CM01 – Concrete and Masonry Structures 1

HW12 – Pad footing

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Task 6

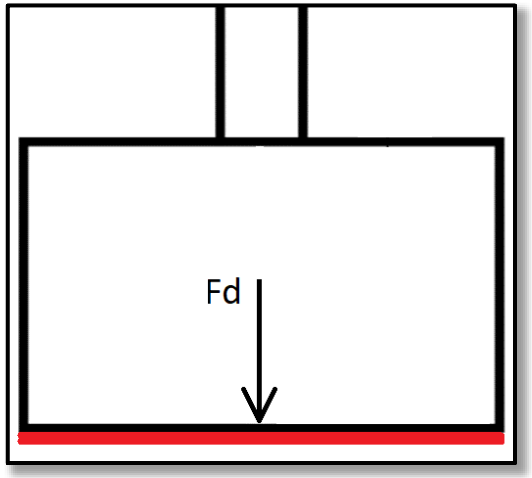
Task 6 – Staircase

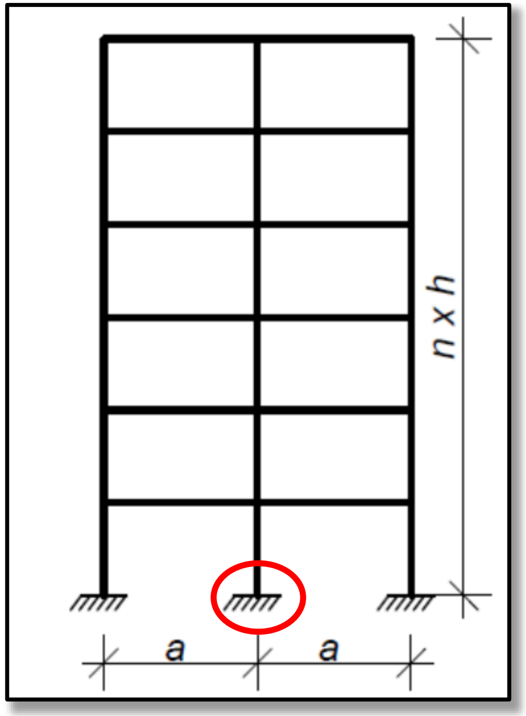
In Task 6, pad (isolated) footing will be designed for a bottom middle column of the structure from Task 1.

Design a footing for the inner column of the frame from Task 1. Use the resistance of the soil $R_d = 400$ kPa.

Please work out:

1. **Plain concrete footing.** Sketch of the footing shape.
2. **Reinforced concrete footing**. Drawing of the shape of the footing and layout of reinforcement including list of reinforcement.





Task 6 – Assignment goals

Our goal will be to:

1) Design and assess a footing made of:
   a) plain concrete,
   b) reinforced concrete.
2) Draw a sketch of:
   a) shape the plain concrete footing,
   b) reinforcement of the reinforced concrete footing.

Theory

Th1) Stresses in footings

Theory

Stresses in footings

Unlike in most other concrete structures, the **compressive stresses in the footing are not important**.

The **tensile stress** in the footing **are important**!
→ In PC footings, the tensile strength of the concrete is important.
→ In RC footings, it is necessary to design tensile reinforcement.

Additionally, the **compressive resistance of the soil** under the footing must be assessed.

Theory

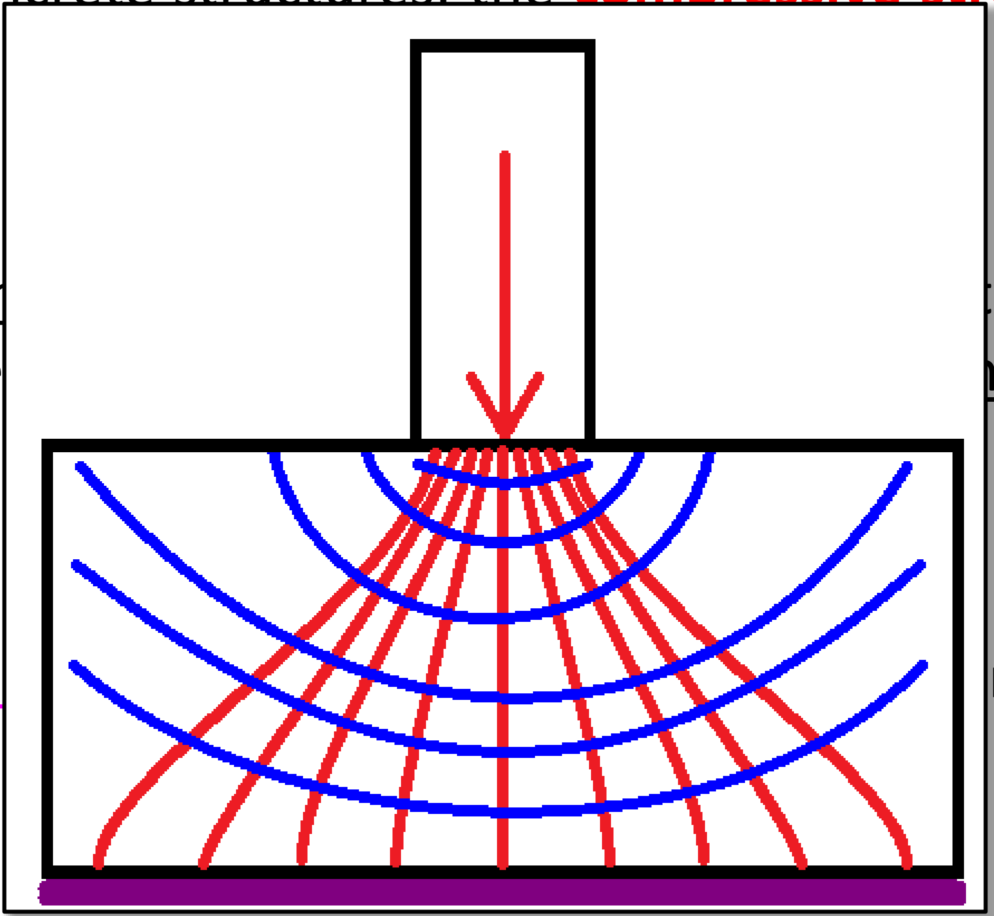
Stresses in footings

Unlike in most other concrete structures, the **compressive stresses in the footing** **are not important**.

The **tensile stress** in the

➞ In PC footings, the ter ant.

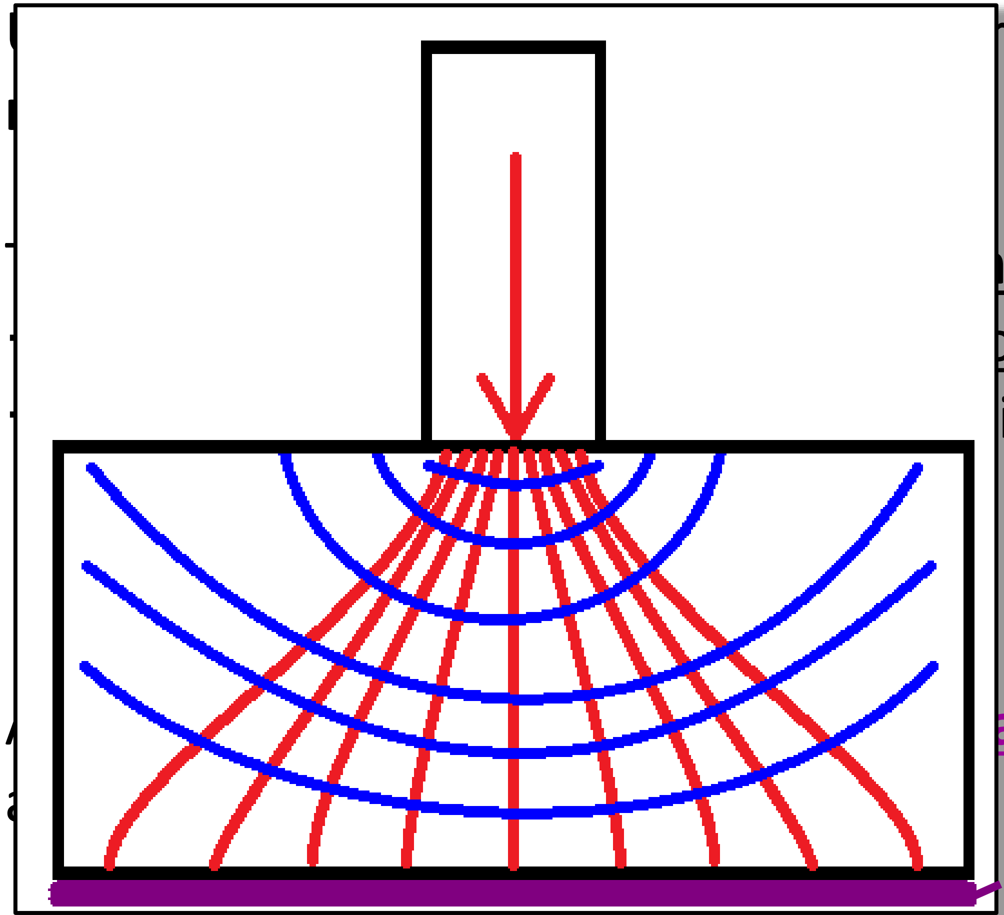
➞ In RC footings, it is ne nt.



Additionally, the **comp** r the footing must be assessed.

Theory

Stresses in footings



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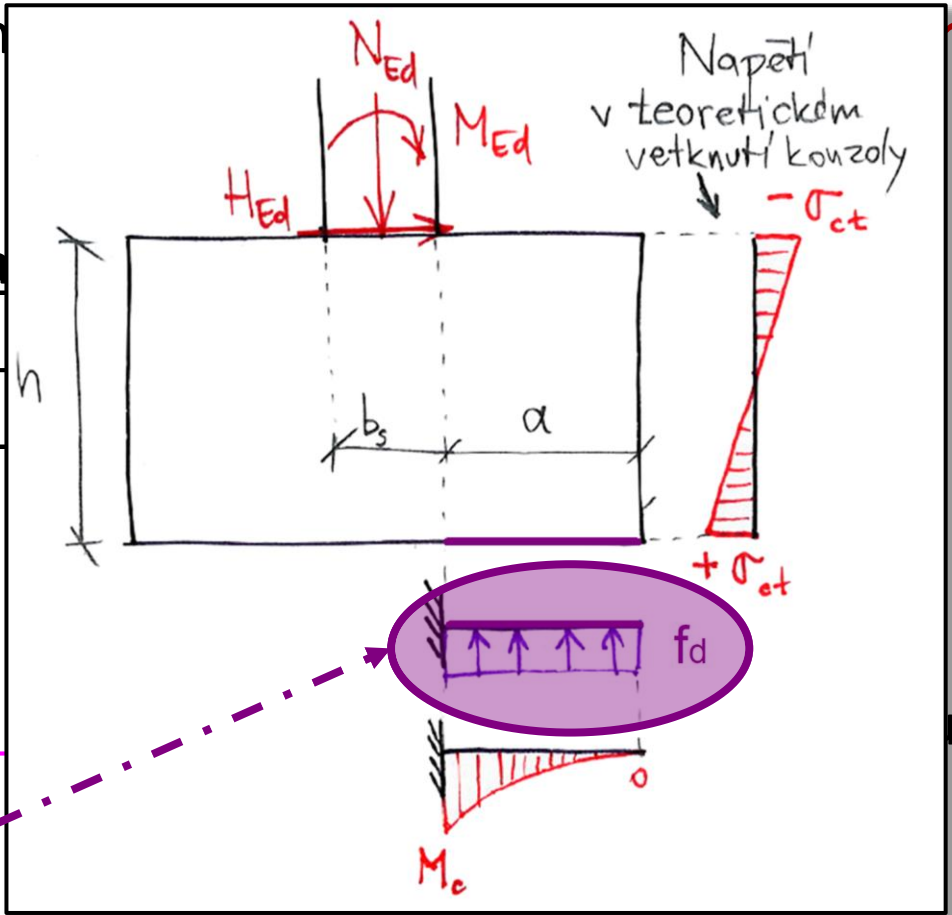


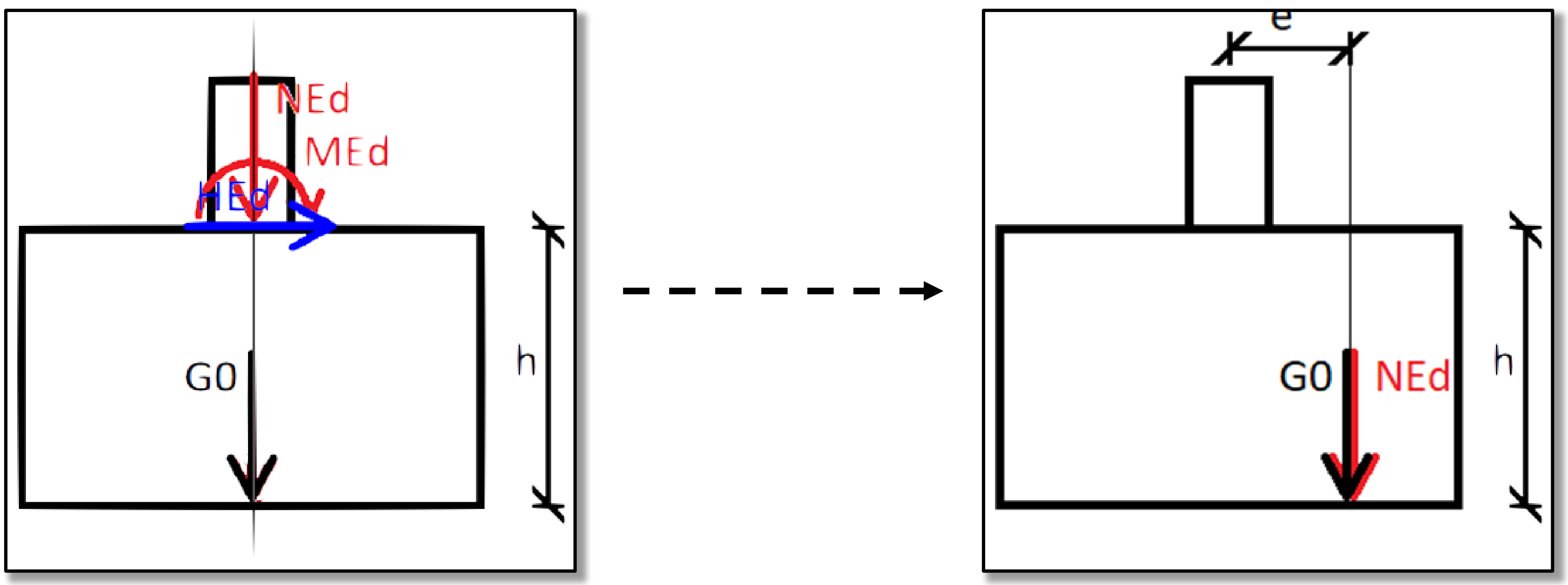
Theory

Th2) Eccentricity of the normal force

Theory

Eccentricity of the normal force

The footing is loaded by **normal force** as well as **bending moment** and **shear force** (see Task 1), which can also be expressed as a **normal force with eccentricity**



Theory

Eccentricity of the normal force

The eccentricity can be calculated as

$$e = \frac{M}{N},$$

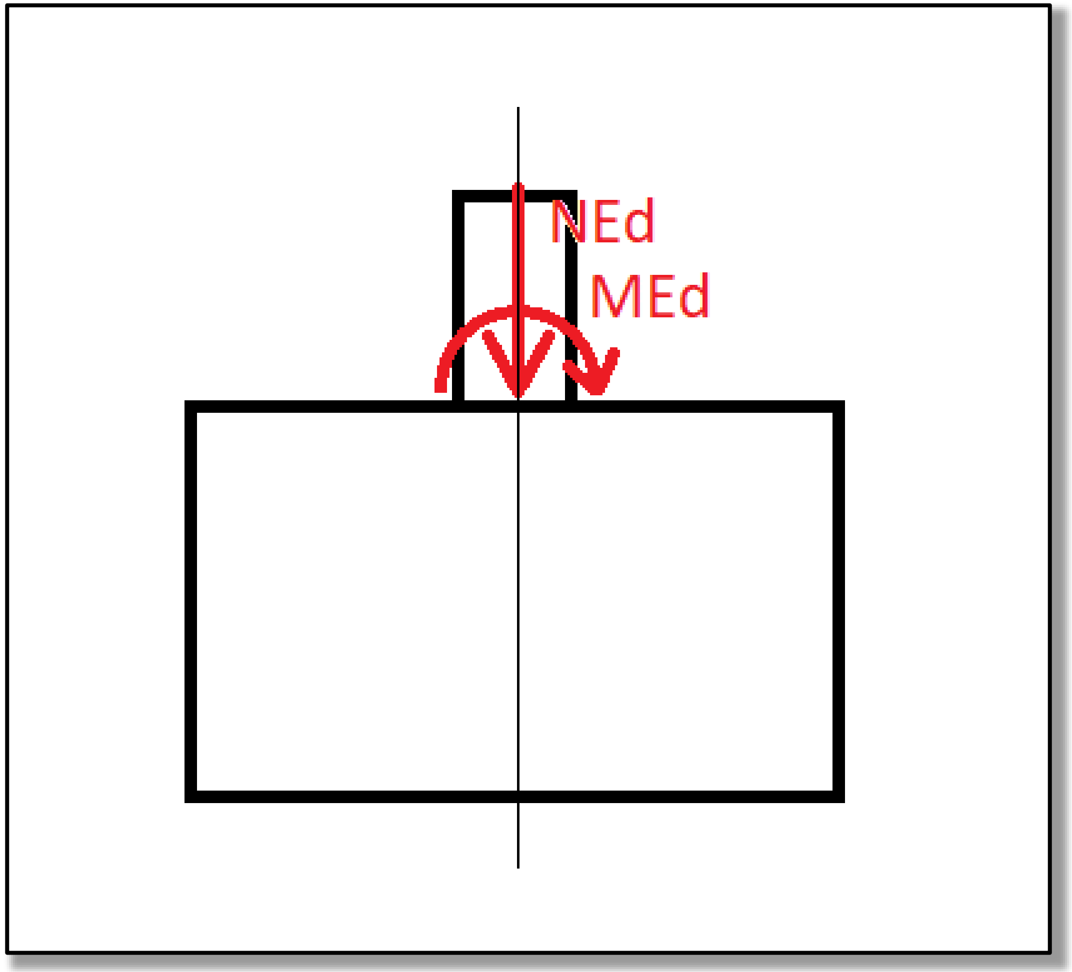
where:

M consists of a **bending moment from the upper structure** and **bending moment induced by the shear force the upper structure**,

N consists of **normal force from the upper structure** and **self-weight of the footing**.

Theory

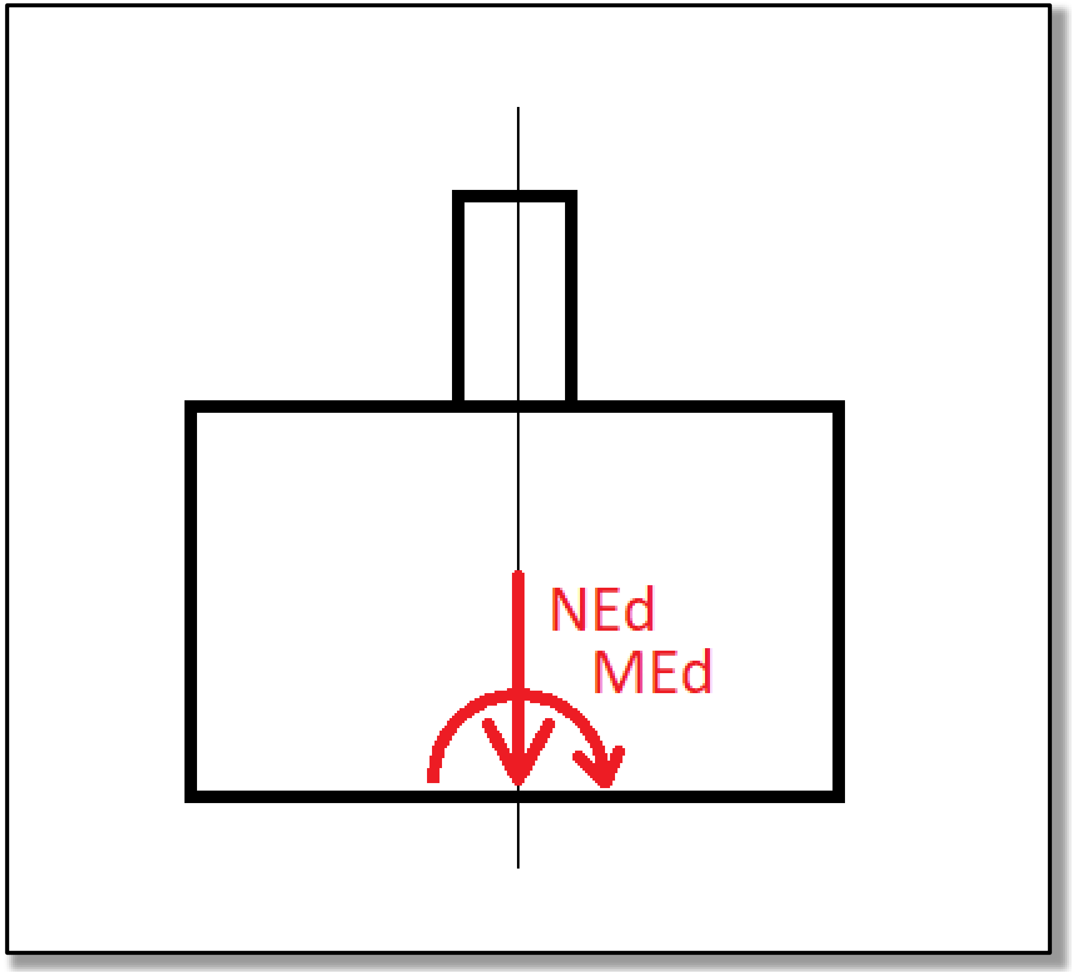
Eccentricity of the normal force



Normal force and bending moment from the upper structure acting at the base of the column.

Theory

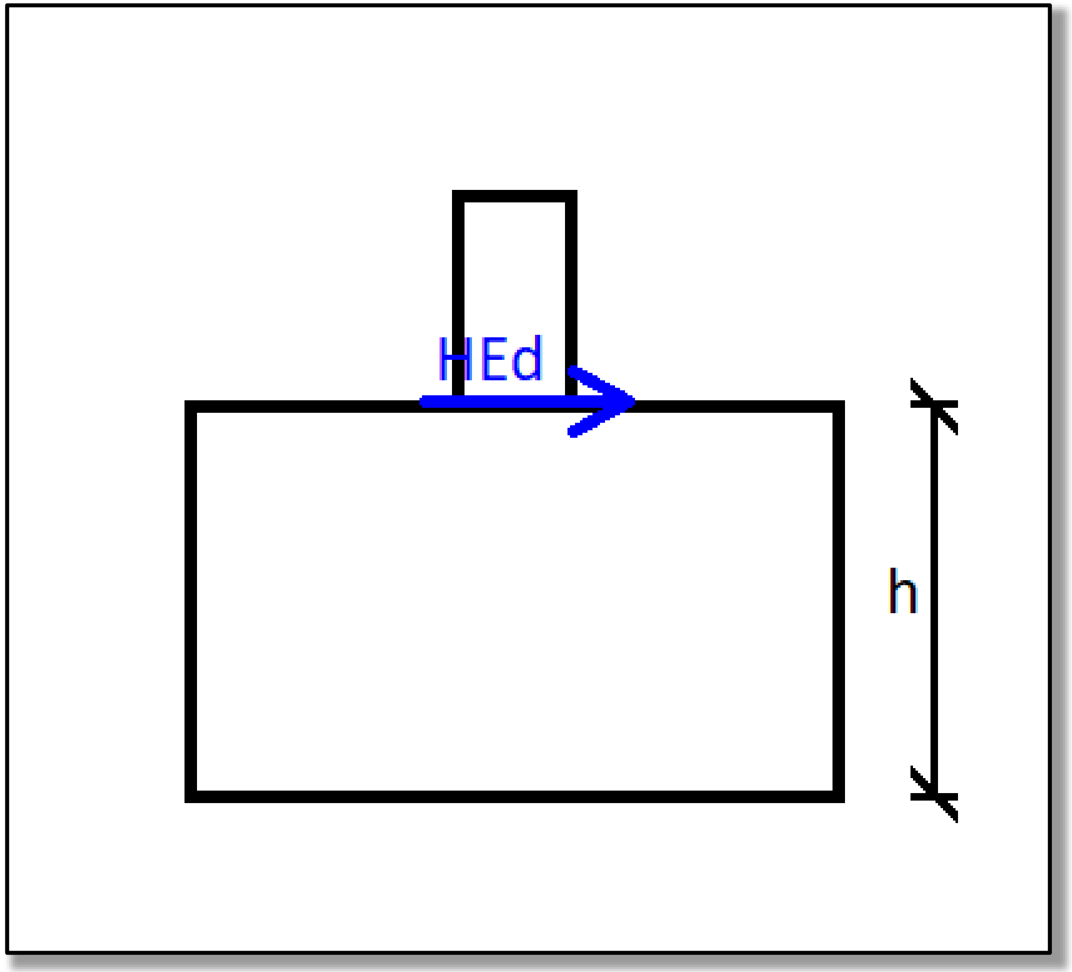
Eccentricity of the normal force



Normal force and bending moment from the upper structure acting on the soil.

Theory

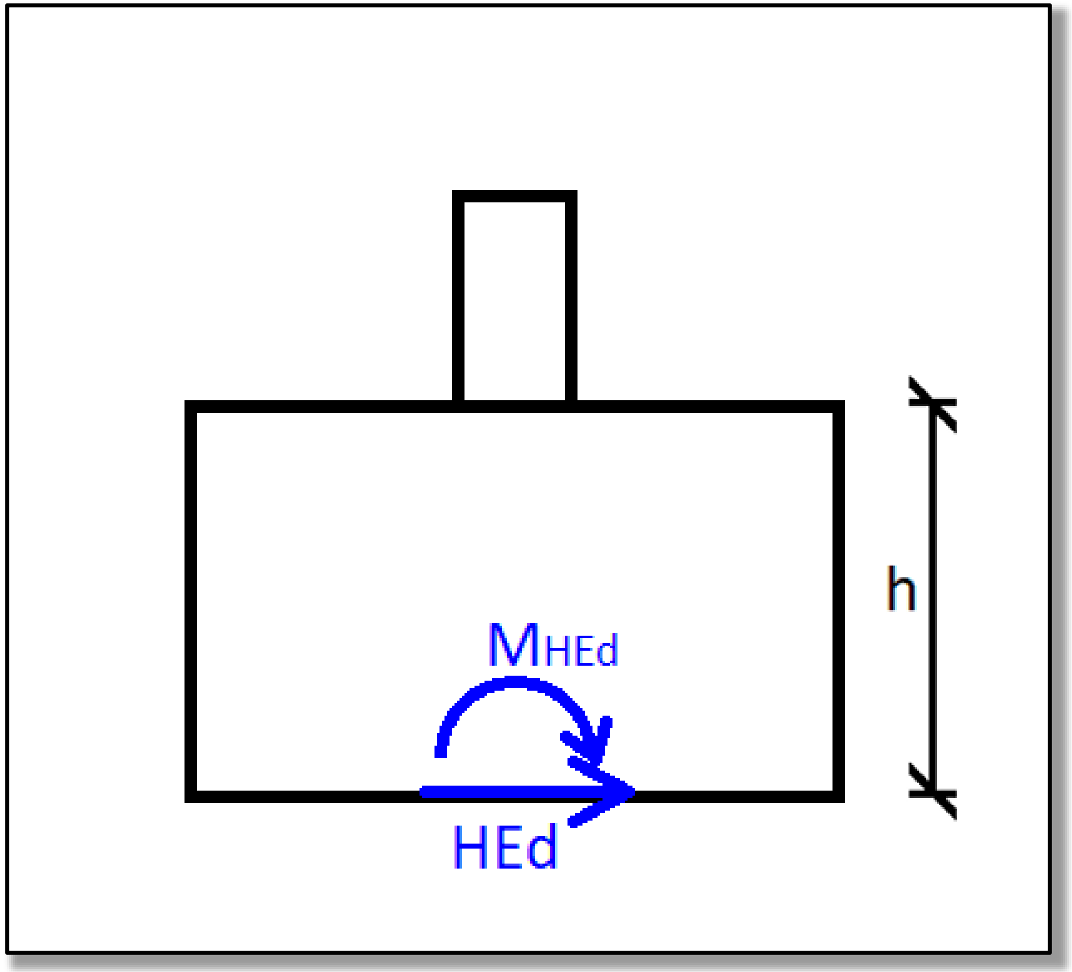
Eccentricity of the normal force



The shear force acting at the base of the column.

Theory

Eccentricity of the normal force

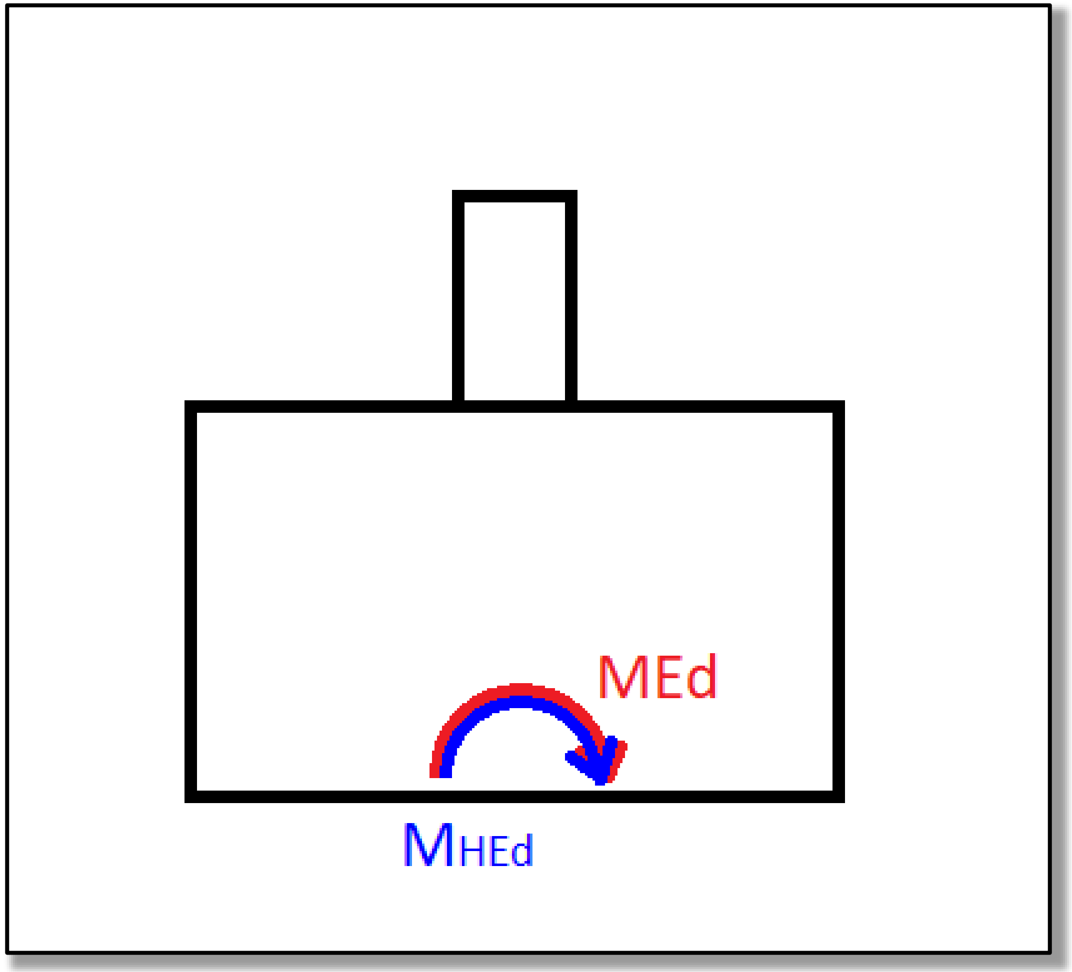


The shear force **induces a bending moment** at the soil level:

$$\boldsymbol{M = H_{Ed} h}$$

Theory

Eccentricity of the normal force

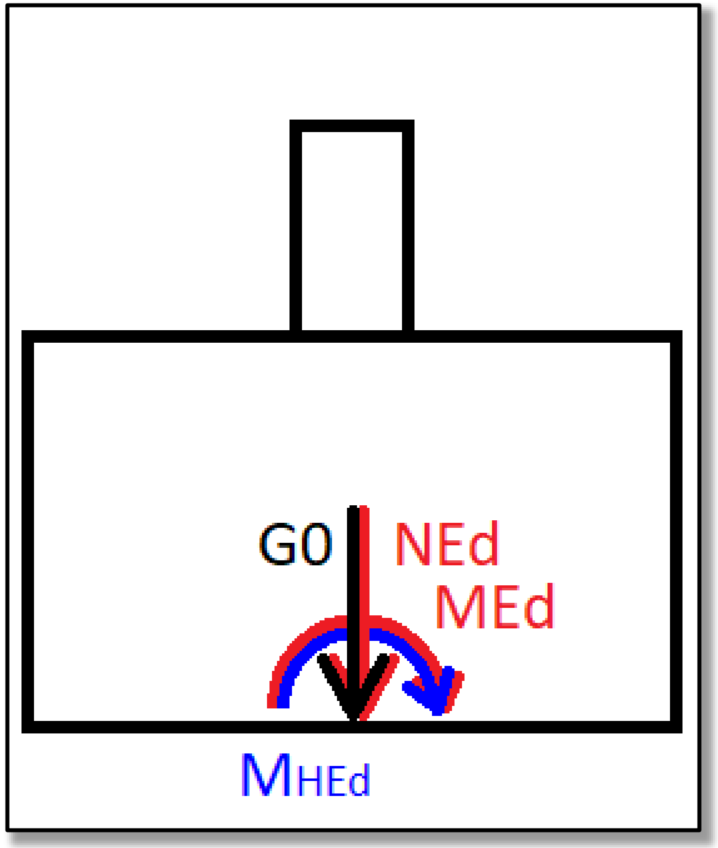


Overall bending moment at the soil level is a sum of:

- **bending moment from the upper structure**
- **bending moment from the shear force**.

Theory

Eccentricity of the normal force

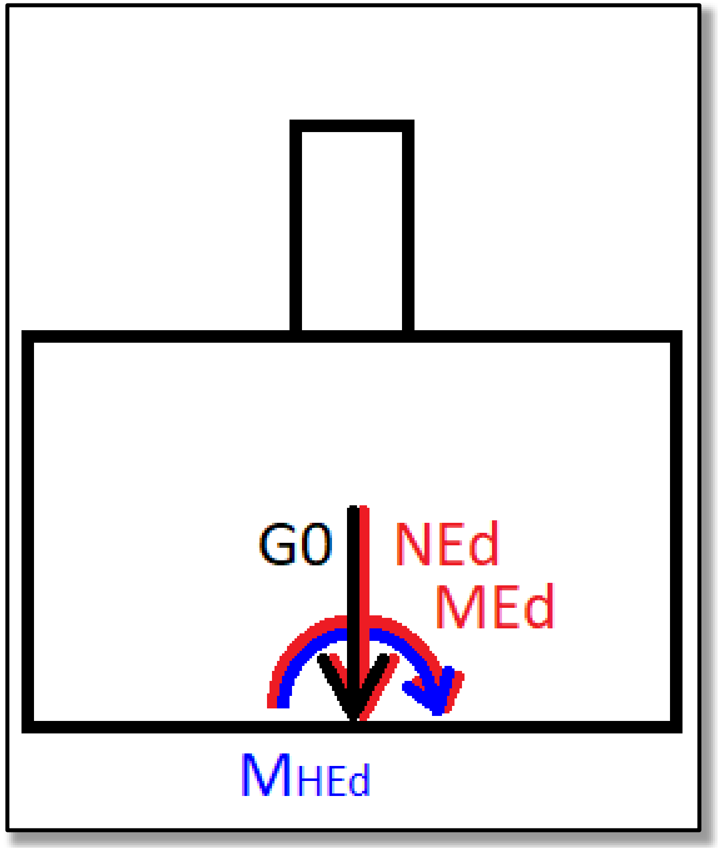


Overall normal force at the soil level is the sum of:

- **normal force from the upper structure**
- **self-weight of the footing.**

Theory

Eccentricity of the normal force



The total eccentricity of the normal force is determined by the **the overall moment** and **overall normal force**.

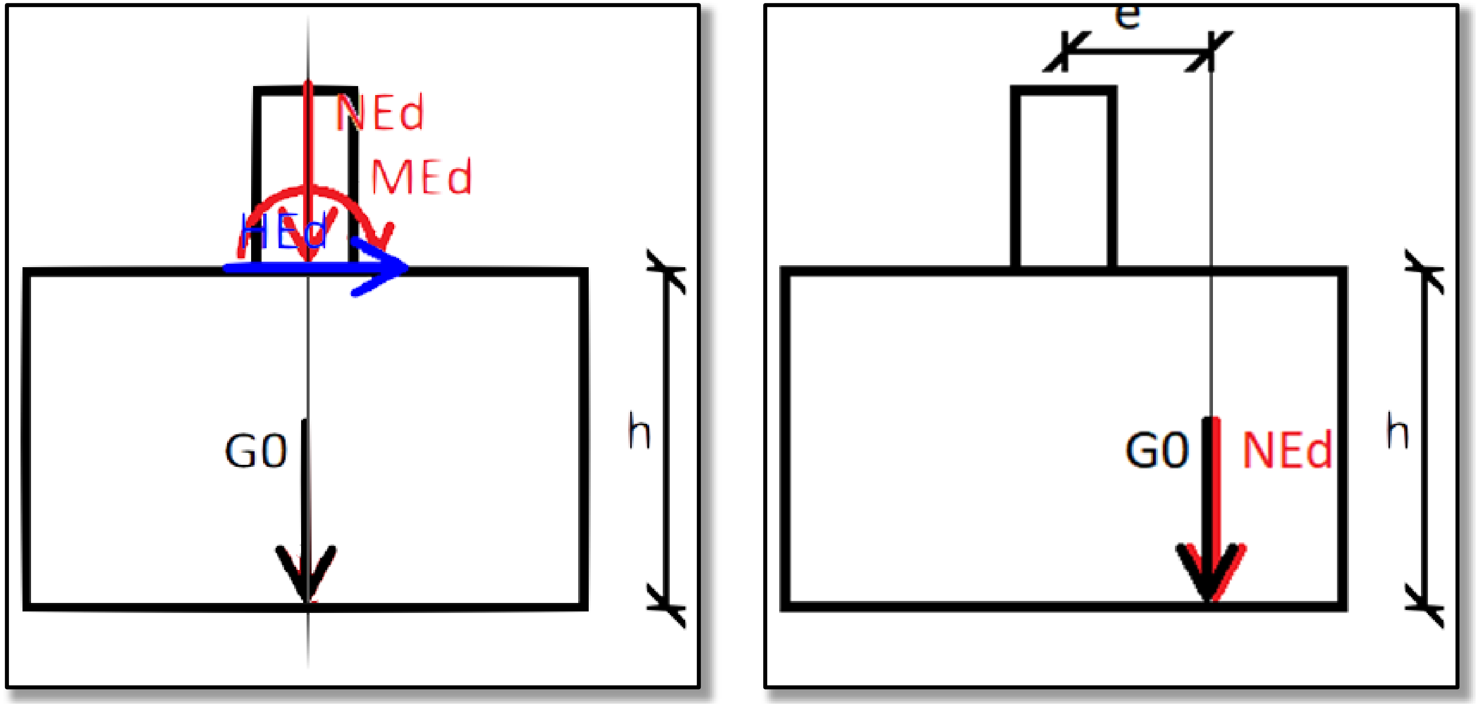
Theory

Eccentricity of the normal force

The total eccentricity of the normal force is determined by the overall moment and overall normal force:

$$e = \frac{M_{Ed} + H_{Ed}h}{N_{Ed} + G_{0,d}}$$

$$G_{0,d} = b^2h \cdot 25 \cdot 1.35$$



Remember!
The eccentricity depends on the height of the footing.
Therefore, **if the height changes** during the design process, **the eccentricity will change also!**

Theory

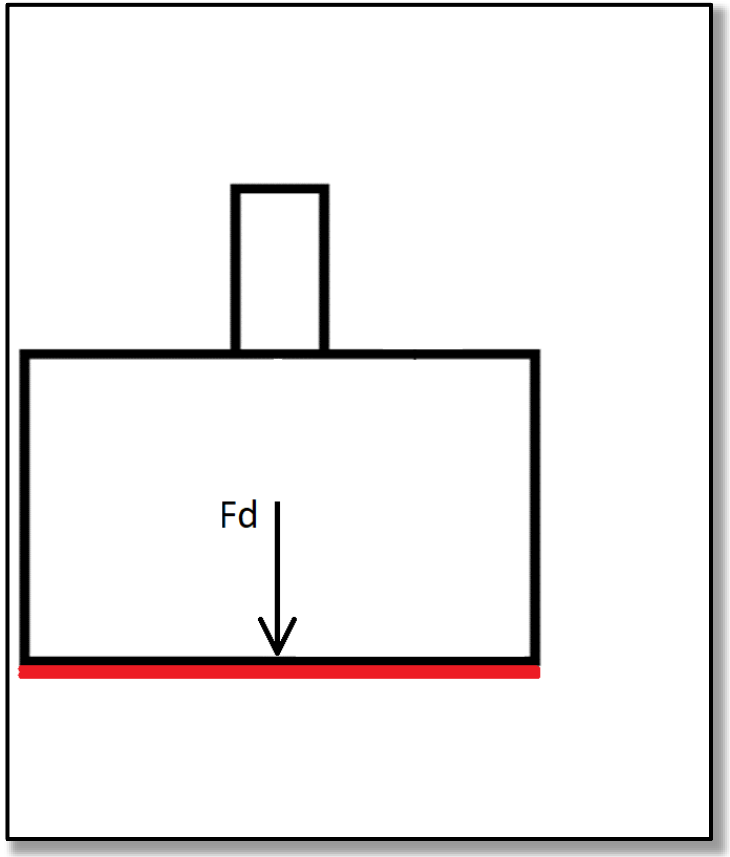
Th3) Effective loading area

Theory

Effective loading area

If the **normal force were applied in the column axis** ($e = 0$), the load area of the footing would correspond to the **floor area of the footing**

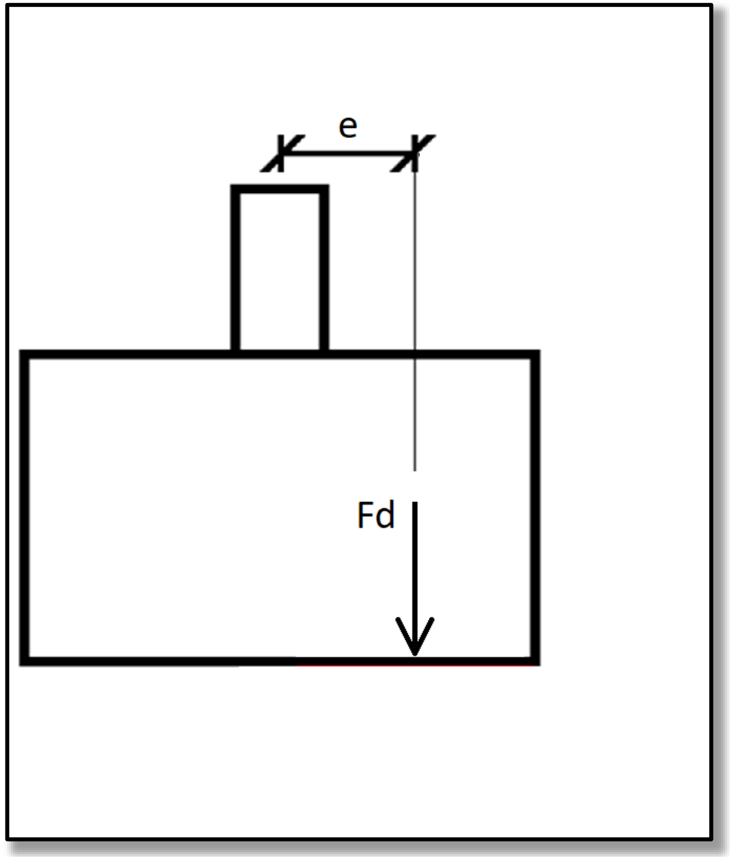
$$A_{eff} = A_c = b^2.$$



Theory

Effective loading area

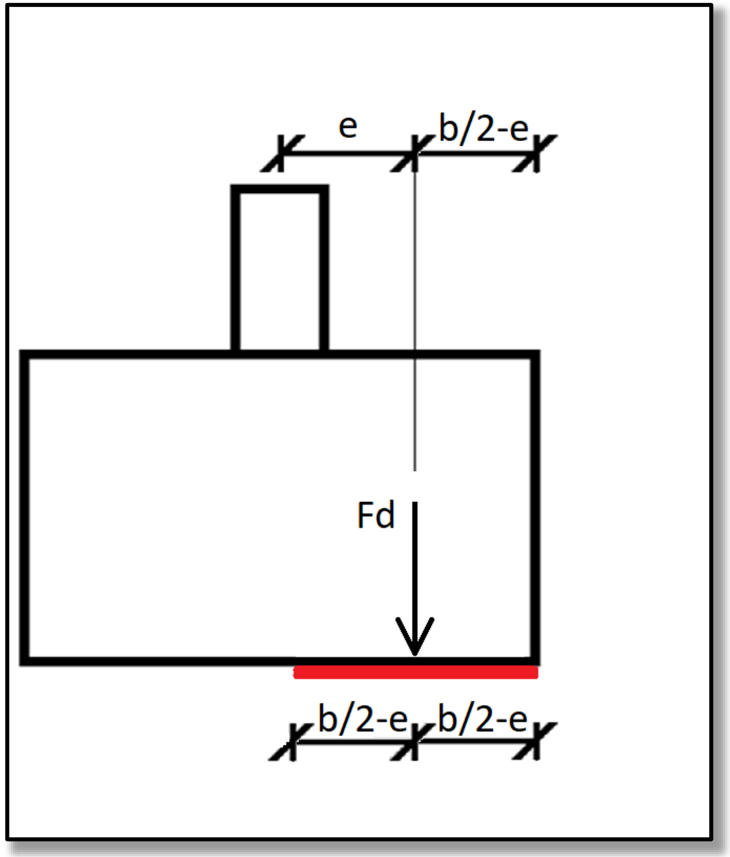
However, **in reality**, **the normal force does not act in the column axis** but acts at a certain eccentricity.



Theory

Effective loading area

Therefore, only a certain part of the soil is loaded by the footing, and this part is known as the **effective loading area of the footing**, and this area depends on the eccentricity of the normal force.



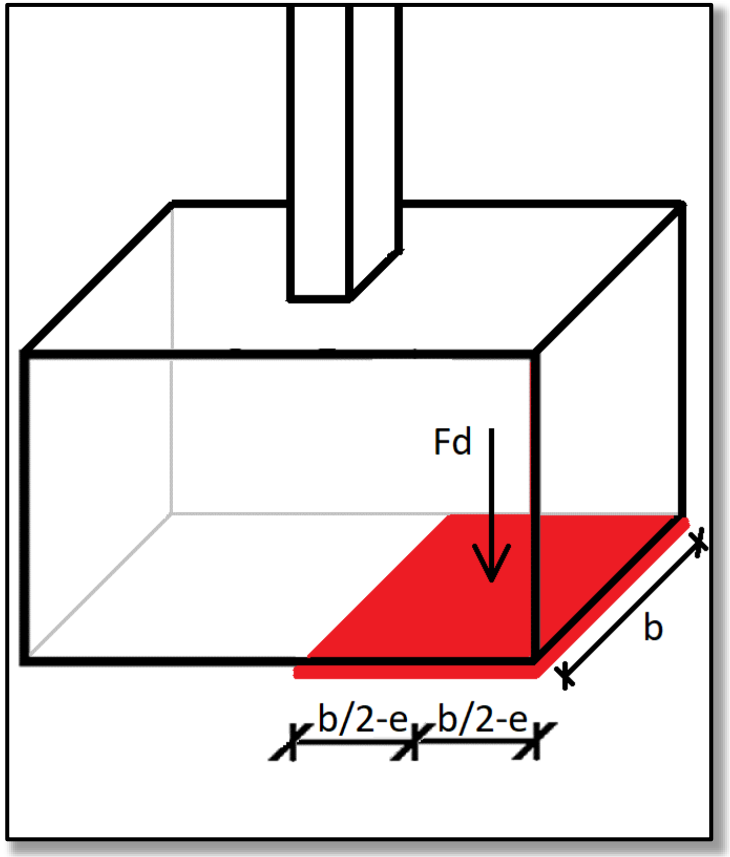
Theory

Effective loading area

The effective loading area can be calculated using the equation

$$A_{eff} = 2(b/2 - e) \cdot b = (\boldsymbol{b} - \boldsymbol{2e}) \cdot \boldsymbol{b}$$

Remember!
The effective loading area depends on the eccentricity, which depends on the height of the footing.
Therefore, **if the height changes** during the design process, **the effective loading area will change also!**



Task steps

Task steps

The task will consist of the following steps:

1) design of floor dimensions (**width** of the footing),
2) design and assessment of **plain concrete** (PC) footing
3) design and assessment of **reinforced concrete** (RC) footing
4) drawing of **sketches**.

The width of the footing will be the same for PC and RC footing!

1) Design of footing width

Design of footing width – design

For simplicity, we will design a **square foot** with a width of b.

When designing the footing width, we use the condition that the stress in the soil must be smaller than the soil resistance

$$\frac{N_{Ed} + G_{0,d}}{A_{eff}} \leq R_d.$$

$$\sigma \leq R_d$$

Design of footing width – design

From the condition

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \leq R_d$$

we obtain an equation for the calculation of the required effective load area

$$\boldsymbol{A_{eff,req} = \frac{N_{Ed} + G_{0,d}}{R_d}}$$

where N_{Ed} is the normal force from the upper structure (see Task 1),

$G_{0,d}$ is the self-weight of the footing (we do not know the exact value right now, so we will estimate it as $0.1N_{Ed}$),

R_d is the soil resistance (assigned as 400 kPa).

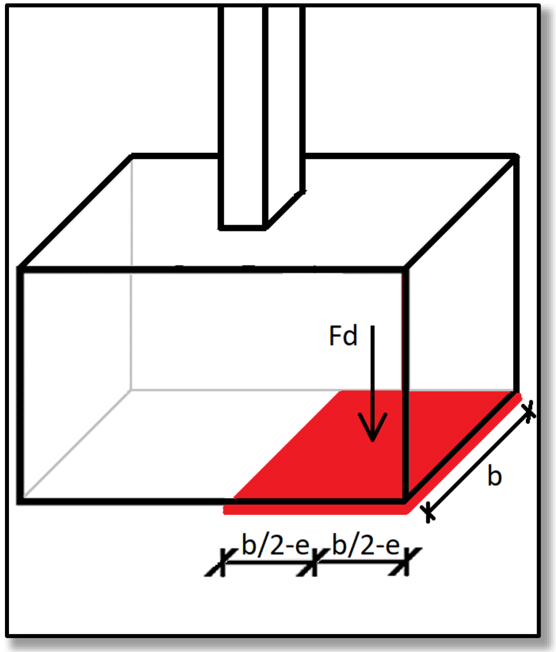
Design of footing width – design

We know that **the effective area is always smaller the plan area**.

Therefore, when designing the footing width, we want the **floor area to be "slightly larger"*** **than the required effective area**

$$b^2 \geq 1.25 \cdot A_{eff,req}$$

$$A_{floor} \geq 1.25 \cdot A_{eff,req}$$



* We will estimate that the floor plan needs to be approximately 25% larger. This is only an "experience estimate", which is not supported by any calculations. Therefore, we must verify the design width later!

Design of footing width – design

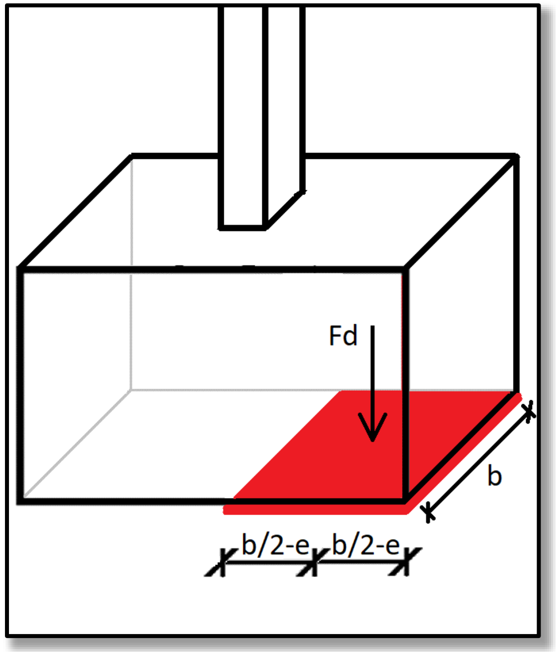
From the condition

$$b^2 \geq 1.25 \cdot A_{eff,req}$$

we obtain an equation for the calculation of the footing width

$$\boldsymbol{b \geq \sqrt{1.25 \cdot A_{eff,req}}}$$

The width of the footing must be a multiple of 50 mm!



Design of footing width – verification

After we design the footing width, we must verify that the stress in the soil is smaller than the soil resistance:

$$\sigma = \frac{N_{Ed} + G_{0_d}}{A_{eff}} \leq R_d.$$

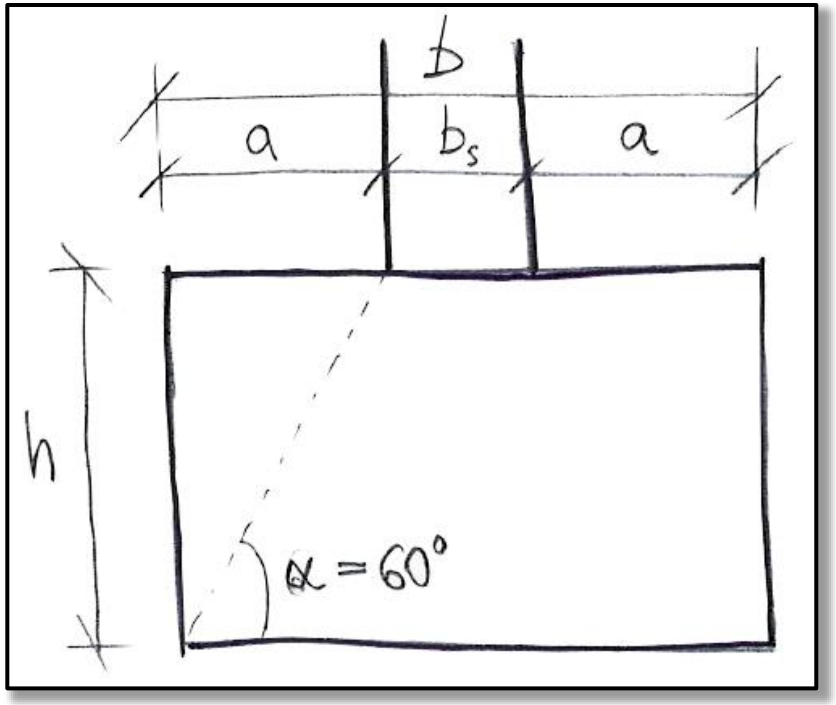
However, in order to determine the stress, we must first determine

- the **height** of the foot,
- the **self-weigh**t of the foot,
- **effective loading area**.

Design of footing width – verification

When verifying the design, we **estimate the height of the footing** by assuming that the load-bearing angle should be at least 60°*,

$$\boldsymbol{h} \geq a \tan \alpha = \frac{b - b_s}{2} \tan 60^{\circ}.$$



The height of the footing must be a multiple of 50 mm!

* This is approximately true for plain concrete footings.

Design of footing width – verification

After we estimate the height of the footing, we can calculate the self-weight of the footing as

$$\boldsymbol{G_{0,d} = 1.35 \cdot 25 \cdot b^2 h},$$

where b is the footing width and h the footing height.

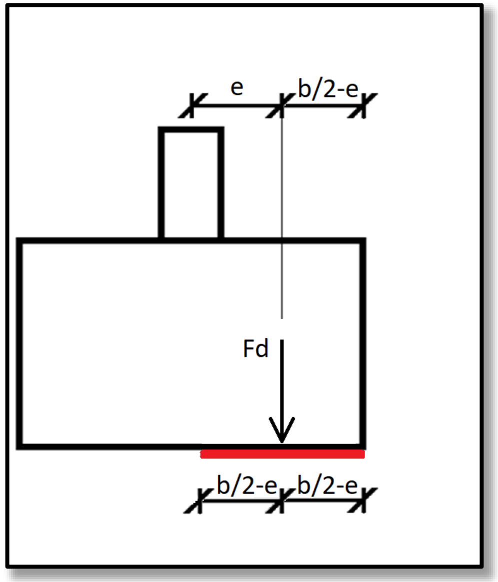
Design of footing width – verification

Using the height and the self-weight of the footing, we can calculate the **eccentricity** of the normal force

$$e = \frac{M_{Ed} + H_{Ed}h}{N_{Ed} + G_{0,d}}.$$

And using the eccentricity, we can calculate the **effective loading area** of the footing

$$A_{eff} = b(b - 2e).$$



Design of footing width – verification

Finally, we can assess the **condition for the stress in the soil**:

$$\sigma = \frac{N_{Ed} + G_{0_d}}{A_{eff}} \leq R_d.$$

If this **condition is not satisfied**, the footing **width is too small and must be increased**.

If this **condition is satisfied by a large margin**, the footing **width is too big and should be decreased**.

If you change the width (increased or decreased), the **verification*** **must be done again!**

* Calculation of h, $G_{0,d}$, e, A_{eff}, and $\sigma \leq R_d$.

Design of footing width – summary

$$A_{eff,req} = \frac{N_{Ed} + 0.1N_{Ed}}{R_d}$$

$$b \geq \sqrt{1.25 \cdot A_{eff,req}}$$

$$\boldsymbol{b = \ ... \ \mathbf{mm}}$$

$$h \geq \frac{b - b_s}{2} \operatorname{tg} 60°$$

$$h = \ ... \ \mathrm{mm}$$

$$G_{0,d} = 1.35 \cdot 25 \cdot b^2 h$$

$$e = \frac{M_{Ed} + H_{Ed} h}{N_{Ed} + G_{0,d}}$$

$$A_{eff} = b(b - 2e)$$

$$\boldsymbol{\sigma = \frac{N_{Ed} + G_{0_d}}{A_{eff}} \leq R_d}$$

If the condition is not satisfied, we change the width and repeat the verification.

2) Design of plain concrete footing

Plain concrete footing

The **width of the foot is already determined** from the previous calculation **and is not changed in any way**.

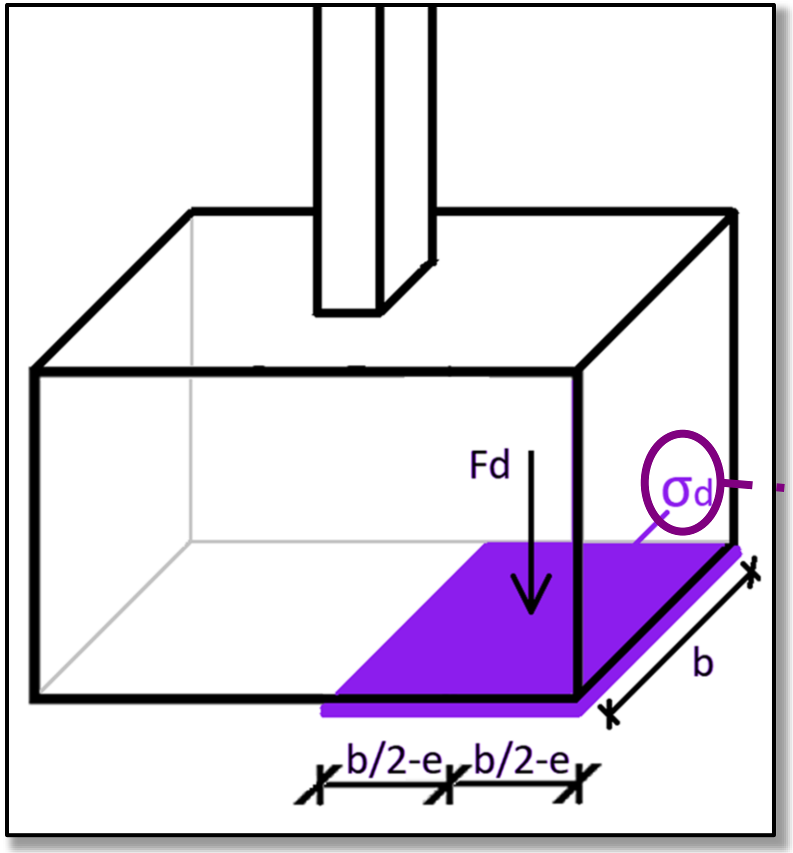
The **height** of the footing **will be designed more accurately**.

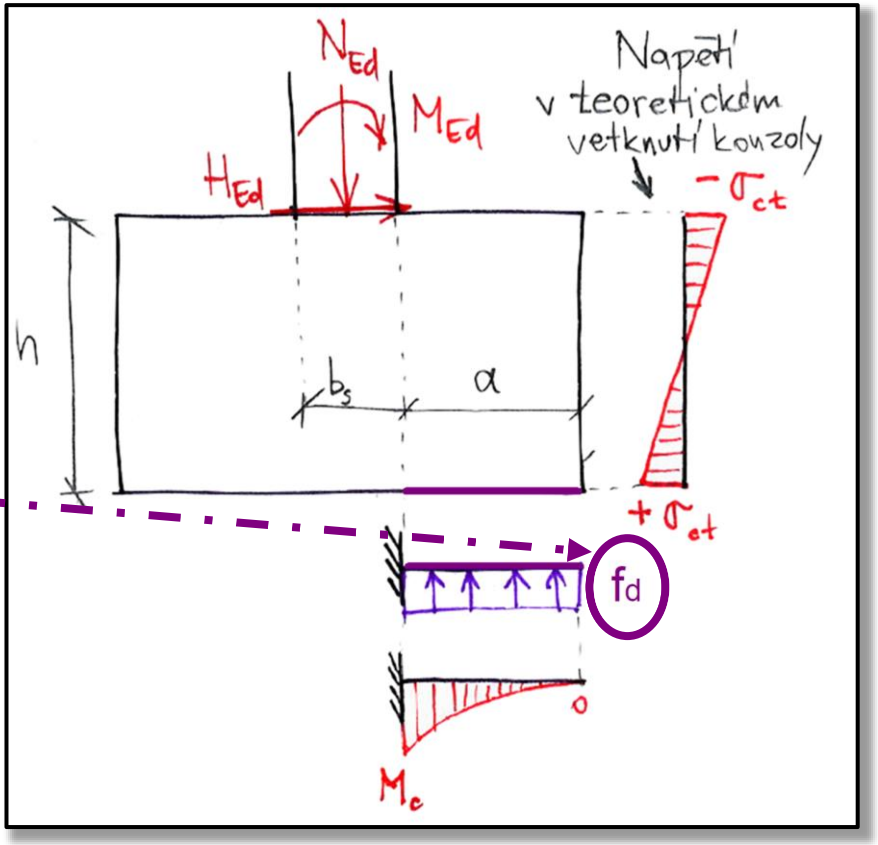
We will assess:

- **tensile** stresses **in concrete**
- **compressive** stresses **in soil**.

Plain concrete footing

For the calculations, the footing is modelled as a **cantilever** loaded from the load from soil – i.e. the **stress in the soil** induced by the normal force.





Design – Height of the footing

We design the height from the condition the that the most extreme tension in concrete must not exceed tensile strength of concrete

$$\sigma_{ct} = \frac{m_c}{W} = \frac{\frac{1}{2} f_d a^2}{\frac{1}{6} b h^2} \leq f_{ctd} = \frac{\alpha_{ct} f_{ctk,0.05}}{\gamma_c}$$

$\alpha_{ct} = 0.8$

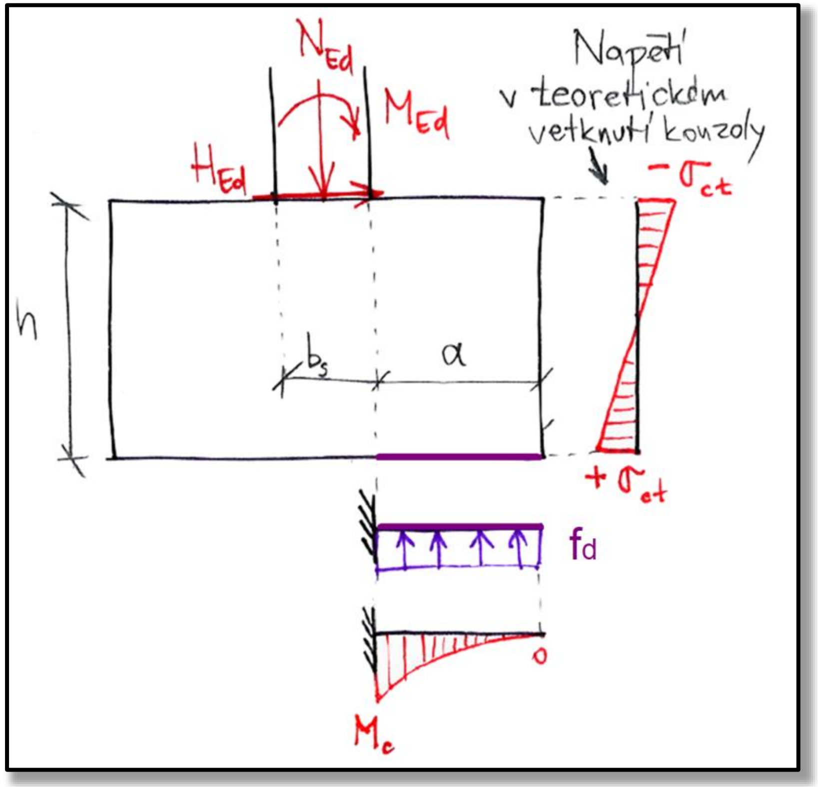
$\gamma_c = 1.5$

$a = (b - b_s)/2$

$f_d = b\sigma_d$

$\sigma_d = \frac{N_{Ed}}{A_{eff}}$

We do not know the exact value of A_{eff} for the PC footing.
Thus, we will use the value calculated during the verification of the footing width.



Design – Height of the footing

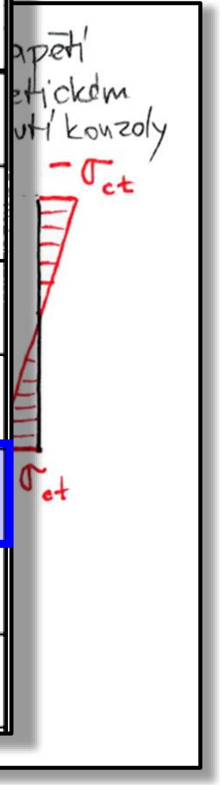
We design the height from the condition the that the most extreme tension in concre

Tabulka 3.1 – Pevnostní a deformační charakteristiky betonu

	Pevnostní třídy betonu														Analytické vztahy/ vysvětlivky
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck} + 8$ (MPa)
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5	$f_{ctm} = 0,30 \times f_{ck}^{(2/3)} \leq$ C50/60 $f_{ctm} = 2,12 \cdot \ln(1+(f_{cm}/10)) >$ C50/60
$f_{ctk,0,05}$ (MPa)	1,1	1,3	1,5	1,8	2	2,2	2,5	2,7	2,9	3	3,1	3,2	3,4	3,5	$f_{ctk,0,05} = 0,7 \times f_{ctm}$ 5% kvantil
$f_{ctk,0,95}$ (MPa)	2	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6	6,3	6,6	$f_{ctk;0,95} = 1,3 \times f_{ctm}$ 95% kvantil
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{cm} = 22(f_{cm}/10)^{0,3}$ (f_{cm} v MPa)

σ_{ct}

$f_d = b\sigma_d$



during the verification of the footing width.

Design – Height of the footing

We design the height from the condition the that the **most extreme tension in concrete must not exceed tensile strength of concrete**

$$\sigma_{ct} = \frac{m_c}{W} = \frac{\frac{1}{2} f_d a^2}{\frac{1}{6} b h^2} \leq f_{ctd} = \frac{\alpha_{ct} f_{ctk,0.05}}{\gamma_c}$$

By modifying the condition above, we obtain the **equation for the design of the footing height**

$$\boldsymbol{h \geq a \sqrt{\frac{3 f_d}{b f_{ctd}}}}$$

The height of the footing must be a multiple of 50 mm!

Design – Effective loading area

After we design the footing height, we can calculate the real effective loading area

$$\boldsymbol{A_{eff} = b(b - 2e)}$$

$$e = \frac{M_{Ed} + H_{Ed}h}{N_{Ed} + G_{0,d}}$$

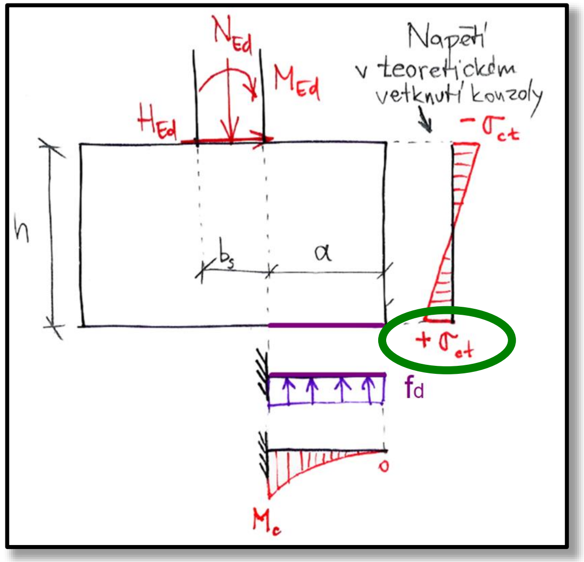
$$G_{0,d} = b^2 h \cdot 25 \cdot 1.35$$

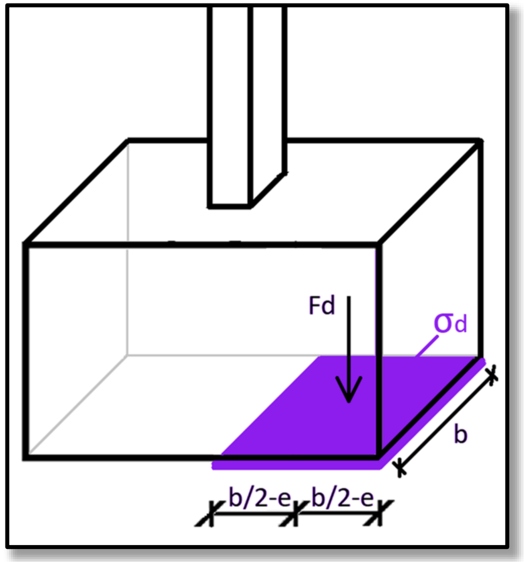
Use the height designed for the PC footing (see previous slide) and NOT the estimation of height used in the design of width ($a \cdot \mathrm{tg}\, 60°$)!

Assessment

The designed footing must be assessed using two conditions.

- the most extreme tensile stress **must be smaller than the tensile strength** of the concrete.
- the stress in the soil **must be smaller than the soil resistance**.





Assessment – concrete stress

The most extreme **tensile stress in concrete** must be smaller than the tensile strength of the concrete

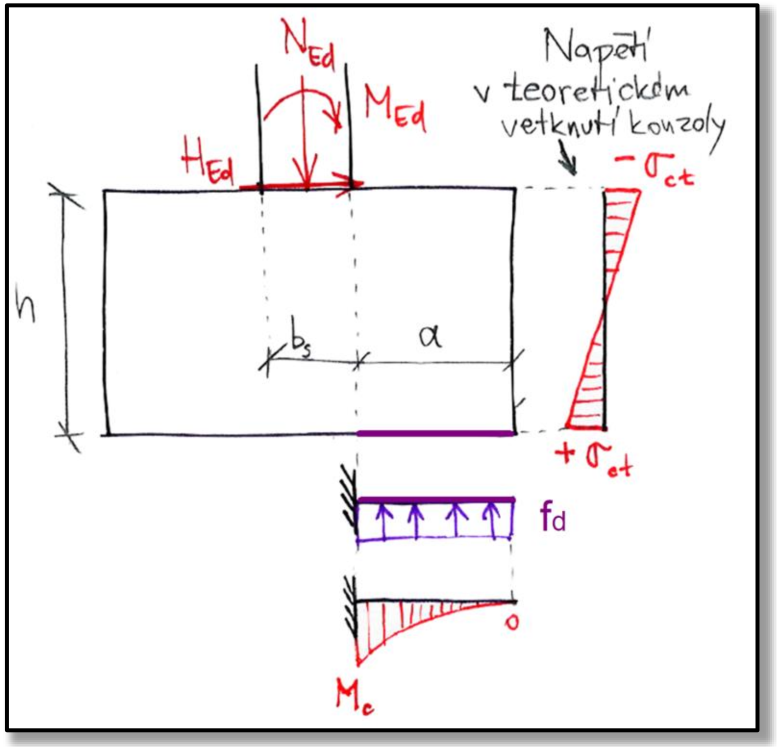
$$\sigma_{ct} = \frac{M_c}{W} = \frac{\frac{1}{2} f_d a^2}{\frac{1}{6} b h^2} \leq f_{ctd}$$

$$f_d = b\sigma_d$$

$$\sigma_d = \frac{N_{Ed}}{A_{eff}}$$

Use the height designed for the PC footing. Do NOT use the estimation of height used in the design of the width ($a \cdot \text{tg}\, 60°$)!

We must use the real effective loading area calculated for the PC footing. Do NOT use the value calculated during the design of the footing width!

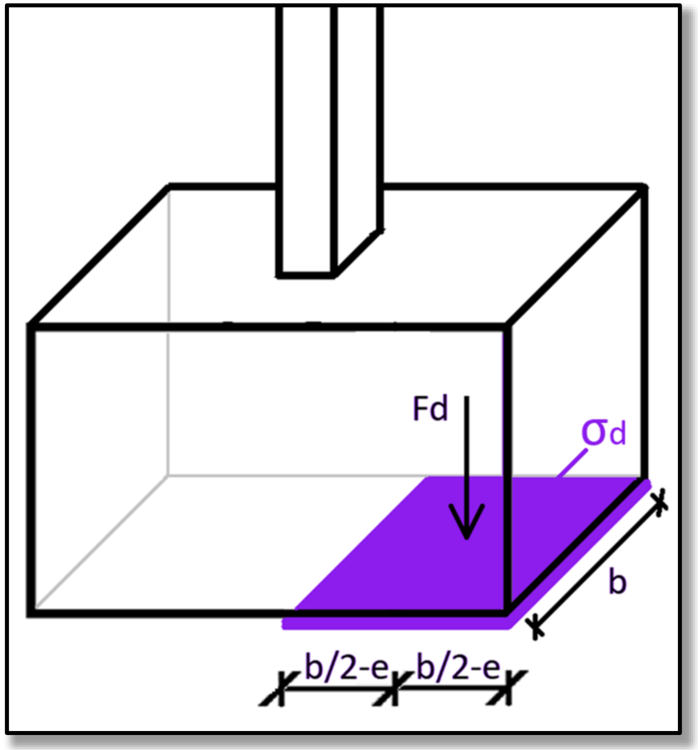


Assessment – soil stress

The compressive **stress in the soil** must be smaller than the soil resistance

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \leq R_d.$$

We must use the real self-weight and real effective loading area of the PC footing. Do NOT use the values calculated during the design of the footing width!



Assessment

If any of the **conditions are not satisfied**, the **footing should be redesigned**.

In the homework, only propose how you would change the design. Do not recalculate the HW.

3) Design of reinforced concrete footing

Reinforced concrete footing

The **width of the foot is already determined** from the previous calculation **and is not changed in any way**.

The **height** of the footing **will be designed more accurately**.

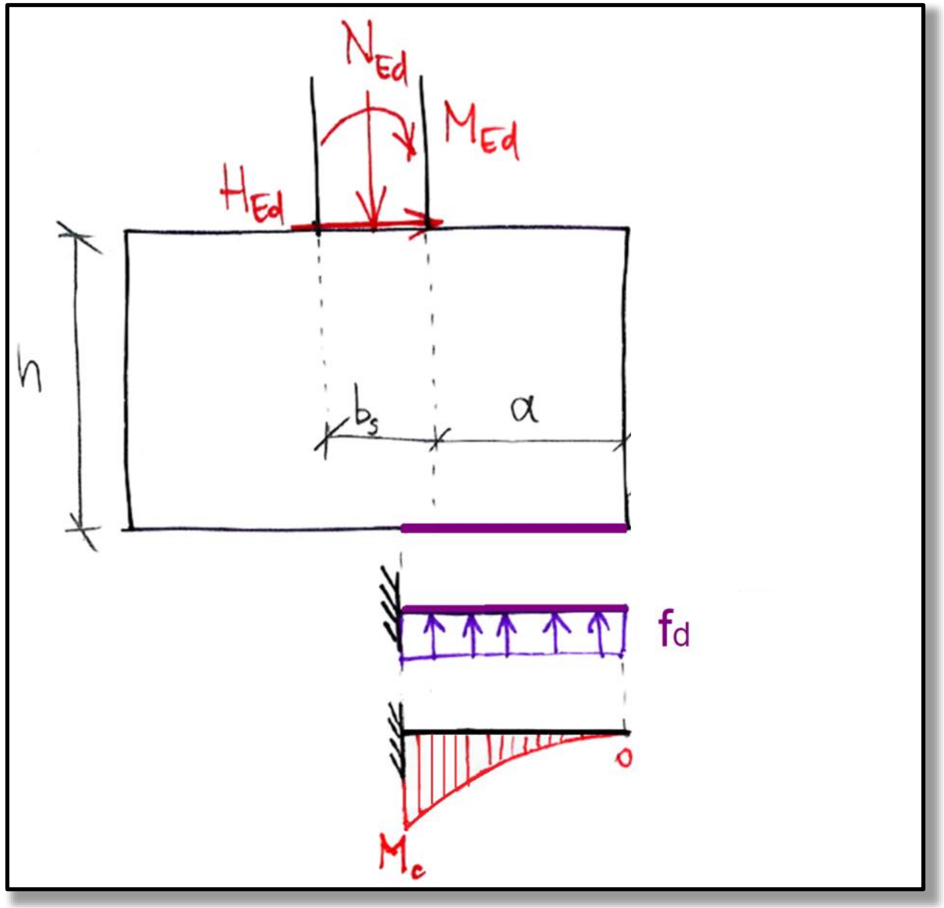
We will design/assess:

- **tensile reinforcement**,
- **compressive** stresses **in soil**.

Reinforced concrete footing

The reinforced footing is again modelled as a cantilever, but now with an **effective length**

$$l_k = a + 0.15 b_s$$



Design – Height of the footing

We can **choose the height** anywhere from 200 mm (puncing limit) to the height for plain concrete footing.

A good (safe and economic) height of the RC footing is a **half of the height for plain concrete footing**

$$\boldsymbol{h}_{\mathbf{RC}} \cong \boldsymbol{h}_{\mathbf{PC}}/\mathbf{2}$$

Design – Effective loading area

After we design the footing height, we can calculate the real effective loading area

$$\boldsymbol{A_{eff} = b(b - 2e)}$$

$$e = \frac{M_{Ed} + H_{Ed}h}{N_{Ed} + G_{0,d}}$$

$$G_{0,d} = b^2h \cdot 25 \cdot 1.35$$

Use the height designed for the RC footing (see previous slide) and NOT the height of the PC footing nor the estimation of height used in the design of width!

Design – Reinforcement

We design the reinforcement in the same way as in beams.

$$A_{s,req} = \frac{M_{Ed}}{0.9d}$$

$$M_{Ed} = \frac{1}{2} f_d l_k^2$$

$$f_d = b\sigma_d$$

We must use the real effective loading area calculated **for the RC footing!** Do NOT use the value for PC footing nor the value used during the design of the footing width!

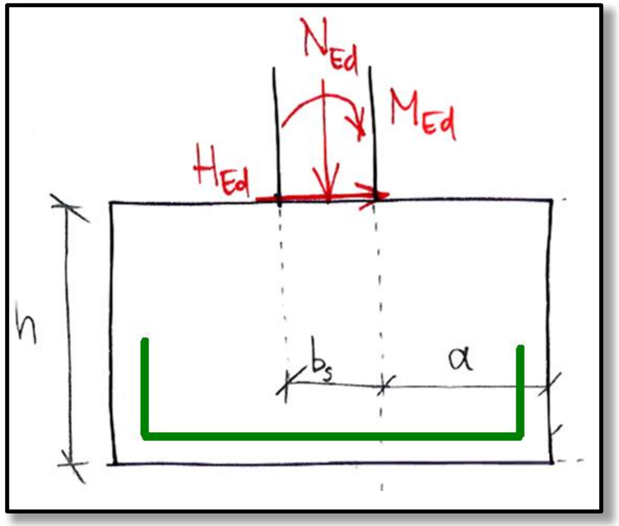
$$\sigma_d = \frac{N_{Ed}}{A_{eff}}$$

$$d = h - c - \emptyset/2$$

$c = 50$ mm

Use diameter 14 to 20 mm.

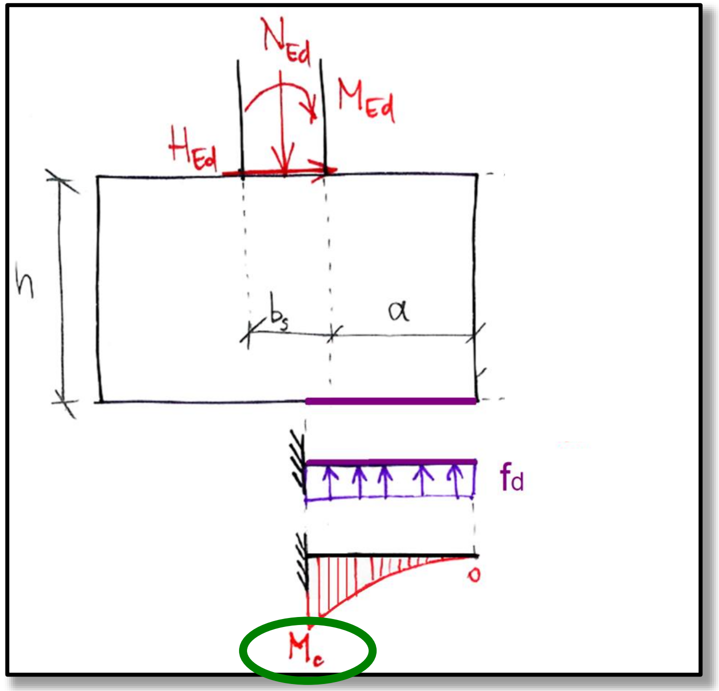
Design: $X \times \emptyset Y \ (A_{s,prov} = \ \dots)$

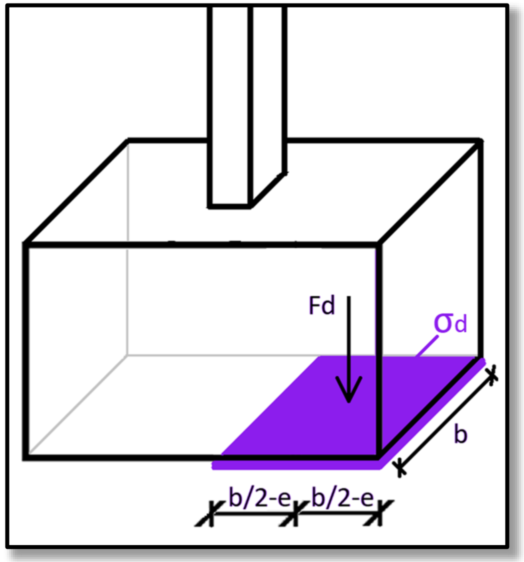


Assessment

The designed footing must be assessed using two conditions.

- the **bending moment** **must be smaller than the load-bearing capacity of reinforcement**.
- the **stress in the soil** **must be smaller than the soil resistance**.





Assessment – Reinforcement

We assess the bearing capacity of the footing cross-section in the same way as in beams

$$x = \frac{A_{s,prov} f_{yd}}{0.8 b f_{cd}},$$

$$z = d - 0.4x,$$

$$M_{Rd} = A_{s,prov} f_{yd} z.$$

We verify the footing by assessing

$$\boldsymbol{M_{Ed} \leq M_{Rd}}.$$

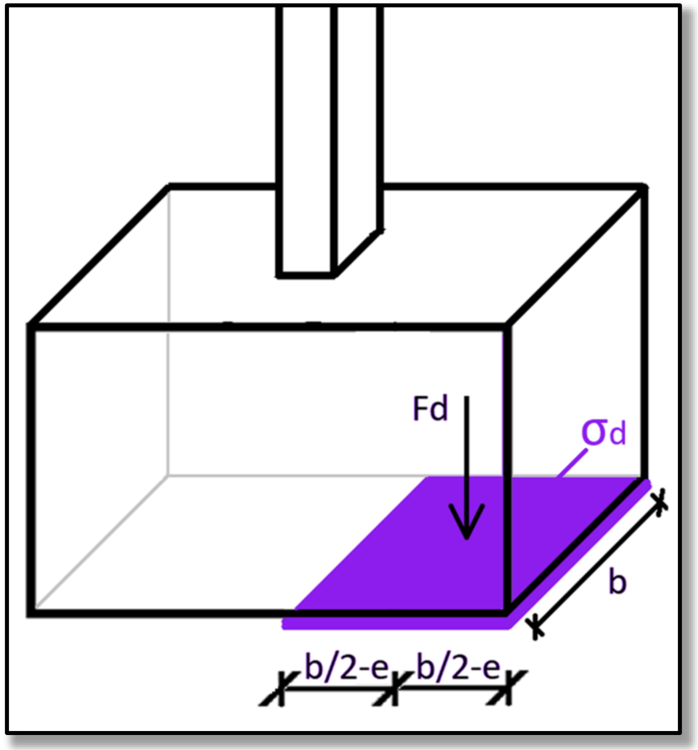
Use the bending moment calculated **for the RC footing!** Do NOT use the moment calculated for the PC footing.

Assessment – soil stress

The compressive **stress in the soil** must be smaller than the soil resistance

$$\sigma = \frac{N_{Ed} + G_{0,d}}{A_{eff}} \leq R_d.$$

We must use the real self-weight and real effective loading area **of the RC footing!** Do NOT use the values calculated for PC footing nor the values used during the design of the footing width!



Assessment

If any of the **conditions are not satisfied**, the **footing should be redesigned**.

In the homework, only propose how you would change the design. Do not recalculate the HW.

4) Drawings

Drawings

For the **plain concrete footing**, draw the **shape of the footing** (in scale with dimensions).

For the **reinforced concrete footing**, draw the **sketch of reinforcement** – see next slide.

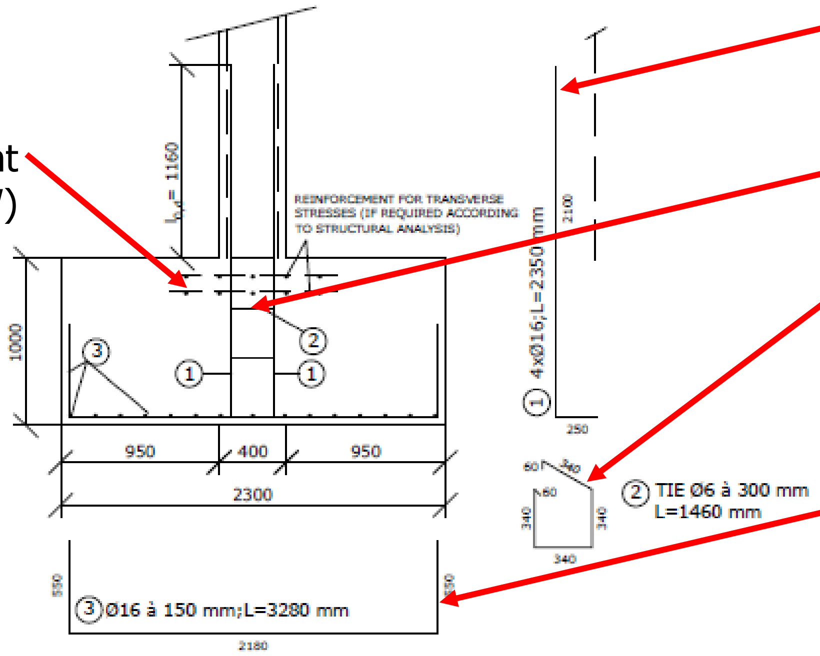
Drawings - reinforcement

Transverse reinforcement
(not needed in the HW)

Starting reinforcement for columns
(same as the column reinforcement)

Ties – see column reinforcement (middle part)

Shapes of the designed rebars



LIST OF REINFORCEMENT

Item	Rebar	Length [mm]	Pieces [pcs]	Total length of rebars [m] Ø6	Total length of rebars [m] Ø16
1	Ø16	2 350	8		18,80
2	Ø6	1 460	3	4,38	
3	Ø16	3 280	30		98,40
Total length [m]				4,38	117,20
Unit weight [kg/m]				0,22	1,58
Weight of steel [kg]				0,97	184,94
Total weight of steel				185,91	

List of reinforcement (not needed in the HW)

Notes

MATERIALS:
CONCRETE C20/25
STEEL B500

COVER DEPTH 50 mm

AXIAL DIMENSIONS OF REBARS

REINFORCEMENT DRAWING – REINFORCED CONCRETE PAD FOOTING	
Prepared by: JÁRA CIMRMAN	WT 1911/12
Checked by: prof. KLOENER	1:25

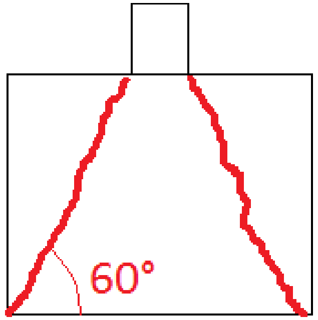
Drawing title

Theory

Th4) PC vs RC footing

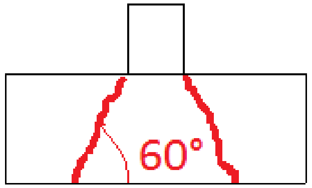
PC vs RC footing

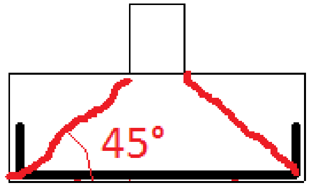
In a PC footing, the height is bigger, load-bearing angle is approximately 60°, and the **tensile stresses are small** (smaller than the tensile strength of concrete).



In a RC footing, the height is smaller, load-bearing angle is approximately 30° to 45°, and the **tensile stresses are big** (bigger than the tensile strength of concrete), and therefore, we must design a reinforcement to transfer the tensile stresses.

Without reinforcement, the angle would be 60°.





The reinforcement changes the load-bearing angle.

thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.