Assignment 3

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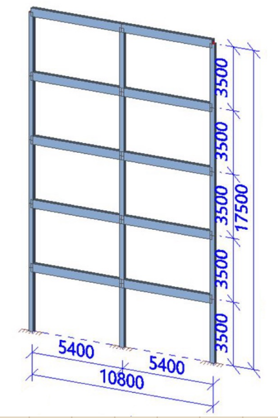
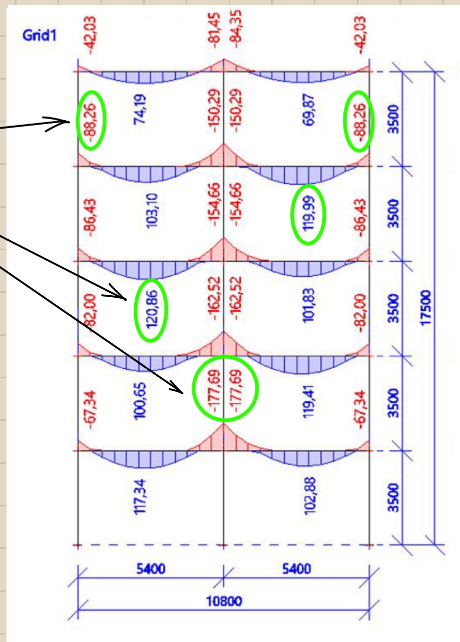
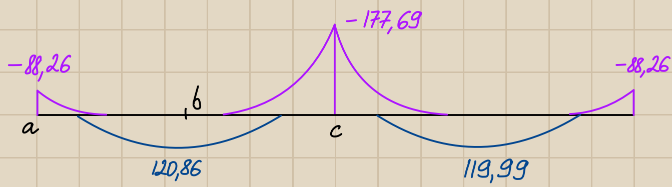


Bending reinforcement

We must design tensile reinforcement in 3 CS-ns, taking the highest values.



So the final envelope would be:

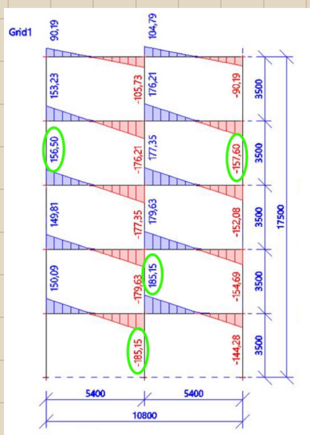


Max values in the supports should be reduced:

$$|M_{Ed,red}| = |M_{Ed,FEM}| - |V_{Ed,FEM}| \cdot 0{,}5 \cdot b_{sup}$$

$$|M_{Ed,red}|_a = 88{,}26 - 0{,}5 \cdot 0{,}25 \cdot 156{,}50 = 68{,}7 \text{ kNm}$$

$$|M_{Ed,red}|_c = 177{,}69 - 0{,}5 \cdot 0{,}25 \cdot 185{,}15 = 154{,}55 \text{ kNm}$$



beam:

$h_b = 450$

$b_b = 250$

$c = 30$ mm

$f_{yd} = 434{,}78$ MPa

$f_{cd} = 16{,}67$ MPa

Design of bending reinforcement:

Assume $M_{Rd} = M_{Ed}$

$M_{Ed,a} = 68{,}7$ kNm

$M_{Ed,b} = 120{,}86$ kNm

$M_{Ed,c} = 154{,}55$ kNm

$M_{Rd} = F_s \cdot z = A_{s,rqd} \cdot f_{yd} \cdot z$; $\quad z = 0{,}9 d_b$

$$\Rightarrow \quad A_{s,rqd} = \frac{M_{Ed}}{0{,}9 \cdot d_b \cdot f_{yd}}$$

Design $\varnothing = 16$ mm, $\varnothing_{sw} = 8$ mm (c was calc-d in hw1 and doesn't need to be adjusted)

$d_b = 450 - 30 - 8 - 8 = 404$ mm

$A_{s,prov} \geq A_{s,rqd}$

to convert into mm^2

a) $A_{s,rqd} = \frac{68{,}7 \cdot 10^6}{0{,}9 \cdot 0{,}404 \cdot 434{,}78 \cdot 10^3} = 434{,}65\, mm^2$

Design $3 \phi 16$

$A_{s,prov} = 3\pi \cdot 8^2 = \boxed{603\ mm^2} > A_{s,rqd}$

b) $A_{s,rqd} = \frac{120{,}86 \cdot 10^6}{0{,}9 \cdot 0{,}404 \cdot 434{,}78 \cdot 10^3} = 764{,}52\, mm^2$

Design $4 \phi 16$

$A_{s,prov} = 4\pi \cdot 8^2 = \boxed{804\ mm^2} > A_{s,rqd}$

c) $A_{s,rqd} = \frac{154{,}55 \cdot 10^6}{0{,}9 \cdot 0{,}404 \cdot 434{,}78 \cdot 10^3} = 978\ mm^2$

Design $\cancel{6 \phi 16}$ $4 \phi 20$ $d_b(c) = 402\, mm$

$A_{s,prov} = \cancel{6}\,4\,\pi \cdot \cancel{8}^2_{10} = \cancel{1206}\ 1257\ mm^2 > A_{s,rqd}$

*purple recalculations are due to change of the design to $4\phi 20$ later in the solution

Assessment of bending reinforcement:

$F_c = F_s$

$A_c \cdot f_{cd} = A_s \cdot f_{yd}$

$0{,}8x \cdot b \cdot f_{cd} = A_{s,prov} \cdot f_{yd} \Rightarrow x = \frac{A_{s,prov} \cdot f_{yd}}{0{,}8\, b \cdot f_{cd}}$

Lever arm: $z = d_b - 0{,}4x$

$M_{Rd} = A_{s,prov} \cdot f_{yd} \cdot z$

It must satisfy the condition: $M_{Rd} \geq M_{Ed}$

a) $A_{s,prov} = 603\, mm^2$; $M_{Ed} = 68{,}7\, kNm$

$\frac{mm^2 \cdot \cancel{MPa}}{\cancel{mm} \cdot \cancel{MPa}} = mm$

$x = \frac{603 \cdot 434{,}78}{0{,}8 \cdot 250 \cdot 16{,}67} = 78{,}6\, mm$

$z = 404 - 0{,}4 \cdot 78{,}6 = 372{,}6\, mm$

$\frac{N}{m^2}$

$m^2 \cdot Pa \cdot m = Nm$

$M_{Rd} = 603 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^3 \cdot 0{,}3726 = 97{,}69\, kNm$

$m^2 \cdot kPa \cdot m = kNm$

$97{,}69 > 68{,}7$ ✓

c) $A_{s,prov} = \overset{1257}{\cancel{1206}}\ mm^2 ;\ M_{Ed} = 154,55\ kNm$

$$x = \frac{\overset{1257}{\cancel{1206}} \cdot 434,78}{0,8 \cdot 250 \cdot 16,67} = \overset{163,9}{\cancel{157,3}}\ mm$$

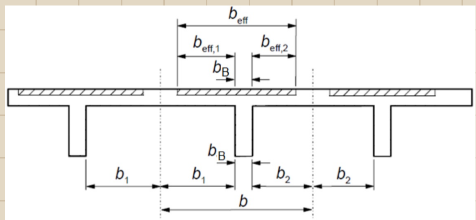
$$z = \overset{402}{\cancel{404}} - 0,4 \cdot \overset{163,9}{\cancel{157,3}} = \overset{336,4}{\cancel{341,1}}\ mm$$

$$M_{Rd} = \overset{1257}{\cancel{1206}} \cdot 10^{-6} \cdot 434,78 \cdot 10^{3} \cdot \overset{0,3364}{\cancel{0,3411}} = \overset{183,85}{\cancel{178,85}}\ kNm$$

$\overset{183,85}{\cancel{178,85}} > 154,55$ ✓✓

b) $A_{s,prov} = 804\ mm^2;\ M_{Ed} = 120,86\ kNm$

point b – T-section



$$b_{eff} = \sum b_{eff,i} + b_b \leq b$$

$$b_{eff,i} = 0,2 b_i + 0,1 \ell_0 \leq 0,2 \ell_0 ;\ \ell_0 = 0,85 \ell$$

$$b_{eff,i} \leq b_i$$

$$b = 4\ m ;\ \ell = 5,4\ m$$

$$b_{eff,1} = b_{eff,2} = 0,2 \overbrace{(4-0,25) \cdot 0,5}^{b_1 = b_2} + 0,1 \cdot \overbrace{0,85 \cdot 5,4}^{\ell_0} = 0,834\ m < b_i = 1,875\ m \checkmark$$

$$0,2 \ell_0 = 0,918\ m$$

$0,834 < 0,918$ ✓

$$b_{eff} = 2 \cdot 0,834 + 0,25 = 1,918\ m < 4\ m \checkmark$$

$$X = \frac{804 \cdot 434,78}{0,8 \cdot 1918 \cdot 16,67} = 13,7\ mm$$

$$z = 404 - 0,4 \cdot 13,7 = 398,5\ mm$$

$$M_{Rd} = 804 \cdot 10^{-6} \cdot 434,78 \cdot 10^{3} \cdot 0,3985 = 139,3\ kNm$$

$139,3 > 120,86$ ✓

Detailing rules

1. Relative compressive height:

$$\xi = \frac{x}{d_b} \leq \min\left(\xi_{bal,1} = \frac{700}{700 + f_{yd}} ;\ 0,45\right)$$

$f_{ctm} = 2{,}6\ MPa$

2. Minimal rebar area:

$$A_{s,prov} \geq A_{s,min} = max\left(0{,}26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_b \cdot d_b;\ 0{,}0013 b_b \cdot d_b\right)$$

3. Maximal rebar area: $A_{s,prov} \leq A_{s,max} = 0{,}04 b_b \cdot d_b$

4. Maximal axial spacing of rebars: $S_a \leq S_{a,max} = min(2h_b;\ 250\ mm)$

5. Minimal clear spacing of rebars: $S_c \geq S_{c,min} = max(20\ mm;\ 1{,}2\varnothing)$

a) $A_{s,prov} = 603\ mm^2;\quad x = 78{,}6\ mm$

1. $\xi = \dfrac{78{,}6}{404} = 0{,}195;\quad \xi_{bal,1} = \dfrac{700}{700 + 434{,}78} = 0{,}617 > 0{,}45$

$0{,}195 < 0{,}45$ ✓

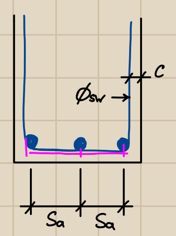
2. $A_{s,min} = max\left(0{,}26 \cdot 250 \cdot 404 \cdot \dfrac{2{,}6}{500};\ 0{,}0013 \cdot 250 \cdot 404\right) =$

$= max(136{,}55;\ 131{,}3) = 136{,}55$

$603\ mm^2 > 136{,}55\ mm^2$ ✓

3. $A_{s,max} = 0{,}04 \cdot 250 \cdot 404 = 4040$

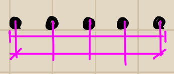
$603\ mm^2 < 4040\ mm^2$



4. $S_{a,max} = min(2 \cdot 450;\ 250\ mm) = 250\ mm$

$$S_a = \frac{b_b - 2c - 2\varnothing_{sw} - \varnothing}{n-1} = \frac{250 - 2 \cdot 30 - 2 \cdot 8 - 16}{2} = 79\ mm$$

$79\ mm < 250\ mm$ ✓



5. $S_{c,min} = max(20\ mm;\ 1{,}2\varnothing) = max(20\ mm;\ 19{,}2\ mm) = 20\ mm$

$$S_c = \frac{b_b - 2c - 2\varnothing_{sw} - n \cdot \varnothing}{n-1} = 63\ mm$$

$63\ mm > 20\ mm$ ✓

The first group of rebars passed the check ✓

b) $A_{s,prov} = 804\ mm^2;\ X = 13,7\ mm$

1. $\xi = \frac{13,7}{404} = 0,034$

$0,034 < 0,45$ ✓

2. $804\ mm^2 > 136,55\ mm^2$ ✓

3. $804\ mm^2 < 4040\ mm^2$ ✓

4. $S_a = \frac{250 - 2\cdot30 - 2\cdot8 - 16}{3} = 52,7\ mm$

$52,7\ mm < 250\ mm$ ✓

5. $S_c = \frac{250 - 2\cdot30 - 2\cdot8 - 4\cdot16}{3} = 36,7\ mm$

$36,7\ mm > 20\ mm$ ✓

Second group of rebars passed the check ✓✓

c) $A_{s,prov} = \cancel{1206}\ 1257\ mm^2;\ x = \cancel{157,3}\ 163,9\ mm;\ d_{b(c)} = 402\ mm$

1. $\xi = \frac{\cancel{157,3}\ 163,9}{\cancel{404}\ 402} = \cancel{0,389}\ 0,407$

$\cancel{0,389}\ 0,407 < 0,45$ ✓✓

2. $\cancel{1206}\ 1257\ mm^2 > 136,55\ mm^2$ ✓✓

3. $\cancel{1206}\ 1257\ mm^2 < 4040\ mm^2$ ✓✓

didn't recalculate these values for new d_b because difference would be insignificant and the gap is very big

4. $S_a = \frac{250 - 2\cdot30 - 2\cdot8 - \cancel{16}\ 20}{\cancel{5}\ 3} = \cancel{31,6}\ 51,3\ mm$

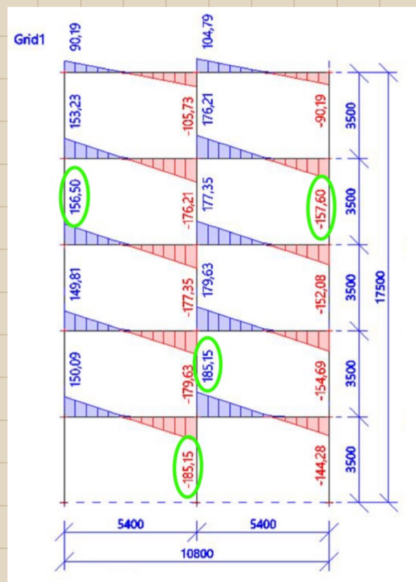
$\cancel{31,6}\ 51,3\ mm < 250\ mm$ ✓✓

5. $S_c = \frac{250 - 2\cdot30 - 2\cdot8 - \cancel{6\cdot16}\ 4\cdot20}{\cancel{5}\ 3} = \cancel{15,6}\ 31,3\ mm$

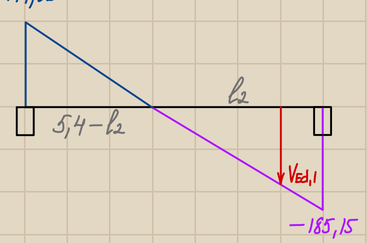
$\cancel{15,6}\ 31,3 \not> 20\ mm$ check failed!

I can now either increase the diameter of the bars and reduce their number or place them in two layers. I have no experience with bundles, so I will choose the first option.

I can increase the ϕ up to 20 mm without changing the c.
So, new design: $4\phi 20$. I will replace the values in the above calculations with purple.
After changing the rebars, the third group passed the check ✓



Design of shear reinforcement:



$$\frac{144,28}{5,4-l_2} = \frac{185,15}{l_2}$$

$$\Rightarrow l_2 = 3,03\,m$$

Load-bearing capacity of stirrups:

$$V_{Rd} = \frac{\Delta l}{S} \cdot A_{sw} \cdot f_{yd}$$

$$V_{Ed} = V_{Rd}$$

$$V_{Ed} = \frac{\Delta l}{S} \cdot A_{sw} \cdot f_{yd} \Rightarrow S = \frac{\Delta l}{V_{Ed}} \cdot A_{sw} \cdot f_{yd}$$

Design shear force:

$$|V_{Ed,1}| = |V_{Ed,sup2}| \cdot \frac{l_2 - (0,5 b_{sup} + d_b)}{l_2} = 185,15 \cdot \frac{3,03 - (0,5 \cdot 0,25 + 0,402)}{3,03} =$$

$$= 152,95\ kN$$

Spacing of stirrups:

$$A_{sw} = \frac{n \pi \cdot \phi_{sw}^2}{4} = \frac{2\pi \cdot 8^2}{4} = 100,5\ mm^2$$

$$\Delta l = z \cdot ctg\vartheta = 336,4 \cdot 1,5 = 504,6\ mm$$

$$S_1 \leq \frac{A_{sw} \cdot f_{yd}}{V_{Ed,1}} \cdot \Delta l = \frac{100,5 \cdot 10^{-6} \cdot 434,78 \cdot 10^{6} \cdot 0,5046}{152,95 \cdot 10^3} = 0,144\,m = 144\,mm$$

And at the same time:

$$\frac{mm^2 \cdot MPa \, / \, \frac{MN}{m^2} \cdot m}{kN}$$

$$\cancel{m^2}\ \frac{MN \cdot m}{kN}$$

$S_1 \leq 0{,}75\, d_b = 0{,}75 \cdot 402 = 301{,}5\,mm$

$S_1 \leq 400\ mm$

$S_1 \geq 100\ mm$

Design: $\phi_{sw} = 8mm$ per $120\ mm$

Now we must assess the shear resistance:

$$V_{Rd,sw,1} = \frac{A_{sw} \cdot f_{yd} \cdot \Delta l}{S_1} = \frac{100{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{6} \cdot 0{,}5046}{0{,}12 \cdot 10^{3}} = 183{,}739\ kN$$

$183{,}739 > 152{,}95$ ✓

Checking the shear reinforcement ratio:

$$\rho_{sw,min} \leq \rho_{sw,1} \leq \rho_{sw,max}$$

$$\rho_{sw,min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} = \frac{0{,}08\sqrt{25}}{500} = 0{,}0008$$

$$\rho_{sw,max} = \frac{0{,}5\,\nu \cdot f_{cd}}{f_{yd}} ; \quad \nu = 0{,}6\left(1 - \frac{f_{ck}}{250}\right) = 0{,}6\left(1 - \frac{25}{250}\right) = 0{,}54$$

$$\rho_{sw,max} = \frac{0{,}5 \cdot 0{,}54 \cdot 16{,}67}{434{,}78} = 0{,}0104$$

$$\rho_{sw,1} = \frac{A_{sw}}{b_b \cdot S_1} = \frac{100{,}5}{250 \cdot 120} = 0{,}00335$$

$0{,}0008 < 0{,}0035 < 0{,}0104$ ✓

Location of support shear reinforcement – we must use stirrups with spacing s_1 for at least the distance Δl from the support.

$\Delta l = z \cdot ctg\,\Theta = 336{,}4 \cdot 1{,}5 = 504{,}6\ mm$ – will return back to this value

Design of mid-span shear reinforcement – spacing S_{max}

The condition is: $S_{max} \leq \min(0{,}75 d_b ; 400\ mm)$

$0{,}75\, d_b = 0{,}75 \cdot 404 = 303\ mm$

$S_{max} \leq 303\,mm$

Design: $\phi_{sw} = 8mm$ per 250 mm

Check:

$\rho_{sw,min} = 0{,}0008$

$\rho_{sw,max} = 0{,}0104$

$$\rho_{sw,2} = \frac{A_{sw}}{b_b \cdot S_{max}} = \frac{100{,}5}{250 \cdot 250} = 0{,}0016$$

$0{,}0008 < 0{,}0016 < 0{,}0104$ ✓

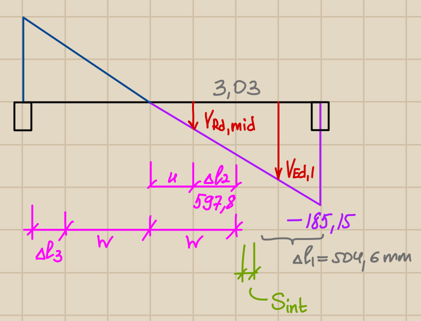
Load-bearing capacity of mid-span stirrups:

$$V_{Rd,mid} = \frac{A_{sw} \cdot f_{yd} \cdot \Delta l}{S_{max}}$$

$$\Delta l_{mid} = z \cdot ctg\,\theta = 398{,}5 \cdot 1{,}5 = 597{,}75\ mm$$

$$V_{Rd,mid} = \frac{100{,}5 \cdot 10^{-6} \cdot 434{,}78 \cdot 10^{6} \cdot 0{,}5978}{0{,}25 \cdot 10^{3}} = 104{,}48\ kN$$

Location of mid-span shear reinforcement:



$$\frac{V_{Ed,sup2}}{l_2} = \frac{V_{Rd,mid}}{u}$$

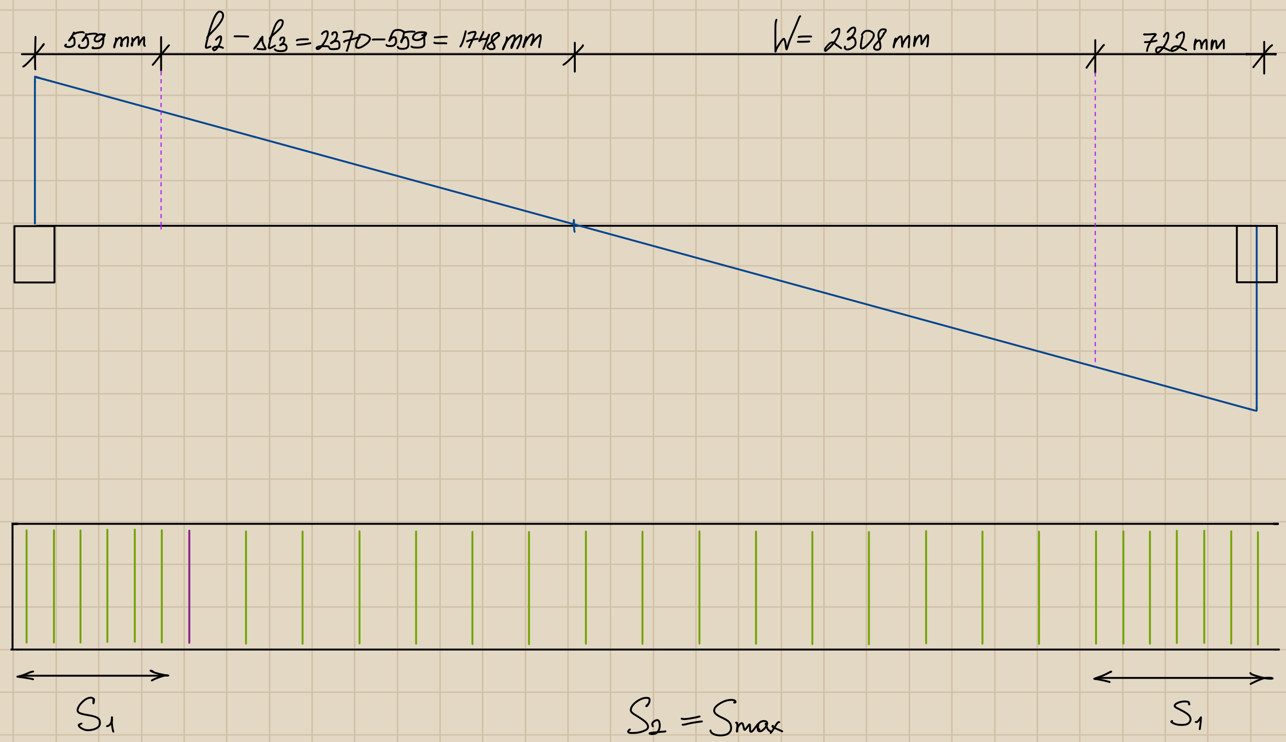
$$u = \frac{V_{Rd,mid} \cdot l_2}{V_{Ed,sup2}} = \frac{104{,}48 \cdot 3{,}03}{185{,}15} =$$

$$= 1{,}71\ m = 1710\ mm$$

$$W = u + \Delta l = 1710 + 597{,}8 = 2307{,}8\ mm$$

Spacing in the intermediate part – we will use s_1

$$\Delta l_3 = 372{,}6 \cdot 1{,}5 = 558{,}9\ mm$$



559 mm
$l_2 - \Delta l_3 = 2370 - 559 = 1748$ mm
$W = 2308$ mm
722 mm
S_1
$S_2 = S_{max}$
S_1