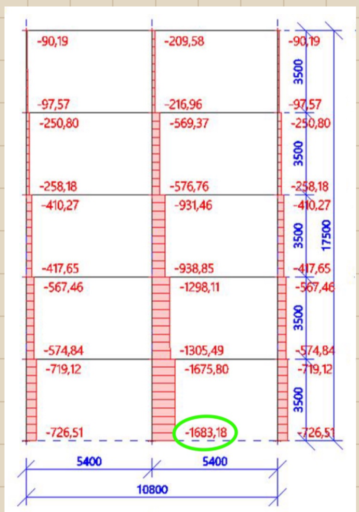


Assignment 4

Darina Ponomarenko

1. Geometric imperfections

$$e_i = \theta_0 \cdot \alpha_h \cdot \alpha_m \cdot \frac{l_0}{2}$$

$$\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{3{,}5-0{,}45}} = 1{,}145, \quad \frac{2}{3} \leq \alpha_h \leq 1 \implies \text{take } \alpha_h = 1$$

$$\alpha_m = \sqrt{\frac{1}{2}\left(1+\frac{1}{m}\right)} = \sqrt{\frac{1}{2}\left(1+\frac{1}{m}\right)} = 0{,}816$$

$$l_0 = 0{,}8h = 0{,}8(3{,}5-0{,}45) = 2{,}44\,m$$

$$e_i = \frac{1}{200} \cdot 1 \cdot 0{,}816 \cdot 2{,}44 \cdot 0{,}5 = 0{,}005\,m$$

Additional moment due to geometric imperfections:

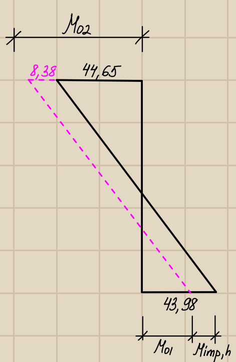
$$M_{imp} = N_{Ed} \cdot e_i$$

$$M_{imp,f} = 1683{,}18 \cdot 0{,}005 = 8{,}42\,kNm$$

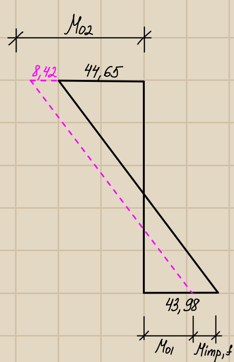
$$M_{imp,h} = 1675{,}8 \cdot 0{,}005 = 8{,}38\,kNm$$

Design moments:

$M_{imp,h}$:



$M_{imp,f}$:



$$M_{01}^{h} = M_f - M_{imp,h} = 35{,}6\,kNm \quad > \quad M_{01}^{f} = M_f - M_{imp,f} = 35{,}56\,kNm$$

$$M_{02}^{h} = M_h + M_{imp,h} = 53{,}03\,kNm \quad < \quad M_{02}^{f} = M_h + M_{imp,f} = 53{,}07\,kNm$$

2. Slenderness of the column

must satisfy the condition: $\lambda \leq \lambda_{lim}$

*purple recalculations are due to having to increase h_c to 350 mm later in the solution

$$\lambda = \frac{l_0}{i}$$

$$i = \sqrt{\frac{I}{A_c}}$$

$$I = \frac{1}{12} b_c h_c^3 = \frac{\cancel{0{,}25^4}\ \ 0{,}25 \cdot 0{,}35^3}{12} = \cancel{3{,}255 \cdot 10^{-4}}\ \ 8{,}9323 \cdot 10^{-4}\ m^4$$

$$A_c = \cancel{0{,}25^2}\ \ 0{,}25 \cdot 0{,}35 = \cancel{625}\ \ 875 \cdot 10^{-4}\ m^2$$

$$i = \sqrt{\frac{3{,}255 \cdot 10^{-4}}{625 \cdot 10^{-4}}} = \cancel{0{,}072}\ \ 0{,}101\ m$$

$$\lambda = \frac{2{,}44}{\cancel{0{,}072}\ \ 0{,}101} = \cancel{33{,}9}\ \ 24{,}16\ m$$

Limiting slenderness: $\lambda_{lim} = \frac{20ABC}{\sqrt{n}}$

A(effect of creep) $= 0{,}7$

B(effect of reinforcement ratio) $= 1{,}1$

C(effect of bending moments) $= 1{,}7 - r_m$

$$r_m = \frac{M_{01}}{M_{02}} = \frac{35{,}56}{53{,}03} = 0{,}67 \Rightarrow C = 1{,}7 - 0{,}67 = 1{,}03$$

$$n \text{ (relative normal force)} = \frac{N_{Ed}}{A_c \cdot f_{cd}} = \frac{1683{,}18}{\cancel{0{,}25^2}\ \ 0{,}0875 \cdot 16{,}67 \cdot 10^3} = \cancel{1{,}616}\ \ 1{,}154$$

$$\lambda_{lim} = \frac{20 \cdot 0{,}7 \cdot 1{,}1 \cdot 1{,}03}{\sqrt{\cancel{1{,}616}\ \ 1{,}154}} = \cancel{12{,}48}\ \ 14{,}77 < 75$$

$33{,}9 > \cancel{12{,}48}\ 14{,}77 \Rightarrow$ the column is slender

We must increase the bending moments by ~30% (M_{02}^f)

$$M_{Ed,1} = 53{,}07 \cdot 1{,}3 = 68{,}99 \approx 67\ kNm$$

3. Design of reinforcement

We assume $N_{Rd} = N_{Ed}$

$$0{,}8 A_c \cdot f_{cd} + A_s \sigma_s = N_{Ed} \Rightarrow A_{s,req,1} = \frac{N_{Ed} - 0{,}8 A_c \cdot f_{cd}}{\sigma_s}$$

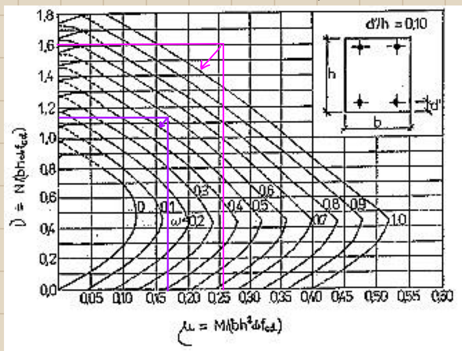
$$A_{s,req,1} = \frac{1683{,}18 - 0{,}8 \cdot \cancel{0{,}25^2}\ 0{,}0875 \cdot 16{,}67 \cdot 10^3}{400 \cdot 10^3} = \cancel{0{,}002124}\ \ 0{,}001291\ m^2 = 2124\ \ 1291\ mm^2$$

Design using the graph:

$$\mu = \frac{M_{Ed,1}}{b_c \cdot h_c^2 \cdot f_{cd}} = \frac{67}{\cancel{0{,}25^3} \; 0{,}25\cdot 0{,}35^2 \cdot 16{,}67\cdot 10^3} = \cancel{0{,}257} \; 0{,}131$$ – relative bending moment

$$\nu = \frac{N_{Ed}}{b_c \cdot h_c \cdot f_{cd}} = \frac{1683{,}18}{\cancel{0{,}25^2} \; 0{,}25\cdot 0{,}35 \cdot 16{,}67\cdot 10^3} = \cancel{1{,}616} \; 1{,}154$$ – relative normal force

Determine ω from the graph – relative exploitation of concrete part of the cross-section



$\omega = \cancel{1} \; 0{,}5$?

$$A_{s,rqd,2} = \frac{\omega A_c \cdot f_{cd}}{f_{yd}} = \frac{\cancel{0{,}25^2} \; 0{,}5\cdot 0{,}25\cdot 0{,}35 \cdot 16{,}67}{434{,}78} =$$

$$= \cancel{0{,}002396} \; 0{,}001677 \; m^2 = \cancel{2396} \; 1677 \; mm^2$$

Design $8\phi 20$ (could also do $6\phi 20$, $A_{s,prov} = 1885 \; mm^2$)

$$A_{s,prov} = 8\pi \cdot 10^2 = 2513 \; mm^2 > A_{s,rqd,1}; A_{s,rqd,2}$$

Detailing rules:

$$A_{s,min} \leq A_{s,prov} \leq A_{s,max}$$

$$A_{s,min} = max\left(\frac{0{,}1 N_{Ed}}{f_{yd}}; 0{,}002 A_c\right) = max\left(\frac{0{,}1\cdot 1638{,}18}{434{,}78\cdot 10^3} m^2; 0{,}002\cdot 0{,}25^2 \, m^2\right) =$$

$$= max(376{,}8 \; mm^2; 125 \; mm^2) = 376{,}8 \; mm^2$$

$$A_{s,max} = 0{,}04 A_c = 2500 \; mm^2$$

$376{,}8 \; mm^2 < 2513 \; mm^2 \not< 2500 \; mm^2$ oopsie!

I can change the design to $6\phi 22$, which will give $A_{s,prov} = 2281 \; mm^2$, but it is less than $A_{s,rqd,2}$, and since this method is more „precise" I don't want to risk it. For my current design $A_{s,prov}$ insignificantly exceeds the limit value, so I will dare to keep it.

However, I want to check the min clear spacing of the rebars:

$$S_c = max(20 \; mm; 1{,}2\phi) = max(20 \; mm; 1{,}2\cdot 20 \; mm) = 24 \; mm$$

— c same as for the beam with $\phi 20$; ϕ_{sw} assume 8 mm

$$S = \frac{250 - 2\cdot 30 - 2\cdot 8 - 4\cdot 20}{3} = 31{,}3 \text{ mm}$$

$31{,}3 \text{ mm} > 24 \text{ mm}$ ✓

4. Check of column – interaction diagram

Column cross-section

$b_c = h_c = 250 \text{ mm}$

$c = 30 \text{ mm}$

$\phi_{sw} = 8 \text{ mm}$

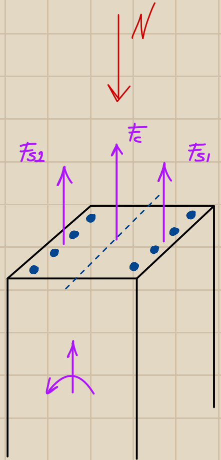
$\phi = 20 \text{ mm}$

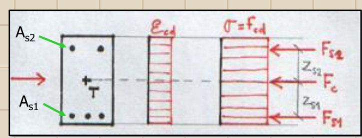
$d = 250 - 30 - 8 - 0{,}5\cdot 20 = 202 \text{ mm}$

$z_{s1} = z_{s2} = 0{,}5\cdot 250 - 30 - 8 - 0{,}5\cdot 20 = 77 \text{ mm}$

$d_1 = d_2 = 0{,}5\cdot 250 - 77 = 48 \text{ mm}$

$A_{s,1} = A_{s,2} = 0{,}5 A_{s,prov} = 1256{,}5 \text{ mm}^2$





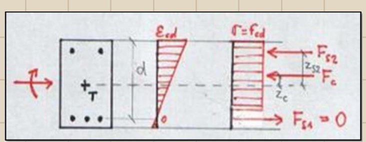
4.1) Point 0 – pure (axial) compression

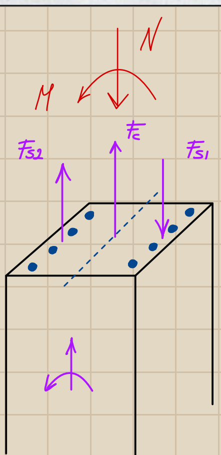
$N_{Rd,0} = F_c + F_{s1} + F_{s2} = b_c \cdot h_c \cdot f_{cd} + A_{s1}\cdot \sigma_s + A_{s2}\cdot \sigma_s =$

$= 0{,}25^2 \cdot 16{,}67\cdot 10^3 + 2513\cdot 10^{-6}\cdot 400\cdot 10^3 = 2047{,}08 \text{ kN}$

$M_{Rd,0} = F_{s2}\cdot z_2 - F_{s1}\cdot z_{s1} = (A_{s2}\cdot z_{s2} - A_{s1}\cdot z_{s1})\,\sigma_s = 0$

4.2) Point 1 – strain in tensile reinforcement is 0 (almost whole CS is compressed)





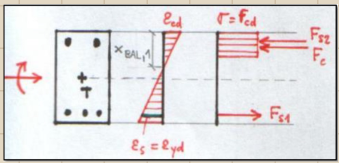
$N_{Rd,1} = F_c + F_{s2} = 0{,}8 b_c \cdot d \cdot f_{cd} + A_{s2}\cdot f_{yd} = 0{,}8\cdot 0{,}25\cdot 0{,}202\cdot 16{,}67\cdot 10^3 +$

$+ 1256{,}5\cdot 10^{-6}\cdot 434{,}78\cdot 10^3 = 1207{,}2 \text{ kN}$

$M_{Rd,1} = F_c\cdot z_c + F_{s2}\cdot z_{s2} = 0{,}8 b_c \cdot d\cdot f_{cd}(0{,}5 h_c - 0{,}4 d) + A_{s2}\cdot f_{yd}\cdot z_{s2} =$

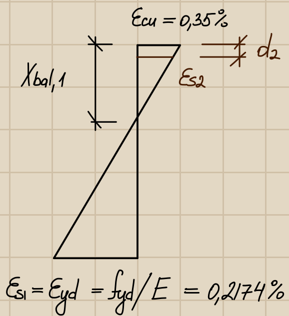
$= 0{,}8\cdot 0{,}25\cdot 0{,}202\cdot 16{,}67\cdot 10^3 (0{,}5\cdot 0{,}25 - 0{,}4\cdot 0{,}202) +$

$+ 1256{,}5\cdot 10^{-6}\cdot 434{,}78\cdot 10^3\cdot 0{,}077 = 71{,}83 \text{ kNm}$



4.3) Point 2 – tensile reinforcement at yield stress

$$N_{Rd,2} = F_c + F_{s2} - F_{s1} = 0{,}8 b_c x_{bal,1} \cdot f_{cd} + A_{s2}\sigma_{s2} - A_{s1} \cdot f_{yd}$$



$$x_{bal,1} = \xi_{bal,1} \cdot d = \frac{700 d}{700 + f_{yd}} = 0{,}125 \text{ m}$$

$$\frac{\varepsilon_{cu}}{x_{bal,1}} = \frac{\varepsilon_{s2}}{x_{bal,1} - d_2} \Rightarrow \varepsilon_{s2} = \frac{\varepsilon_{cu}(x_{bal,1} - d_2)}{x_{bal,1}} = \frac{0{,}0035(0{,}125 - 0{,}048)}{0{,}125} =$$

$$= 0{,}002156 = 0{,}2156\%$$

$$\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{434{,}78}{200 \cdot 10^3} = 0{,}002174 = 0{,}2174\%$$

$$\varepsilon_{s2} < \varepsilon_{yd} \Rightarrow \sigma_{s2} = \varepsilon_{s2} \cdot E_s = 0{,}002156 \cdot 200\, 10^3 = 431{,}2 \text{ MPa}$$

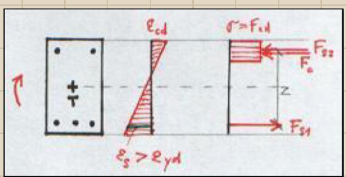
$$\Rightarrow N_{Rd,2} = 0{,}8 \cdot 0{,}25 \cdot 0{,}125 \cdot 16{,}67 \cdot 10^3 + 1256{,}5 \cdot 10^{-6} \cdot 431{,}2 \cdot 10^3 -$$

$$- 1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^3 = 412{,}25 \text{ kN}$$

$$M_{Rd,2} = F_c \cdot z_c + F_{s2} \cdot z_{s2} + F_{s1} \cdot z_{s1} = 0{,}8 b_c \cdot x_{bal,1} \cdot f_{cd}(0{,}5 h_c - 0{,}4 x_{bal,1}) +$$

$$+ A_{s2} \cdot \sigma_{s2} \cdot z_{s2} + A_{s1} \cdot f_{yd} \cdot z_{s1} = 0{,}8 \cdot 0{,}25 \cdot 0{,}125 \cdot 16{,}67 \cdot 10^3 (0{,}5 \cdot 0{,}25 - 0{,}4 \cdot 0{,}125) +$$

$$+ 1256{,}5 \cdot 10^{-6} \cdot 431{,}2 \cdot 10^3 \cdot 0{,}077 + 1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^3 \cdot 0{,}077 = 115{,}04 \text{ kNm}$$



4.4) Point 3 – pure bending

$$N_{Rd,3} = F_c + F_{s2} - F_{s1} = 0$$

$$M_{Rd,3} = F_c \cdot z_c + F_{s2} \cdot z_{s2} + F_{s1} \cdot z_{s1} = 0{,}8 b_c x \cdot f_{cd}(0{,}5 h_c - 0{,}4 x) +$$

$$+ A_{s2} \cdot \sigma_{s2} \cdot z_{s2} + A_{s1} \cdot f_{yd} \cdot z_{s1}$$

From normal force equation: $F_{s1} - F_c - F_{s2} = 0$

$$A_{s1}\sigma_{s1} - 0{,}8 x b_c f_{cd} - A_{s2}\sigma_{s2} = 0 \Rightarrow x = \frac{A_s \cdot f_{yd} - A_s \sigma_{s2}}{0{,}8 b_c \cdot f_{cd}}$$

$$\sigma_{s2} = \frac{(x - d_2) E_s \cdot \varepsilon_{cu}}{x}$$

From these 2 equations we can derive:

$$\sigma_{s2}^2 \cdot A_{s2} - \sigma_{s2}(A_{s1} \cdot f_{yd} + A_{s2} \cdot \varepsilon_{cu} \cdot E_s) + \varepsilon_{cu} E_s (A_{s1} f_{yd} - 0{,}8 b_c \cdot f_{cd} \cdot d_2) = 0$$

$$\sigma_{s2}^2 \cdot 1256{,}5 \cdot 10^{-6} - \sigma_{s2}(1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^3 + 1256{,}5 \cdot 10^{-6} \cdot 0{,}0035 \cdot 200 \cdot 10^3) +$$

$$+ 0{,}0035 \cdot 200 \cdot 10^3 (1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^3 - 0{,}8 \cdot 0{,}25 \cdot 16{,}67 \cdot 10^3 \cdot 0{,}202) = 0$$

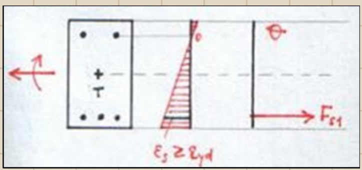
$$\Rightarrow \begin{aligned} \sigma_{s2}^{1} &= 446\,020 \text{ kPa} \\ \sigma_{s2}^{2} &= -158{,}84 \text{ kPa} \end{aligned}$$

σ_{s2}^{1} is greater than f_{yd}, so the other value is valid for us

$$x = \frac{2513 \cdot 10^{-6} (434{,}78 \cdot 10^{3} + 158{,}84)}{0{,}8 \cdot 0{,}25 \cdot 16{,}67 \cdot 10^{3}} = 0{,}164 \text{ m}$$

$$\Rightarrow M_{Rd,3} = 0{,}8 \cdot 0{,}25 \cdot 0{,}164 \cdot 16{,}67 \cdot 10^{3} (0{,}5 \cdot 0{,}25 - 0{,}4 \cdot 0{,}164) - 1256{,}5 \cdot 10^{-6} \cdot 158{,}84 \cdot 0{,}077 +$$

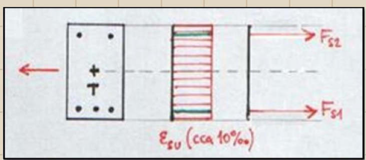
$$+ 1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{3} \cdot 0{,}077 = 74{,}53 \text{ kNm}$$



4.5) Point 4 – strain in compressive reinforcement is 0

$$N_{Rd,4} = F_{s1} = A_{s1} \cdot f_{yd} = 1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{3} = 546{,}3 \text{ kN}$$

$$M_{Rd,4} = F_{s1} \cdot z_{s1} = A_{s1} \cdot f_{yd} \cdot z_{s1} = 1256{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{3} \cdot 0{,}077 = 42{,}07 \text{ kNm}$$



4.6) Point 5 – pure (axial) tension

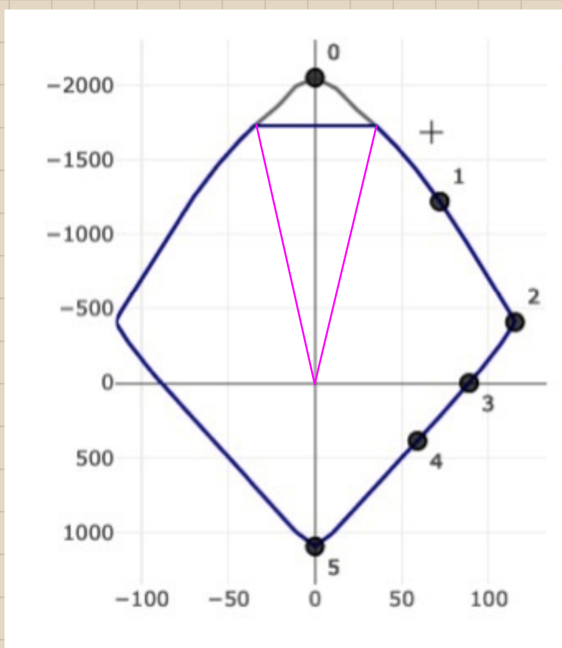
$$N_{Rd,5} = F_{s1} + F_{s2} = A_s \cdot f_{yd} = 2513 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{3} = 1092{,}6 \text{ kN}$$

$$M_{Rd,5} = F_{s1} \cdot z_{s1} - F_{s2} \cdot z_{s2} = 0$$

5) Minimal eccentricity

$$e_0 = \max\left(\frac{h_c}{30}; 20 \text{ mm}\right) = 20 \text{ mm}$$

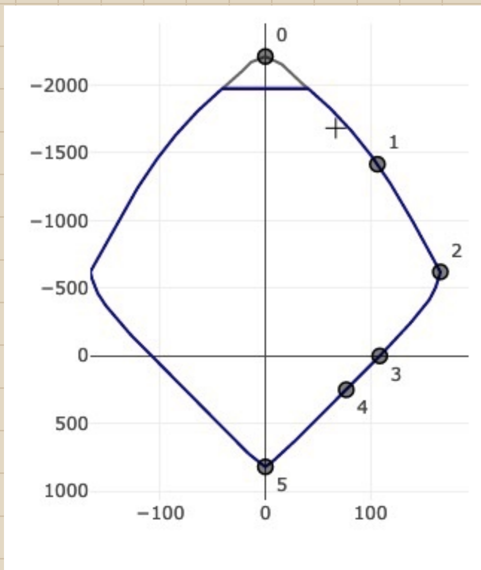
$$M_0 = N_{Rd,0} \cdot e_0 = 2047{,}08 \cdot 0{,}02 = 40{,}94 \text{ kNm}$$



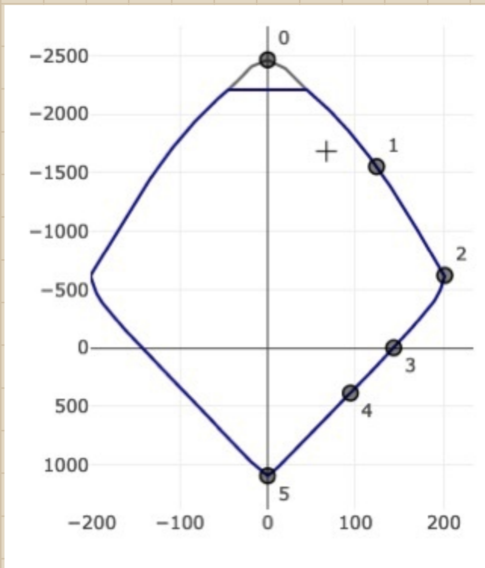
As we can see, the point of internal forces lies outside the diagram => column doesn't pass the assessment.

I will increase $h_c = 350\,mm$, but I won't recalculate the points by hand, but use the software to construct M-N diagram.

$\Downarrow$



6 ϕ20



8 ϕ20 ✓

By increasing the h_c, A_c and $A_{s,max}$ are also increased, so $A_{s,max}$ check will now be satisfied.