CM01 – Concrete and Masonry Structures 1

HW3 – Design of beam reinforcement

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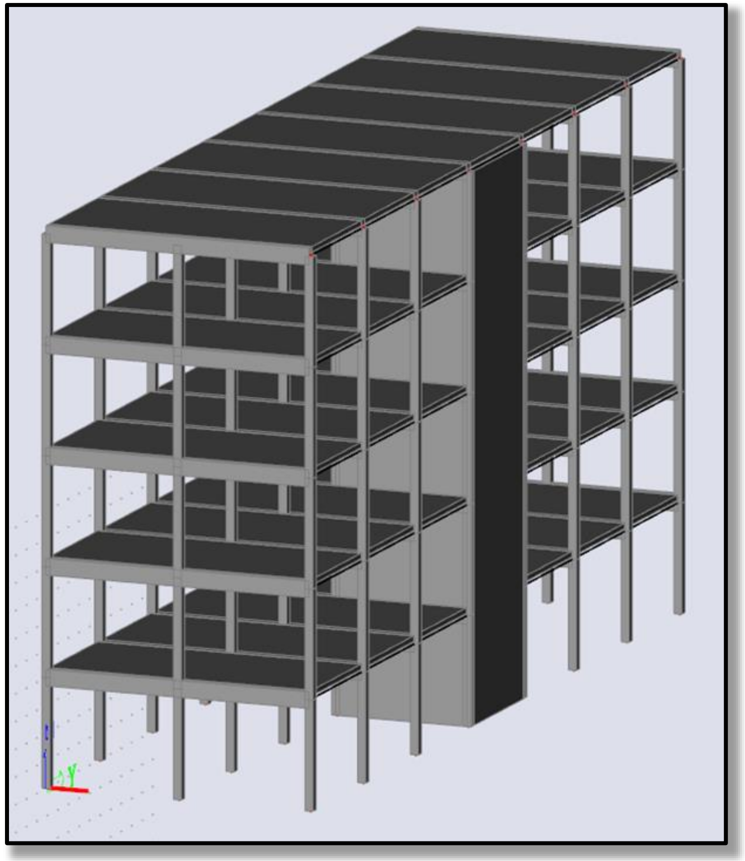
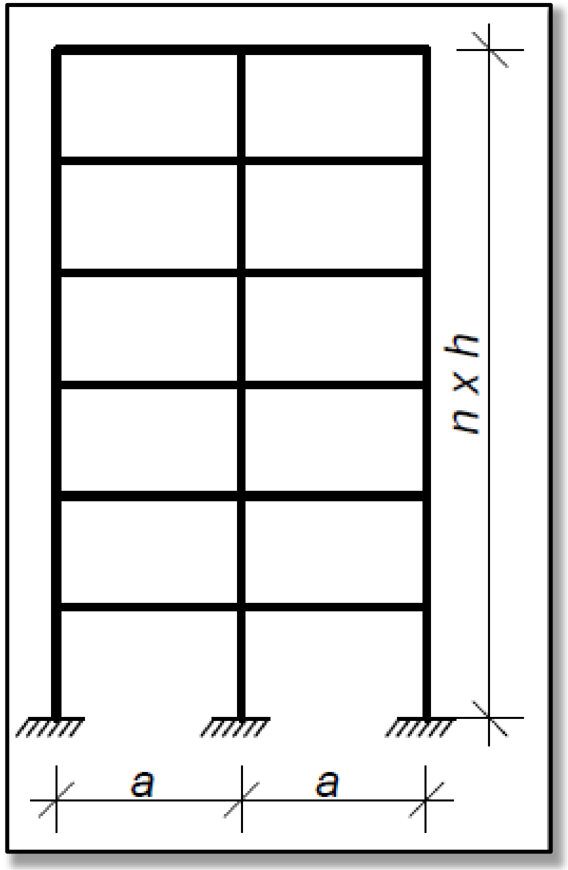
Task 1

Task 1 – Frame structure

In Task 1, frame structure will be designed.





Task 1 – Assignment

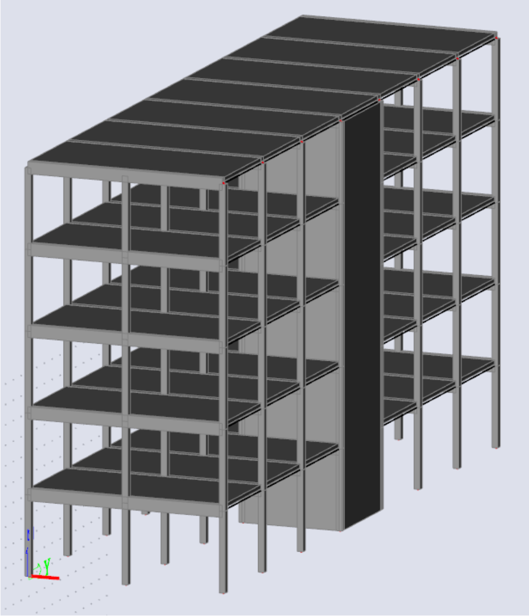
Geometry: ***R*, *a*** [m] – horizontal dimensions, ***h*** [m] – floor height, ***n*** – number of floors

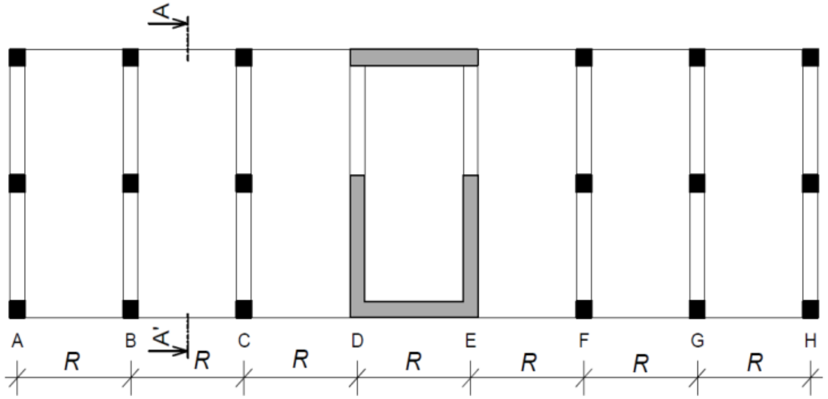
Materials: Concrete – **concrete class**
Steel B 500 B (f_{yk} = 500 MPa)

Loads:
Other permanent load of typical floor $\mathbf{(g\text{-}g_0)_{floor,k}}$ [kN/m^2]
Other permanent load of the roof $\mathbf{(g\text{-}g_0)_{roof,k}}$ [kN/m^2]
Live load of typical floor $\mathbf{q_{floor,k}}$ [kN/m^2]
Live load of the roof $q_{roof,k}$ = 0,75 kN/m^2
Self-weight of the slab $g_{0,k}$ (calculate from the slab depth)

Another parameters: ***S*** – Exposure class related to environmental conditions
Z – Working life of the structure

Parameters in bold are individual parameters, which you can find on the course website.





Your individual parameters:
https://docs.google.com/spreadsheets/d/1uQIuyyKEcG5jaZVLrsmm1ZRRNib_ow3MIwgZSEDgnW8/

Task 1 – Assignment goals

Our goal will be to:

- Design the dimensions of all elements.
- Do detailed calculation of 2D frame – calculation of bending moments, shear and normal forces using FEM software.
- **Design steel reinforcement in the 1st floor members**:
  - **beam**,
  - column.
- Draw layout of the reinforcement.

Design of beam reinforcement

Design of beam reinforcement

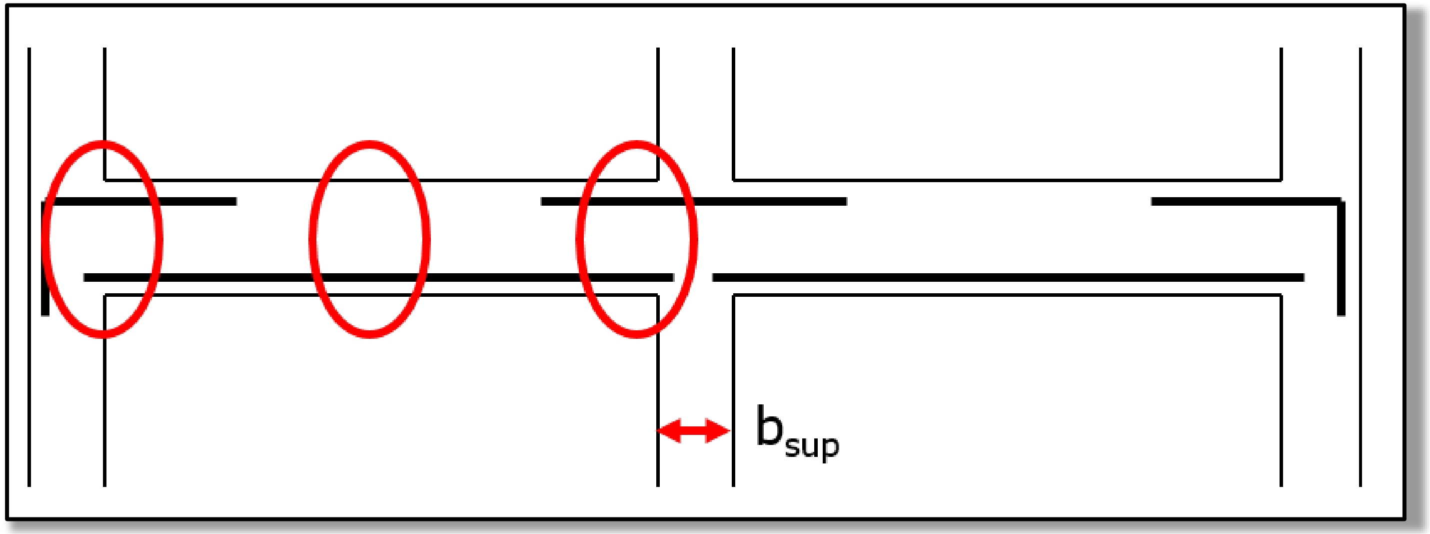
Using the maximal values of internal forces from the „envelope“ of internal forces, we will design and assess:

- **bending reinforcement** of the beam,
- **shear reinforcement** of the beam.

Design of bending reinforcement

Design of bending reinforcement

Design the tensile bending reinforcement in 3 cross-sections:

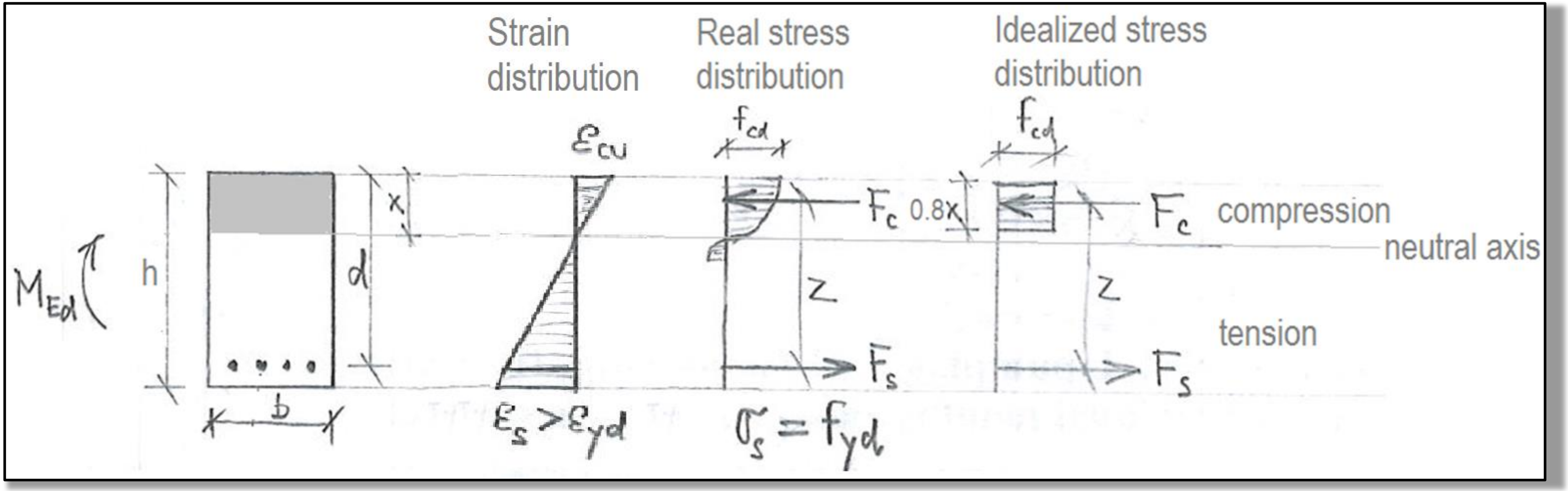


In supports, maximal values from FEM calculation should be reduced to values in the face of the column:

$$\left|M_{\text{Ed,red}}\right| = \left|M_{\text{Ed,FEM}}\right| - \left|V_{\text{Ed,FEM}}\right| \frac{b_{\text{sup}}}{2}$$

Bending

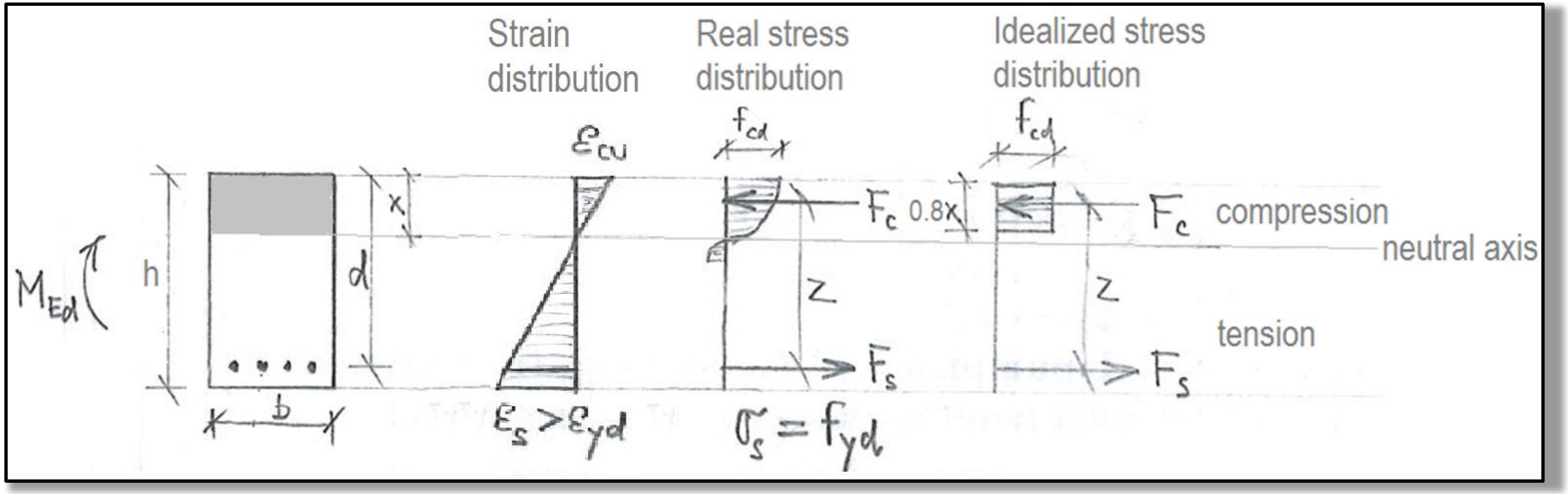
Strain and stress distribution in ultimate limit state (ULS):



Design of bending reinforcement

When designing the reinforcement, we employ the **limit-moment assumption** which means *“assume that the load-bearing capacity will be equal to the bending moment”*:

$$M_{Rd} = M_{Ed}$$



Design of bending reinforcement

Derivation of **required reinforcement area**:

$$M_{Rd} = M_{Ed}$$

$M_{Ed,red}$ in supports; $M_{Ed,FEM}$ in midspan

$$F_s z = M_{Ed}$$

$$A_{s,rqd} f_{yd} z = M_{Ed}$$

$$A_{s,rqd} = \frac{M_{Ed}}{z f_{yd}} = \frac{M_{Ed}}{0.9 d_B f_{yd}} \Rightarrow \boxed{\text{Propose } A_{s,prov} \geq A_{s,rqd}}$$

Effective height of beam: $d_B = h_B - \frac{\varnothing}{2} - \varnothing_{sw} - c$

Stirrup diameter (assume 8 mm)

Bending reinforcement bar diameter - design 16 to 25 mm (more only if necessary)

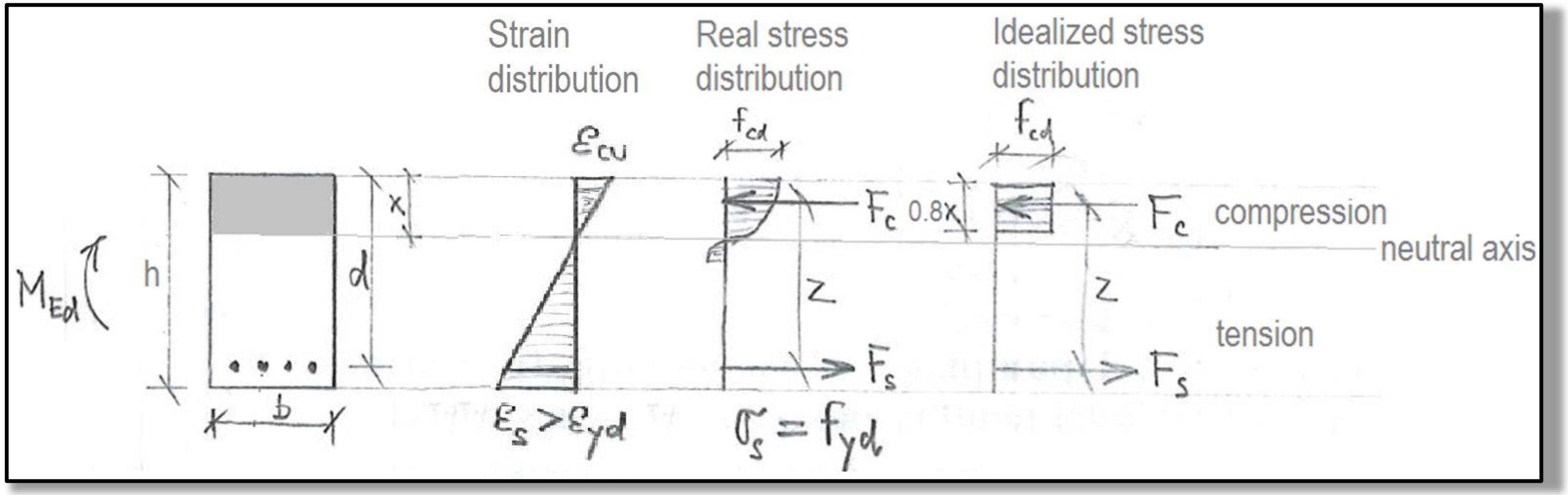
Design **number and diameter of bars**:

Example: **DESIGN**: 3x Ø16 ($A_{s,prov}$ = 603 mm^2)

Assessment of bending reinforcement

For one-side reinforced beam in pure bending, the **partial internal normal forces are equal**:

$$F_c = F_s$$



Assessment of bending reinforcement

Derivation of **compressive height**:

$$F_c = F_s$$

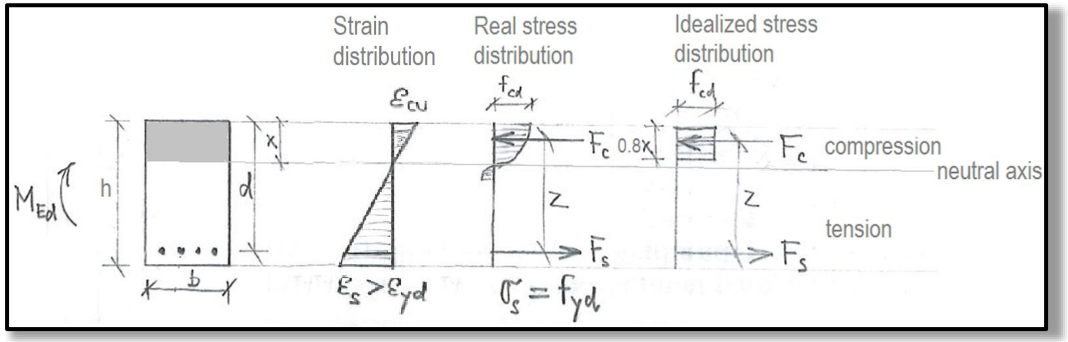
$$A_\mathrm{c} f_\mathrm{cd} = A_\mathrm{s} f_\mathrm{yd}$$

$$0.8xbf_\mathrm{cd} = A_\mathrm{s,prov} f_\mathrm{yd}$$

$$x = \frac{A_\mathrm{s,prov} f_\mathrm{yd}}{0.8bf_\mathrm{cd}}$$

Width of compressed part of the cross-section:

- b_B in supports,
- b_eff in midspan.



Lever arm of internal forces: $z = d_\mathrm{B} - 0.4x$

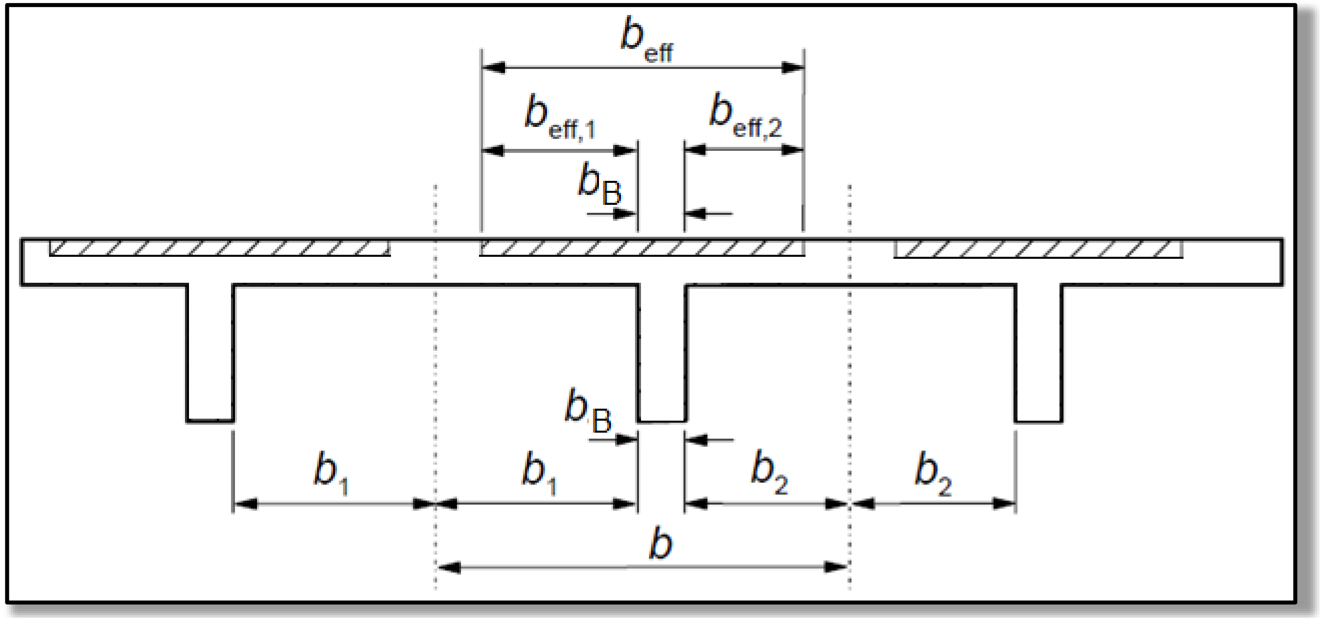
Load-bearing capacity in bending: $\boxed{M_\mathrm{Rd} = A_\mathrm{s,prov} f_\mathrm{yd} z \geq M_\mathrm{Ed}}$

This MUST be satisfied!

Effective width b_{eff}

In mid-span, the slab acts as a part of the beam, and the beam is thus a T-section. The **effective width** is:

$$b_{\mathrm{eff}} = \sum_{\mathrm{i}} b_{\mathrm{eff,i}} + b_{\mathrm{B}} \le b \quad \text{where} \quad b_{\mathrm{eff,i}} = 0.2b_{\mathrm{i}} + 0.1l_0 \le 0.2l_0 \quad \text{and} \quad b_{\mathrm{eff,i}} \le b_{\mathrm{i}}$$



Distance between zero moments on the beam:

- **for outer span of the beam $l_0 \approx 0.85l_B$**
- for inner span of the beam $l_0 \approx 0.7l_B$

Check detailing rules

Relative compressive **height**:

$$\xi = \frac{x}{d_\mathrm{B}} \le \min\left(\xi_{\mathrm{bal},1} = \frac{700}{700 + f_\mathrm{yd}}; 0,45\right)$$

Minimal rebar **area**:

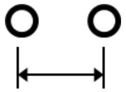
$$A_\mathrm{s,prov} \ge A_\mathrm{s,min} = \max\left(0.26\frac{f_\mathrm{ctm}}{f_\mathrm{yk}} b_\mathrm{B} d_\mathrm{B}; 0.0013 b_\mathrm{B} d_\mathrm{B}\right)$$

Maximal rebar **area**:

$$A_\mathrm{s,prov} \le A_\mathrm{s,max} = 0.04 b_\mathrm{B} d_\mathrm{B}$$

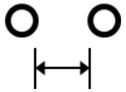
Maximal axial **spacing** of rebars:

$$s_\mathrm{a} \le s_\mathrm{a,max} = \min\left(2h_\mathrm{B};\ 250\ \mathrm{mm}\right)$$



Minimal clear **spacing** of rebars:

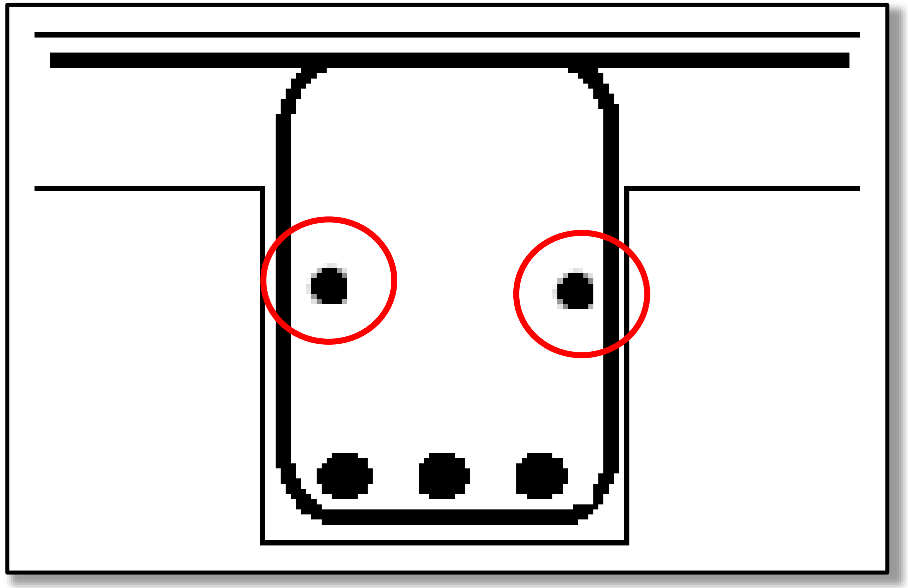
$$s_\mathrm{c} \ge s_\mathrm{c,min} = \max\left(20\ \mathrm{mm};\ 1{,}2\varnothing\right)$$



Mean tensile strength of concrete, see table with properties of concrete classes from 1st class.

Check detailing rules

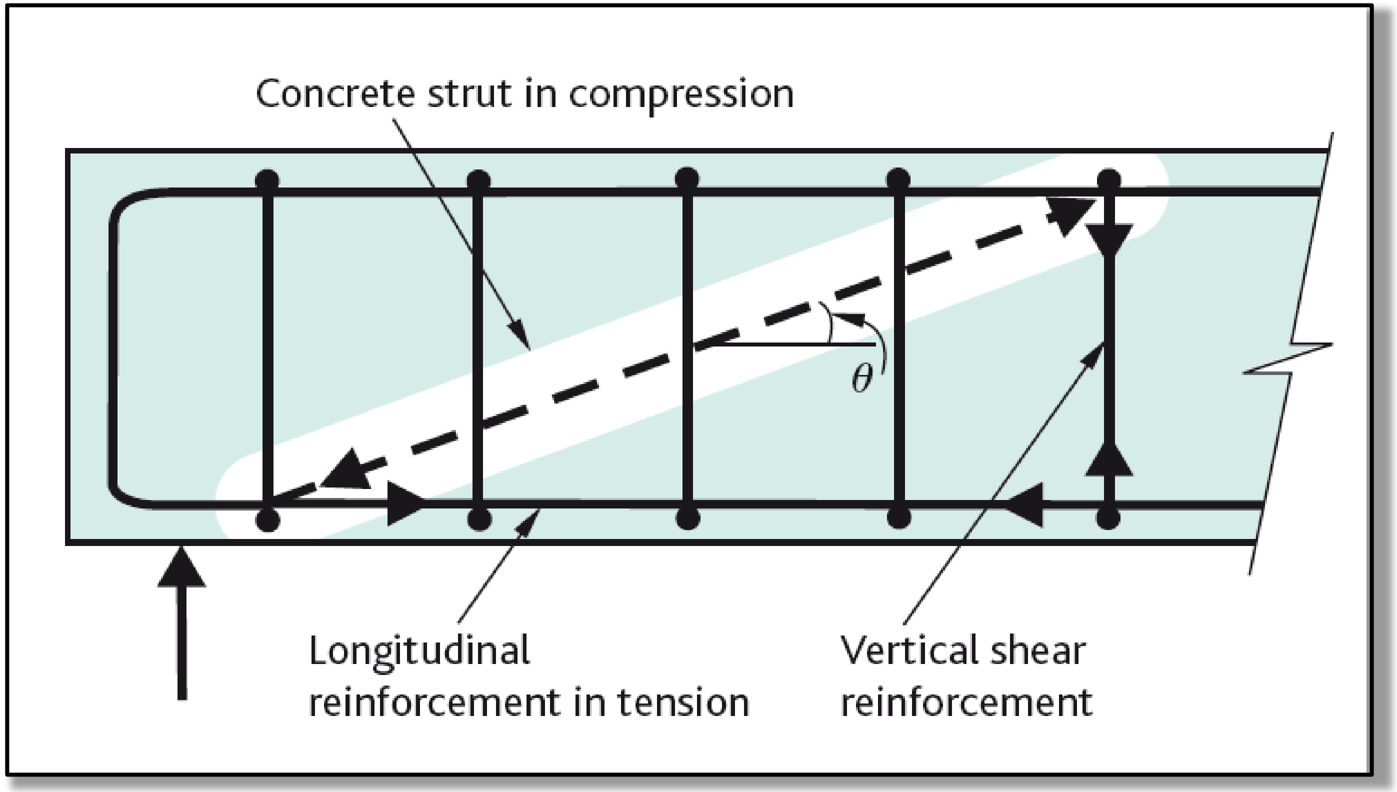
If $h_B \geq 500$ mm, torsion reinforcement is necessary (add two 12 mm rebars to the middle of the beam).



Design of shear reinforcement

Design of shear reinforcement

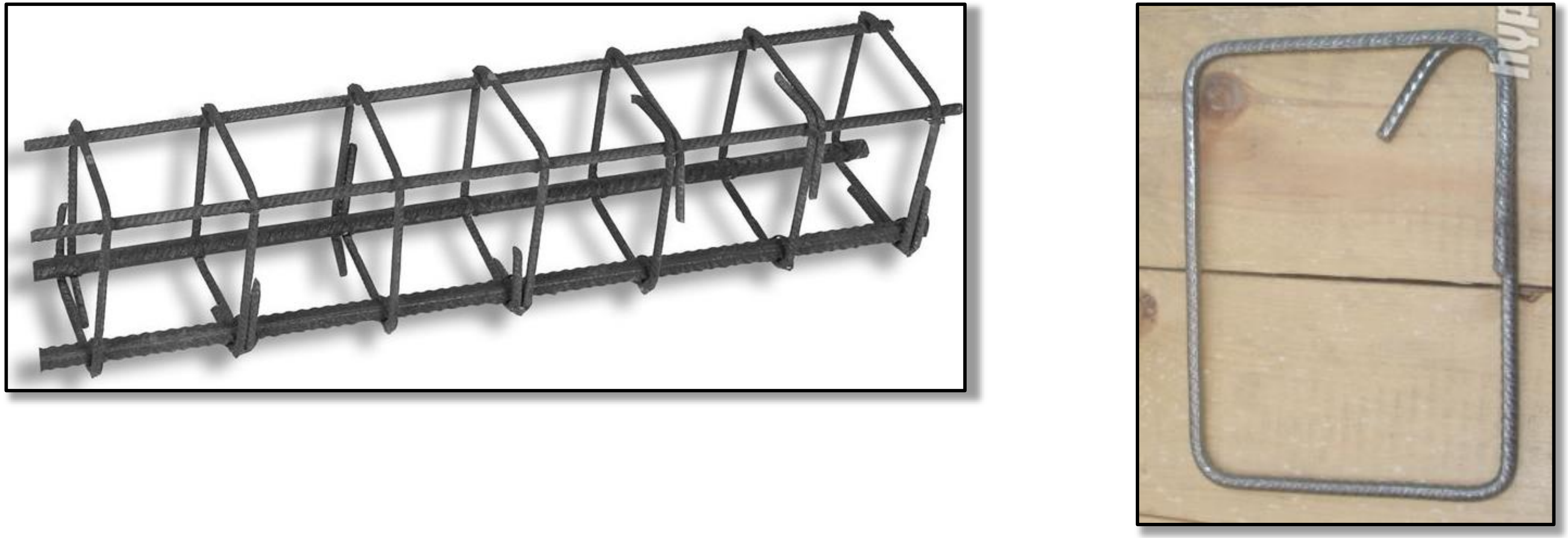
The shear force induces **compression in concrete struts** and **tension in shear reinforcement**.



Design of shear reinforcement

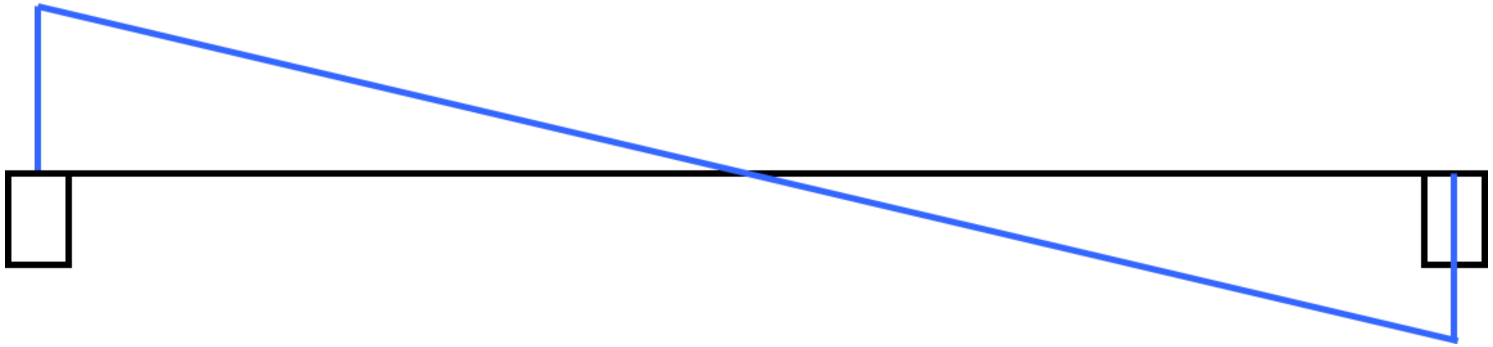
Resistance of compressed concrete struts was already checked in preliminary design ($V_{Rd,max} \geq V_{Ed,max}$).

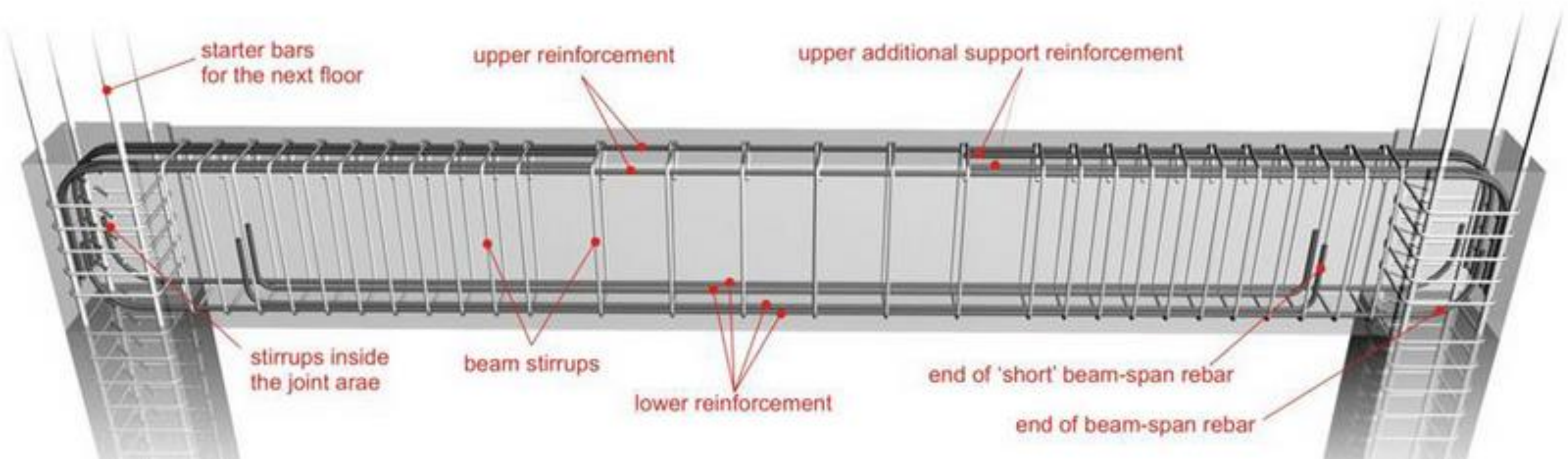
Now, we must design and assess the shear reinforcement (stirrups).



Design of shear reinforcement – principle

The **higher the shear force**, the **denser the stirrups**.



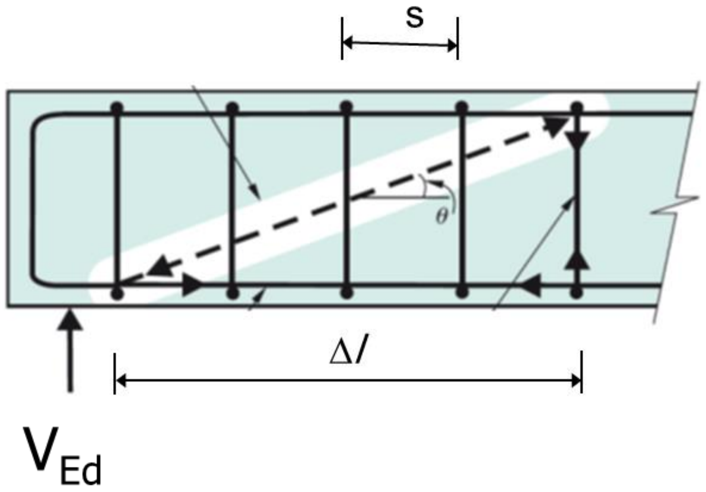


Design of shear reinforcement – principle

The **load-bearing capacity** of stirrups is:

$$V_{Rd} = \boxed{\frac{\Delta l}{s}} A_{sw} f_{yd}$$

Number of stirrups required on the length of Δl.



where:

$A_{sw} f_{yd}$ is the load-bearing capacity of one stirrup,

s is the spacing of stirrups,

Δl is the horizontal projection of the shear crack ($\Delta l = z \cot \theta$).

see HW1

see bending reinforcement

Design of shear reinforcement – principle

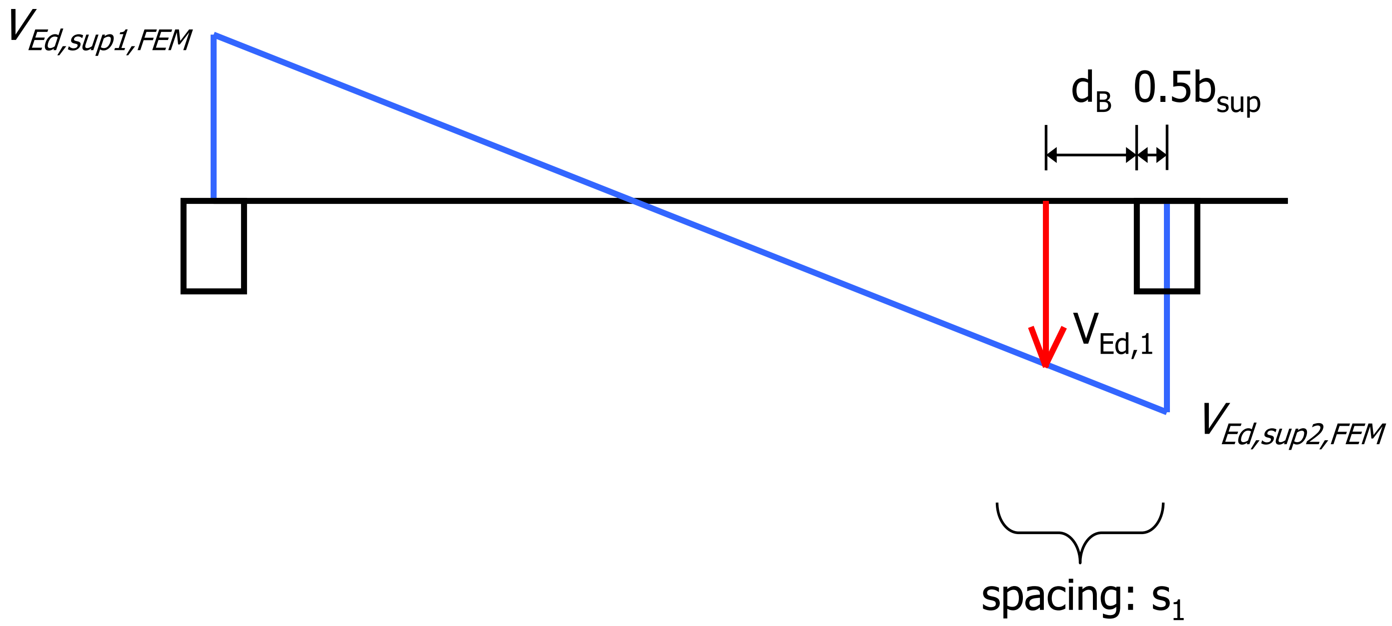
When **designing** the reinforcement, we employ the **limit-force assumption** which means *“assume that the load-bearing capacity will be equal to the shear force”*:

$$V_{Ed} = V_{Rd}$$

$$V_{Ed} = \frac{\Delta l}{s} A_{sw} f_{yd}$$

Design of support shear reinforcement

The stirrups **near the direct support** are designed using the "reduced support shear force $V_{Ed,1}$" in the distance d_B from the face of the column. We will design the stirrups in **spacing s_1 using design force $V_{Ed,1}$**.



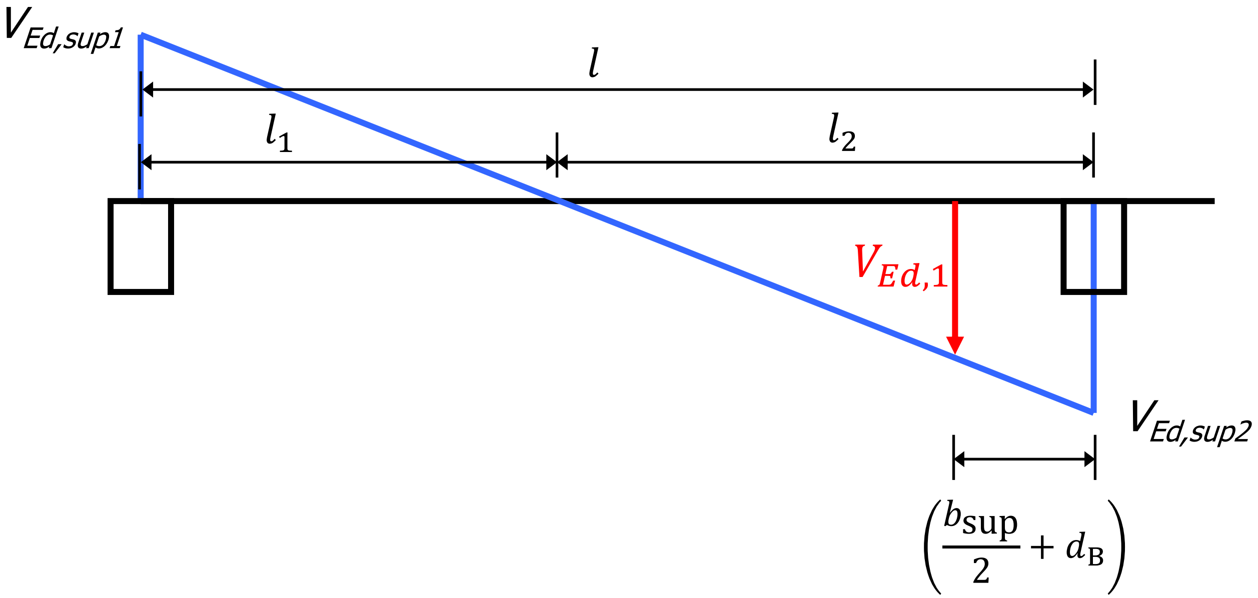
Design of support shear reinforcement

Design shear force (from similar triangles):

$$\left|V_{Ed,1}\right| = \left|V_{Ed,sup2}\right| \frac{l_2 - \left(\frac{b_{\text{sup}}}{2} + d_{\text{B}}\right)}{l_2}$$

$$\frac{V_{Ed,sup1}}{l_1} = \frac{V_{Ed,sup2}}{l_2} = \frac{V_{Ed,sup1} + V_{Ed,sup2}}{l}$$

$$\rightarrow \quad l_2 = \frac{l}{V_{Ed,sup1} + V_{Ed,sup2}} V_{Ed,sup2}$$



Design of support shear reinforcement

Spacing of stirrups:

$$s_1 \le \frac{A_{\text{sw}} f_{\text{yd}}}{V_{\text{Ed,1}}} \Delta l$$

$$\text{and } s_1 \le 0,75 d_{\text{B}}$$

$$\text{and } s_1 \le 400 \text{ mm}$$

$$\text{and } s_1 \ge 100 \text{ mm (recommended)}$$

Cross-sectional area of 1 stirrup:

$$A_{\text{sw}} = \frac{n\pi\varnothing_{\text{sw}}^2}{4}$$

Stirrup diameter (assume 8 mm)

Number of legs of each stirrup (assume n = 2)

$\Delta l = z \cot\theta$

DESIGN: Stirrup $\varnothing_{sw}$ mm per s_1 mm

Assessment of support shear reinforcement

Assess the shear resistance:

$$V_{\mathrm{Rd,sw,1}} = \frac{A_{\mathrm{sw}} f_{\mathrm{yd}}}{s_1} \Delta l \geq V_{\mathrm{Ed,1}}$$

Check the shear reinforcement ratio

$$\rho_{\mathrm{sw,1}} = \frac{A_{\mathrm{sw}}}{b_{\mathrm{B}} s_1} \geq \rho_{\mathrm{sw,min}} = \frac{0,08\sqrt{f_{\mathrm{ck}}}}{f_{\mathrm{yk}}}$$

$$\rho_{\mathrm{sw,1}} = \frac{A_{\mathrm{sw}}}{b_{\mathrm{B}} s_1} \leq \rho_{\mathrm{sw,max}} = \frac{0,5\nu f_{\mathrm{cd}}}{f_{\mathrm{yd}}}$$

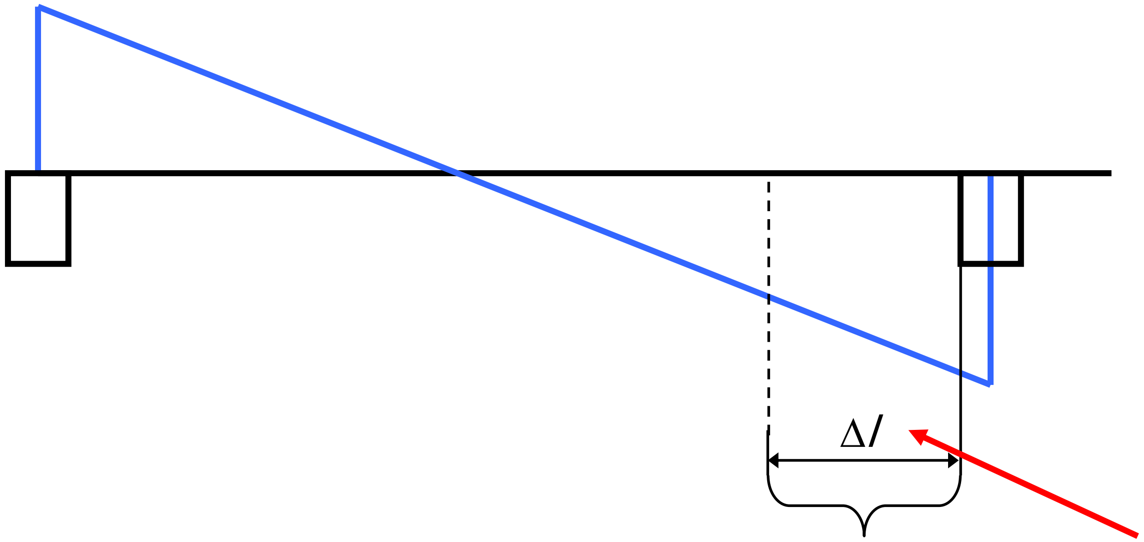
Coefficient expressing effect of shear cracks and transversal deformations:

$$\nu = 0,6\left(1 - \frac{f_{\mathrm{ck}}}{250}\right)$$

If not satisfied, increase ø$_{\mathrm{sw}}$ or decrease s$_1$.

Location of support shear reinforcement

The stirrups with spacing s_1 (i.e., stirrups designed using the “reduced support shear force $V_{Ed,1}$”) must be used in the area close to support – i.e., **at least up to the distance Δl from the support**.



Spacing of stirrups: s_1

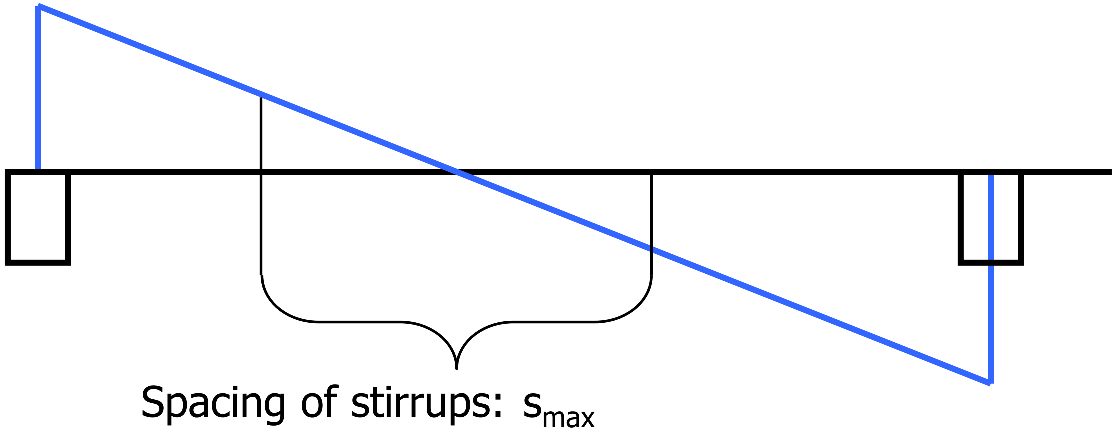
see bending reinforcement

see HW1

$\Delta l = z \cot\theta$

Design of mid-span shear reinforcement

In **middle part** of the beam, shear force is low. Mid-span stirrups will be designed with **maximum possible spacing** s_{max}.



Design the spacing s_{max} according to the condition:

$$s_{\max} \leq \min\left(0,75 d_{\mathrm{B}}; 400\ \mathrm{mm}\right)$$

Check of mid-span shear reinforcement

Check the shear reinforcement ratio:

$$\rho_{\mathrm{sw},2} = \frac{A_{\mathrm{sw}}}{b_{\mathrm{B}} s_{\max}} \geq \rho_{\mathrm{sw,min}} = \frac{0,08\sqrt{f_{\mathrm{ck}}}}{f_{\mathrm{yk}}}$$

$$\rho_{\mathrm{sw},2} = \frac{A_{\mathrm{sw}}}{b_{\mathrm{B}} s_{\max}} \leq \rho_{\mathrm{sw,max}} = \frac{0,5\nu f_{\mathrm{cd}}}{f_{\mathrm{yd}}}$$

If not satisfied, decrease $\boldsymbol{s_{max}}$.

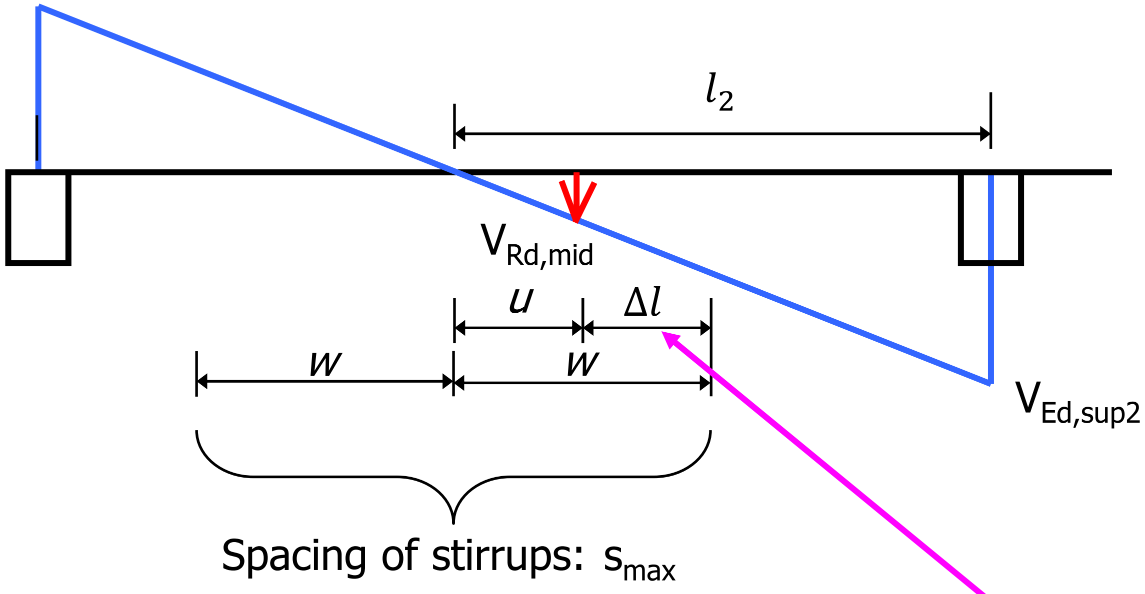
Load-bearing capacity of mid-span shear reinforcement

Load-bearing capacity of s_{max} stirrups:

$$V_{\mathrm{Rd,mid}} = \frac{A_{\mathrm{sw}}\, f_{\mathrm{yd}}}{s_{\mathrm{max}}} \Delta l$$

Location of mid-span shear reinforcement

Position of $V_{Ed} = V_{Rd,mid}$ (from similar triangles) and **length of the regions reinforced by mid-span stirrups**.



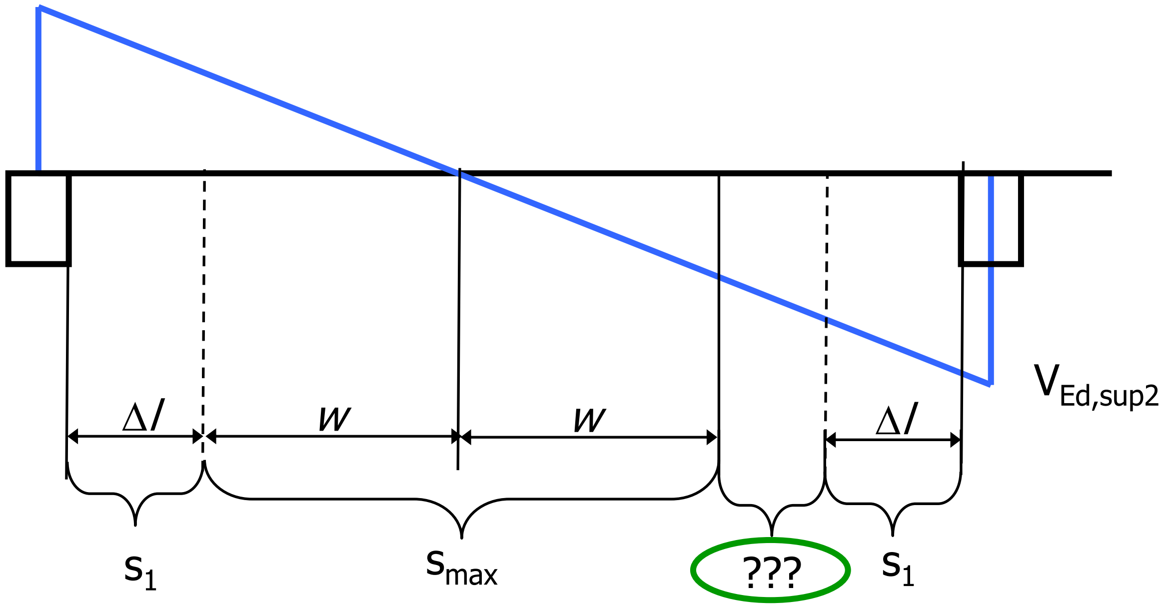
$$\frac{V_{Ed,sup2}}{l_2} = \frac{V_{Rd,min}}{u}$$

$$\rightarrow \quad \boldsymbol{u = \frac{l_2}{V_{Ed,sup2}} V_{Rd,min}}$$

$\Delta l = z \cot\theta$

Design of intermediate shear reinforcement

What spacing should we use in the intermediate part?



Spacing of stirrups:

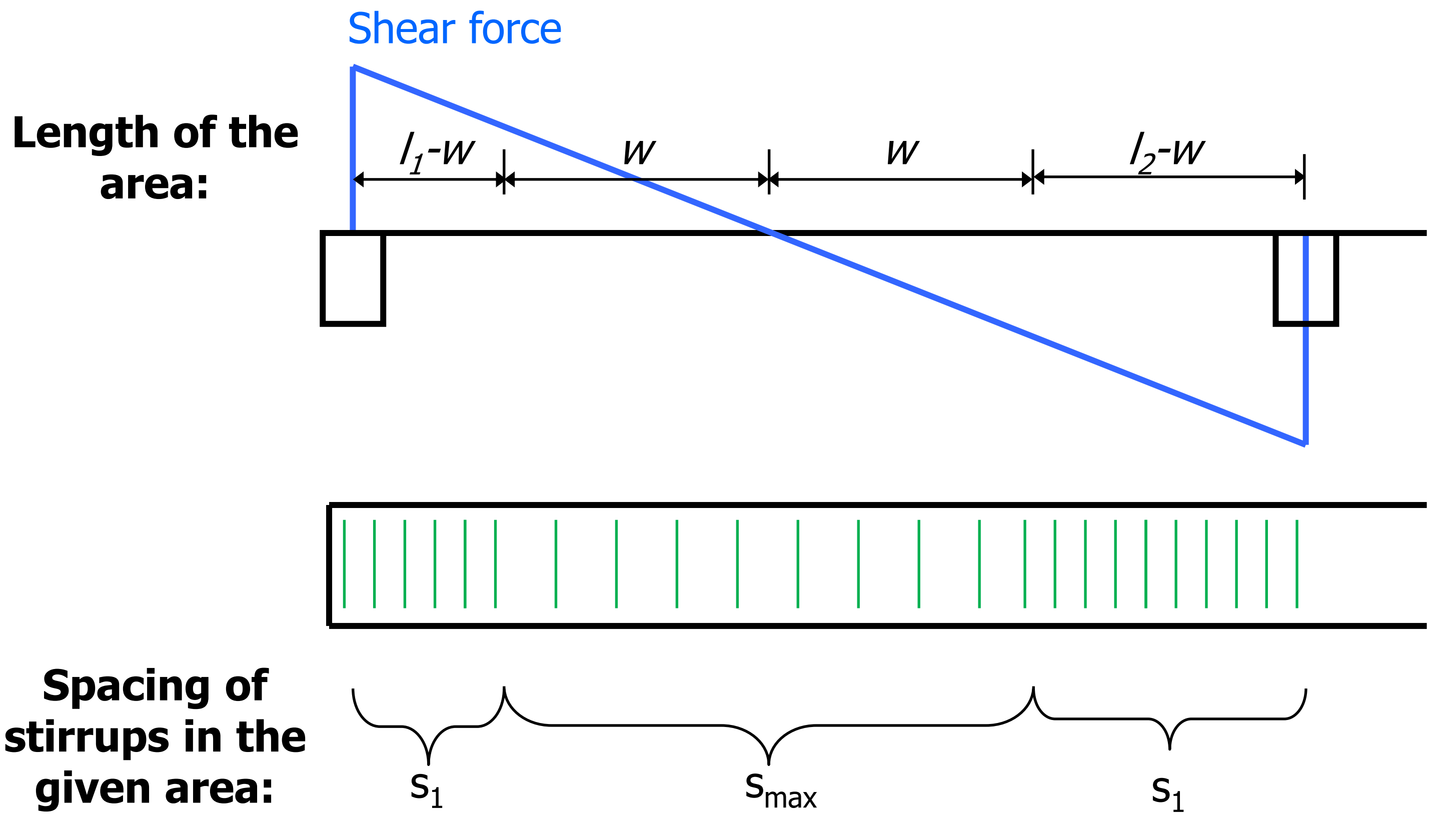
Design of intermediate shear reinforcement

Theoretically, we could calculate $V_{Ed,2}$ (using similar triangles) and design spacing s_2 for the stirrups in the intermediate part ($s_1 < s_2 < s_{max}$).

BUT: This makes sense only for really long beams or beams with point forces.

In our case, we will use s_1 in the intermediate part.

Layout of stirrups



In your homework, draw the scheme in scale using <u>your numerical values</u>.

Next week

Next week

Next week we will focus on **design and assessment of <u>reinforcement of the column</u>**.

thank you for your attention

Recognitions

I thank **Assoc. Prof. Petr Bílý** for his original seminar presentation and other supporting materials from which this presentation was created.