

4th TASK: REINFORCEMENT OF THE COLUMN

1. GEOMETRIC IMPERFECTIONS

$e_0 = 1/200 \rightarrow$ Basic value of imperfection.

$h =$ clear length of the column

$$e_i = e_0 \cdot \alpha_h \cdot \alpha_m \cdot \frac{l_0}{2}$$

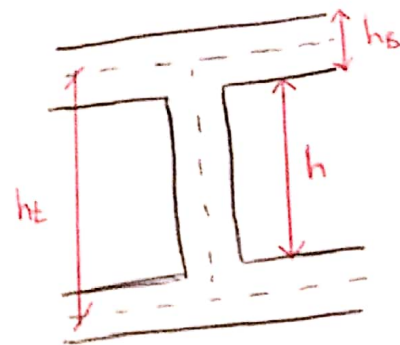
$$\alpha_h = \frac{2}{\sqrt{h}} ; h = h_c - h_b$$

$$h_c = 3.9 \text{ m}$$

$$h_b = 600 \text{ mm}$$

$$h = 3900 - 600$$

$$h = 3300 \text{ mm}$$



$$\alpha_h = \frac{2}{\sqrt{3.3}} = 1.1$$

$m =$ number of columns in one frame

$$\alpha_m = \sqrt{0.5 \left(1 + \frac{1}{m} \right)} \text{ with } m = 3$$

$$\alpha_m = \sqrt{0.5 \left(1 + \frac{1}{3} \right)} = 0.8165$$

$$l_0 = 0.8 \cdot h = 0.8 \cdot 3.3 = 2.64 \text{ m}$$

$$\rightarrow e_i = \frac{1}{200} \cdot 1.1 \cdot 0.8165 \cdot \frac{2.64}{2} = 0.0059 \text{ m}$$

Additional moment due to geometric imperfection:

$$M_{imp} = N_{ed} \cdot e_i$$

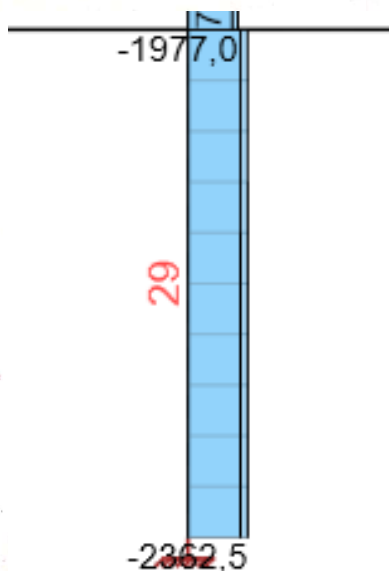
From task 2:

N_{ed} in Head:

$$N_{ed} = -1977 \text{ kN}$$

N_{ed} in the Foot:

$$N_{ed} = -2362.5 \text{ kN}$$



	Head	11'7	13'9
COMB1	Head	0	0
	Head ₁	11'7	13'9
COMB2	Head	4	-2'3
	Head ₁	15'7	16'2

Member	Dx	Combination	N[kN]	Vz[kN]	My[kNm]
29	0,00	CO1(2)	-2362,5	0,0	0,0
29	3,90	CO1(2)	-2354,5	0,0	0,0

COMB1

Member	Dx	Combination	N[kN]	Vz[kN]	My[kNm]
28	3,90	CO2(3)	-128,4	20,2	39,1
29	0,00	CO2(3)	-2075,2	1,6	-2,3
29	3,90	CO2(3)	-2067,1	1,6	4,0

COMB2

2. SLENDERNESS OF THE COLUMN

$$\lambda = \frac{l_0}{i}; \quad i = \sqrt{\frac{I}{A_c}}; \quad I = \frac{1}{12} b_{col} \cdot h_{col}^3$$

$$I = \frac{1}{12} \cdot 0'25 \cdot 0'25^3$$

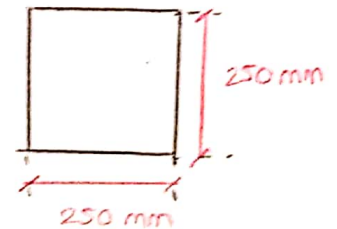
$$I = 3'25 \cdot 10^{-4} \text{ m}^4$$

$$A_c = 0'25 \cdot 0'25 = 6'25 \cdot 10^{-2} \text{ m}^2$$

$$i = \sqrt{\frac{3'25 \cdot 10^{-4}}{6'25 \cdot 10^{-2}}} = 7'5 \cdot 10^{-2} \text{ m}$$

$$\rightarrow \lambda = \frac{2'64 \text{ m}}{7'5 \cdot 10^{-2} \text{ m}} = 35'2 \text{ m}^{-1}$$

Section of my column:
(TASK 1)



LIMITING SLENDERNESS

$$\lambda_{lim} = \frac{20 \cdot A \cdot B \cdot C}{\sqrt{n}} \leq 75; \quad n = \frac{N_{ed}}{A_c \cdot f_{cd}}$$

A = 0'7 → Effect of creep
B = 1'1 → Effect of reinforce

$$f_{cd} = 16'67 \text{ MPa}$$

[C25/30]

$$16'67 \text{ MPa} = 16'67 \cdot 10^3 \text{ kN/m}^2$$

COMB1

$$n = \frac{2362'5 \text{ e}^3}{6'25 \text{ e}^4 \cdot 16'67}$$

$$n = 2'26$$

$$C = 0'7 \text{ (no moments from internal forces)}$$

COMB2

$$n = \frac{2075'2 \text{ e}^3}{6'25 \text{ e}^4 \cdot 16'67}$$

$$n = 1'99$$

$$C = 1'7 - r_m$$

$$r_m = \frac{M_{o1}}{M_{o2}} = \frac{-2'3}{4} = -0'575$$

$$C = 2'275$$

For COMB1:

$$\lambda_{lim} = \frac{20 \cdot 0'7 \cdot 1'1 \cdot 0'7}{\sqrt{2'26}} = 7'17 \leq 75$$

For COMB2:

$$\lambda_{lim} = \frac{20 \cdot 0'7 \cdot 1'1 \cdot 2'275}{\sqrt{1'99}} = 24'83 \leq 75$$

In both cases, $\lambda = 35'2 > \lambda_{lim} = 7'17$ (the lowest)
so my column is slender, so it is supposed we
need to increase the bending moment. a 30%
 $M_{ed, I, COMB1} = 18'07 \text{ KN}\cdot\text{m}$ $M_{ed, I, COMB2} = 21'06 \text{ KN}\cdot\text{m}$

3. DESIGN OF REINFORCEMENT

1st METHOD

$$A_{s, req, d} = \frac{N_{ed} - 0'8 \cdot A_c \cdot f_{cd}}{\sigma_s} ; \sigma_s' = 400 \text{ MPa if } f_{yd} \geq 400 \text{ MPa}$$
$$\sigma_s = f_{yd} \text{ if } f_{yd} < 400 \text{ MPa}$$

In my case, for B500, $f_{yd} = 434'78 \text{ MPa} > 400 \text{ MPa}$
so: $\sigma_s = 400 \text{ MPa}$.

COMB 1

$$A_{s, req, d} = \frac{2362'5 - 0'8 \cdot 6'25 \text{e}^2 \cdot 16'67 \text{e}^3}{400 \text{e}^3}$$

$$A_{s, req, d} = 0'0038 \text{ m}^2 = 3800 \text{ mm}^2$$

COMB 2

$$A_{s, req, d} = \frac{2075'2 - 0'8 \cdot 6'25 \text{e}^2 \cdot 16'67 \text{e}^3}{400 \text{e}^3} = 0'0031 \text{ m}^2$$

2nd METHOD

$$\mu = \frac{M_{ed, I}}{b_{col} \cdot h_{col}^2 \cdot f_{cd}} ; \nu = \frac{N_{ed}}{b_{col} \cdot h_{col} \cdot f_{cd}} \text{ with } \mu \text{ and } \nu \rightarrow \mu \text{ in chart.}$$

COMB 1

$$\mu = \frac{18'07}{0'25 \cdot 0'25^2 \cdot 16'67 \text{e}^3} = 0'069$$

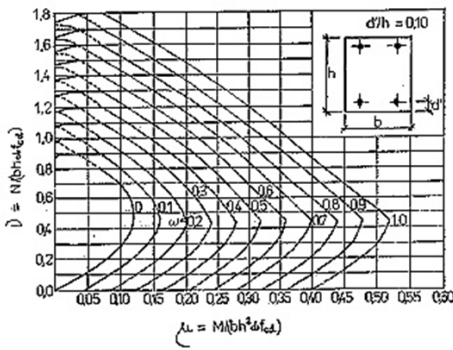
$$\nu = \frac{2362'5}{0'25 \cdot 0'25 \cdot 16'67 \text{e}^3} = 2'26$$

COMB 2

$$\mu = \frac{21'06}{0'25^3 \cdot 16'67 \text{e}^3} = 0'08$$

$$\nu = \frac{2075'2}{0'25^2 \cdot 16'67 \text{e}^3} = 1'99$$

$u=0$ in both cases (out of the graph)



So, in both cases: $A_{s, req, 2} = \frac{u \cdot A_c \cdot f_{cd}}{f_{yd}} = 0 \text{ mm}^2$

For $A_{s, req} < 0 \rightarrow 4 \phi 12 \text{ mm} (452 \text{ mm}^2)$

So, finally: $A_{s, req} = \max(A_{s, req, 1}; A_{s, req, 2})$

$A_{s, req} = \max(3800; 452) \text{ mm}^2$

$A_{s, req} = 3800 \text{ mm}^2 \rightarrow 8 \cdot \phi 25 [A_{s, prov} = 3927 \text{ mm}^2]$

CHECK DETAILING RULES

- $A_{s, prov} \geq A_{s, min} = \max(0.1 \cdot \frac{N_{ed}}{f_{yd}}; 0.002 \cdot A_c)$

$A_{s, min} = \max(0.1 \cdot \frac{2362.5}{434.78 \text{ e}^3}; 0.002 \cdot 6.25 \text{ e}^2)$

$A_{s, min} = \max(0.00054 \text{ m}^2; 0.000125 \text{ m}^2)$

$A_{s, min} = 540 \text{ mm}^2 < A_{s, prov} = 3927 \text{ mm}^2 \checkmark$

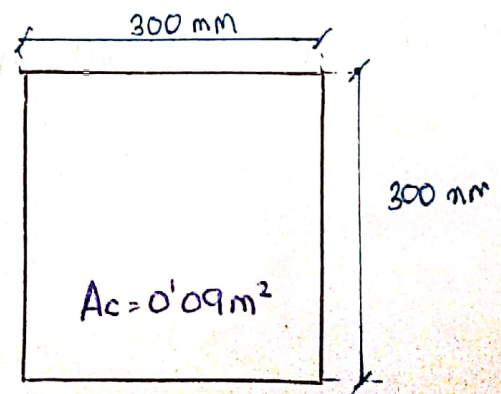
- $A_{s, max} = 0.04 \cdot A_c \geq A_{s, prov}$

$0.04 \cdot 6.25 \text{ e}^2 = 0.0025 \text{ m}^2 = 2500 \text{ mm}^2$

$A_{s, max} = 2500 \text{ mm}^2 \not\geq A_{s, prov} = 3927 \text{ mm}^2$

So, I will increase the cross-section of my column and change the concrete class to a C35/45. And the cross-section to 300 mm x 300 mm.

C35/45
 $f_{ck} = 35 \text{ MPa}$
 $f_{cd} = \frac{35}{1.5} = 23.3 \text{ MPa}$



So, we restart with the calculation of the REINFORCEMENT (I assume with this new section it will be also a slender column).

3.(1) DESIGN OF REINFORCEMENT

1st METHOD $A_c = 0.09 \text{ m}^2 = 9 \text{ e}^4 \text{ mm}^2$

COMB 1

$$A_{s, \text{req}, 1} = \frac{2362.5 - 0.8 \cdot 0.09 \cdot 23.3 \text{ e}^3}{400 \text{ e}^3} = 0.00171 \text{ m}^2$$

$$A_{s, \text{req}, 1} = 1712.2 \text{ mm}^2$$

COMB 2

$$A_{s, \text{req}, 1} = \frac{2075.2 - 0.8 \cdot 0.09 \cdot 23.3 \text{ e}^3}{400 \text{ e}^3} = 0.00099 \text{ m}^2$$

$$A_{s, \text{req}, 1} = 994 \text{ mm}^2$$

2nd METHOD

COMB 1

$$\mu = \frac{18.07}{0.3^3 \cdot 23.3 \text{ e}^3} = 0.028$$

$$v = \frac{2362.5}{0.3^2 \cdot 23.3 \text{ e}^3} = 1.12$$

$$\omega = 0.1$$

$$A_{s, \text{req}, 2} = \frac{0.1 \cdot 0.09 \cdot 23.3 \text{ e}^3}{434.78 \text{ e}^3}$$

$$A_{s, \text{req}, 2} = 0.00048 \text{ m}^2 = 482.3 \text{ mm}^2$$

COMB 2

$$\mu = \frac{21.06}{0.3^3 \cdot 23.3 \text{ e}^3} = 0.033$$

$$v = \frac{2075.2}{0.3^2 \cdot 16.67 \text{ e}^3} = 0.989$$

$$\omega = 0$$

$$A_{s, \text{req}, 2} = 0$$

$$A_{s, \text{req}} = \max(1712.2 \text{ mm}^2; 482.3 \text{ mm}^2) = 1712.2 \text{ mm}^2$$

with $6 \phi 20$ ($A_{s, \text{prov}} = 1885 \text{ mm}^2$)

CHECK DETAILING RULES

$$\bullet A_{s, \text{min}} = \max\left(0.1 \cdot \frac{2362.5}{434.78 \text{ e}^3}; 0.002 \cdot 0.09\right) = \max(543.4; 180) \text{ mm}^2$$

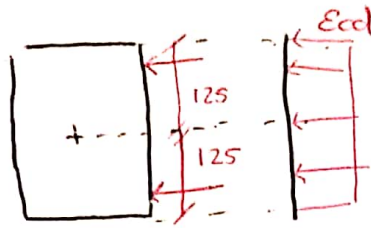
$$A_{s, \text{min}} = 543.4 < A_{s, \text{prov}} = 1885 \text{ mm}^2 \checkmark$$

$$\bullet A_{s, \text{max}} = 0.04 \cdot A_c = 0.04 \cdot 0.09 = 3600 \text{ mm}^2$$

$$A_{s, \text{max}} > A_{s, \text{prov}} = 1885 \text{ mm}^2 \checkmark$$

4. ITERATION DIAGRAM

POINT 0: maximum normal force resistance.



$$N_{rd,0} = b \cdot h \cdot f_{cd} + A_{s1} \sigma_s + A_{s2} \sigma_s$$

$$M_{rd,0} = (A_{s2} z_2 - A_{s1} z_1) \sigma_s$$

$$A_{s1} = A_{s2} \quad \text{and} \quad z_1 = z_2$$

$$N_{rd,0} = 0.3^2 \cdot 23.3e^3 + 2 \cdot 1885e^6 \cdot 400e^3 =$$

$$N_{rd,0} = 3605 \text{ KN}$$

$$M_{rd,0} = 0 \text{ KN}\cdot\text{m}$$

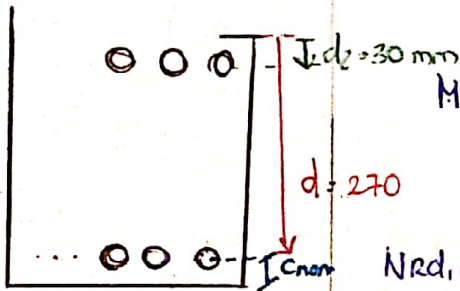
$$N_{rd,0} = 3605 \text{ KN}$$

$$M_{rd,0} = 0 \text{ KN}\cdot\text{m}$$

POINT 1: whole cross-section is compressed

$$N_{rd,1} = 0.8 \cdot b \cdot d \cdot f_{cd} + A_{s2} f_{yd}$$

$$M_{rd,1} = 0.8 \cdot b \cdot d \cdot f_{cd} \left(\frac{h}{2} - 0.4d \right) + A_{s2} f_{yd} \cdot z_{s2}$$



$$M_{rd,1} = 0.8 \cdot 0.3 \cdot 0.27 \cdot 23.3e^3 \left(\frac{0.3}{2} - 0.4 \cdot 0.27 \right) + 1885 \cdot 434.74e^3 \cdot 0.15$$

$$M_{rd,1} = 186.12 \text{ KN}\cdot\text{m}$$

$$N_{rd,1} = 0.8 \cdot 0.3 \cdot 0.27 \cdot 23.3e^3 + 1885e^6 \cdot 434.78e^3$$

$$N_{rd,1} = 2329.4 \text{ KN}$$

$$c_{nom} = c + \phi/2$$

$$c = 20 \text{ mm}$$

$$d = h - c_{nom} = 300 - 20 - \frac{20}{2}$$

$$d = 270$$

$$N_{rd,1} = 2329.4 \text{ KN}$$

$$M_{rd,1} = 186.12 \text{ KN}\cdot\text{m}$$

POINT 2: maximum bending moment resistance

$$N_{rd,2} = 0.8 \cdot b \cdot x_{bal,1} \cdot f_{cd} + A_{s2} \sigma_{s2} - A_{s1} f_{yd}$$

$$M_{rd,2} = 0.8 \cdot b \cdot x_{bal,1} \cdot f_{cd} \left(\frac{h}{2} - 0.4x_{bal,1} \right) + A_{s2} \cdot \sigma_{s2} z_{s2} + A_{s1} f_{yd} z_{s1}$$

$$x_{bal,1} = \frac{700}{700 + f_{yd}} \cdot d = \frac{700}{700 + 434.78} \cdot 0.27 = 0.1665 \text{ m}$$

$$\sigma_{s2} = f_{yd} \quad \text{if} \quad \epsilon_{s2} \geq \epsilon_{yd}$$

$$= \epsilon_{s2} E_s$$

$$\epsilon_{s2} = \epsilon_{cd} \left(1 - \frac{d_2}{x_{bal,1}} \right) = 0.0035 \left(1 - \frac{0.03}{0.1665} \right) = 2.8 \text{ ‰}$$

$$\epsilon_{y} = \frac{f_{yd}}{E_s} = \frac{434.78}{210000} = 2.07 \text{ ‰}$$

$$\epsilon_{cd} = 3.5 \text{ ‰}$$

$$\epsilon_{s2} > \epsilon_y \rightarrow \sigma_{s2} = f_{yd} = 434'78 \text{ MPa}$$

$$N_{rd,2} = 0'8 \cdot 0'3 \cdot 0'1665 \cdot 23'3 e^3 + 1885 e^6 \cdot 434'78 e^3 - 1885 e^6 \cdot 434'78 e^3$$

$$N_{rd,2} = 931 \text{ KN}$$

$$M_{rd,2} = 323'5 \text{ KN}\cdot\text{m}$$

$$N_{rd,2} = 931 \text{ KN}$$

$$M_{rd,2} = 0'8 \cdot 0'3 \cdot 0'1665 \cdot 23'3 e^3 \left(\frac{0'3}{2} - 0'4 \cdot 0'1665 \right) + 1885 e^6 \cdot 434'78 e^3 \cdot 0'15 \cdot 2$$

$$M_{rd,2} = 323'5 \text{ KN}\cdot\text{m}$$

POINT 3: Pure bending

$$N_{rd,3} = 0 \text{ KN}$$

$$M_{rd,3} = 0'8 \cdot b \cdot c \cdot x \cdot f_{cd} \cdot \left(\frac{h}{2} - 0'4x \right) + A_{s2} \sigma_{s2} \cdot z_{s2} + A_{s1} f_{yd} \cdot z_{s1}$$

For σ_{s2} :

$$\sigma_{s2}^2 \cdot A_{s2} - \sigma_{s2} (A_{s1} f_{yd} + A_{s2} E_{cd} E_s) + E_{cd} E_s (A_{s1} f_{yd} - 0'8 b c f_{cd} x) = 0$$

$$1885 e^6 \sigma_{s2}^2 - 820'9 \sigma_{s2} - 630659 = 0$$

$$\sigma_{s2,1} = 436257 \text{ N/m}^2$$

$$\sigma_{s2,2} = -776'9 \text{ N/m}^2$$

$$\sigma_{s2} = 776'9 \text{ N/m}^2$$

$$x = \frac{A_{s1} f_{yd} - A_{s2} \sigma_{s2}}{0'8 \cdot b \cdot c \cdot f_{cd}} = \frac{1885 e^6 \cdot 434'78 e^3 - 1885 e^6 \cdot 776'9}{0'8 \cdot 0'3 \cdot 23'3 e^3}$$

$$x = 0'146 \text{ m}$$

$$N_{rd,3} = 0 \text{ KN}$$

$$M_{rd,3} = 197'9 \text{ KN}\cdot\text{m}$$

$$M_{rd,3} = 0'8 \cdot 0'3 \cdot 0'146 \cdot 23'3 e^3 \left(\frac{0'3}{2} - 0'4 \cdot 0'146 \right) + 1885 e^6 \cdot 0'15 (776'9 + 434'78 e^3) = 197'9 \text{ KN}\cdot\text{m}$$

POINT 4: whole cross section is in tension

$$N_{rd,4} = 819'56 \text{ KN}$$

$$M_{rd,4} = 122'9 \text{ KN}\cdot\text{m}$$

$$N_{rd,4} = A_{s1} f_{yd} = 1885 e^6 \cdot 434'78 e^3 = 819'56 \text{ KN}$$

$$M_{rd,4} = A_{s1} f_{yd} z_{s1} = 1885 e^6 \cdot 434'78 e^3 \cdot 0'15 = 122'9 \text{ KN}\cdot\text{m}$$

POINT 5: Pure tension

$$N_{rd,5} = 1639'12 \text{ KN}$$

$$M_{rd,5} = 0 \text{ KN}\cdot\text{m}$$

$$N_{rd,5} = (A_{s1} + A_{s2}) f_{yd} = 2 \cdot 1885 e^6 \cdot 434'78 e^3 = 1639'12 \text{ KN}$$

$$M_{rd,5} = (A_{s1} z_{s1} - A_{s2} z_{s2}) \cdot f_{yd} = 0 \text{ KN}\cdot\text{m}$$

5. RESTRICTION OF COMPRESSIVE RESISTANCE

MINIMUM EXCENTRICITY

$$e_0 = \max\left(\frac{h_{col}}{30}; 20 \text{ mm}\right)$$

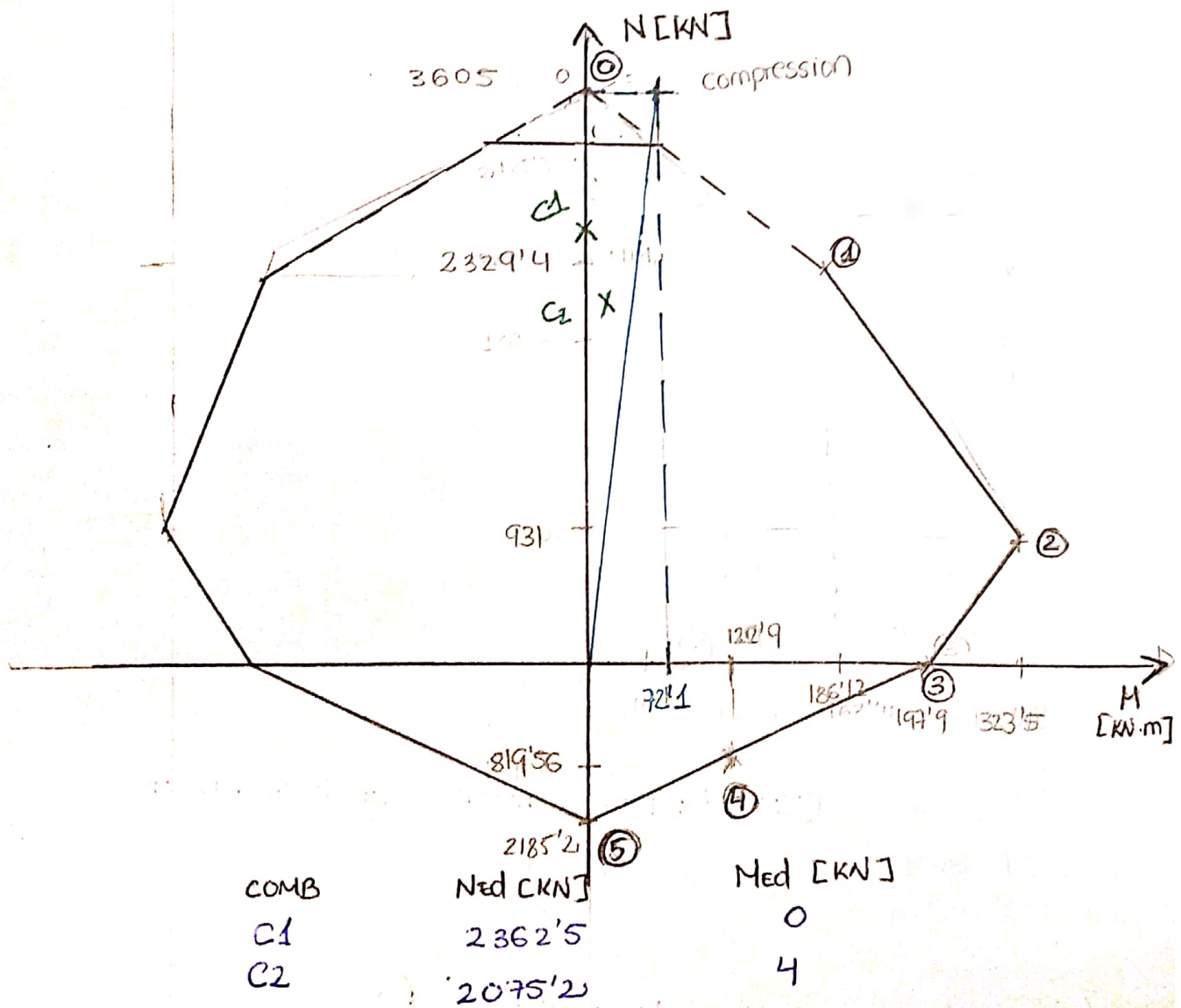
$$e_0 = \max\left(\frac{300}{30}; 20\right)$$

$$e_0 = \max(10; 20)$$

$$e_0 = 20 \text{ mm}$$

MINIMUM BENDING MOMENT

$$M_0 = N_{red,0} \cdot e_0 = 3605 \cdot 0.02 = 72.1 \text{ kN}\cdot\text{m}$$



Column is checked, all extreme values of the internal forces are inside the interaction diagram.

CONCLUSION

It is needed rebars $6 \phi 20$.