.$ 

\*In determining the height of the footing, it can also be assumed that the load spreading angle should be approximately 45°. For our application, however, the footing often comes out unnecessarily high at this angle. Therefore, for the purpose of the exercise, it is better to choose a

## 2) Centricity and effective area

The effective load area can again be determined using the relationship

$$A_{eff} = b(b - 2e),$$

Where

$$e = \frac{M_{Ed} + H_{Ed}h}{N_{Ed} + G_{0,d}}.$$

Note: Use the height and deadweight as the actual current one (previous slide) and not the values for the plain concrete footing.

## 3) Stress and load acting on the footing

The stress exerted by the substrate on the footing is deterelation

 $\sigma_d = \frac{N_{Ed}}{A_{eff}},$ 

Where see previous slide.

The load that the subgrade exerts on the foo

$$f_d = b\sigma_d$$
.





#### 4) Moment in theoretical bracket insertion

The moment in the console insertion is determined using the relationship

$$M_c = \frac{1}{2} f_d l_k^2,$$

Where .



The bending reinforcement is designed in the same way as the bending reinforcement in the beam.

#### First determine the **effective cross-sectional height**

 $d=h-c-\varnothing^*-\varnothing/2,$ 

where (we consider that there is an underlying concrete under the footing),

vote to

\* It is safe to assume that the binders will place the reinforcement further away from the surface.

Estimate\* the arm of internal forces as

z = 0.9d,

the required area is obtained as

$$A_{s,req} = \frac{M_c}{zf_{yd}}$$

and design the reinforcement from the required area in the shape of NÁVRH:  $n \times øY (A_{s,prov} = Z \text{ mm}^2)$ .

We will verify the design in terms of design principles:\*

- minimum reinforcement area:
- Maximum reinforcement area: ,
- Minimum reinforcement spacing: ,
- Maximum reinforcement spacing: .

\* For an explanation of the relationships, see previous tasks.

\*\* There will be two vertical members on the edges of the foot (similar to the stirrups on a
#### Reinforced concrete footing Assessment

# Assessment of reinforced concrete footing

The design of the footing must be considered from two perspectives.

- 1) Footing stress: the load capacity of the **bending reinforcement must be greater than the moment on the cantilever**.
- 2) Soil stress: the stress in the **foundation joint must be less than the strength of the soil**.

3) Pressing the foot\*.





\*In practice, in the case of a low foot it would be necessary to assess the foot for extrusion. The process of extrusion design is similar to that of column extrusion, but it is iterative and lengthy. In practice, it is not necessary to assess extrusion.

# Assessment of the cross-section of the footing

Determine the bearing capacity of the footing cross-section using the classical procedure

$$\begin{aligned} x &= \frac{A_{s,prov} f_{yd}}{0.8 b f_{cd}}, \\ z &= d - 0.4 x, \\ M_{Rd} &= A_{s,prov} f_{yd} z. \end{aligned}$$

We'll verify the foot by assessing  $M_c \leq M_{Rd}$ ,

where is the moment in the theoretical bracket insertion (see slide 68).

# Stress assessment in the foundation joint

The stress in the foundation joint must be less than the strength of the soil

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \le R_d,$$

where and are the "most recently" calculated values (see slides 65 and 66).

# Stress assessment in the foundation joint

The stress in the foundation joint must be less than the strength of the soil

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \le R_d,$$

where and are the "most recently" calculated values (see slides 65 and 66).

#### **NOTICE**

After and we insert the most recently calculated values (slide 65 and 66). We do not use the values calculated when designing the plan dimensions or when calculating the footing from plain concrete.

Make a sketch for both the plain concrete footing and the reinforced concrete footing.

The reinforcement sketch of the reinforced concrete footing should include:

- the designed **main bending reinforcement of** the footing,
- starter reinforcement for columns,
- structural reinforcement of the footing,
  - upper in the cut in question,
  - upper and lower in the other direction,
  - Horizontal ("fringes")

```
. . . .
```













## The Council concludes

### The Council concludes

When calculating the task, the calculations of the individual values (especially and ) are very repetitive.

#### Always use the "most current" values in every calculation! \*

For example, when calculating for a reinforced concrete footing, use and for a reinforced concrete footing (and not and for a plain concrete footing).

\* The easiest way to find the correct value is to go **backwards** in the state from the current step until you find the value you are looking for.

# Thanks for your attention

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