CM01 – Concrete and Masonry Structures 1

HW8 – Flat slab (reinforcements and drawings)

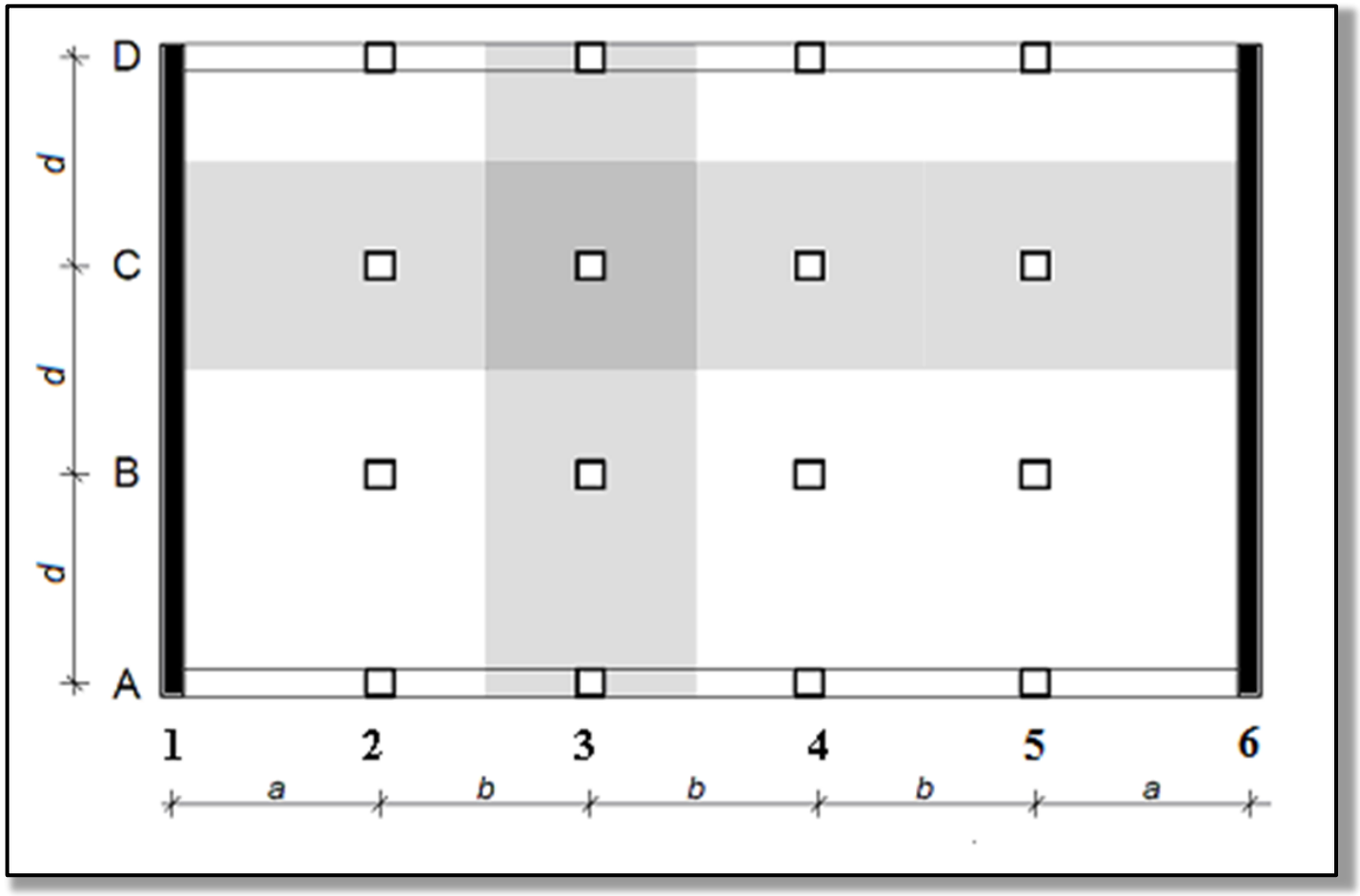
Author: Jakub Holan

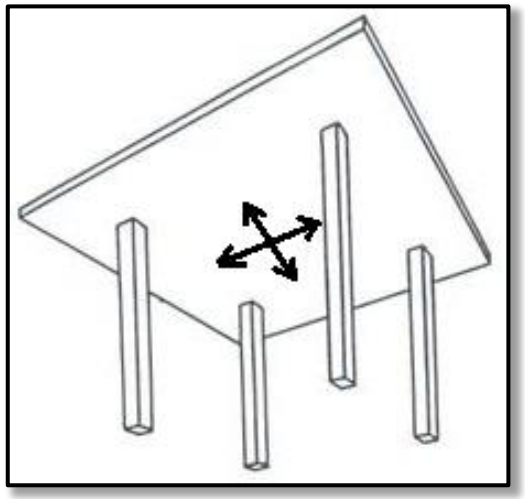
Last update: 16.11.2023 11:28

Task 3

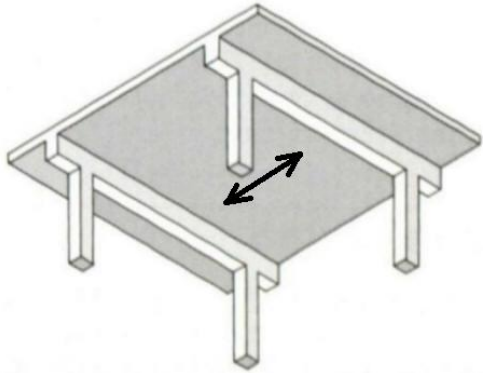
Task 3 – Flat slab

In Task 3, a two-way flat slab (slab supported by columns) will be designed.

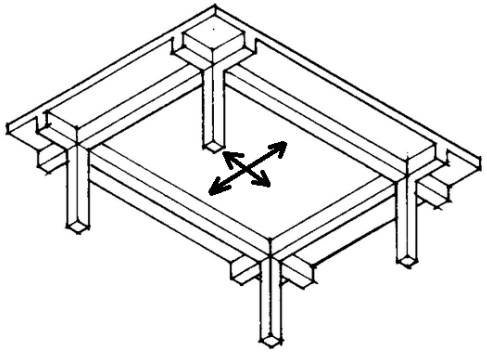




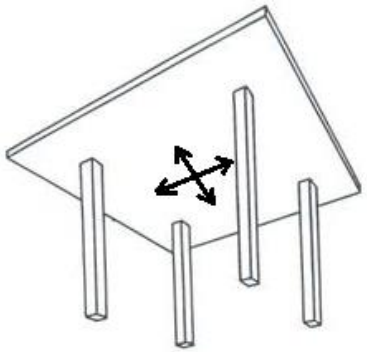
Comparison of Tasks 1 to 3



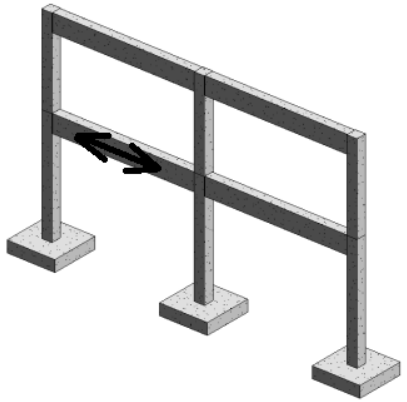
One-way slab – **Task 1**



Two-way slab supported on 4 sides – **Task 2**



Two-way flat slab – **Task 3**



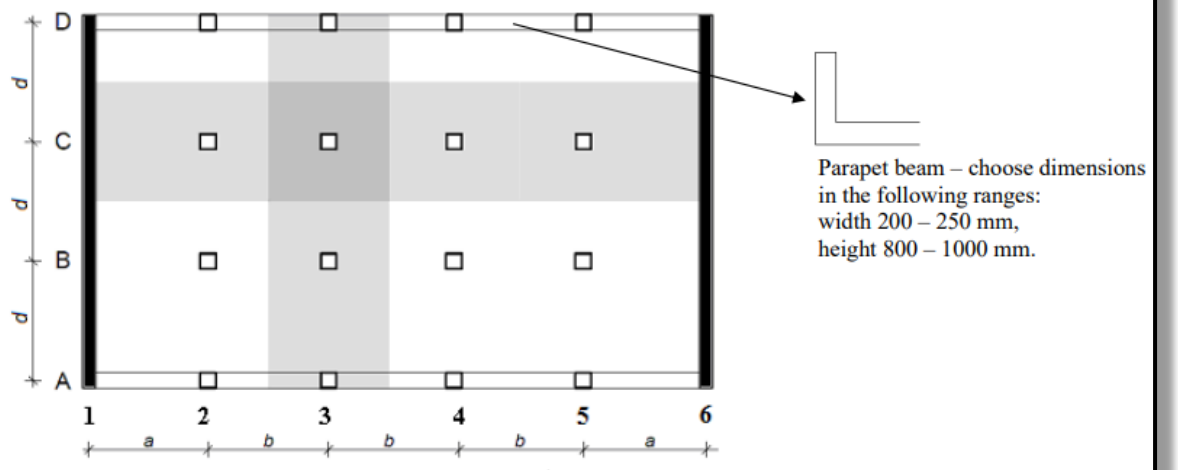
Beam (frame) – **Task 1**

Task 3 – Assignment

Task 3: Two-way slab supported on columns

Two-way slab supported on columns, edge beam in axes A and D, walls in axes 1 and 6.

Scheme of the structure:



Parapet beam – choose dimensions in the following ranges:
width 200 – 250 mm,
height 800 – 1000 mm.

Individual parameters (parameters in **bold** you can find on teacher's website):

Geometry: ***a***, ***b***, ***d*** [m] – horizontal dimensions of the structure (***a*** see 1st task, ***b*** see 2nd task), ***n*** – number of floors (see 1st task)

Materials: **see 1st task**

Loads: **see 1st task**, values for typical floor (except the self-weight, which will be different)

Please work out:

1. **Design of the dimensions of the load-bearing elements** (slab, columns). Choose the thickness of the wall as 200 mm or 250 mm.
2. **Sketch of the structure.**
3. **Structural analysis of the slab**:
   - Bending moments in strips C and 3 using the *Direct design method*.
   - Draw moment curves for both strips.
   - Design slab reinforcements (rebars) for the calculated moments.
   - Design the punching reinforcement for column C3.
4. **Layout of reinforcement** (separately for upper and lower layer of reinforcement).

Task 3 – Assignment goals

1) Design of the **dimensions** of the load-bearing elements and **sketch the structure**.
2) Preliminarily **check punching**.
3) Calculate **bending moments** in lanes C and 3.
4) Design slab **bending reinforcements** (rebars) for the calculated moments.
5) Draw a sketch of the **layout of the bending reinforcement**.
6) Design the **punching reinforcement** for column C3.
7) Draw a sketch of the **layout of the punching reinforcement**.

4) Design of bending reinforcement

Task 3 – Assignment goals

1) Design of the dimensions of the load-bearing elements and sketch the structure.
2) Preliminarily check punching.
3) Calculate bending moments in lanes C and 3.
4) **Design slab bending reinforcements (rebars) for the calculated moments.**
5) Draw a sketch of the layout of the bending reinforcement.
6) Design the punching reinforcement for column C3.
7) Draw a sketch of the layout of the punching reinforcement.

Bending reinforcement

Perform the design **for all the moments** calculated in previous HW.

For **one cross-section do the calculation by hand**, the others in an Excel spreadsheet.

Apart from some specific steps, design and assessment of slab reinforcement is **almost the same as for beams**.

Bending reinforcement

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Design and check of bending reinforcement of the slab																
			Design							Check						
Panel	Cross-section	Strip	m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0,45$	spacing
			[kNm/m]	mm	mm	[mm²]	[mm²]		[mm²]	[mm]		[mm]	[kNm/m]			of bars
C_o	1 (left support)	no division	31,02	169	152	469	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK
	2 (midspan)	Column	34,46	169	152	521	220	Ø12 á 200 mm	566	23,07	0,137	160	39,32	OK	OK	OK
		Middle	22,97	169	152	347	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
	3 (right support)	Column	60,31	169	152	912	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK
		Middle	20,10	169	152	304	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
C_{in}	1 (left support)	Column	70,08	169	152	1060	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK
		Middle	23,36	169	152	353	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
	2 (midspan)	Column	30,19	169	152	456	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK
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3_o	1 (left support)	Column	36,14	156	140	592	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK
		Middle	12,65	156	140	207	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
	2 (midspan)	Column	29,78	156	140	488	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK
		Middle	10,69	156	140	175	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
	3 (right support)	Column	50,12	156	140	821	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK
		Middle	9,00	156	140	147	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
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		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
	2 (midspan)	Column	24,39	156	140	400	204	Ø12 á 250 mm	452	18,42	0,117	149	29,41	OK	OK	OK
		Middle	11,50	156	140	188	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
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		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK

Effective depth of slab

The effective depth of slab is different in each direction based on the chosen rebar positions (chose higher d in the direction with most extreme m_{Ed}):

$$d_1 = h_s - c - \varnothing_2 - \varnothing_1/2$$
$$d_2 = h_s - c - \varnothing_2/2$$

Design rebars with diameter 8, 10, 12, or 14 mm.

Use cover depth from Task 1.

Effective depth of slab

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d_1

d_2

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Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0{,}45$	spacing
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Required cross-sectional area of reinforcement

The required cross-sectional area of reinforcement can be estimated using:

$$a_{s,rqd} = \frac{m_{Ed}}{z f_{yd}}$$

→ From previous HW

→ $f_{yd} = f_{yk}/1.15$

→ Lever arm of internal forces – estimated as $z = 0.9\ d$

Required cross-sectional area of reinforcement

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Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0{,}45$	spacing
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Minimum reinforcement area

When designing the reinforcement, check all of the conditions for minimum reinforcement area:

$$a_{s,min,1} = 0.0013bd,$$

Width of the slab: 1 m

$$a_{s,min,2} = \frac{0.26f_{ctm}}{f_{yk}}bd,$$

Mean tensile strength of concrete (see the table with concrete classes).

$$a_{s,min,3} = \frac{k_c k f_{ct,eff} a_{ct}}{\sigma_s} = \frac{0.4 \cdot 1 \cdot f_{ctm} \cdot (1 \cdot h_d/2)}{f_{yk}}$$

Maximum stress permitted in the reinforcement immediately after formation of the crack:
$\sigma_s = f_{yk} = 500$ MPa

	Pevnostní třídy betonu														Analytické vztahy/ vysvětlivky
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck\,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck} + 8$ (MPa)
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5	$f_{ctm} = 0,30 \times f_{ck}^{(2/3)} \leq$ C50/60 $f_{ctm} = 2,12 \cdot \ln(1+(f_{cm}/10)) >$ C50/60

Minimum reinforcement area

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Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0{,}45$	spacing
			[kNm/m]	mm	mm	[mm²]	[mm²]		[mm²]	[mm]		[mm]	[kNm/m]			of bars
C_o	1 (left support)	no division	**31,02**	169	152	469	220	**Ø12 á 250 mm**	452	18,42	0,109	162	**31,76**	OK	OK	OK
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C_{in}	1 (left support)	Column	**70,08**	169	152	1060	220	**Ø12 á 100 mm**	1131	46,10	0,273	151	**74,04**	OK	OK	OK
		Middle	**23,36**	169	152	353	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
	2 (midspan)	Column	**30,19**	169	152	456	220	**Ø12 á 250 mm**	452	18,42	0,109	162	**31,76**	OK	OK	OK
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Reinforcement design

Reinforcements in a slab are designed as **ø$\mathbf{X}$ per $\mathbf{Y}$ mm** (e.g., ø10 per 150 mm).

Design the reinforcement so that $a_{s,prov}$ is approx. **20 - 30% larger than $a_{s,req}$** and also **larger than $a_{s,min}$.**

Reinforcement design

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Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0,45$	spacing
			[kNm/m]	mm	mm	[mm²]	[mm²]		[mm²]	[mm]		[mm]	[kNm/m]			of bars
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	2 (midspan)	Column	34,46	169	152	521	220	Ø12 á 200 mm	566	23,07	0,137	160	39,32	OK	OK	OK
		Middle	22,97	169	152	347	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
	3 (right support)	Column	60,31	169	152	912	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK
		Middle	20,10	169	152	304	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
C_{in}	1 (left support)	Column	70,08	169	152	1060	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK
		Middle	23,36	169	152	353	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
	2 (midspan)	Column	30,19	169	152	456	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK
		Middle	20,12	169	152	304	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK
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3_o	1 (left support)	Column	36,14	156	140	592	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK
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	2 (midspan)	Column	29,78	156	140	488	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK
		Middle	10,69	156	140	175	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
	3 (right support)	Column	50,12	156	140	821	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK
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3_{in}	1 (left support)	Column	56,62	156	140	927	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK
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	2 (midspan)	Column	24,39	156	140	400	204	Ø12 á 250 mm	452	18,42	0,117	149	29,41	OK	OK	OK
		Middle	11,50	156	140	188	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK
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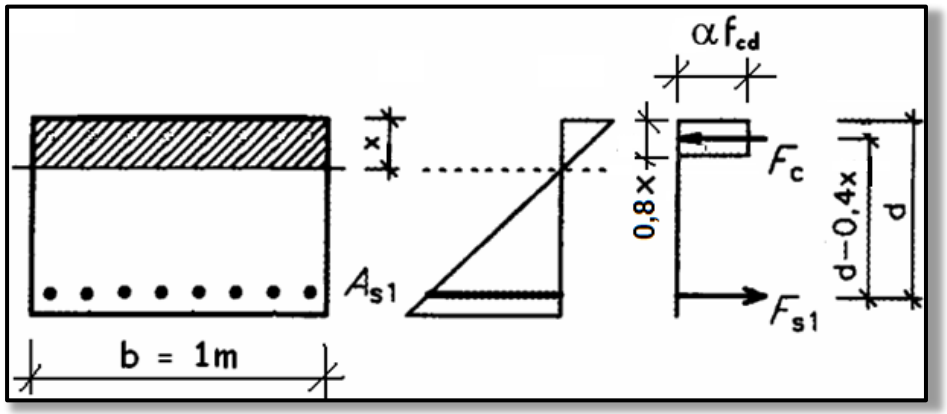
		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK

Assessment of the design

Height of the compressed zone of concrete cross-section:

$$x = \frac{a_{\mathrm{s,prov}} f_{\mathrm{yd}}}{0.8 b f_{\mathrm{cd}}}$$



Lever arm of internal forces (exact value):

$$z = d - 0.4x$$

Load-bearing capacity of the cross-section:

$$m_{\mathrm{Rd}} = a_{\mathrm{s,prov}} f_{\mathrm{yd}} z$$

Assessment:

$$m_{\mathrm{Rd}} \geq m_{\mathrm{Ed}}$$

Assessment of the design

Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0,45$	spacing
			[kNm/m]	mm	mm	[mm²]	[mm²]		[mm²]	[mm]		[mm]	[kNm/m]			of bars
C_o	1 (left support)	no division	31,02	169	152	469	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK
	2 (midspan)	Column	34,46	169	152	521	220	Ø12 á 200 mm	566	23,07	0,137	160	39,32	OK	OK	OK
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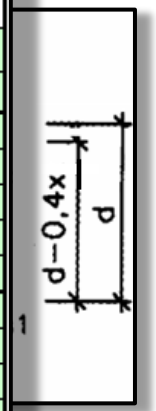

Check of detailing rules

Relative height of compressed zone:

$$\xi = \frac{x}{d} \leq 0.45$$

Spacing of rebars:

$$s \leq \min\left(2h_s;\ 250\ \text{mm}\right)$$

Recommendation: Do not design spacing smaller than 100 mm.

RULE: It is always **better to use higher number of smaller bars** than lower number of bigger bars.

Check of detailing rules

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Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0,45$	spacing
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3_o	1 (left support)	Column	36,14	156	140	592	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK
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	3 (right support)	Column	56,62	156	140	927	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK
		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK

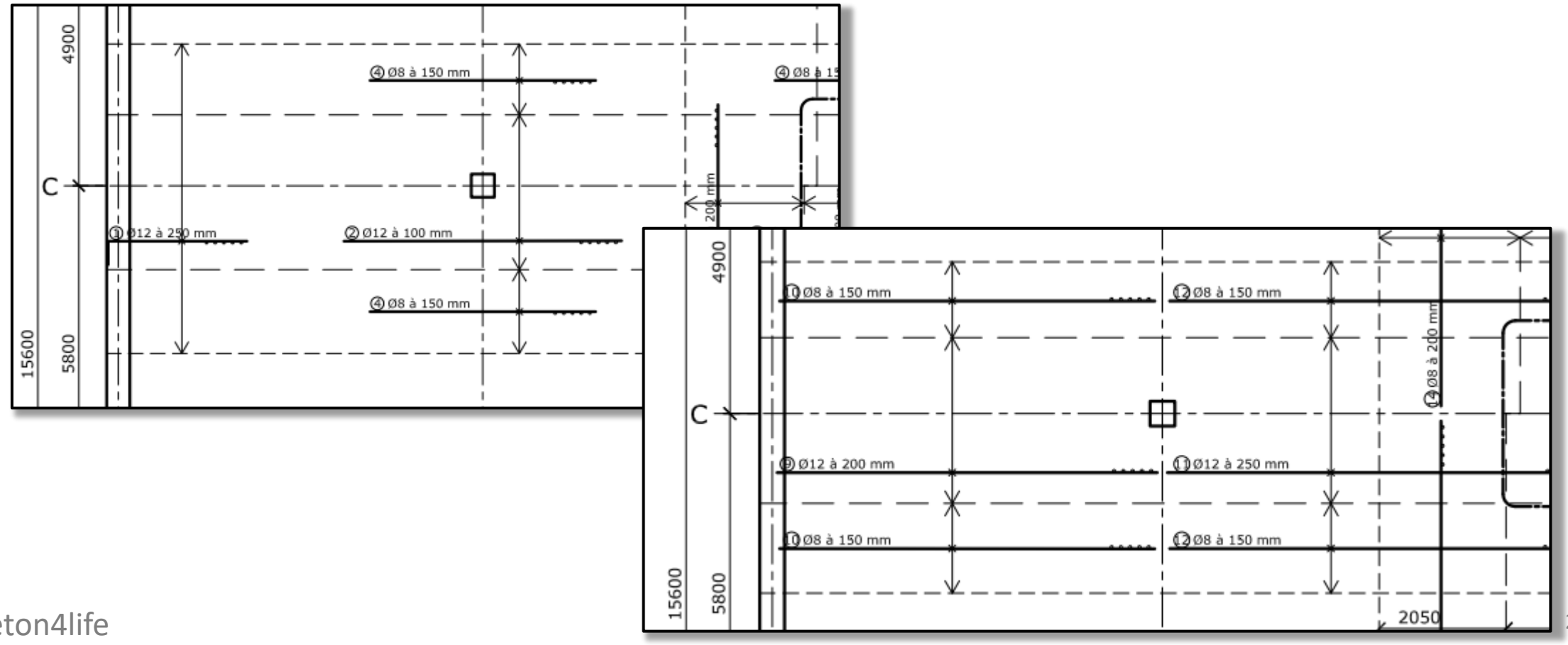
5) Drawing of the bending reinforcement

Task 3 – Assignment goals

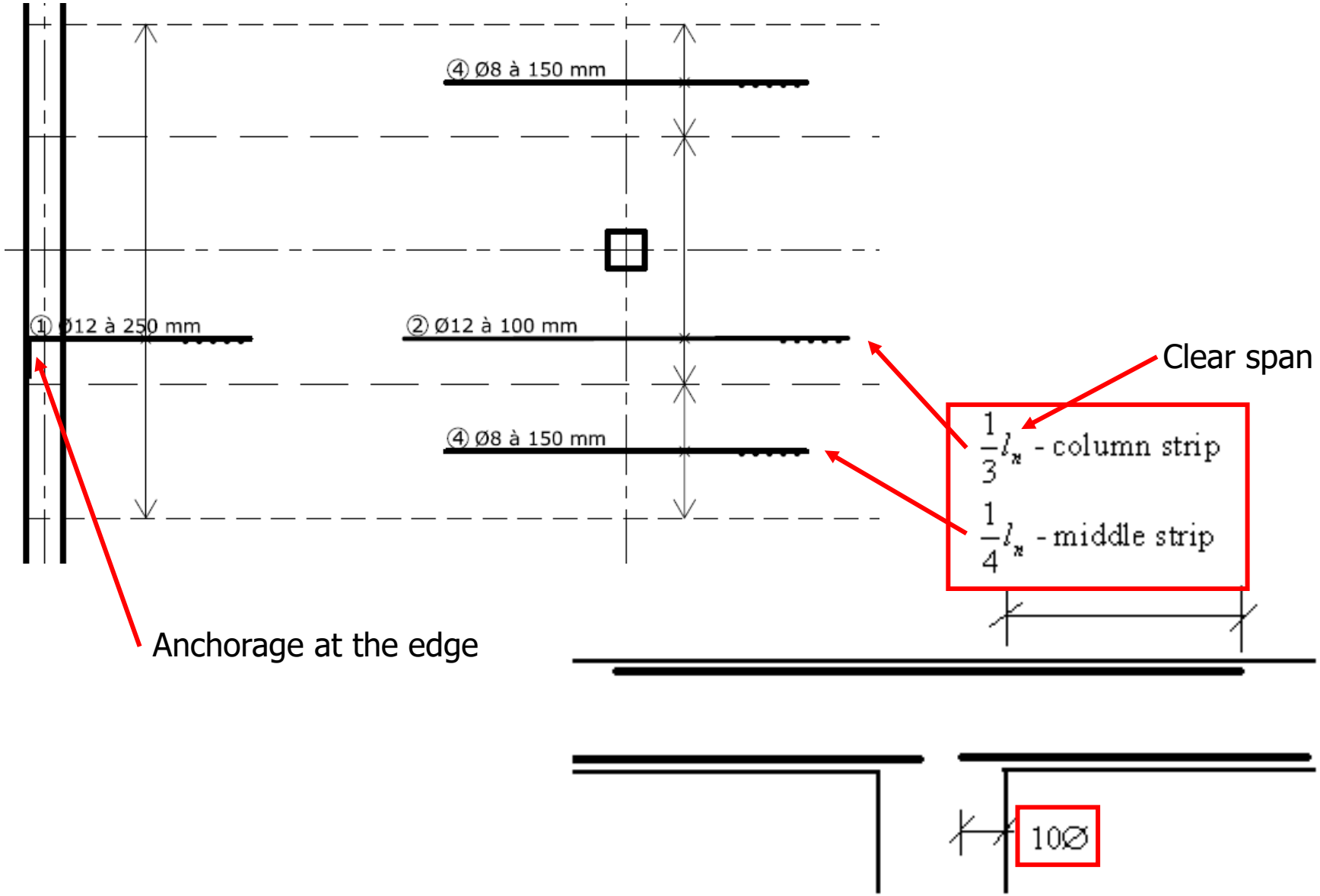
1) Design of the dimensions of the load-bearing elements and sketch the structure.
2) Preliminarily check punching.
3) Calculate bending moments in lanes C and 3.
4) Design slab bending reinforcements (rebars) for the calculated moments.
5) **Draw a sketch of the layout of the bending reinforcement.**
6) Design the punching reinforcement for column C3.
7) Draw a sketch of the layout of the punching reinforcement.

Sketch of bending reinforcement

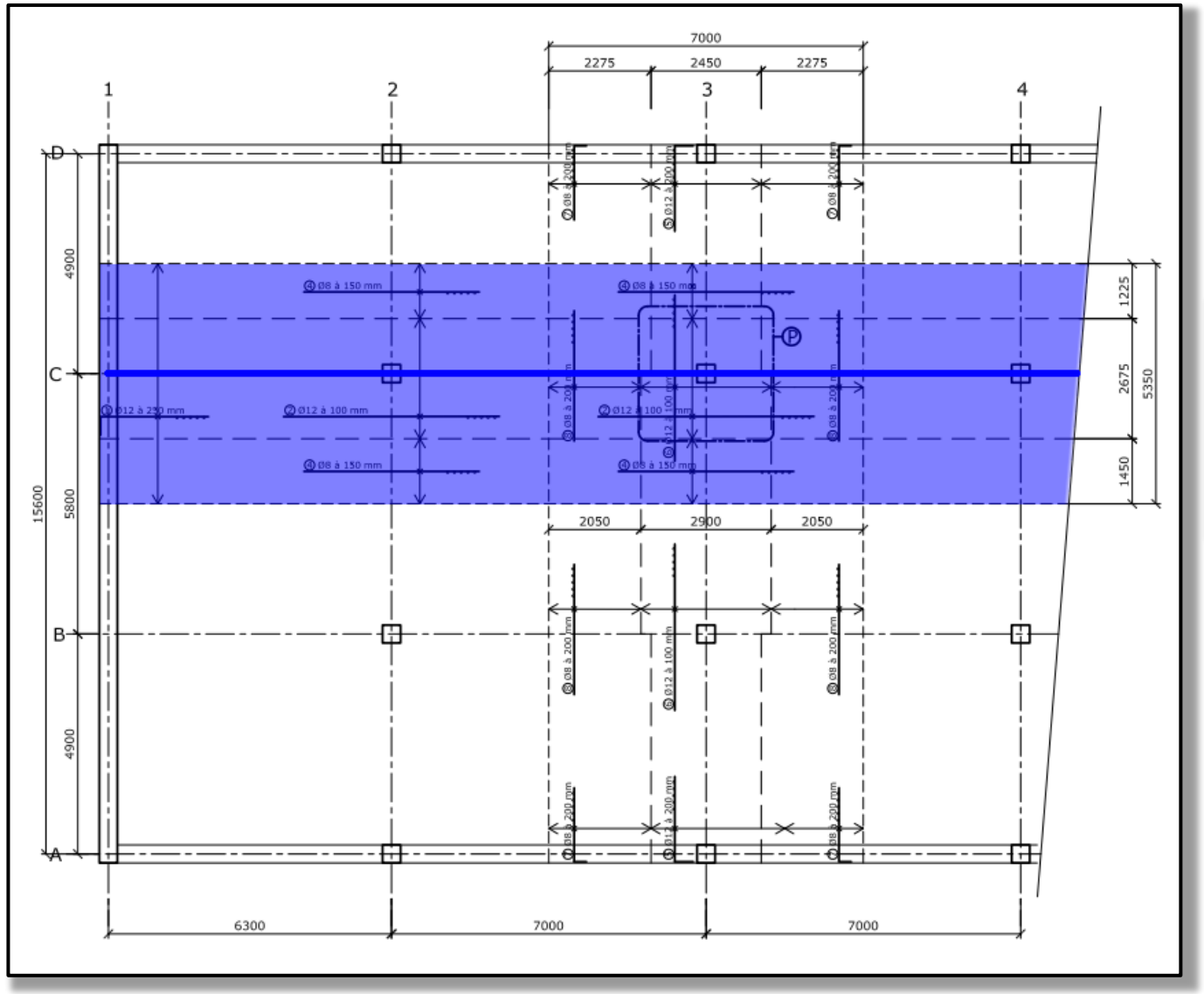
We must do **2 separate drawings** – 1 for **upper** reinforcement and 1 for **lower** reinforcement .



Anchorage

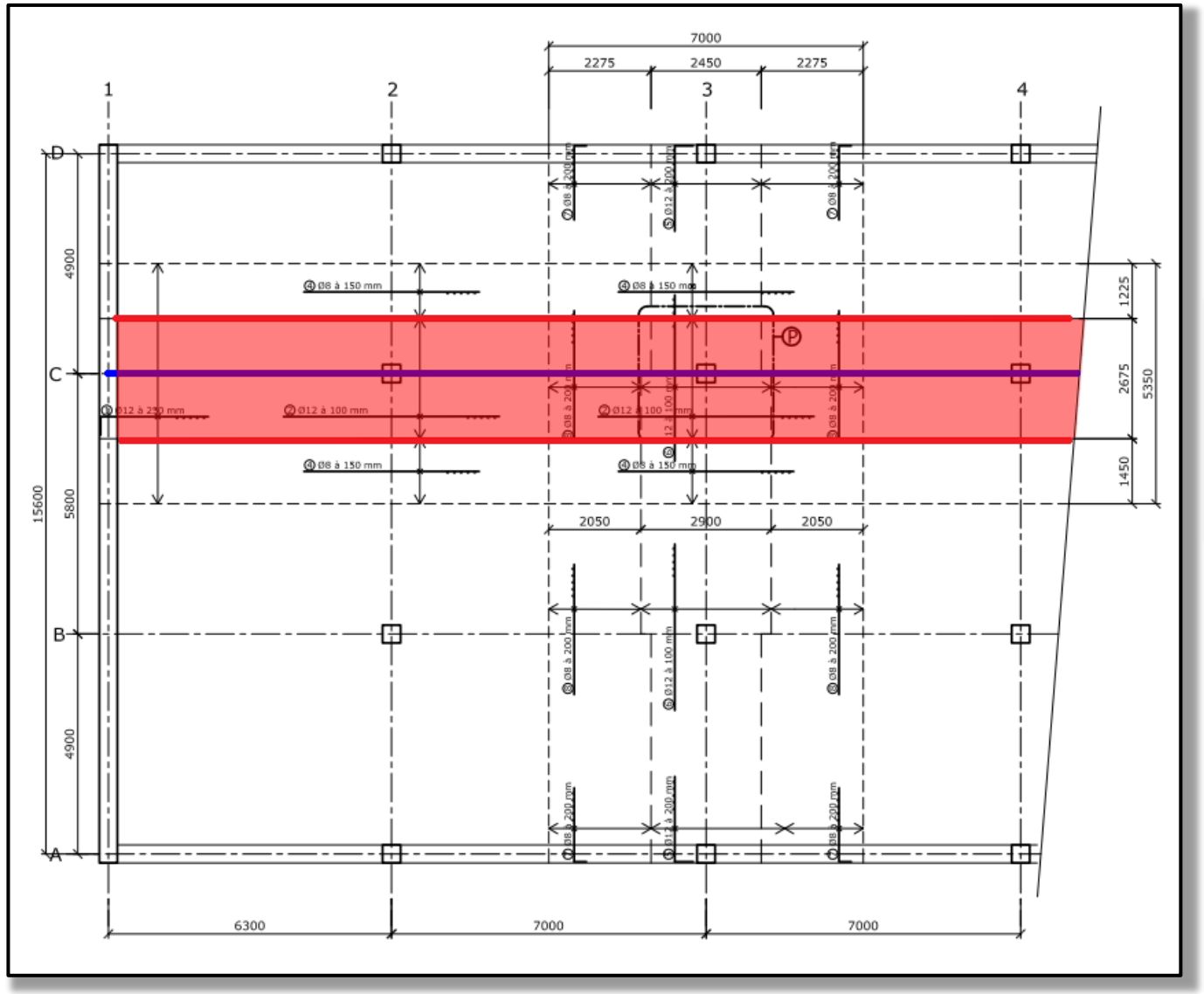

④ Ø8 à 150 mm
① Ø12 à 250 mm
② Ø12 à 100 mm
④ Ø8 à 150 mm
Clear span
$\frac{1}{3}l_n$ - column strip
$\frac{1}{4}l_n$ - middle strip
Anchorage at the edge
10∅

Sketch of bending reinforcement



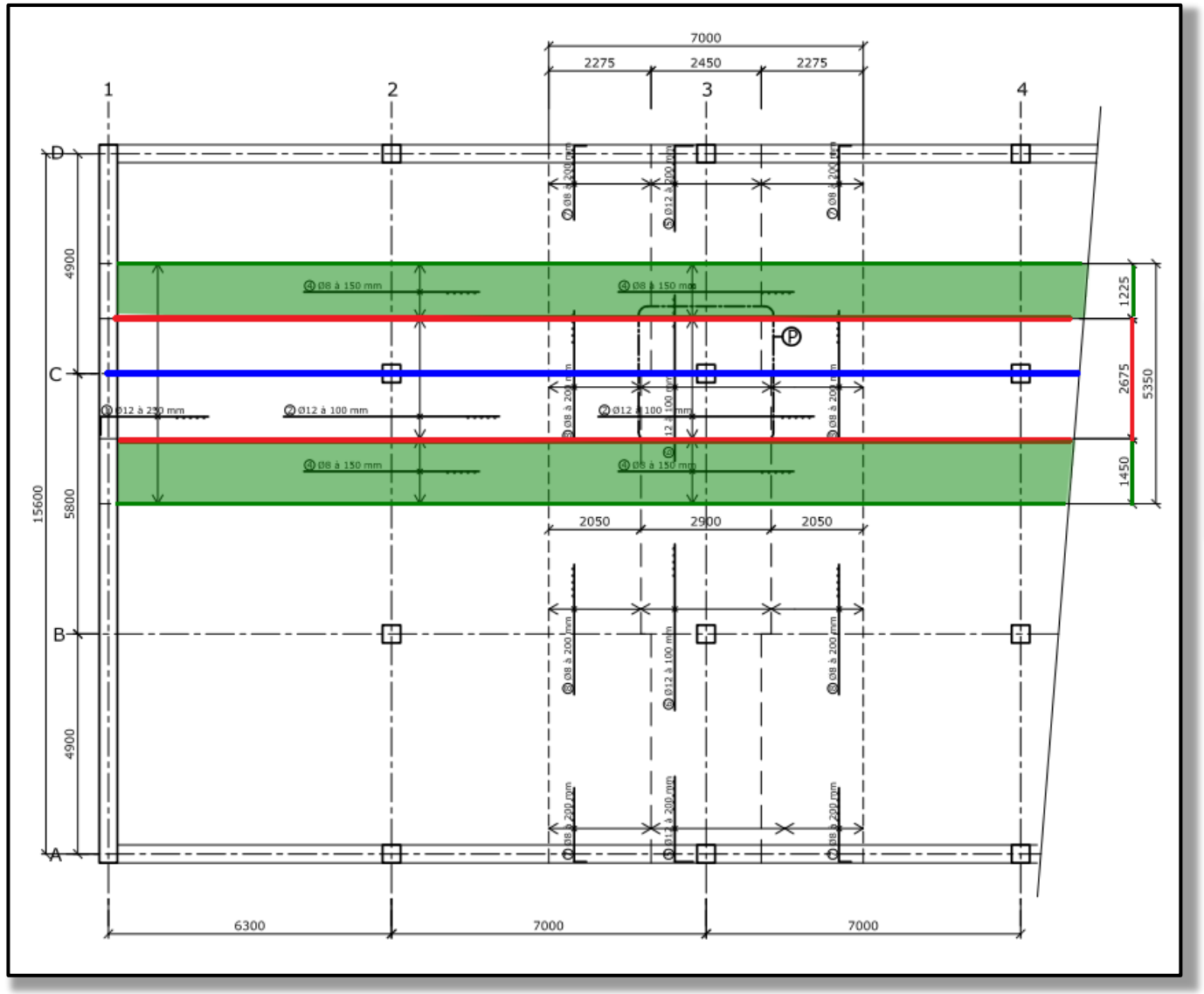
The dashed lines indicate the given lane.

Sketch of bending reinforcement



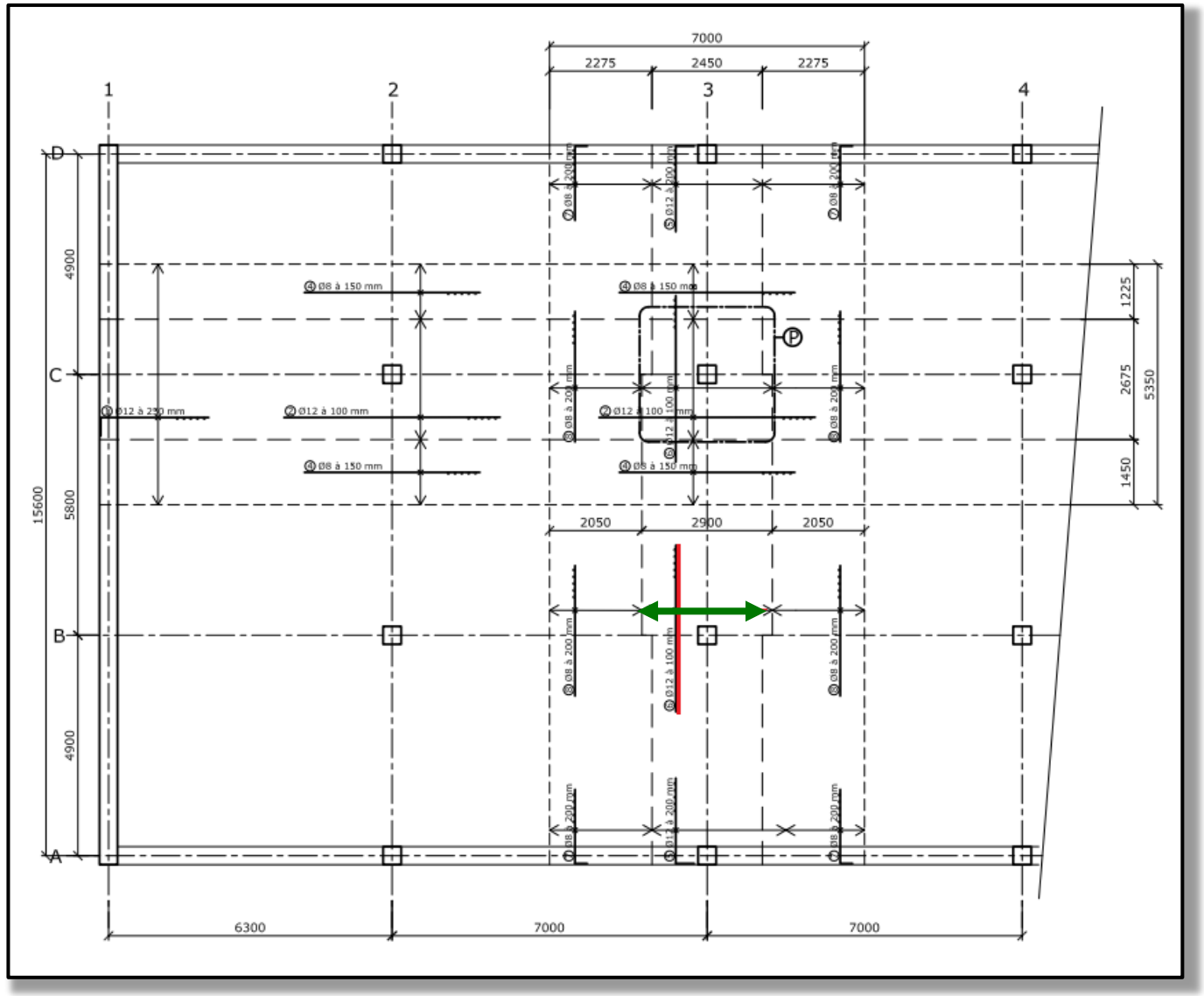
Long dashed lines separate the column and centre strip.

Sketch of bending reinforcement



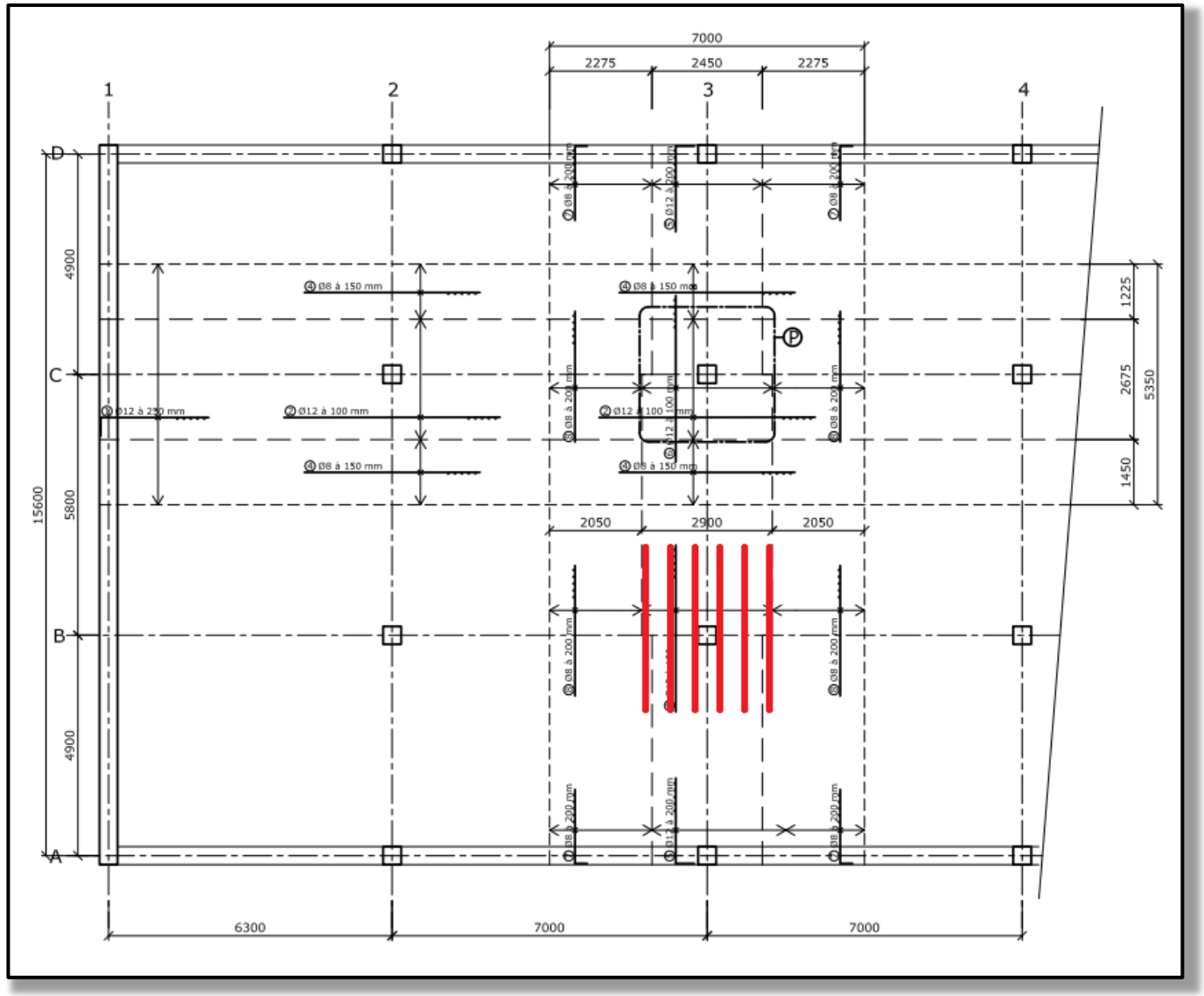
Long dashed lines separate the column and centre strip.

Sketch of bending reinforcement



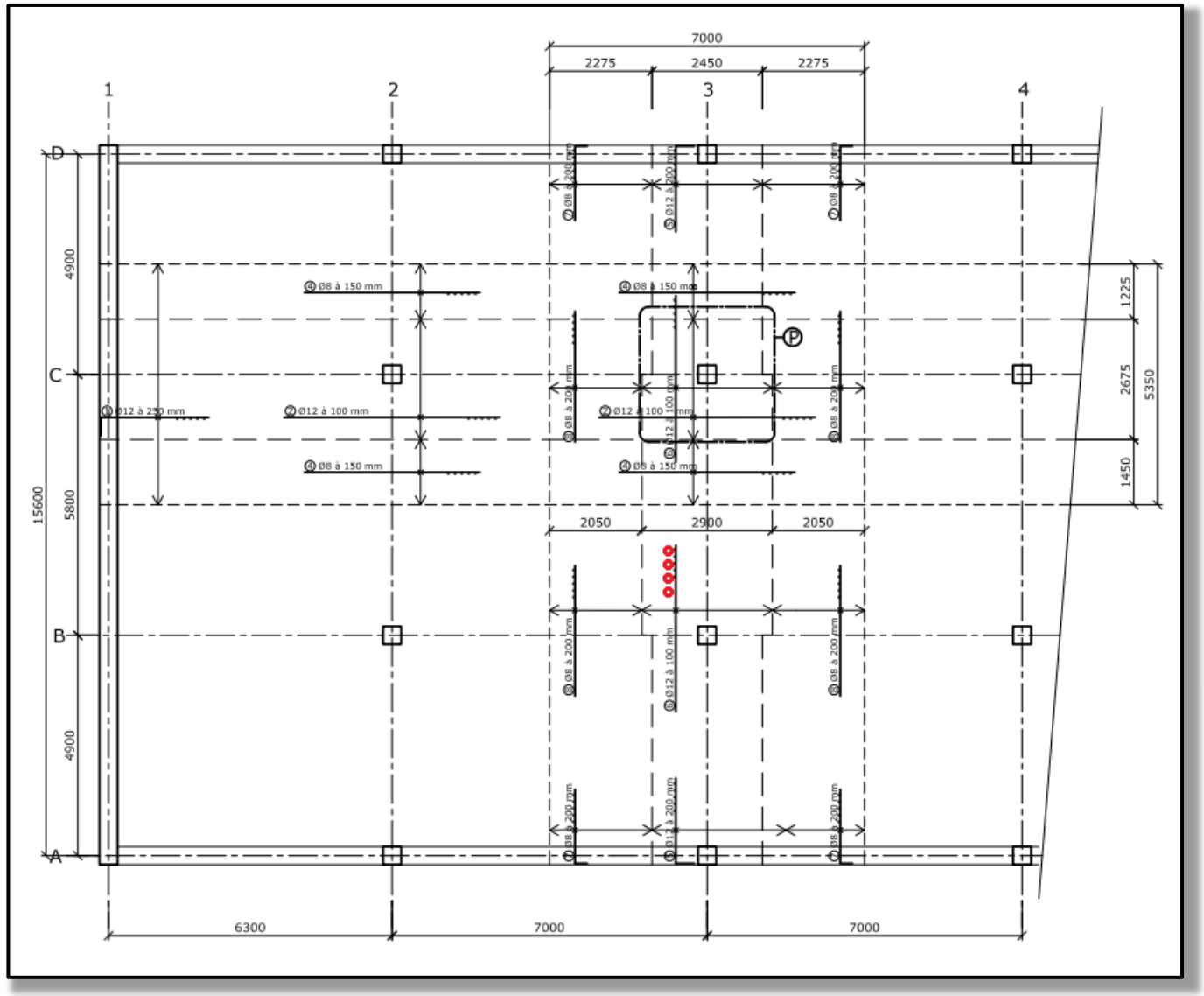
The thick lines indicate the rebars, and the dimensions mark the widths over which the rebars of the same type are spread.

Sketch of bending reinforcement



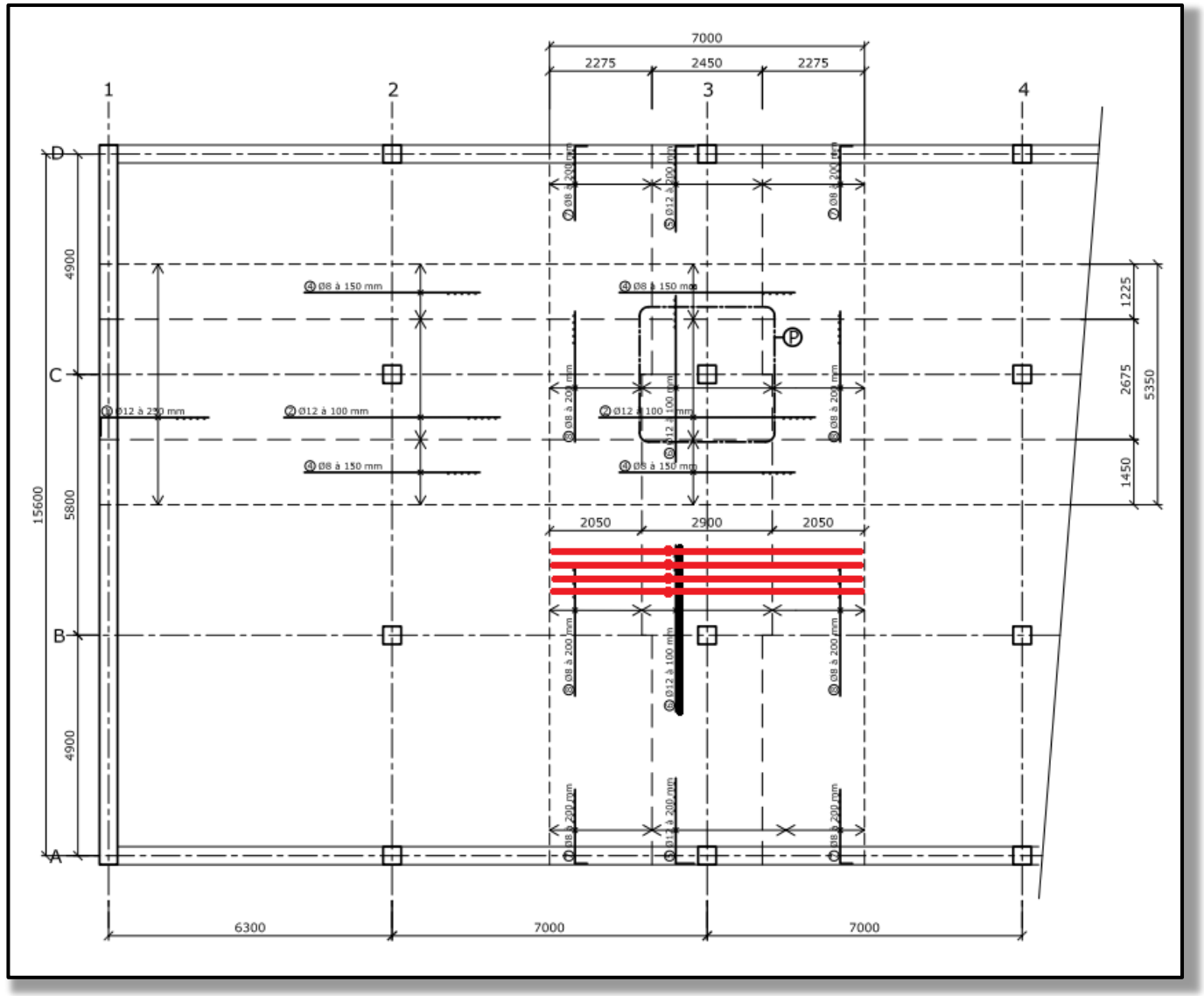
We do not draw each individual rebars!

Sketch of bending reinforcement



For each member, draw circles to indicate the location of the transverse reinforcement in the other direction.

Sketch of bending reinforcement



For each member, draw circles to indicate the location of the transverse reinforcement in the other direction.

Sketch of bending reinforcement

The sketch should contain the **designed bending reinforcement**. The drawing will not include any additional structural reinforcement. Choose the scale so that the drawing fits on **A3 format**.

The drawing should include:

- **floor plan** with rebar locations,
- simplified **reinforcement list** with the shapes of the rebars,
- overview of **materials**, **anchor lengths**, and **concrete cover**,
- drawing **title**.

(see sample on the web)

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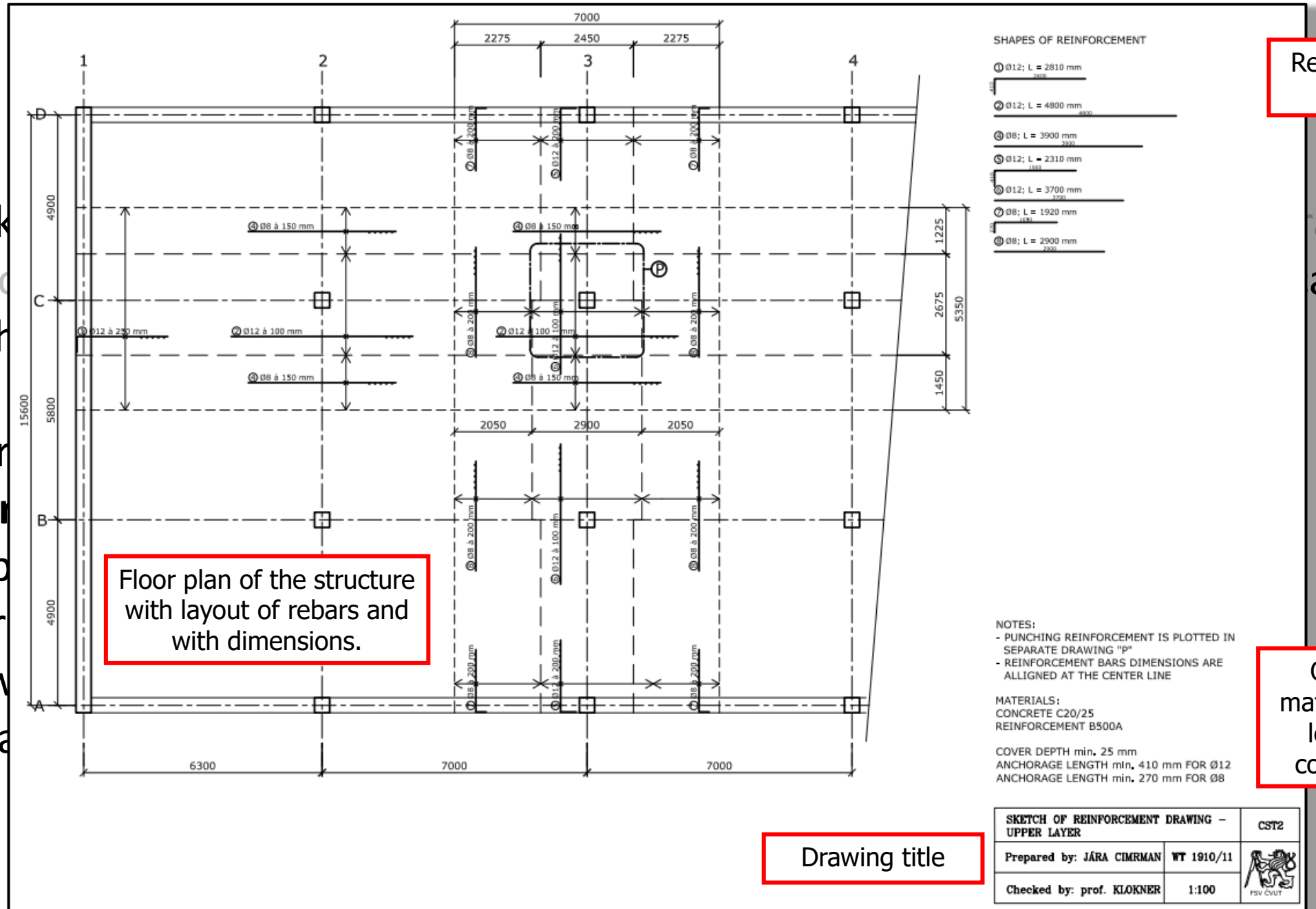

Reinforcement list

Floor plan of the structure with layout of rebars and with dimensions.

Overview of materials, anchor lengths, and concrete cover

Drawing title

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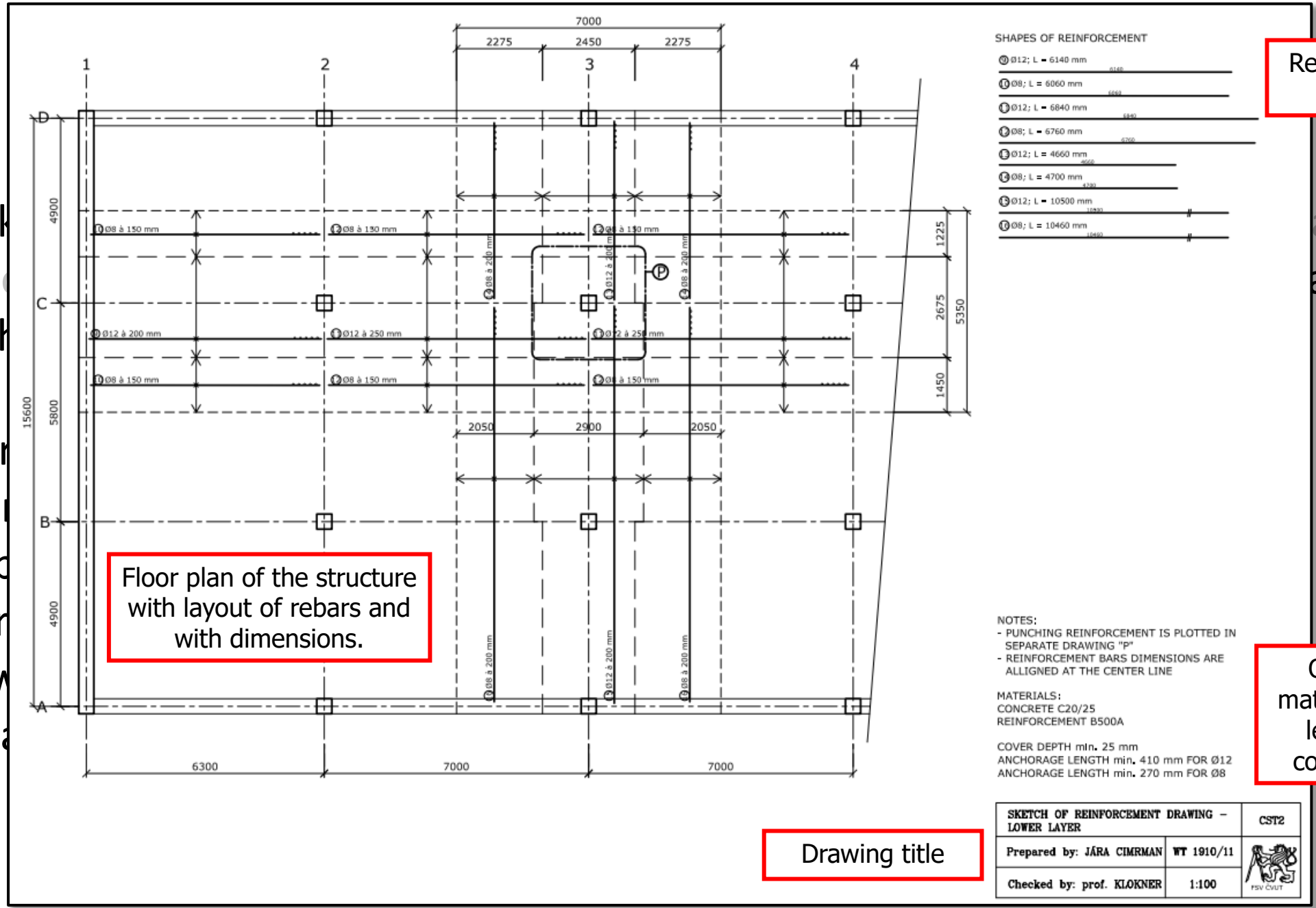


Floor plan of the structure with layout of rebars and with dimensions.

Reinforcement list

Overview of materials, anchor lengths, and concrete cover

Drawing title

5) Design of punching reinforcement

Task 3 – Assignment goals

1) Design of the dimensions of the load-bearing elements and sketch the structure.
2) Preliminarily check punching.
3) Calculate bending moments in lanes C and 3.
4) Design slab bending reinforcements (rebars) for the calculated moments.
5) Draw a sketch of the layout of the bending reinforcement.
6) **Design the punching reinforcement for column C3.**
7) Draw a sketch of the layout of the punching reinforcement.

Types of punching failure

There are **two types of punching failure which can occur** in the slab.

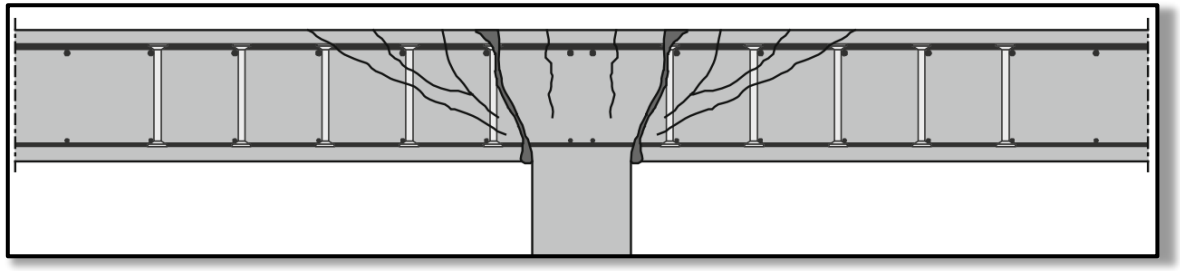
Failure of compressed concrete

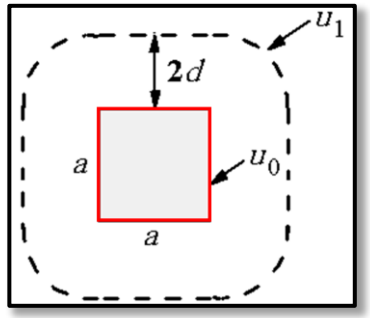
Failure of punching reinforcement

Types of punching failure

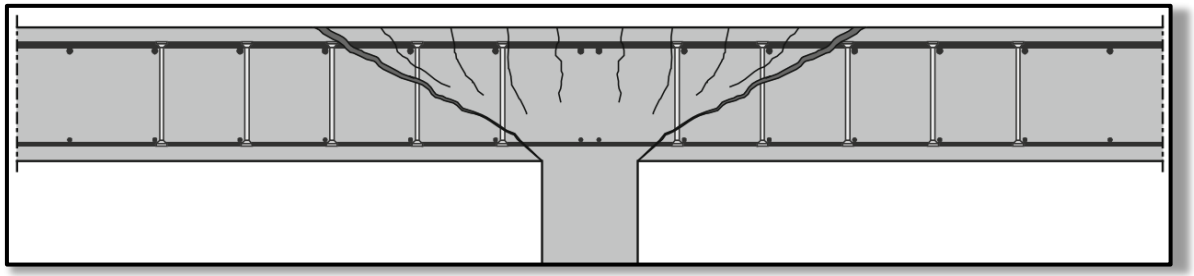
There are **two types of punching failure which can occur** in the slab.

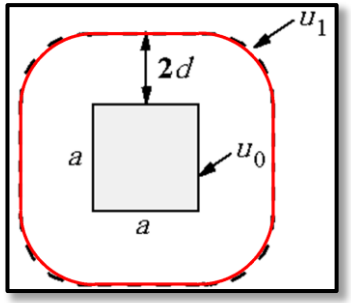
Failure at u_0 perimeter





Failure at u_1 perimeter





Failure at u_0 perimeter

In perimeter u_0, the shear force induces high compressive stress in concrete

We must check whether the **load-bearing capacity of compressed concrete** (i.e., the "*Maximum punching shear resistance*") is sufficient:

$$v_{\mathrm{Ed},0} = \frac{\beta V_{\mathrm{Ed}}}{u_0 d} \leq v_{\mathrm{Rd,max}} = 0.4 \nu f_{\mathrm{cd}}$$

We have already checked this in the preliminary design!

Failure at u_1 perimeter

In perimeter u_1, the shear force induces high tensile stress in concrete.

We must check whether the **slab is able to carry the load without punching reinforcement**:

$$v_{Ed,1} \leq v_{Rd,c}.$$

If the condition above is not satisfied, we must **design punching reinforcement** and check whether the **slab is able to carry the load with punching reinforcement**:

$$v_{Ed,1} \leq v_{Rd,cs}.$$

Load-bearing capacity without reinforcement

The load-bearing capacity in punching of the **slab <u>without</u> punching reinforcement** is assessed using equation:

$$v_{\mathrm{Ed},1} = \frac{\beta V_{\mathrm{Ed}}}{u_1 d} \le v_{\mathrm{Rd,c}} = \max\left[C_{\mathrm{Rd,c}} \cdot k \cdot \sqrt[3]{(100\rho_l \cdot f_{\mathrm{ck}})};\quad 0{,}035\sqrt{k^3 f_{\mathrm{ck}}} \right]$$

If you used steel caps, use u_1 calculated with the cap.

$$d = \frac{d_C + d_3}{2}$$

Reduction factor (0.12)

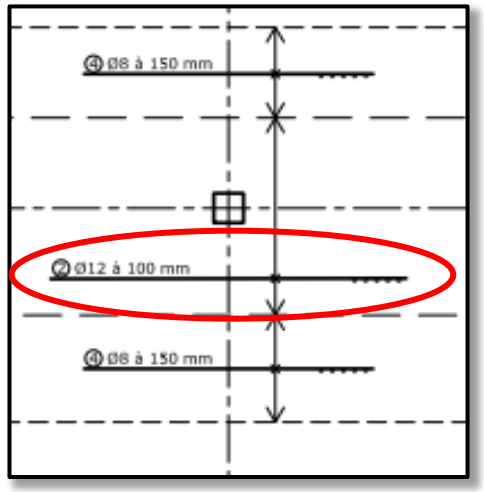
Effect of depth: $k = 1 + \sqrt{\frac{200}{d}} \le 2{,}0$

Reinforcement ratio for the bending reinforcement

$$\rho_l = \sqrt{\rho_{lC} \cdot \rho_{l3}} \le 0{,}02$$

$$\rho_{lC} = \frac{a_{\mathrm{sC}}}{1000 d_C},\ \rho_{l3} = \frac{a_{\mathrm{s3}}}{1000 d_3}$$

a_{sC} $[\mathrm{mm}^2/\mathrm{m}]$ is the cross-sectional area of the upper reinforcement per 1 m width of the slab in lane C.



Load-bearing capacity without reinforcement

If the condition

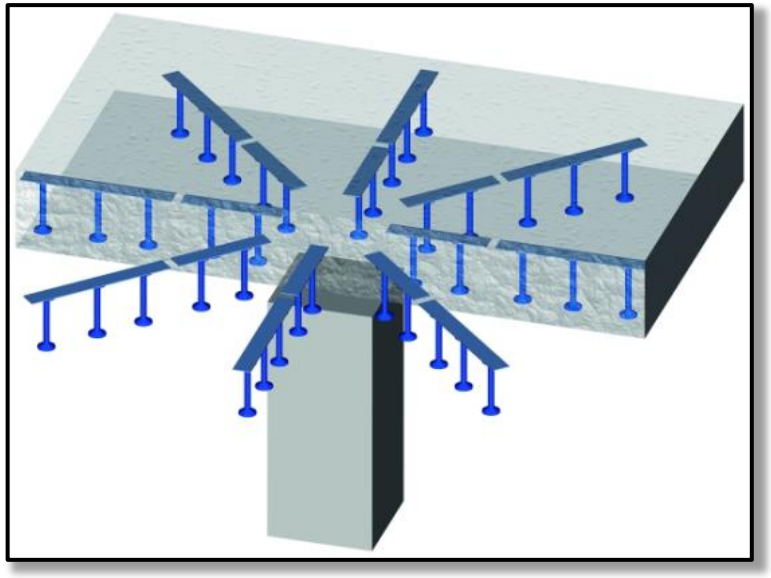
$$v_{\mathrm{Ed},1} = \frac{\beta V_{\mathrm{Ed}}}{u_1 d} \leq v_{\mathrm{Rd,c}} = \max\left[C_{\mathrm{Rd,c}} \cdot k \cdot \sqrt[3]{(100 \rho_l \cdot f_{\mathrm{ck}})};\quad 0{,}035 \sqrt{k^3 f_{\mathrm{ck}}} \right]$$

is satisfied, then no punching reinforcement is needed.

If the **condition is not satisfied**, we **must design punching reinforcement**.

Design of punching reinforcement

We will use **double-headed studs connected by a spacer rail** placed **radially** as a punching reinforcement.



We need to design the **number of rails**, **number of studs** in each rail, and **spacing of the studs**.

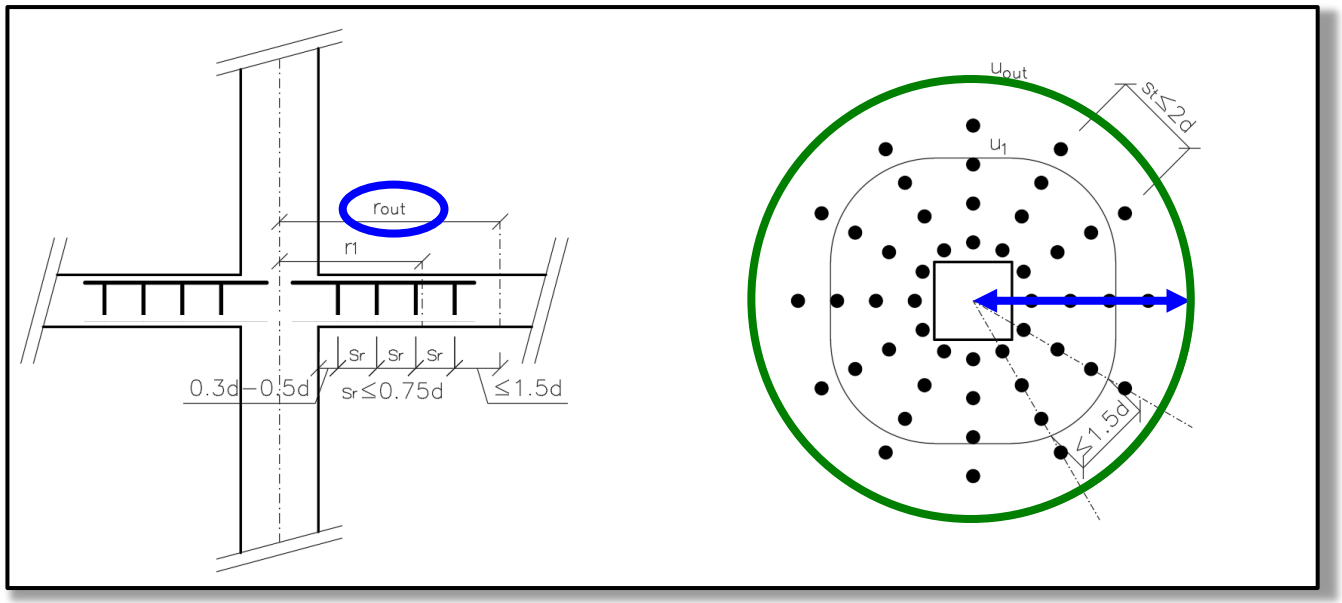
Design of punching reinforcement

When we design the punching reinforcement, we first determine the length of the controlled perimeter in which shear reinforcement is no longer required (i.e., the perimeter where $v_{Ed,1} = v_{Rd,c}$)

$$u_{out} = \frac{\beta V_{Ed}}{v_{Rd,c} d},$$

and its diameter

$$r_{out} = \frac{u_{out}}{2\pi}.$$



Design of punching reinforcement

The number of rails and studs must be determined form the following rules:

- **first stud** must be less than $(0.3–0.5)d$ from the face of the column,
- **last stud** must be less than $1.5d$ from u_{out},
- **radial spacing** of studs must be $s_r \leq 0.75d$,
- **tangential spacing** of **last studs** must be $s_t \leq 2d$,
- **tangential spacing** of **studs in perimeter $\boldsymbol{u_1}$** must be less than $1.5d$.

Design of punching reinforcement

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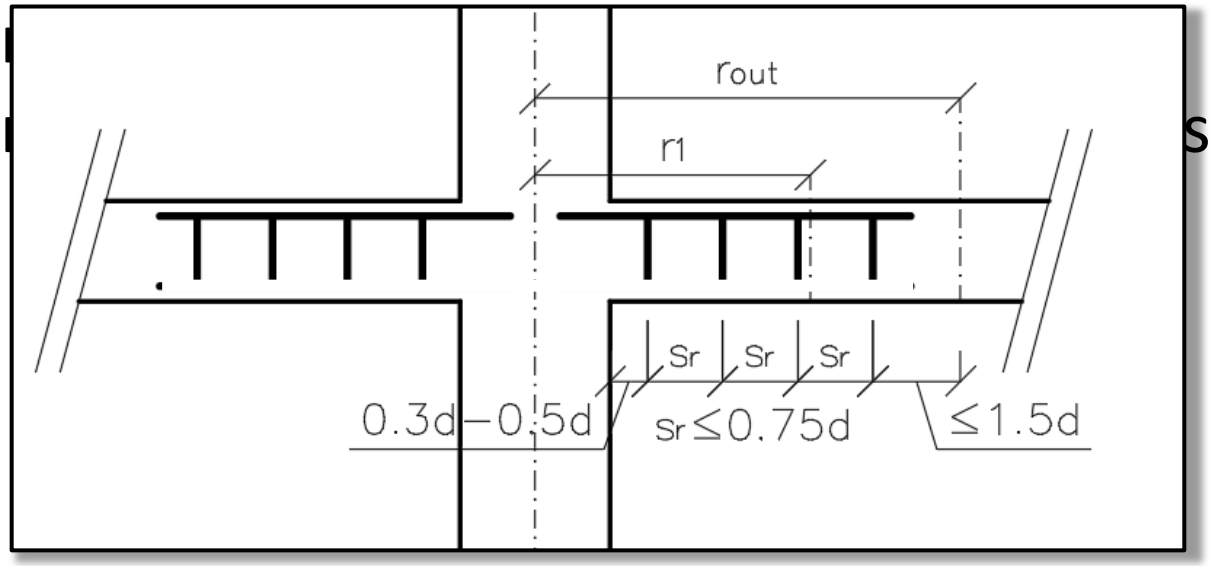
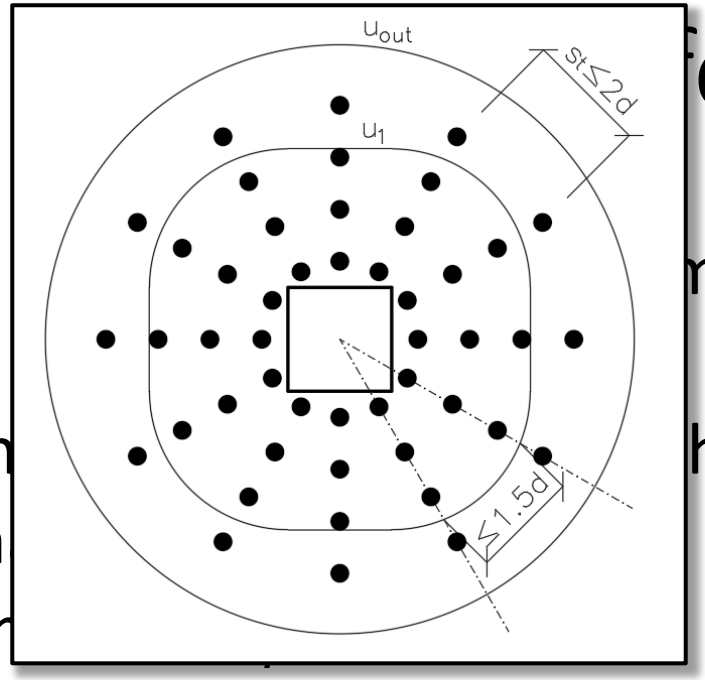


Design of [illegible]forcement



The number of rails and [illegible]mined form the following rules:

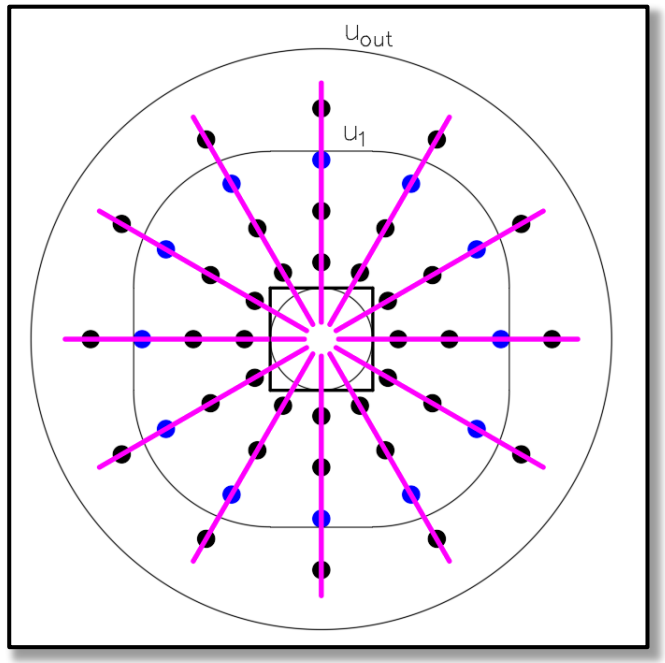
- first stud must be less th[illegible]he face of the column,
- last stud must be less th[illegible]
- radial spacing of studs m[illegible]
- **tangential spacing** of **last studs** must be $s_t \leq 2d$,
- **tangential spacing** of **studs in perimeter u_1** must be less than $1.5d$.

Design of punching reinforcement

The number of bars can be calculated from the rules for maximum distances in the tangential direction

$$n \geq \max\left(\frac{2\pi(r_{out} - 1.5d)}{2d}; \frac{u_1}{1.5d}\right)$$

If you used steel caps, use u_1 calculated with the cap.



The cross-sectional area of the reinforcement profiles **in one circumference** is then

$$A_{sw} = n(\pi \varnothing_{sw}^2/4),$$

where the profile of the studs ($\varnothing_{sw}$) is 10 to 14 mm.

Load-bearing capacity with reinforcement

Finally, we can assess the **load-bearing capacity in punching of the slab <u>with</u> punching reinforcement** using equation

$$v_{Rd,cs} = \min\left(0.75 v_{Rd,c} + 1.5\frac{A_{sw} f_{yd}}{s_r u_1}; k_{max} v_{Rd,c}\right).$$

All of the parameters have been calculated or designed above (k_{max} was determined in the previous HW).

Assessment of punching resistance

The assessment of punching is carried out by comparing the load effects with the load-bearing capacity

$$v_{Ed,1} = \frac{\beta V_{Ed}}{u_1 d} \leq v_{Rd,cs}.$$

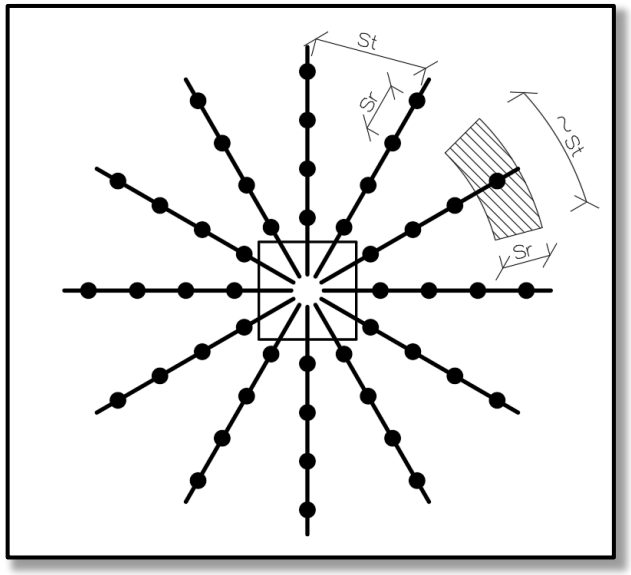
If the **condition is not satisfied**, we must redesign the punching reinforcement – i.e., **increase stud diameter** or **number of rails**.

Detailing rules

Finally, we must check the detailing rules punching – i.e, the punching reinforcement ratio

$$\rho_{sw} \geq \rho_{sw,min}$$

$$1.5\frac{\pi \varnothing_{sw}^2/4}{s_r s_t} \geq 0.08\frac{\sqrt{f_{ck}}}{f_{yk}}$$



If the **condition is not satisfied**, we must redesign the punching reinforcement – i.e., increase **stud diameter** or increase **number of rails** or reduce the **radial spacing** of the studs.

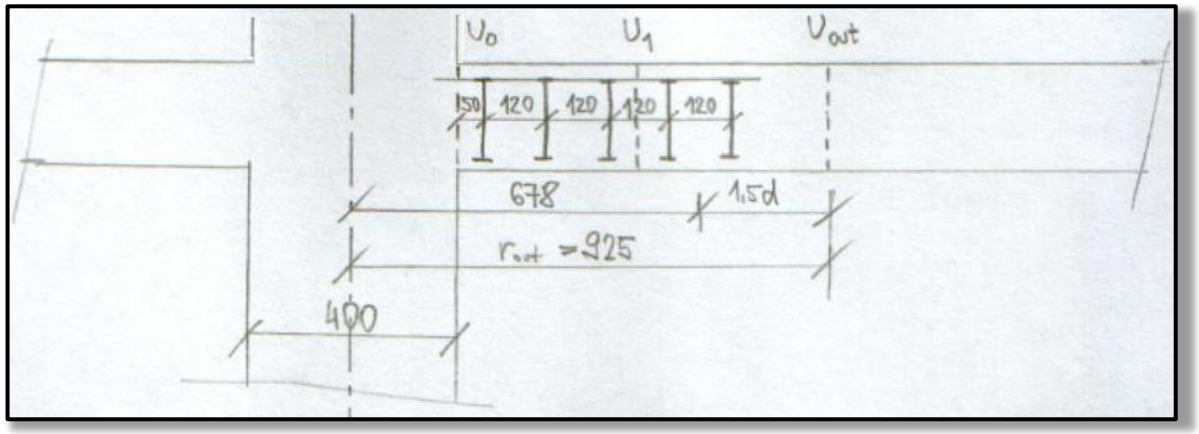
6) Drawing of the punching reinforcement

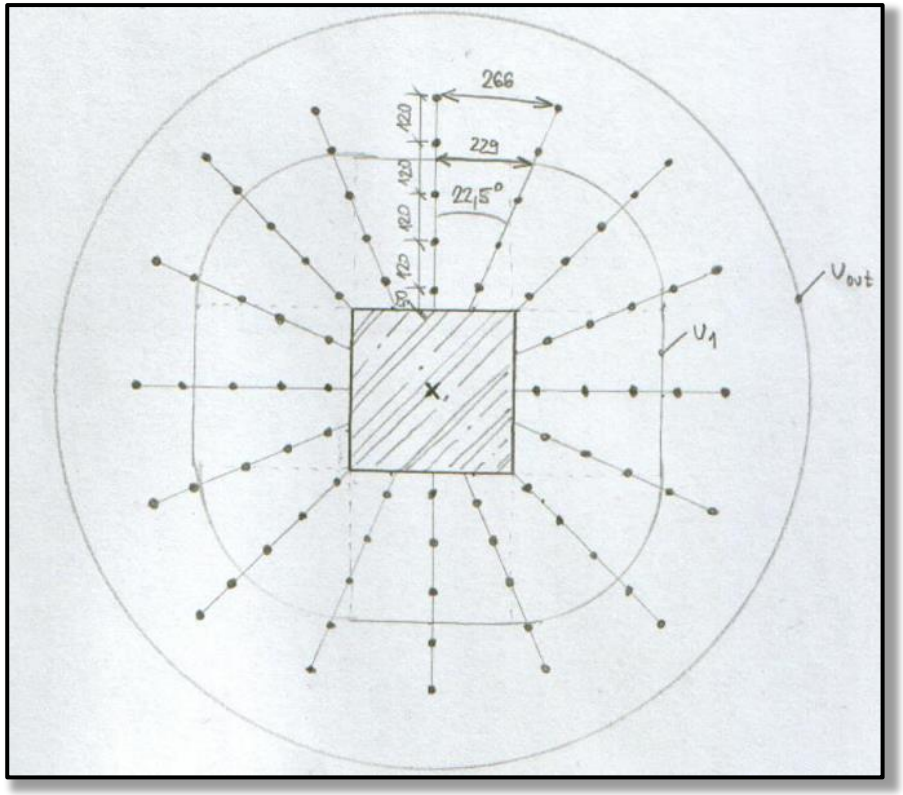
Task 3 – Assignment goals

1) Design of the dimensions of the load-bearing elements and sketch the structure.
2) Preliminarily check punching.
3) Calculate bending moments in lanes C and 3.
4) Design slab bending reinforcements (rebars) for the calculated moments.
5) Draw a sketch of the layout of the bending reinforcement.
6) Design the punching reinforcement for column C3.
7) **Draw a sketch of the layout of the punching reinforcement.**

Drawing of the punching reinforcement

We will draw the punching reinforcement with all relevant dimensions and notes. Use scale of 1:10 (or 1:20).





thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.