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BUCKLING OF STIFFENED AND UNSTIFFENED PLATES
in compression

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INTRODUCTION

The Eurocode EN 1993-1-5 provides reduction factors for both unstiffened and stiffened plates under compression and other loadings, which slightly differ from values given in former ENV 1993-1-5 and other codes in past. Therefore, it may be interesting to know what these new values mean in reality. Many years ago author presented extensive numerical and experimental studies in this area, covering strength and reduction factors based on large-deflection elastic-plastic analysis taking into account initial deflections, residual stresses, various geometric parameters including continuity influence over several bays and various boundary conditions. The reduction factors were derived both for collapse (ultimate) loading and for elastic loading (insuring elastic condition of the plates including stiffeners, which were supposed to be used in bridge design). The results were presented in charts and design formulas for easy use (see e.g. [1], [2], [3], [4], [5]).
At the time the values were compared with other results by Winter, Frieze, Crisfield, Webb, Wolchuk, Moxham, Carlson, Kvocak, Maquoi, Usami etc. E.g. ultimate reduction factors by Winter (1968) were found fully identical with author's values of unstiffened plates with amplitude of initial deflection *b*/200 and without any residual stresses. Other comparisons of author's formulas with available tests on unstiffened and stiffened plates were excellent. Without giving full details in this paper, the formulas are used to evaluate the new Standard values and procedures concerning strengths of unstiffened and stiffened plates in uniform compression.

1 UNSTIFFENED PLATING

Only basic elastic-plastic numerical values concerning square plates under compression and with imperfections shown in *Fig. 1* are presented in this paper. The amplitude of initial deflections are *b*/200 as recommended in EN 1993-1-5 and residual stresses were modelled by residual uniaxial strains with given compression level (a slight change of the pattern was employed after introductory procedure restoring equilibrium of the deflected plate). The elastic-plastic value is given either by collapse of the plate or by reaching excessive equivalent plastic strain at the mid-plane of the plate, see *Fig. 2* (ε_{pl}^{L} = 0.002, for the criterion covering all membrane stresses see more details in [2]).

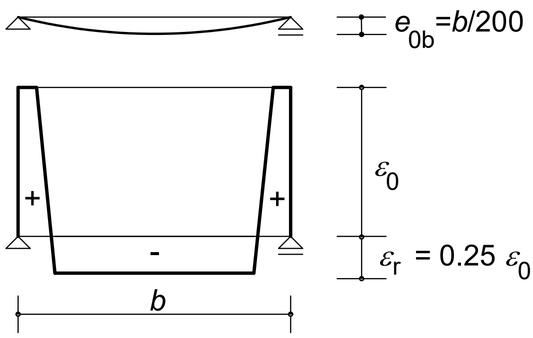

Fig. 1. Initial deflections and strains

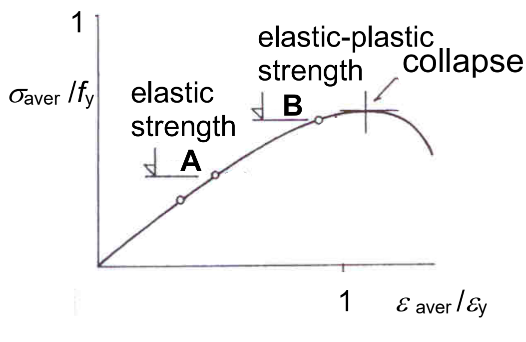

Fig. 2. Average stress-average strain curve

Some results of the numerical simulations are shown in *Fig. 3* (instead of residual strains the level of residual stresses denotes all the next results).

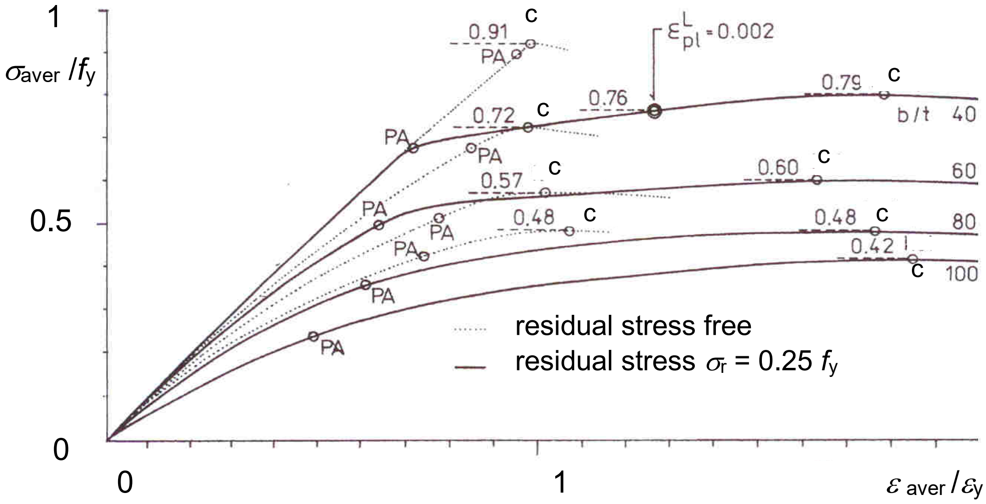

Fig. 3. Average stress-average strain curves of plates with various slenderness *b*/*t* with and without residual stresses

Resulting formulas for elastic-plastic reduction factors (limited by ε^{L}_{pl}) are as follows:

$\beta > 1.55$: $\rho = 0.0582\, r_1 \beta^2 - 0.489\, r_2 \beta + 1.471\, r_3$ (1)

where $r_1 = 1 + 0.00282\, r$ $r_2 = 1 - 0.00123\, r$ $r_3 = 1 - 0.00451\, r$

$\beta \leq 1.55$: $\rho = -0.1653\, r_1 \beta^2 + 0.200\, r_2 \beta + 940\, r_3$ (2)

where $r_1 = 1 - 0.10209\, r$ $r_2 = 1 - 0.21520\, r$ $r_3 = 1 + 0.02217\, r$

(The basic parameters: level of residual stresses $r = \dfrac{\sigma_r}{f_y} 100$ [%] and slenderness $\beta = \dfrac{b}{t}\sqrt{\dfrac{f_y}{E}}$).

Comparison of the elastic-plastic numerical values with reduction factors given in Eurocodes is shown in *Fig. 4*. Apart from ENV and EN values for internal parts of plates also values for pure bending and outstand parts in uniform compression are shown.

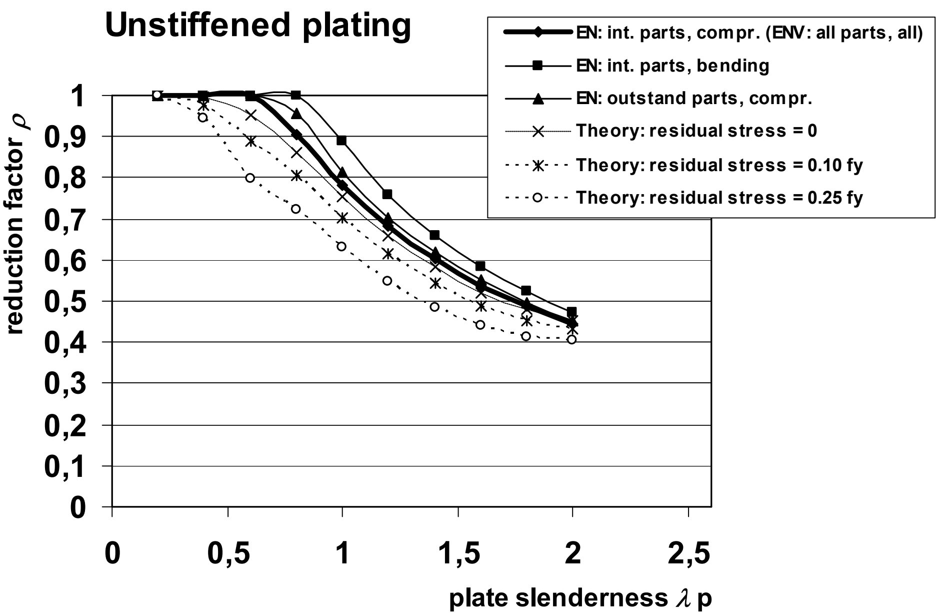

Fig. 4. Comparison of Eurocode's reduction factors with numerical values respecting various level of compression residual stresses due to welding

From the comparison follows simple conclusion that the Eurocode values do not consider any residual stresses and are even higher than Winter's traditional ones. It may be, of course, argued with little impact of these values on capacity of 4-class girder in bending, as in example shown in *Fig. 6*. In spite of introduction of favourable reduction factor values respecting bending in accordance with EN 1993-1-5 and therefore different effective sections (see *Fig. 5*) the cross-section modules resulting from calculations according to EN differ from those of ENV just for 1.5 % (- 1,5 % at compression flange and + 0.3 % at tension flange).

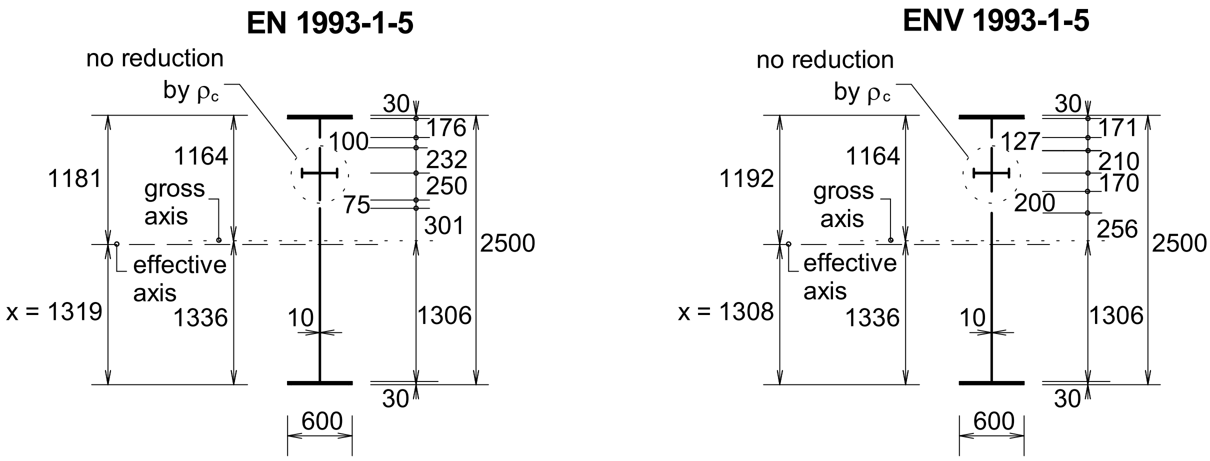

Fig. 5. Results of effective cross-section calculations

2 STIFFENED PLATING

The Eurocode procedure for determination of reduction factor concerning stiffened plates is based on combination of orthotropic theory (shape orthotropy in case of more than 2 stiffeners or stiffeners elastically supported by plate for 1-2 stiffeners) and column type behaviour. The procedure is clear and educative, nevertheless very tedious and lengthy. As mentioned in the introduction, author analysed non-linearly hundreds of stiffened plates and evaluated in accordance with *Fig. 7* both elastic and elastic-plastic strength (= reduction factors ρ_c). The analysis resulted into formulas respecting initial deflections of panels between stiffeners (amplitude $e_{0b} = b/200$), initial deflections of stiffeners (amplitude $e_{0L} = L/500$; Eurocode recommends $e_{0L} = L/400$) and arbitrary level of residual stresses both in panels and stiffeners between < 0; $0.25 f_y$ > (more details e.g. in [2] and full results in [5]).

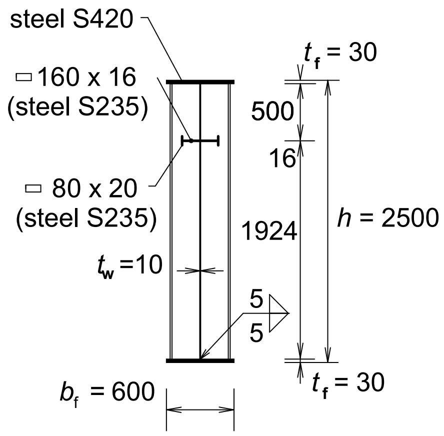

Fig. 6. 4-class girder in bending

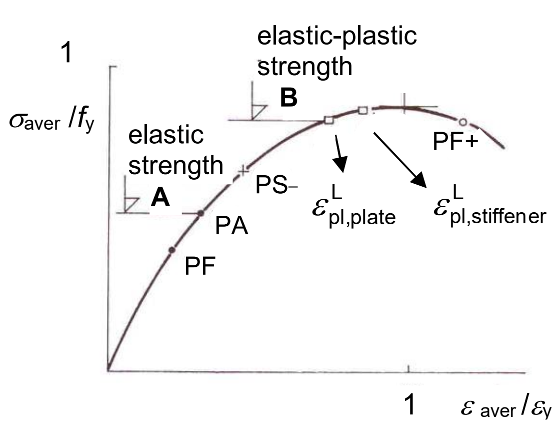

Fig. 7. Average stress-average strain curve for stiffened plates in compression

Again, only basic elastic-plastic numerical values concerning stiffened plates under compression are presented here. Similarly to unstiffened plates the elastic-plastic value is given either by collapse of the stiffened plate or by reaching excessive equivalent plastic strain (ε^{L}_{pl} = 0.002) at the mid-plane of the plate or at the stiffener extreme fibre ($\varepsilon^{L}_{pl,stiffener}$ = 0.002), the lowest decides. At the *Fig. 7* other points (PF = fibre panel plasticity, PA = area panel plasticity, PS- = stiffener extreme fibre plasticity in compression, PS+ = stiffener extreme fibre plasticity in tension,) are shown, which are important for determining of elastic capacity of the stiffened plate.

The numerical analysis in accordance with tests embraced various shapes of initial deflections both of panels between stiffeners and stiffeners themselves. Very important proved to be direction of initial bows of stiffeners (either "positive" i.e. towards the stiffener tips or "negative", towards the panels). Superimposed panel and stiffener initial deflections are shown in *Figs 8, 9*.

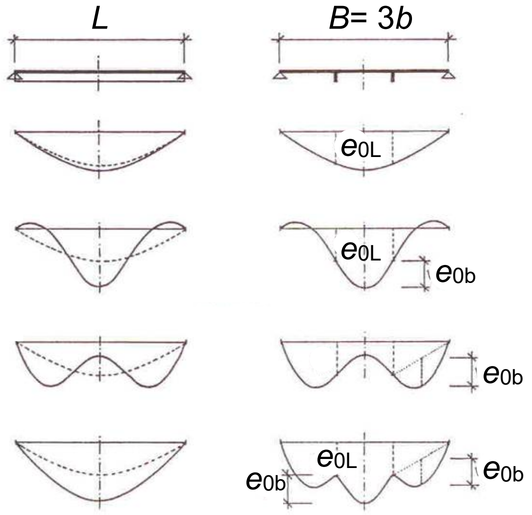

Fig. 8. Positive initial deflections

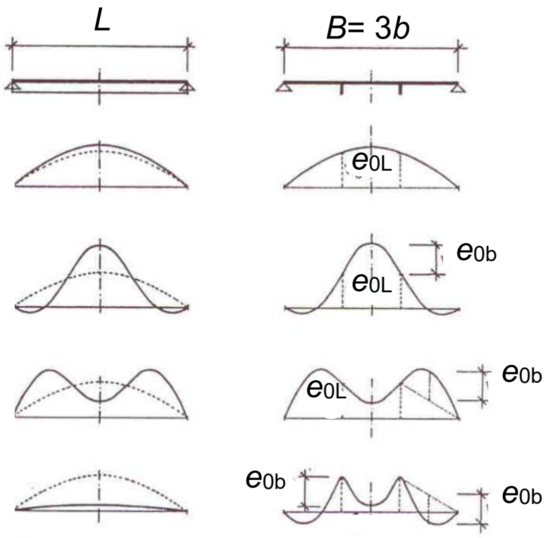

Fig. 9. Negative initial deflections

In accord with tests and in-situ measurements the positive ones were "assigned" to flats and welding strain free angle stiffeners, while negative ones prevail for T-section welded stiffeners, *Fig. 10*. Following vast parametrical study [3] on effect of longitudinal continuity of stiffeners covering various shapes of initial deflections, including common "hungry horse" shape, the single bay results were recommended to be used in design, respecting the classification "rolled" and "welded" stiffeners to distinguish the possible direction of stiffener initial deflection.

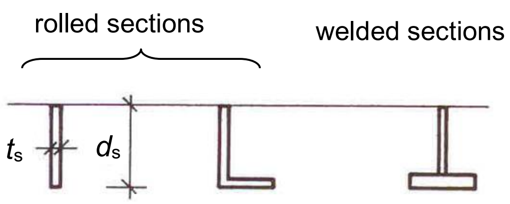

Fig. 10. Stiffeners considered in the study

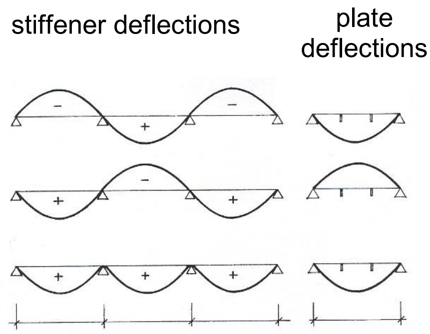

Fig. 11. Modes of stiffener continuity

Example of average stress-strain curves for single and continuous stiffened plates with various initial deflections is shown in *Fig. 12*.

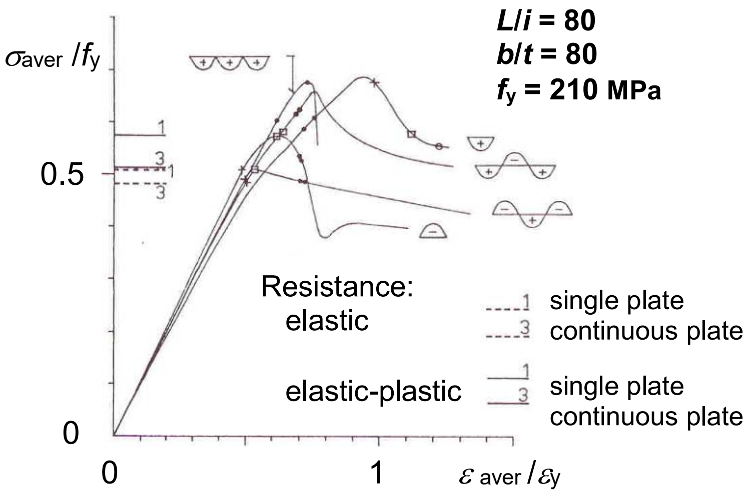

Fig. 12 Average stress-average strain curves for plates with $L/i = b/t = 80$

The resulting formulas for elastic-plastic reduction factors of stiffened plates in compression are based on following non-dimensional slendernesses (i = radius of gyration of a stiffener cross section with associated panel of width b; t = plate thickness):

- stiffener slenderness $\alpha = (L/i)\sqrt{f_y / E}$ formulas valid for $\alpha \in \langle 0.95; 3.16\rangle$
- plate slenderness $\beta = (b/t)\sqrt{f_y / E}$ formulas valid for $\beta \in \langle 0.95; 2.53\rangle$

i) Plates with 2 stiffeners (arbitrary level of residual stresses may be linearly interpolated):
Rolled stiffeners:

$$\rho_{c(\varepsilon_r = 0\,\%)} = 0.77 - 0.101(\alpha - 1.9) - 0.066(\alpha - 1.9)^2 - [0.15 - 0.006(\alpha - 1.9)](\beta - 1.9) \quad (3)$$

$$\rho_{c(\varepsilon_r = 25\,\%)} = 0.67 - 0.097(\alpha - 1.9) - 0.018(\alpha - 1.9)^2 - [0.165 - 0.031(\alpha - 1.9) - 0.011(\alpha - 1.9)^2](\beta - 1.9) \quad (4)$$

Welded stiffeners (arbitrary level of residual stresses may be linearly interpolated):

$$\rho_{c(\varepsilon_r = 0\,\%)} = 0.684 - 0.131(\alpha - 1.9) - 0.023(\alpha - 1.9)^2 - [0.09 - 0.062(\alpha - 1.9) + 0.025(\alpha - 1.9)^2](\beta - 1.9) \quad (5)$$

$$\rho_{c(\varepsilon_r = 25\,\%)} = 0.625 - 0.110(\alpha - 1.9) - [0.1 - 0.037(\alpha - 1.9)](\beta - 1.9) \quad (6)$$

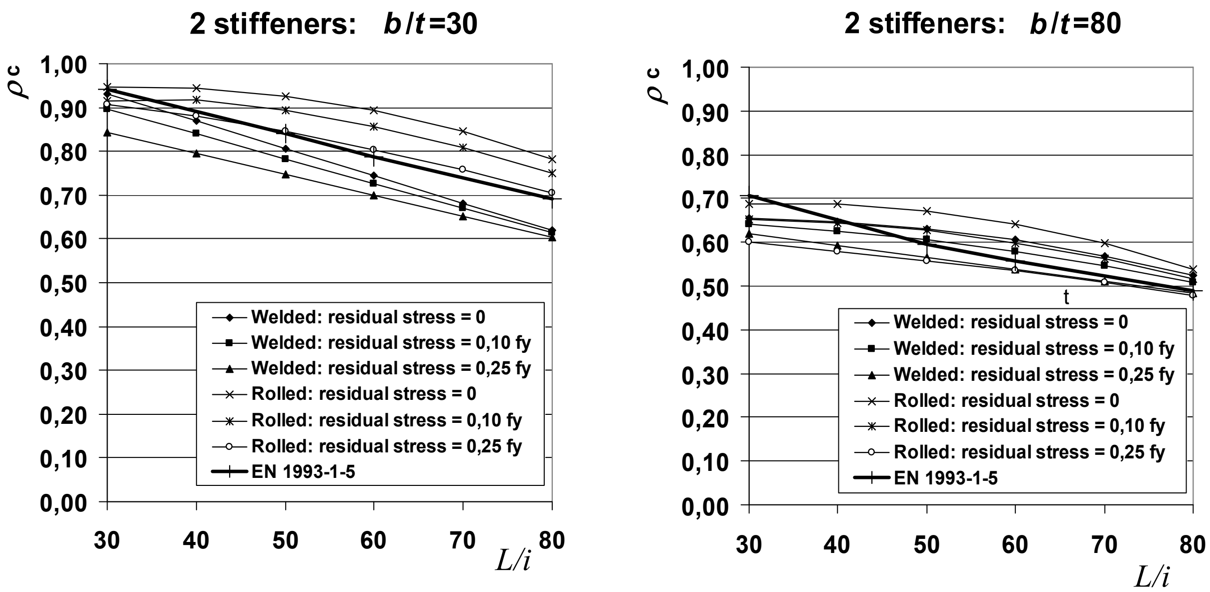

Fig. 13. Comparison of reduction factors for 2 stiffeners (f_y = 235 MPa)

ii) Plates with 4 and more stiffeners (for 3 stiffeners linear interpolation may be used):
The values for 2 stiffeners are multiplied with coefficient $K \leq 1$ due to missing plate behaviour:
Rolled stiffeners:

$$K = 0.98 - 0.046(\alpha - 1.9) - 0.046(\alpha - 1.9)^2 + [-0.035 + 0.052(\alpha - 1.9) + 0.073(\alpha - 1.9)^2](\beta - 1.9)$$

Welded stiffeners:

$$K = 0.96 - 0.043(\alpha - 1.9) + [0.042 + 0.044(\alpha - 1.9)](\beta - 1.9) \qquad (7)$$

Comparisons of the values of several extreme stiffened plates with Eurocode are shown in *Figs 13*, *14*. Generally the Eurocode solution exhibits values for rolled stiffeners with heavy welds (residual compression stresses approx. 0.25 f_y) and employed initial deflections $b/200$ and $L/500$.

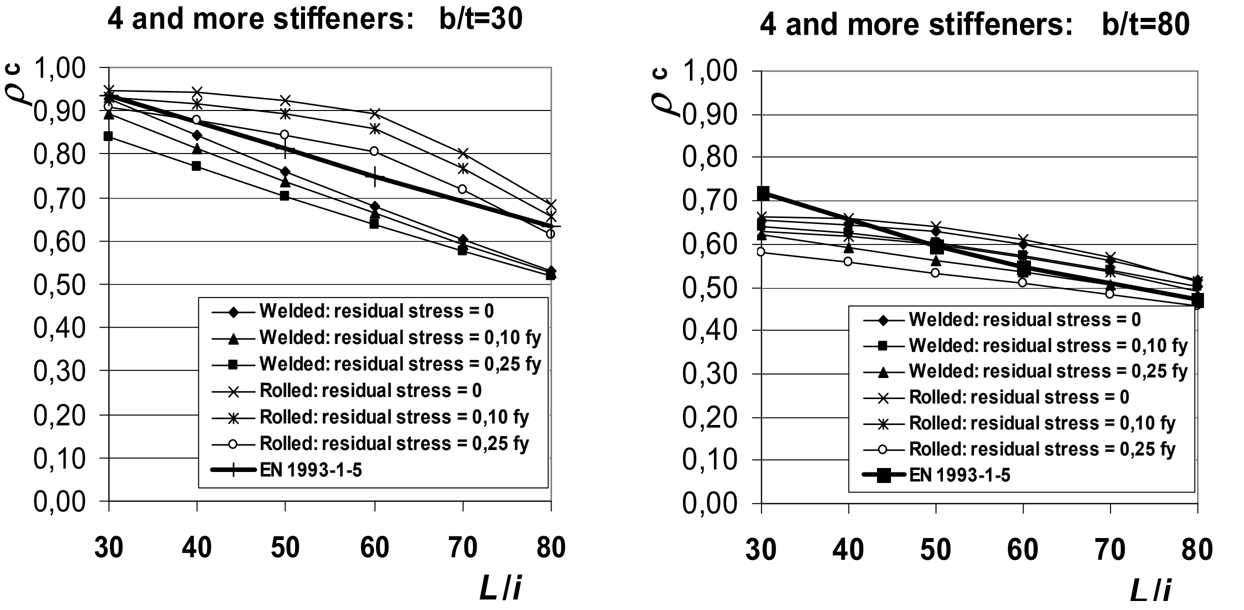

Fig. 14. Comparison of reduction factors for 4 and more stiffeners (f_y = 235 MPa)

3 SUMMARY AND ACKNOWLEDGMENT

The aim of the paper was to clarify the EN 1993-1-5 values of reduction factors for unstiffened and stiffened plates under uniform compression. Based on rigorous numerical and experimental research there has been shown, that the values for unstiffened plating are rather venturous (representing plates without residual stresses) while procedure for stiffened plates gives reasonable values for plates with rolled stiffeners and heavy welds (but without spreadsheet the calculations are very tedious).

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