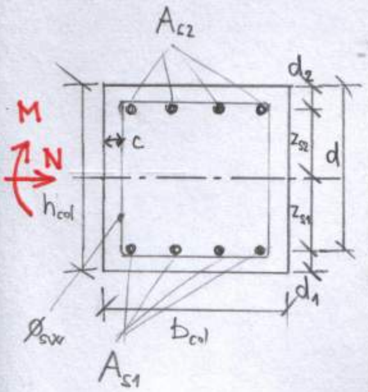
CHECK OF COLUMN – INTERACTION DIAGRAM



Lever arms of internal forces are always considered to the axis of the cross-section

DIMENSIONS OF CROSS-SECTION

$b_{col} = h_{col} = 400\ mm$

$c = 25\ mm$ (cover depth)

$\varnothing_{sw} = 6\ mm$ (estimated diameter of stirrups)

$\varnothing_s = 12\ mm$ (main reinforcement)

$$d = h_{col} - c - \varnothing_{sw} - \frac{\varnothing_s}{2} =$$
$$= 400 - 25 - 6 - \frac{12}{2} = 363\ mm$$

(effective height)

$$z_{s1} = z_{s2} = \frac{1}{2}\left(h_{col} - 2c - 2\varnothing_{sw} - \varnothing_s\right) =$$
$$= \frac{1}{2}\left(400 - 2\cdot 25 - 2\cdot 6 - 12\right) = 163\ mm$$

(lever arm of internal forces)

$$d_1 = d_2 = \frac{h_{col}}{2} - z_{s1} = 200 - 163 = 37\ mm$$

CROSS-SECTIONAL AND MATERIAL PROPERTIES

$A_{s1} = A_{s2} = \frac{1}{2} A_s = 226\ mm^2$ (see your design)

$f_{cd} = 13{,}3\ MPa$

$A_c = 160000\ mm^2$

$f_{yd} = 435\ MPa$

$\sigma_s = 400\ MPa$

$\varepsilon_{cd} = 0{,}0035$ (limit strain of concrete)

$E_s = 200000\ MPa$ (Young's modulus of steel)

POINT „0" (PURE COMPRESSION)

$$N_{Rd,0} = b_{col} h_{col} f_{cd} + A_{s1}\sigma_s + A_{s2}\sigma_s = 400.400.13{,}3 + 226.400 + 226.400 = 2309\ kN$$

$$M_{Rd,0} = \left(A_{s2} z_{s2} - A_{s1} z_{s1}\right)\sigma_s = 0$$

POINT „1" (STRAIN IN TENSILE REINFORCEMENT $\varepsilon_{s1} = 0$)

$$N_{Rd,1} = 0{,}8 b_{col} d f_{cd} + A_{s2} f_{yd} = 0{,}8 .400.13{,}3\cdot 363 + 226.435 = 1642\ kN$$

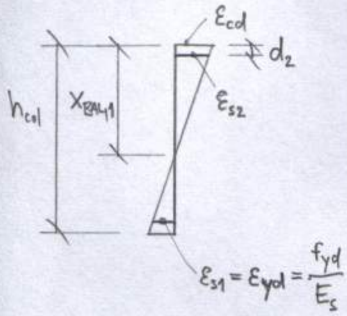
$$M_{Rd,1} = 0{,}8 b_{col} d f_{cd}\left(\frac{h_{col}}{2} - 0{,}4 d\right) + A_{s2} z_{s2} f_{yd} = 0{,}8.400.363.13{,}3\left(\frac{400}{2} - 0{,}4.363\right) +$$
$$+ 226.163\cdot 435 = 100{,}7\ kNm$$

POINT „2" (STRESS IN TENSILE REINFORCEMENT IS ON YIELD LIMIT – $\sigma_{s1} = f_{yd}$)

$$\xi_{BAL,1} = \frac{700}{700 + f_{yd}} = \frac{700}{700 + 435} = 0{,}617$$

$$x_{BAL,1} = \xi_{BAL,1}\cdot d = 0{,}617\cdot 363 = 224\ mm$$

Strains in the cross-section:



σ_{s2} can be obtained by applying rules for similarity of triangles

$$\frac{\varepsilon_{cd}}{x_{BAL,1}} = \frac{\varepsilon_{s2}}{x_{BAL,1} - d_2}$$

$$\varepsilon_{s2} = \varepsilon_{cd}\left(1 - \frac{d_2}{x_{BAL,1}}\right) = 0{,}0035\left(1 - \frac{37}{224}\right) = 0{,}00292$$

$$\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{435}{200000} = 0{,}002175\ (= 2{,}18\ ‰)$$

Because $\varepsilon_{s2} > \varepsilon_{yd}$, we can assume $\sigma_{s2} = f_{yd}$ (in case of $\varepsilon_{s2} < \varepsilon_{yd}$, $\sigma_{s2} = E_s \varepsilon_{s2}$ should be assumed)

$N_{Rd,2} = 0{,}8\, b_{col}\, x_{BAL,1}\, f_{cd} + \cancel{A_{s2} f_{yd} - A_{s1} f_{yd}} = 0{,}8 \cdot 400 \cdot 224 \cdot 13{,}3 = 953{,}3\ kN$

$M_{Rd,2} = 0{,}8\, b_{col}\, x_{BAL,1}\, f_{cd} \left(\frac{h_{col}}{2} - 0{,}4\, x_{BAL,1}\right) + A_{s2} \cdot f_{yd}\, z_{s2} + A_{s1} \cdot f_{yd}\, z_{s1} =$

$= 0{,}8 \cdot 400 \cdot 224 \cdot 13{,}3 \cdot \left(\frac{400}{2} - 0{,}4 \cdot 224\right) + 226 \cdot 435 \cdot 163 + 226 \cdot 435 \cdot 163 =$

$= 137{,}3\ kNm$

POINT „3" (PURE BENDING)

Quadratic equation for σ_{s2}:

$$\sigma_{s2}^2 \cdot A_{s2} - \sigma_{s2}\left(A_{s1} f_{yd} + A_{s2}\, \varepsilon_{cd}\, E_s\right) + \varepsilon_{cd}\, E_s \left(A_{s1} f_{yd} - 0{,}8\, b_{col}\, f_{cd}\, d_2\right) = 0$$

$$\sigma_{s2}^2 \cdot 226 - \sigma_{s2}\left(226 \cdot 435 + 226 \cdot 0{,}0035 \cdot 210000\right) + 0{,}0035 \cdot 210000 \left(226 \cdot 435 - 0{,}8 \cdot 400 \cdot 13{,}3 \cdot 37\right) = 0$$

$$\sigma_{s2}^2 \cdot 226 - \sigma_{s2} \cdot 264420 - 43484070 = 0$$

The equation has two roots: $\sigma_{s2}^1 = 1316\ MPa$

$\sigma_{s2}^2 = -146{,}2\ MPa$

The reinforcement has characteristic strength of 500 MPa. Therefore, σ_{s2}^1 has no sense for our calculation and we use $\sigma_{s2} = -146{,}2\, MPa$ for further calculations.

Equation for x:

$$x = \frac{A_{s1} f_{yd} - A_{s2}\, \sigma_{s2}}{0{,}8\, b_{col}\, f_{cd}} = \frac{226 \cdot 435 + 226 \cdot 146{,}2}{0{,}8 \cdot 400 \cdot 13{,}3} = 30{,}863\ mm$$

$N_{Rd,3} = 0$

$M_{Rd,3} = 0{,}8\, b_{col}\, x\, f_{cd} \left(\frac{h_{col}}{2} - 0{,}4\, x\right) + A_{s2}\, \sigma_{s2}\, z_{s2} + A_{s1}\, f_{yd}\, z_{s1} =$

$= 0{,}8 \cdot 400 \cdot 30{,}863 \cdot 13{,}3 \cdot \left(\frac{400}{2} - 0{,}4 \cdot 30{,}863\right) - 226 \cdot 146{,}2 \cdot 163 +$

$+ 226 \cdot 435 \cdot 163 = 35{,}3\ kNm$

POINT „4" (STRAIN IN COMPRESSED REINFORCEMENT $\varepsilon_{s2} = 0$)

$N_{Rd,4} = A_{s1} f_{yd} = 226 \cdot 435 = 98{,}3\ kN$

$M_{Rd,4} = A_{s1} f_{yd}\, z_{s1} = 226 \cdot 435 \cdot 163 = 16{,}0\ kNm$

POINT „5" (PURE TENSION)

$N_{Rd,5} = (A_{s1} + A_{s2})\, f_{yd} = (226 + 226) \cdot 435 = 196{,}62\ kN$

$M_{Rd,5} = 0$

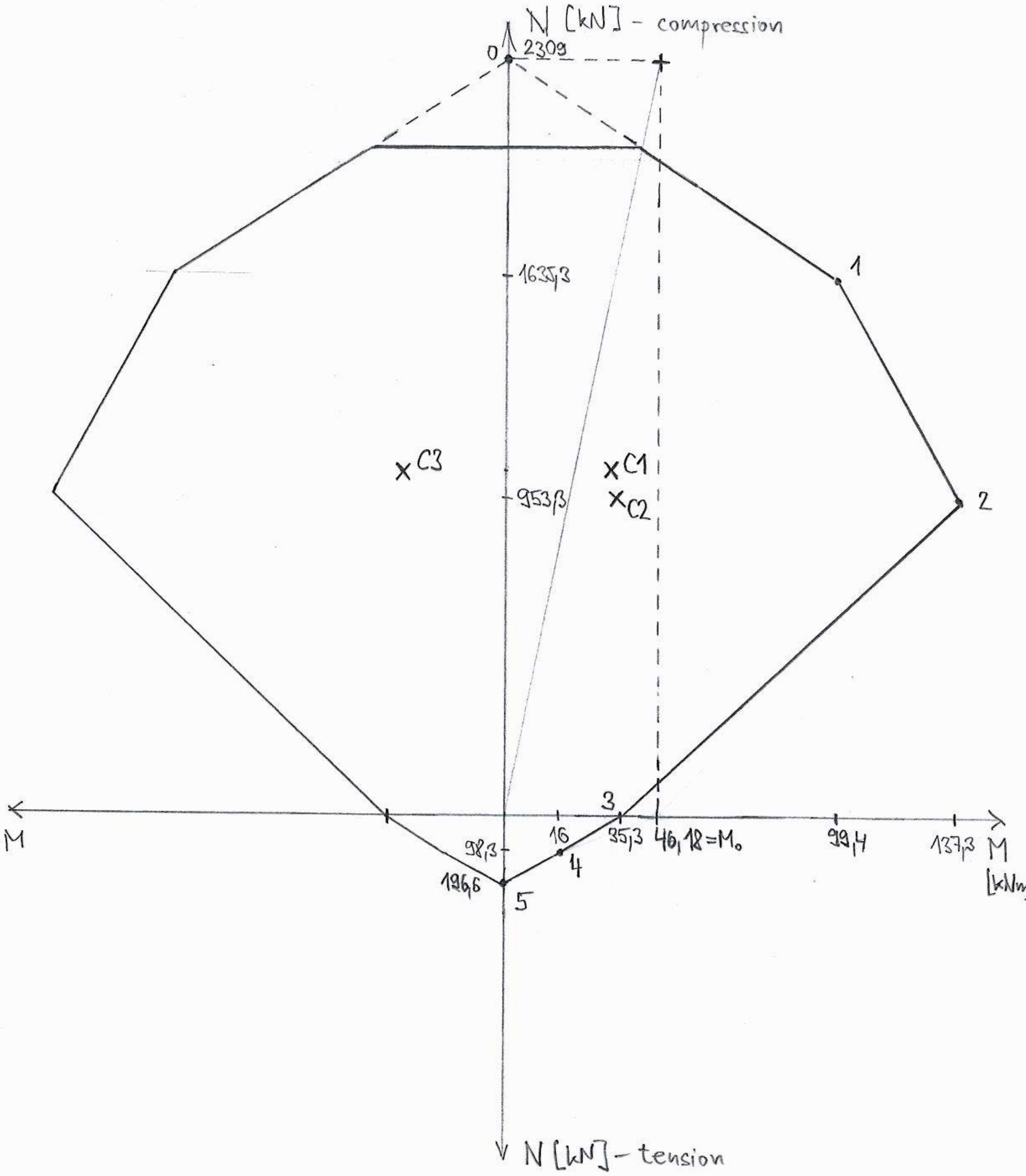
RESTRICTION OF COMPRESSIVE RESISTANCE

Minimum eccentricity : $e_0 = \max\left(\frac{h_{col}}{30}; 20\ mm\right) = \max\left(\frac{400}{30}; 20\right) =$

$= \max\left(13{,}\overline{3}; 20\right) = 20\ mm$

Minimum bending moment : $M_0 = N_{Rd,0}\, e_0 = 2309 \cdot 0{,}02 = 46{,}18\ kNm$

INTERACTION DIAGRAM



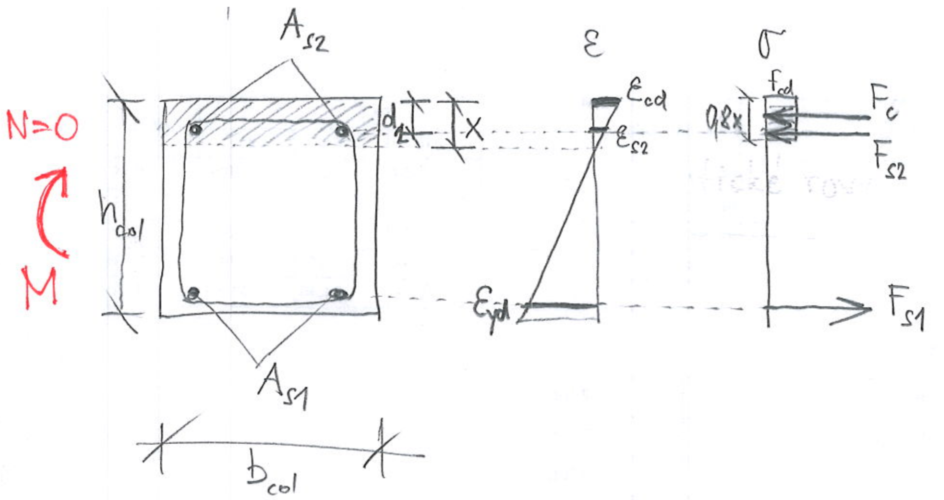
LOAD COMBINATIONS	N_{Ed} [kN]	M_{Ed} [kN]
C1	1053	31,2
C2	949	34,3
C3	1056	−30,8

All the extreme combinations of internal forces lay inside interaction diagram

⇒ COLUMN CHECKED

Appendix 1 - derivation of quadratic equation for sigma s2

Scheme of the cross-section:



Unknowns:

- height x
- strain of steel ε_{s2} and

therefore stress $\sigma_{s2} = \varepsilon_{s2} E_s$

① Equilibrium:

$$F_c + F_{s2} = F_{s1}$$

$$0{,}8 b_{col} x f_{cd} + A_{s2}\sigma_{s2} = A_{s1} f_{yd}$$

$$x = \frac{A_{s1} f_{yd} - A_{s2}\sigma_{s2}}{0{,}8 b_{col} f_{cd}}$$

② Strain distribution
- similar triangles:

$$\frac{\varepsilon_{cd}}{x} = \frac{\varepsilon_{s2}}{x - d_2}$$

$$x = \frac{\varepsilon_{cd} d_2}{\varepsilon_{cd} - \varepsilon_{s2}} \qquad \Big/ \quad \varepsilon_{s2} = \frac{\sigma_{s2}}{E_s}$$

$$x = \frac{E_s \varepsilon_{cd} d_2}{E_s \varepsilon_{cd} - \sigma_{s2}}$$

① × ② :

$$\frac{A_{s1} f_{yd} - A_{s2}\sigma_{s2}}{0{,}8 b_{col} f_{cd}} = \frac{E_s \varepsilon_{cd} d_2}{E_s \varepsilon_{cd} - \sigma_{s2}}$$

$$(A_{s1} f_{yd} - A_{s2}\sigma_{s2}) \cdot (E_s \varepsilon_{cd} - \sigma_{s2}) = 0{,}8 b_{col} f_{cd} E_s \varepsilon_{cd} d_2$$

$$\boxed{\sigma_{s2}^2 \cdot A_{s1} - \sigma_{s2} \cdot (A_{s1} f_{yd} - A_{s2} E_s \varepsilon_{cd}) + E_s \varepsilon_{cd} (A_{s1} f_{yd} - 0{,}8 b_{col} f_{cd} d_2) = 0}$$

⟹ we can calculate sigma s2 from the quadratic equation, then calculate x from one of the equations 1 or 2 and finally calculate bending moment capacity MRd,3