CM01 – Concrete and Masonry Structures 1

HW4 – Design of column reinforcement

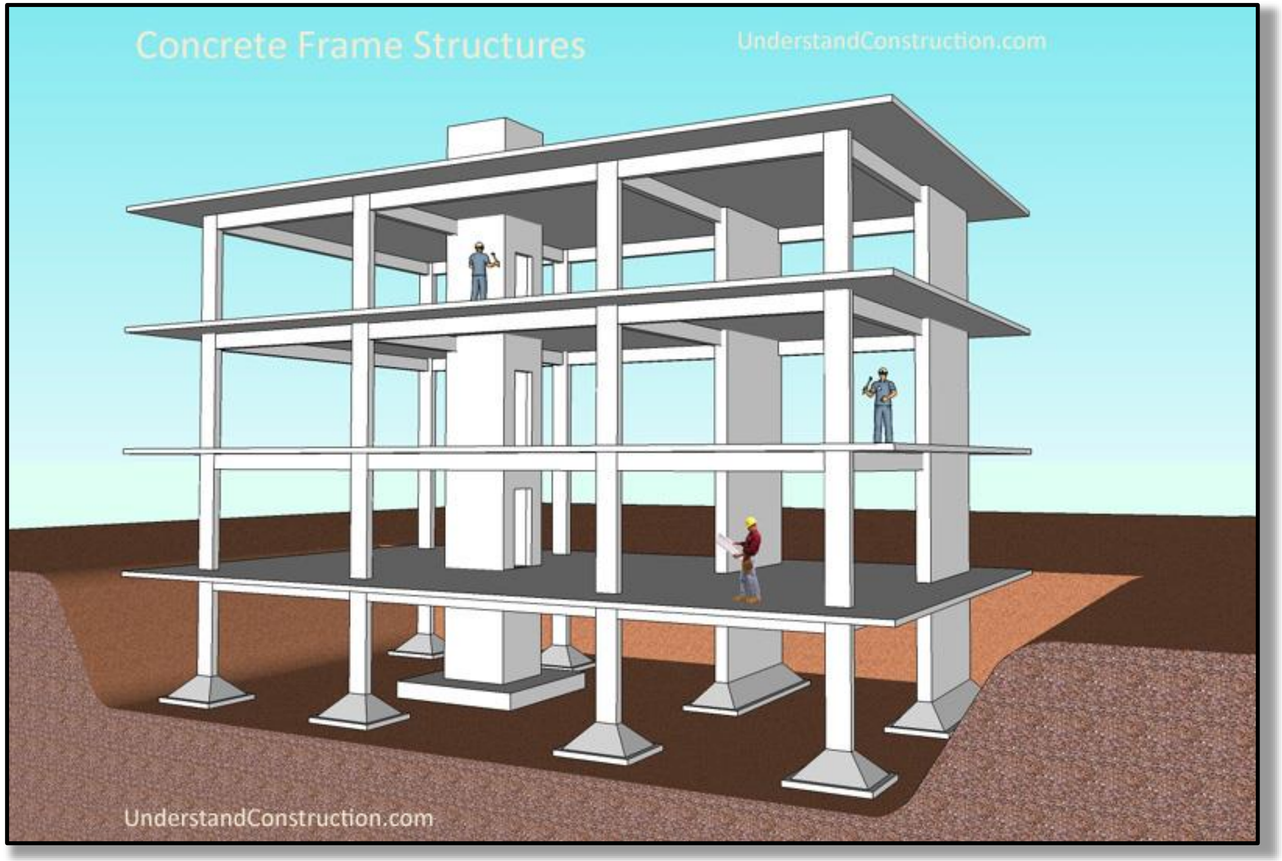
Author: Jakub Holan

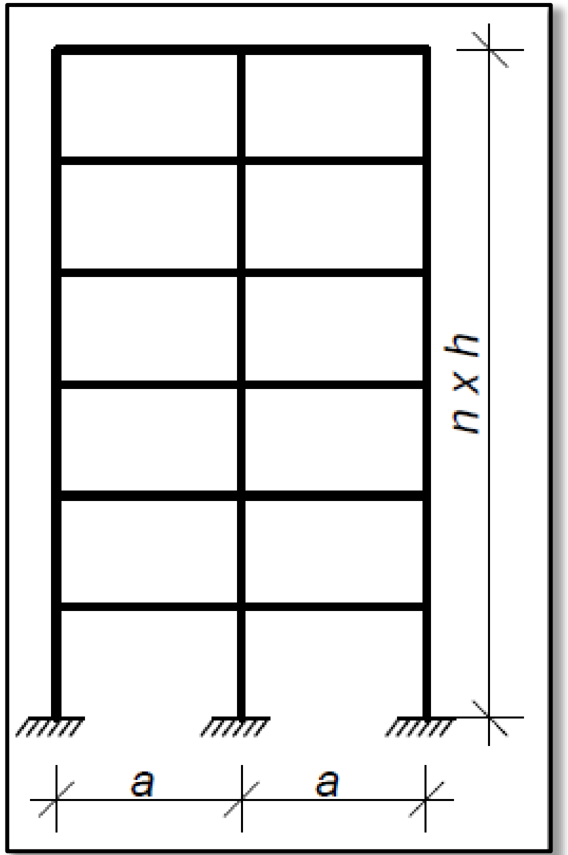
Last update: 13.10.2022 20:51

Task 1

Task 1 – Frame structure

In Task 1, frame structure will be designed.

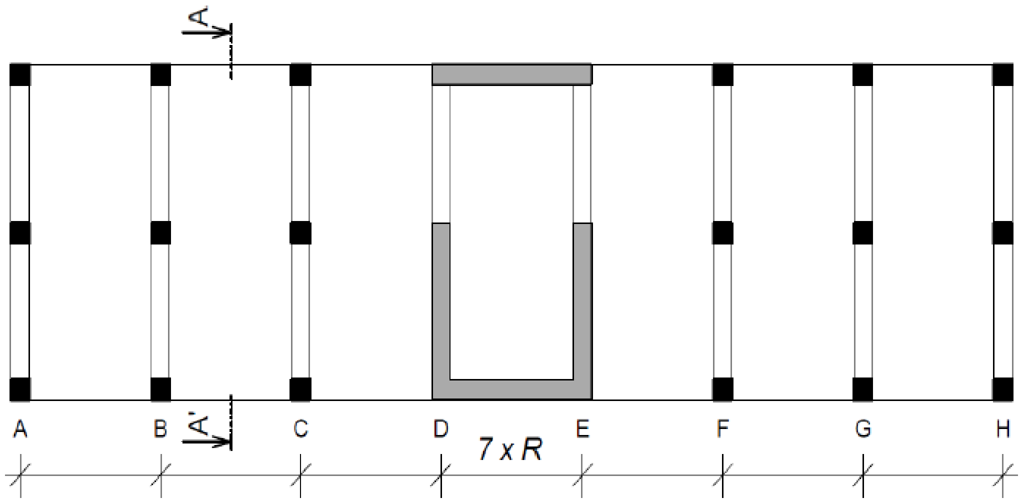




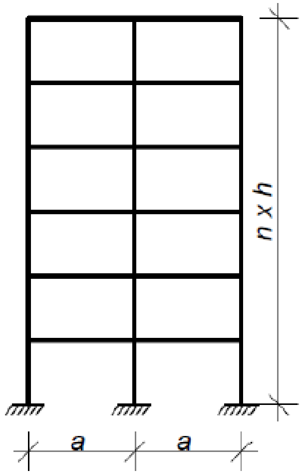
Task 1 – Assignment

Scheme of the stucture:

Plan:



Section A -A´:



Individual parameters (parameters in **bold** you can find on teacher's website):

Geometry: ***R***, ***a*** [m] – horizontal dimensions, ***h*** [m] – floor height, ***n*** – number of floors

Materials: Concrete – **concrete class**
Steel B 500 B (f_{yk} = 500 MPa)

Loads: Other permanent load of typical floor $\boldsymbol{(g\text{-}g_0)_{floor,k}}$ [kN/m^2]
Other permanent load of the roof $\boldsymbol{(g\text{-}g_0)_{roof,k}}$ [kN/m^2]
Live load of typical floor $\boldsymbol{q_{floor,k}}$ [kN/m^2]
Live load of the roof $q_{roof,k}$ = 0,75 kN/m^2
Self-weight of the slab according to calculated depth

Another parameters: ***S*** – Exposure class related to environmental conditions
Z – Working life of the structure

Task 1 – Assignment goals

<u>Our goal</u> will be to:

- Design the dimensions of all elements.
- Do detailed calculation of 2D frame – calculation of bending moments, shear and normal forces using FEM software.
- **Design steel reinforcement in the 1st floor members:**
  - beam,
  - **column**.
- Draw layout of the reinforcement.

Design of column reinforcement

Design of column reinforcement

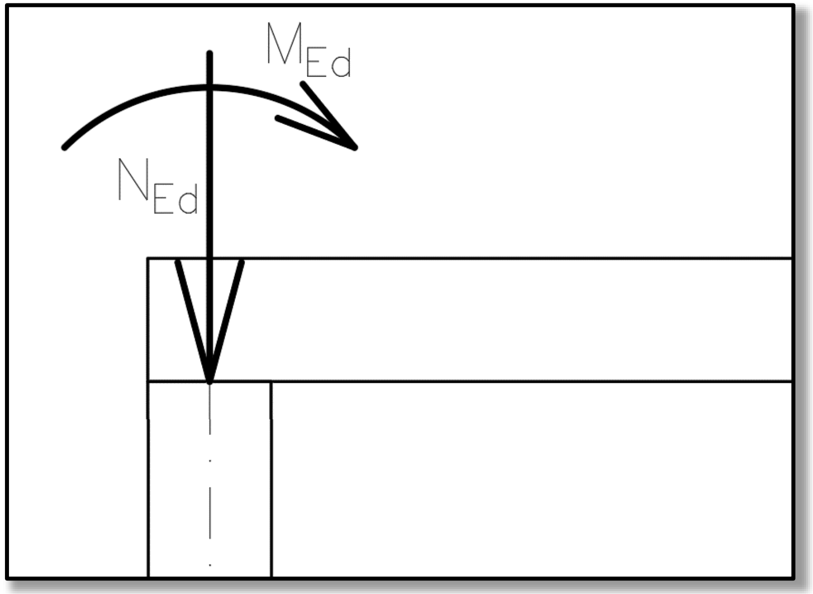
Using the maximal values of internal forces from the „envelope“ of internal forces, we will design and assess **longitudinal reinforcement** of the column using these steps:

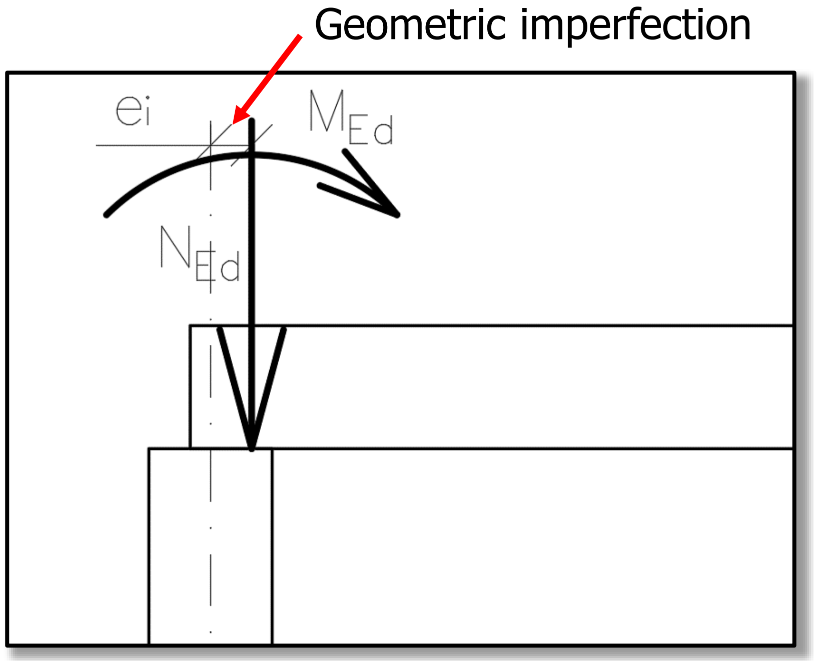
1) Calculate **geometric imperfections** and **design moments**.
2) Assess **slenderness** of the column.
3) **Design** reinforcement.
4) **Assess** the column with reinforcement.

Geometrical imperfections and design moments

Geometrical imperfections

We calculated moments on ideal model of frame structure, but **real structures are not perfect**. Geometric imperfections **cause additional bending moments**.





Geometrical imperfections

Geometrical imperfections:

$$e_i = \theta_0 \cdot \alpha_h \cdot \alpha_m \cdot \frac{l_0}{2}$$

Effective length of the column, in our case $l_0 = 0{,}8h$

Basic value of imperfection: 1/200

Reduction factor for height:

$$\alpha_h = \frac{2}{\sqrt{h}}$$

$$\frac{2}{3} \leq \alpha_h \leq 1{,}0$$

Clear length of the column in 1 floor.

Reduction factor for number of members:

$$\alpha_m = \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)}$$

Number of columns in one frame: m = 3

Geometrical imperfections

Additional moment due to geometrical imperfection:

$$M_{\text{imp}} = N_{\text{Ed}} e_{\text{i}}$$

Normal force in given cross-section (head or foot of column)

Design moments

Calculate **bending moments with the effect of geometric imperfections ($M_{Ed,I}$)** in the head and foot of the column for both combinations:

		Head of the column	Foot of the column
	$\lvert M_{imp} \rvert$	8	8.3
CO1	M_{Ed}	7	-17
	$M_{Ed} + M_{imp}$	**15**	**-8.7**
	$M_{Ed} - M_{imp}$	**-1**	**-25.3**
CO2	M_{Ed}	15	-11
	$M_{Ed} + M_{imp}$	**23**	**-2.7**
	$M_{Ed} - M_{imp}$	**7**	**-19.3**

We will use these values later to check the load-bearing capacity.

Slenderness of the column

Slenderness of the column

We must check if the column is slender or massive using the condition:

$$\lambda \leq \lambda_{lim}$$

where λ is the slenderness of the column,

λ_{lim} is the limiting slenderness.

Slenderness of the column

Slenderness of the column:

$$\lambda = \frac{l_0}{i}$$

$$i = \sqrt{\frac{I}{A_c}}$$

$$I = \frac{1}{12} b_{col} h_{col}^3$$

Cross-sectional area of the column

Dimensions of the **cross-section** of the column

Radius of gyration

Moment of inertia

Limiting slenderness:

Effect of creep, A = 0.7

Effect of reinforcement ratio, B = 1.1

Effect of bending moments

$$\lambda_{lim} = \frac{20ABC}{\sqrt{n}} \leq 75$$

Relative normal force:

$$n = \frac{N_{Ed}}{A_c f_{cd}}$$

You can't consider λ_{lim} to be more than 75.

Effect of bending moments

Effect of bending moments:

$$C = 1,7 - r_m$$

$$r_m = \frac{M_{01}}{M_{02}}$$

M_{01} and M_{02} are bending moments in the head and foot of the column, where $|M_{02}| > |M_{01}|$.
(Compare absolute values, but use values WITH the sign in the equation for r_m.)

		Head of the column	Foot of the column	M01	M02	r_m (M01/M02)
	$\|M_{imp}\|$	8	8.3	-	-	-
CO1	M_{Ed}	7	-17	-	-	-
	$M_{Ed} + M_{imp}$	**15**	**-8.7**	**-8.7**	**15**	**-0.580**
	$M_{Ed} - M_{imp}$	**-1**	**-25.3**	**-1**	**-25.3**	**0.040**
CO2	M_{Ed}	15	-11	-	-	-
	$M_{Ed} + M_{imp}$	**23**	**-2.7**	**-2.7**	**23**	**-0.117**
	$M_{Ed} - M_{imp}$	**7**	**-19.3**	**7**	**-19.3**	**-0.363**

Use the **highest** $\boldsymbol{r_m}$ in the check of slenderness.

Effect of bending moments

If the bending moments are caused predominantly by the imperfections (i.e., $M_{imp} > M_{Ed}$), we should always assume **C = 0.7**.

Slenderness of the column

We must check if the column is slender or massive using the condition:

$$\lambda \leq \lambda_{lim}$$

where λ is the slenderness of the column,

λ_{lim} is the limiting slenderness.

If $\lambda \leq \lambda_{lim}$, the column is robust.

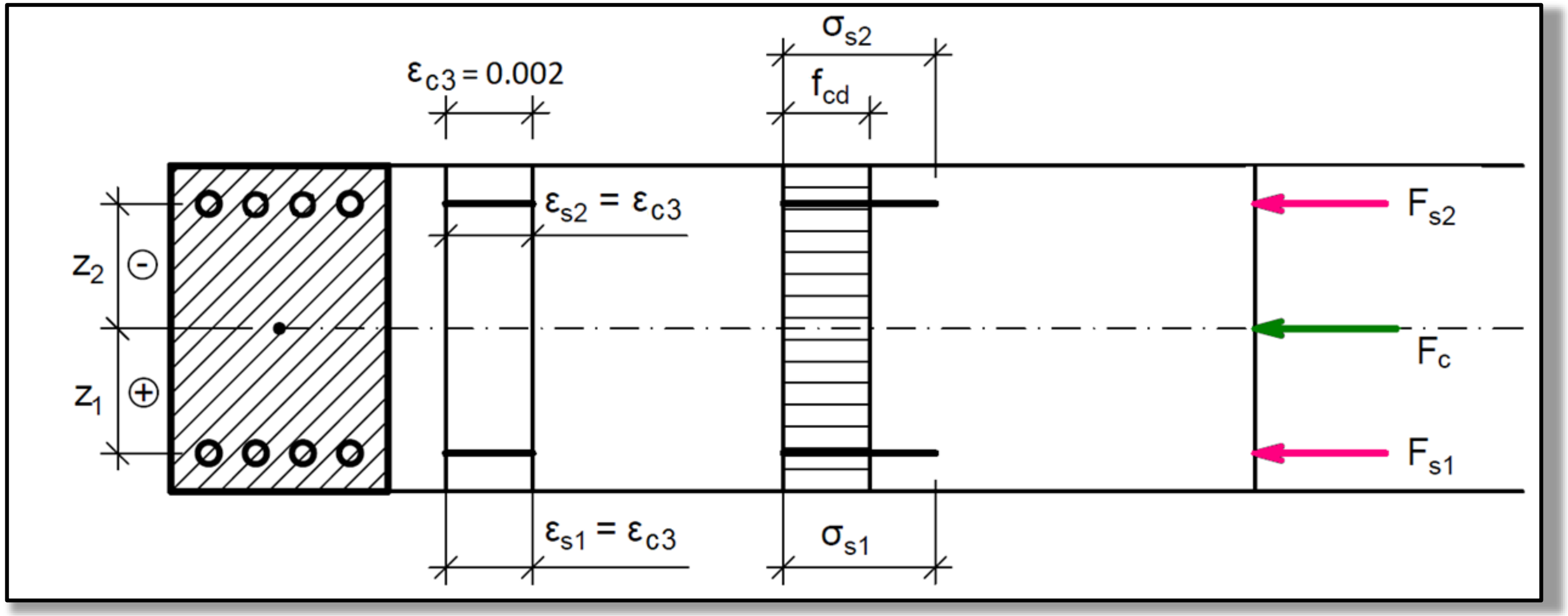
If $\lambda > \lambda_{lim}$, the column is slender.

If your column is slender, increase bending moments by approximately 30 % (simplification).

Design of reinforcement

Design of reinforcement

When designing the reinforcement, we use an estimation based on the the presumption of pure compression (uniformly distributed compression over the whole cross-section).



Design of reinforcement

We employ the limit-force assumption which means *"assume that the load-bearing capacity will be equal to the acting normal force"*:

$$N_{Rd} = N_{Ed}$$

$$0.8A_c f_{cf} + A_s f_{yd} = N_{Ed}$$

From this equation, we can derive equation for required reinforcement:

$$A_{s,req,1} = \frac{N_{Ed} - 0,8 A_c f_{cd}}{\sigma_s}$$

Stress in reinforcement in pure compression:

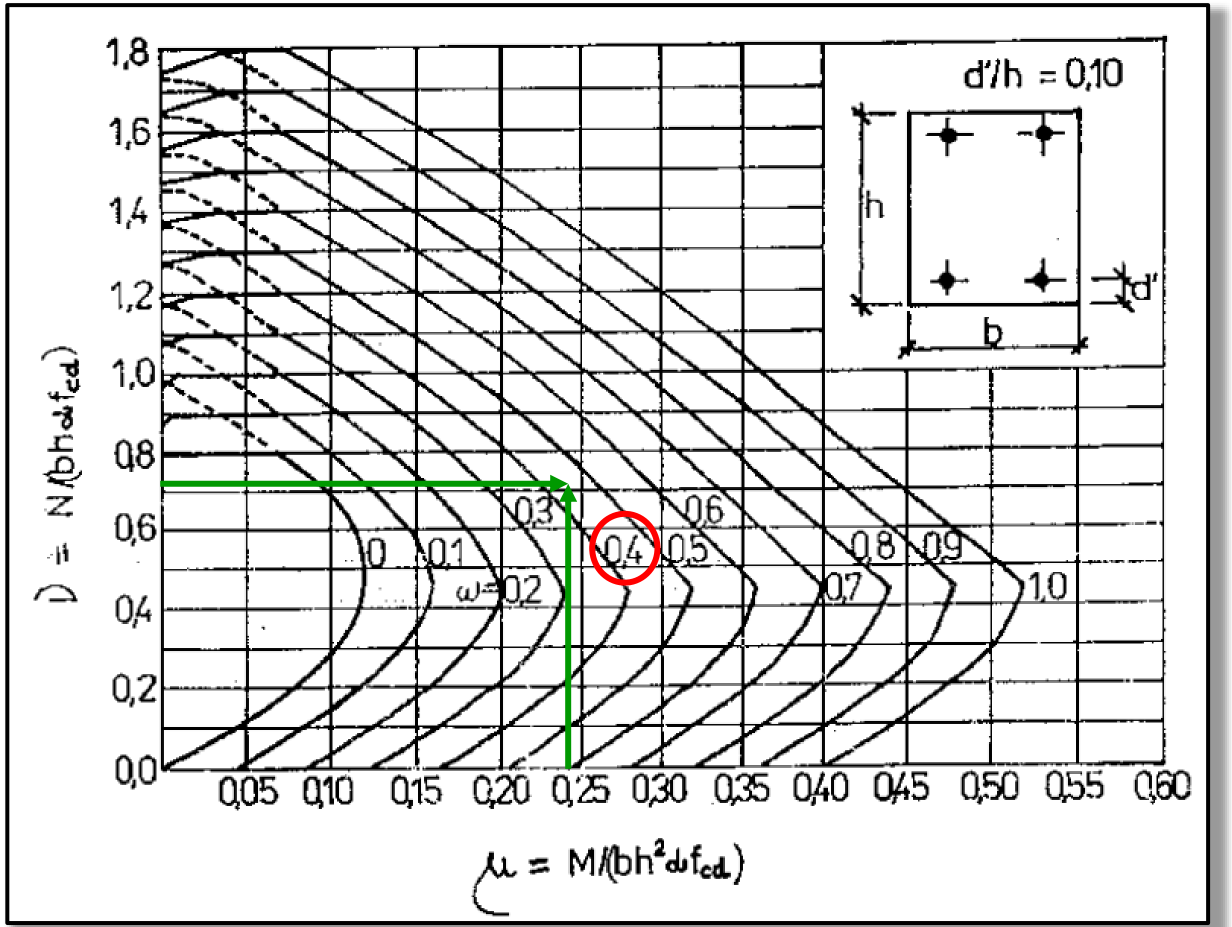
$\sigma_s = 400\ \text{MPa}$ if $f_{yd} \geq 400\ \text{MPa}$

$\sigma_s = f_{yd}$ if $f_{yd} < 400\ \text{MPa}$

If the equation gives As,req,1 < 0, minimum reinforcement of 4 ø12 mm should be designed.

Design of reinforcement

For the design, you can also employ a more complex but more precise method using a graph for design of symmetrical reinforcement.



Design of reinforcement

For the design, you can also employ a more complex but more precise method using a graph for design of symmetrical reinforcement.

$$\mu = \frac{M_{\mathrm{Ed,I}}}{b_{\mathrm{col}} h_{\mathrm{col}}^2 f_{\mathrm{cd}}} \qquad \nu = \frac{N_{\mathrm{Ed}}}{b_{\mathrm{col}} h_{\mathrm{col}} f_{\mathrm{cd}}} \quad \xrightarrow{\text{chart}} \quad \omega$$

Relative bending moment (μ)

Relative normal force (ν)

Relative exploitation of concrete part of the cross-section (ω)

Required reinforcement area:

$$\rightarrow A_{\mathrm{s,req,2}} = \frac{\omega A_{\mathrm{c}} f_{\mathrm{cd}}}{f_{\mathrm{yd}}}$$

Design of reinforcement

Design number and diameter of bars:

Example: **DESIGN**: 6x Ø16 ($A_{s,prov}$ = 1206 mm^2)

The design must satisfy:

$$A_{s,prov} \geq A_{s,req}.$$

Also, the cross-section must be symmetrically reinforced (i.e., same number of bars on each side) – that means that we **must design odd number of bars** (4, 6, 8 etc.).

Design of reinforcement

Check detailing rules for the designed reinforcement:

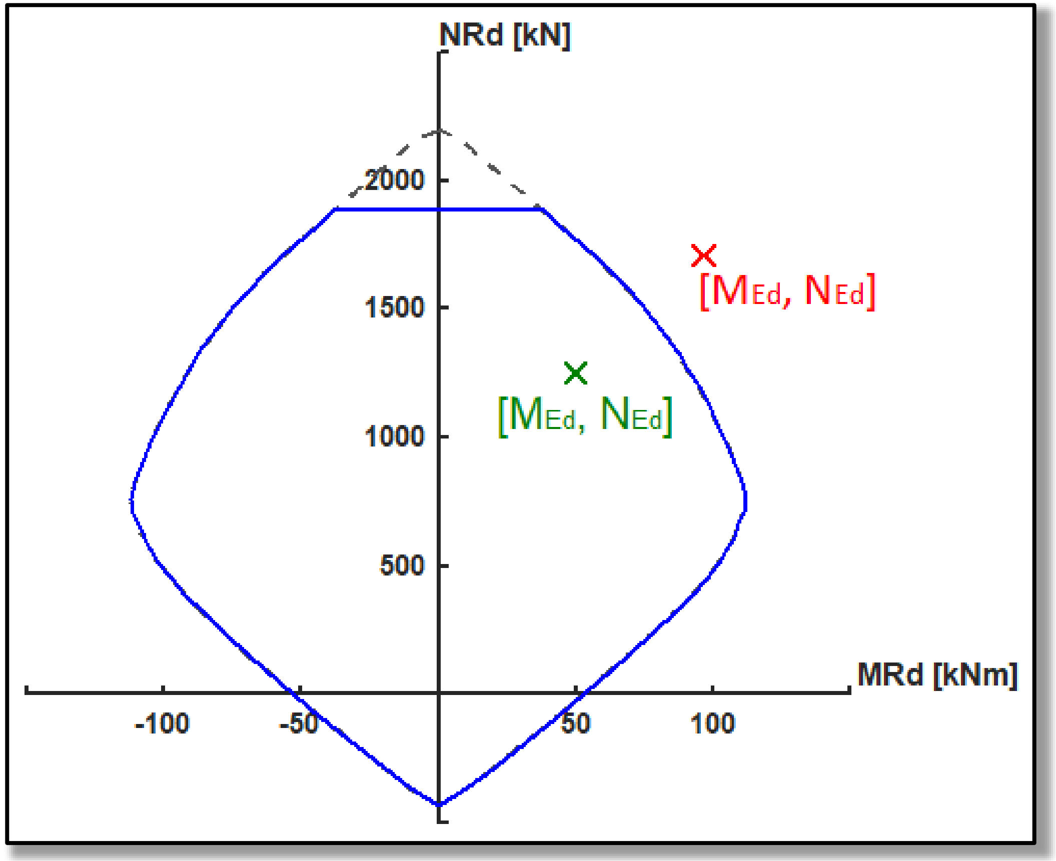
$$A_{\mathrm{s,prov}} \geq A_{\mathrm{s,min}} = \max\left(0.1\frac{N_{\mathrm{Ed}}}{f_{\mathrm{yd}}};0.002A_{\mathrm{c}}\right)$$

$$A_{\mathrm{s,prov}} \leq A_{\mathrm{s,max}} = 0,04A_{\mathrm{c}}$$

Check of column

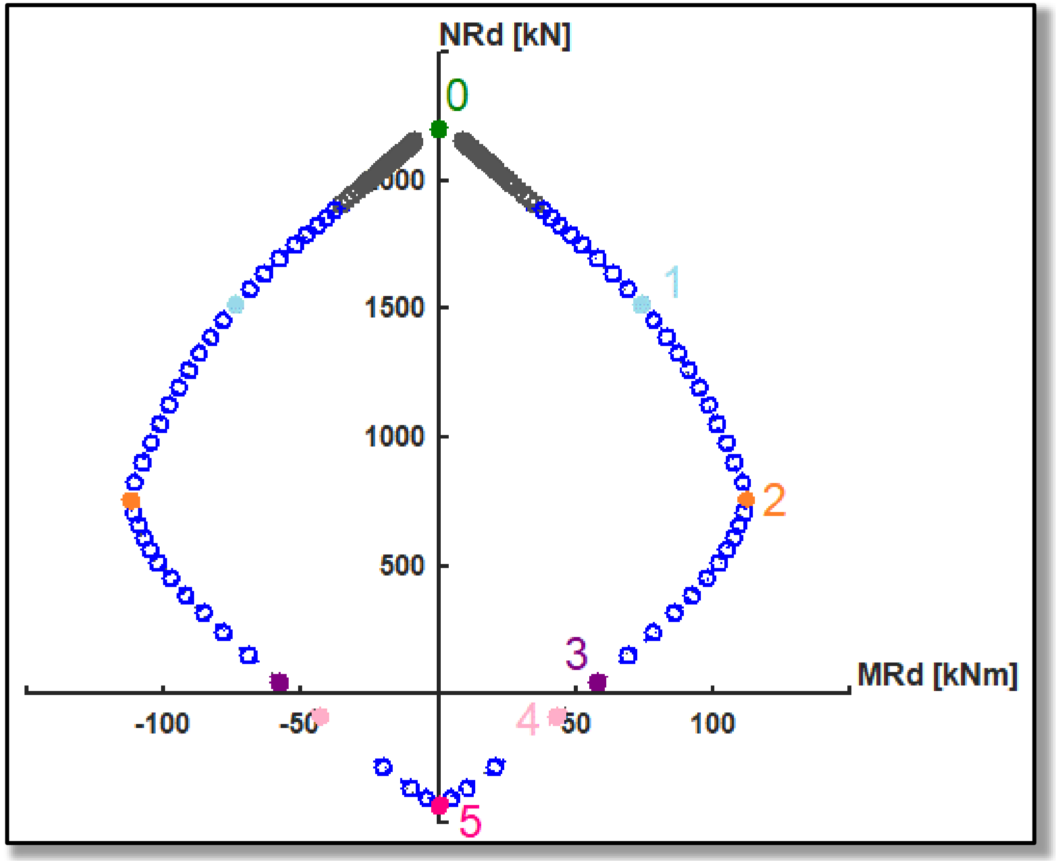
Check of column

We check the column using a “M-N interaction diagram (ID)”.



Check of column

The ID is made of many “load-bearing capacity” points. We will calculate only few of them.



Check of column

The ID is created by:

1) Calculating main points of interaction diagram (0 to 6) – see below.
2) Connecting points by lines (simplification).
3) Calculating minimum bending moment M_0.
4) Restricting axial resistance using M_0.

If internal forces lay inside the curve, the condition for the assessment of the column is satisfied. If not, adjust the design (but you don't have to recalculate the ID).

See the example of ID calculation on CM01 website.

Interaction diagram – all points

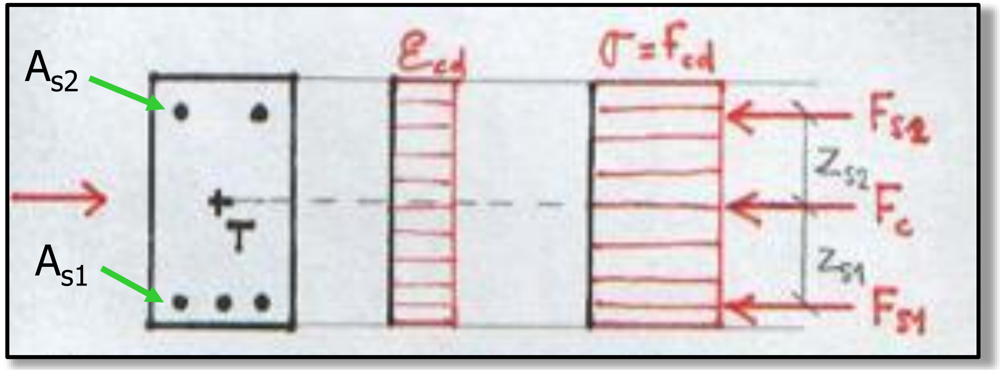
For each calculated point, the following is true.

The **normal force** load-bearing **capacity** is **the sum the partial internal forces**.

The **bending moment** load-bearing **capacity** is **the sum the moments generated by the partial internal forces**.

Point 0 – pure (axial) compression

Axial compression (maximum normal load-bearing capacity in compr.):



$$N_{Rd,0} = F_c + F_{s1} + F_{s2} = b_{col} h_{col} f_{cd} + A_{s1}\sigma_s + A_{s2}\sigma_s$$

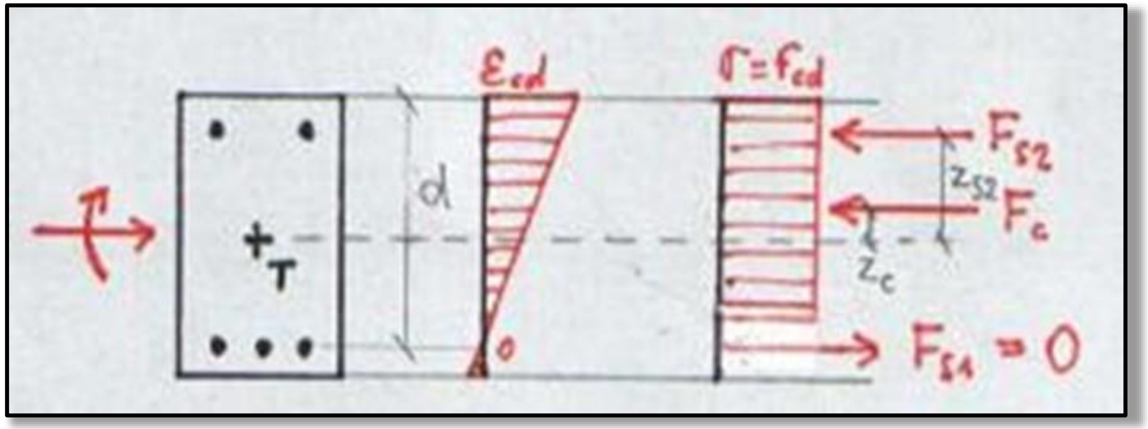
$$M_{Rd,0} = F_{s2} z_{s2} - F_{s1} z_{s1} = (A_{s2} z_{s2} - A_{s1} z_{s1})\sigma_s$$

See design of reinforcement

In our case, $A_{s1} = A_{s2}$ $(= A_{s,prov}/2)$ and $z_{s1} = z_{s2}$ $(= d - h/2)$ because we have symmetrical reinforcement.

Point 1 – strain in tensile reinforcement is 0

Strain in tensile reinforcement is 0 (almost whole cross-section is compressed):



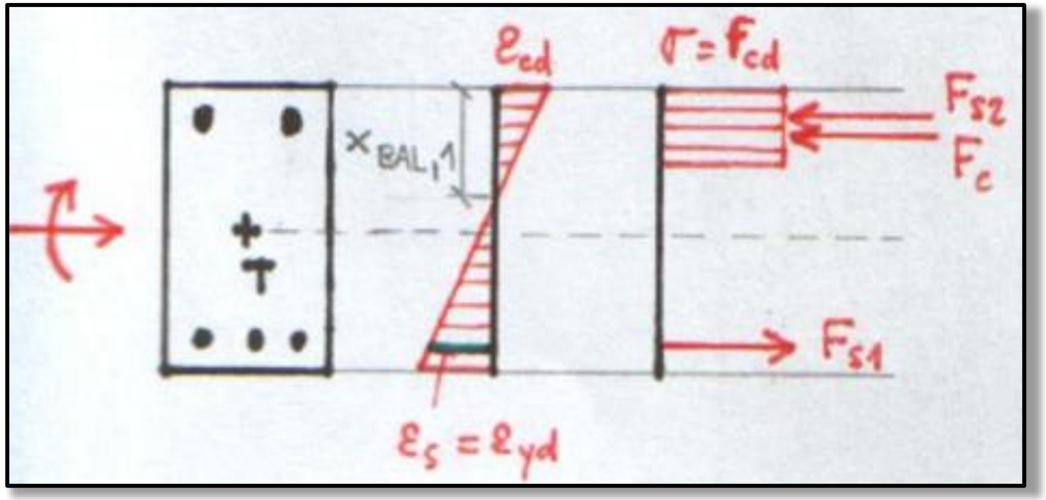
$$N_{\mathrm{Rd,1}} = F_{\mathrm{c}} + F_{\mathrm{c2}} = 0.8 b_{\mathrm{col}} d f_{\mathrm{cd}} + A_{\mathrm{s2}} f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd,1}} = F_{\mathrm{c}} z_{\mathrm{c}} + F_{\mathrm{s2}} z_{\mathrm{s2}} = 0.8 b_{\mathrm{col}} d f_{\mathrm{cd}} \left(\frac{h}{2} - 0.4d \right) + A_{\mathrm{s2}} f_{\mathrm{yd}} z_{\mathrm{s2}}$$

Factor expressing the difference between real and idealized stress distribution, see HW3.

Point 2 – tensile reinforcement at yield stress

Stress in tensile reinforcement is $\sigma_{s1} = f_{yd}$ (maximum bending moment resistance):



$$N_{Rd,2} = F_c + F_{s2} - F_{s1} = 0.8 b_{col} x_{bal,1} f_{cd} + A_{s2}\sigma_{s2} - A_{s1} f_{yd}$$

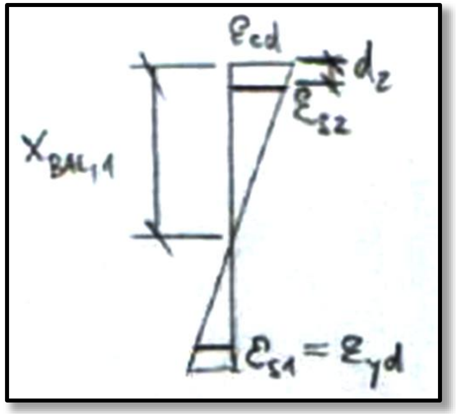
$$M_{Rd,2} = F_c z_c + F_{s2} z_{s2} + F_{s1} z_{s1} = 0.8 b_{col} x_{bal,1} f_{cd}\left(\frac{h}{2} - 0.4 x_{bal,1}\right) + A_{s2}\sigma_{s2} z_{s2} + A_{s1} f_{yd} z_{s1}$$

$$x_{bal,1} = \xi_{bal,1} d = \frac{700}{700 + f_{yd}} d$$

?

Point 2 – tensile reinforcement at yield stress

How to find stress in compressed reinforcement (σ_{s2})?



$$\rightarrow \quad \varepsilon_{s2} = \varepsilon_{cd}\left(1 - \frac{d_2}{x_{bal,1}}\right)$$

Distance from surface of the column to the centroid of compressed reinforcement.

Limit strain of concrete:
$\varepsilon_{cd} = 0.0035$

$$\text{If} \quad \varepsilon_{s2} \geq \varepsilon_{yd} = \frac{f_{yd}}{E_s} \quad \rightarrow \quad \boldsymbol{\sigma_{s2} = f_{yd}}$$

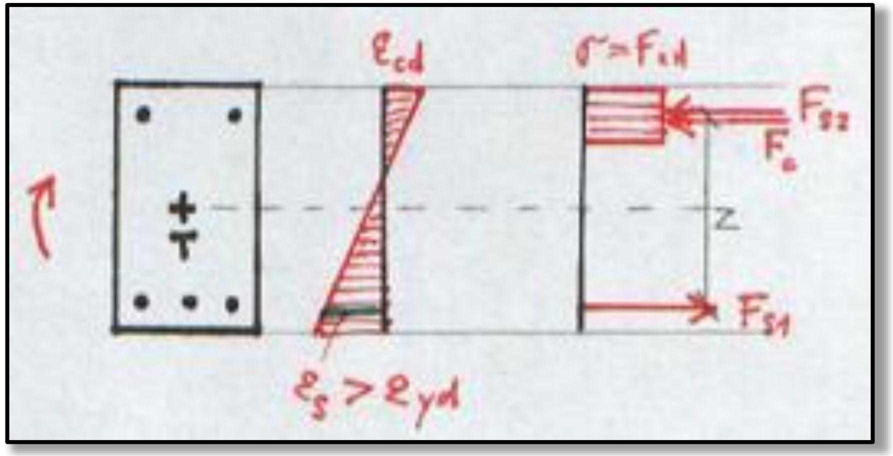
$$\text{else} \quad \boldsymbol{\sigma_{s2} = \varepsilon_{s2} E_s}$$

Elastic modulus of steel reinforcement:

$E_s = 200\,000$ MPa

Point 3 – pure bending

Pure bending (no normal force):



$$N_{Rd,3} = F_c + F_{s2} - F_{s1} = 0$$

$$M_{Rd,3} = F_c z_c + F_{s2} z_{s2} + F_{s1} z_{s1} = 0.8 b_{col} x f_{cd} \left(\frac{h}{2} - 0.4x \right) + A_{s2} \sigma_{s2} z_{s2} + A_{s1} f_{yd} z_{s1}$$

We have 2 unknowns:

- height of compressed part of concrete cross section (x),
- stress in compressed reinforcement (σ_{s2})

Point 3 – pure bending

From **“zero normal force” equation**

$$A_{s1}\sigma_{s1} - 0.8xbf_{cd} - A_{s2}\sigma_{s2} = 0,$$

an equation for compressive height can be derived:

$$x = \frac{A_s f_{yd} - A_s \sigma_{s2}}{0.8bf_{cd}}.$$

From **Hook’s law and similar triangles** of strain, an equation for stress in compressed reinforcement can be derive:

$$\sigma_{s2} = \frac{0.0035}{x}(x - d_2)E_s.$$

Point 3 – pure bending

From

$$x = \frac{A_s f_{yd} - A_s \sigma_{s2}}{0.8 b_{col} f_{cd}}.$$

and

$$\sigma_{s2} = \frac{0.0035}{x}(x - d_2)E_s.$$

a quadratic equation σ_{s2} for can be derived

$$\sigma_{s2}^2 A_{s2} - \sigma_{s2}\left(A_{s1} f_{yd} + A_{s2}\varepsilon_{cd}E_s\right) + \varepsilon_{cd}E_s\left(A_{s1} f_{yd} - 0.8 b_{col} f_{cd} d_2\right) = 0$$

Point 3 – pure bending

By solving equation

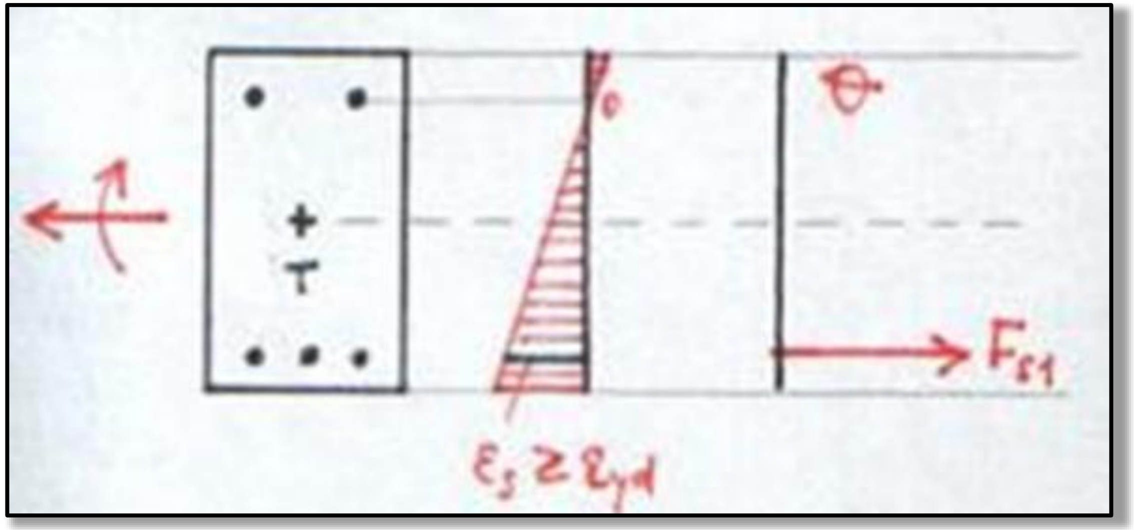
$$\sigma_{s2}^2 A_{s2} - \sigma_{s2}\left(A_{s1} f_{yd} + A_{s2}\varepsilon_{cd}E_s\right) + \varepsilon_{cd}E_s\left(A_{s1} f_{yd} - 0.8 b_{col} f_{cd} d_2\right) = 0$$

we will receive 2 values, but only one of them will “make sense” – we will use that one to calculate x:

$$\boldsymbol{x = \frac{A_s f_{yd} - A_s \sigma_{s2}}{0.8 b_{col} f_{cd}}}.$$

Point 4 – strain in compressive reinforcement is 0

Strain in compressive reinforcement is 0 (almost whole cross-section is in tension):

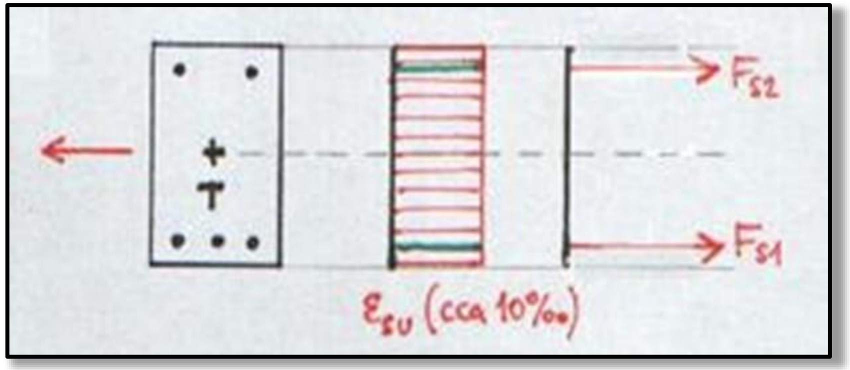


$$N_{\mathrm{Rd},4} = F_{\mathrm{s1}} = A_{\mathrm{s1}} f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd},4} = F_{\mathrm{s1}} z_{\mathrm{s1}} = A_{\mathrm{s1}} f_{\mathrm{yd}} z_{\mathrm{s1}}$$

Point 5 – pure (axial) tension

Axial tension (maximum normal load-bearing capacity in tension):

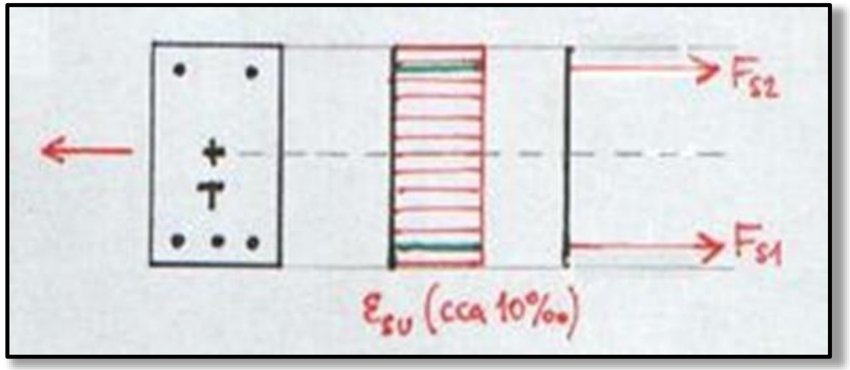


$$N_{\mathrm{Rd,5}} = F_{\mathrm{s1}} + F_{\mathrm{s2}} = \left(A_{\mathrm{s1}} + A_{\mathrm{s2}}\right) f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd,5}} = F_{\mathrm{s1}} z_{\mathrm{s1}} - F_{\mathrm{s2}} z_{\mathrm{s2}} = \left(A_{\mathrm{s1}} z_{\mathrm{s1}} - A_{\mathrm{s2}} z_{\mathrm{s2}}\right) f_{\mathrm{yd}}$$

Point 5 – pure (axial) tension

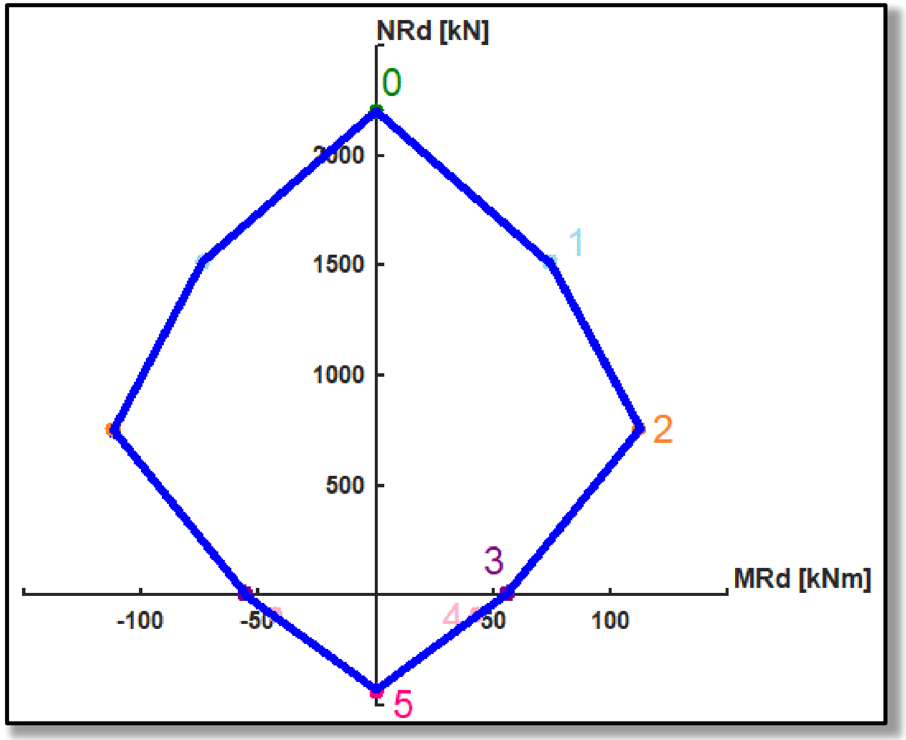
Axial tension (maximum normal load-bearing capacity in tension):



$$N_{Rd,5} = F_{s1} + F_{s2} = (A_{s1} + A_{s2}) f_{yd}$$
$$M_{Rd,5} = F_{s1} z_{s1} - F_{s2} z_{s2} = (A_{s1} z_{s1} - A_{s2} z_{s2}) f_{yd}$$

Interaction diagram

Using the calculated points 0 to 5, we create the ID



Minimal eccentricity

We must consider minimal eccentricity

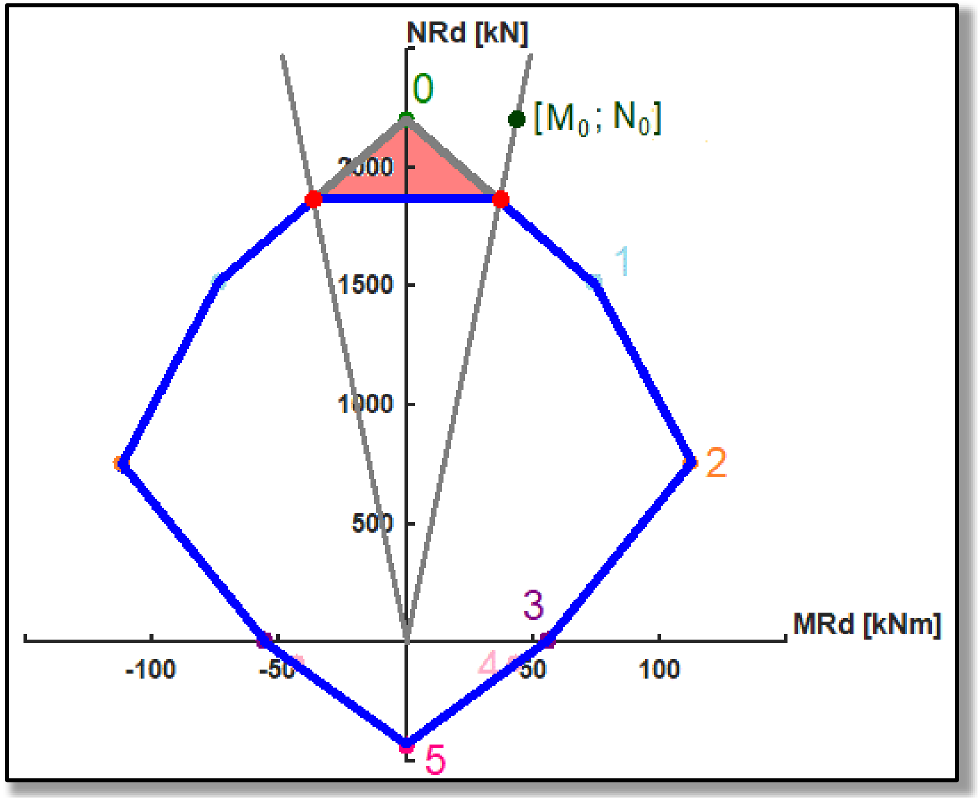
$$e_0 = \max\left(\frac{h_{\mathrm{col}}}{30}; 20\ \mathrm{mm}\right)$$

and calculate minimal bending moment

$$M_0 = N_{\mathrm{Rd,0}} e_0$$

Minimal eccentricity

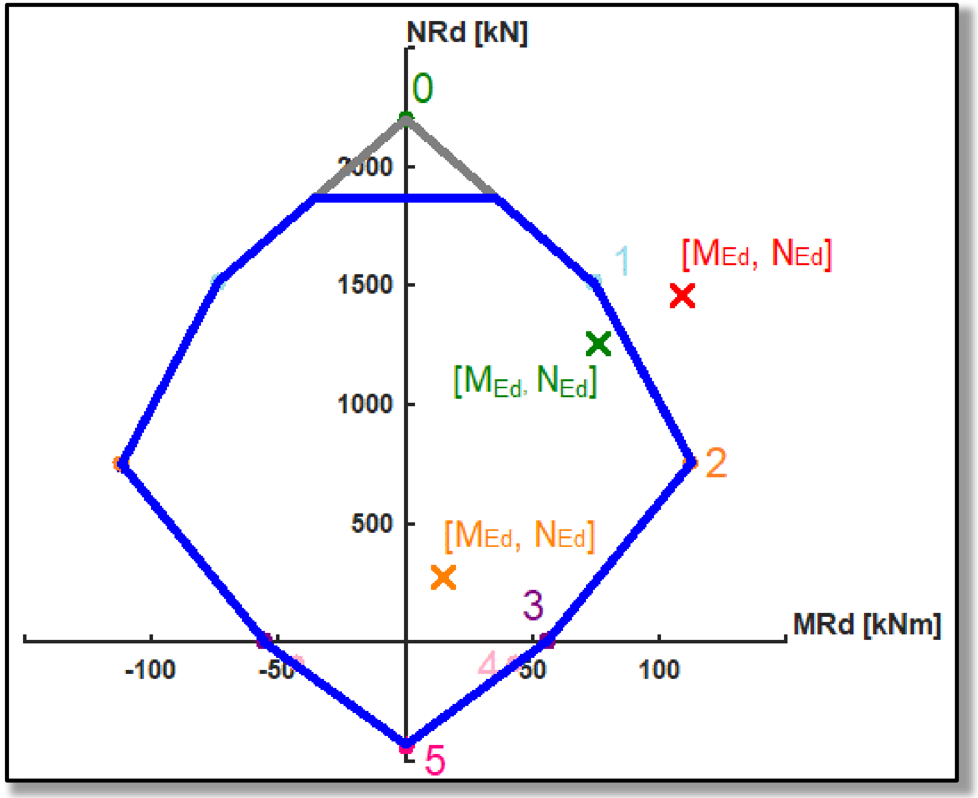
Using minimal bending moment, we restrict the ID (pure compression can never occur).



Column assessment

Using the ID, we can assess the column.

- If the point of internal forces lies outside the ID – column **does not satisfy** the assessment.
- If the point of internal forces lies inside the ID near its border – column **does satisfy** the assessment and is **economic**.
- If the point of internal forces lies inside the ID far from its border – column **does satisfy** the assessment but is **not economic**.



Next week

Next week

Next week we will focus on **reinforcement drawings of the beam and column**.

thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.