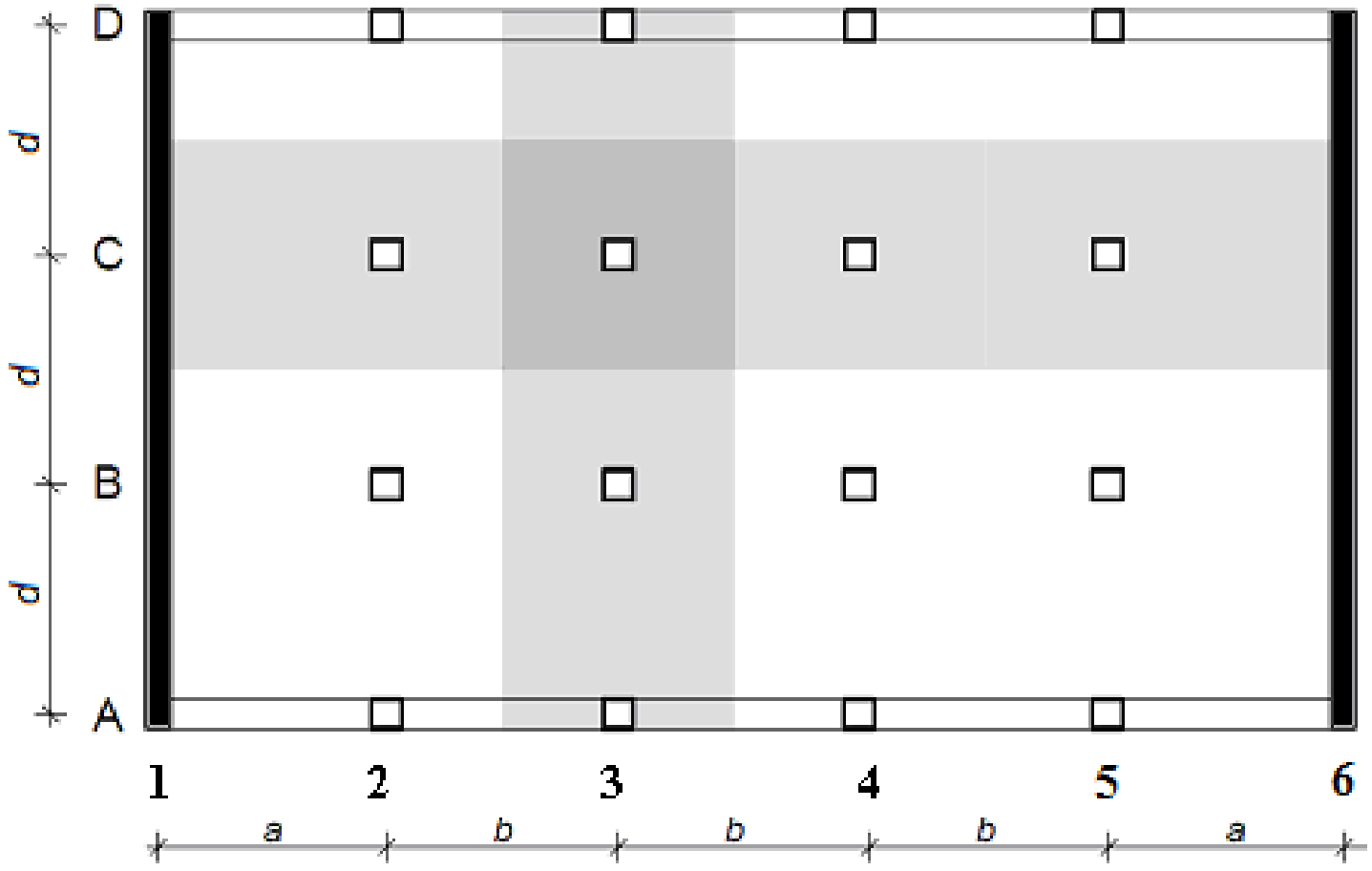
3rd task:

Two-way slab supported by columns (flat slab)



Reinforcement of the slab

- Design of bending reinforcement
- Drawing of bending reinforcement
- Detailed check of punching
- Design of punching reinforcement (if necessary)
- Sketch of punching reinforcement

Design of bending reinforcement

- Generally, it is the same as for beams
- Do the calculation for the moments calculated in 7th seminar
- Do the calculation in a table like this:

Design and check of bending reinforcement of the slab

Panel	Cross-section	Strip	Design							Check						
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0,45$	spacing
			[kNm/m]	mm	mm	[mm^2]	[mm^2]		[mm^2]	[mm]		[mm]	[kNm/m]			of bars
C_o	1 (left support)	no division	**31,02**	169	152	469	220	**Ø12 á 250 mm**	452	18,42	0,109	162	**31,76**	OK	OK	OK
	2 (midspan)	Column	**34,46**	169	152	521	220	**Ø12 á 200 mm**	566	23,07	0,137	160	**39,32**	OK	OK	OK
		Middle	**22,97**	169	152	347	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
	3 (right support)	Column	**60,31**	169	152	912	220	**Ø12 á 100 mm**	1131	46,10	0,273	151	**74,04**	OK	OK	OK
		Middle	**20,10**	169	152	304	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
C_{in}	1 (left support)	Column	**70,08**	169	152	1060	220	**Ø12 á 100 mm**	1131	46,10	0,273	151	**74,04**	OK	OK	OK
		Middle	**23,36**	169	152	353	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
	2 (midspan)	Column	**30,19**	169	152	456	220	**Ø12 á 250 mm**	452	18,42	0,109	162	**31,76**	OK	OK	OK
		Middle	**20,12**	169	152	304	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
	3 (right support)	Column	**70,08**	169	152	1060	220	**Ø12 á 100 mm**	1131	46,10	0,273	151	**74,04**	OK	OK	OK
		Middle	**23,36**	169	152	353	220	**Ø8 á 150 mm**	335	13,65	0,081	164	**23,82**	OK	OK	OK
3_o	1 (left support)	Column	**36,14**	156	140	592	204	**Ø12 á 200 mm**	566	23,07	0,147	147	**36,36**	OK	OK	OK
		Middle	**12,65**	156	140	207	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK
	2 (midspan)	Column	**29,78**	156	140	488	204	**Ø12 á 200 mm**	566	23,07	0,147	147	**36,36**	OK	OK	OK
		Middle	**10,69**	156	140	175	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK
	3 (right support)	Column	**50,12**	156	140	821	204	**Ø12 á 100 mm**	1131	46,10	0,294	138	**68,14**	OK	OK	OK
		Middle	**9,00**	156	140	147	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK
3_{in}	1 (left support)	Column	**56,62**	156	140	927	204	**Ø12 á 100 mm**	1131	46,10	0,294	138	**68,14**	OK	OK	OK
		Middle	**13,35**	156	140	219	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK
	2 (midspan)	Column	**24,39**	156	140	400	204	**Ø12 á 250 mm**	452	18,42	0,117	149	**29,41**	OK	OK	OK
		Middle	**11,50**	156	140	188	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK
	3 (right support)	Column	**56,62**	156	140	927	204	**Ø12 á 100 mm**	1131	46,10	0,294	138	**68,14**	OK	OK	OK
		Middle	**13,35**	156	140	219	204	**Ø8 á 200 mm**	251	10,23	0,065	152	**16,69**	OK	OK	OK

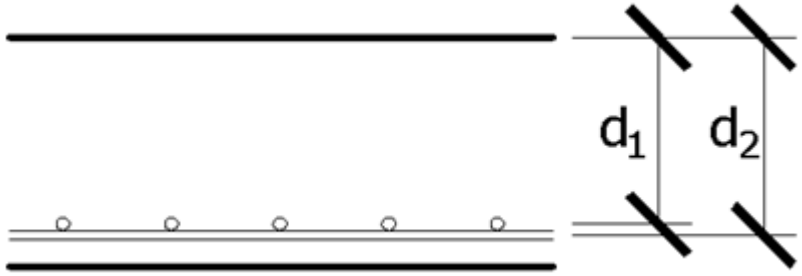
Design of bending reinforcement

- Cross-sectional area of reinforcement:

$$a_{s,prov} \geq a_{s,rqd} = \frac{m_{Ed}}{z f_{yd}}$$

$f_{yd} = f_{yk}/1.15$

Lever arm of internal forces, estimation: $z = 0.9\ d$



- Two-way slab = two values of *d* !!!
- Use higher *d* in the direction of higher m_{Ed}
- Use 8 – 14 mm bars
- Use cover depth from the 1st task

Minimum reinforcement

- Brittle failure precaution:

$$a_{s,prov} \geq a_{s,min,1} = \max\left(0.26\frac{f_{ctm}}{f_{yk}}bd; 0.0013bd\right)$$

Mean tensile strength of concrete, see table

Width of the slab, 1 m

- Excessive cracking precaution:

$$a_{s,prov} \geq a_{s,min,2} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s}$$

In our case, $f_{ct,eff} = f_{ctm}$

Coefficients describing stress distribution in the cross-section, $k_c = 0.4$ and $k = 1.0$

Area of concrete within tensile zone at the first crack, $A_{ct} = 0.5bd$

Mean value of the tensile strength of the concrete effective at the time when the first cracks may be expected to occur; $f_{ct,eff} = f_{ctm}$

Maximum stress permitted in the reinforcement immediately after formation of the crack, $\sigma_s = f_{yk} = 500$ MPa

Check of the design

Height of compressed zone of concrete cross-section →

$$x = \frac{a_{s,prov} f_{yd}}{0.8 b f_{cd}}$$

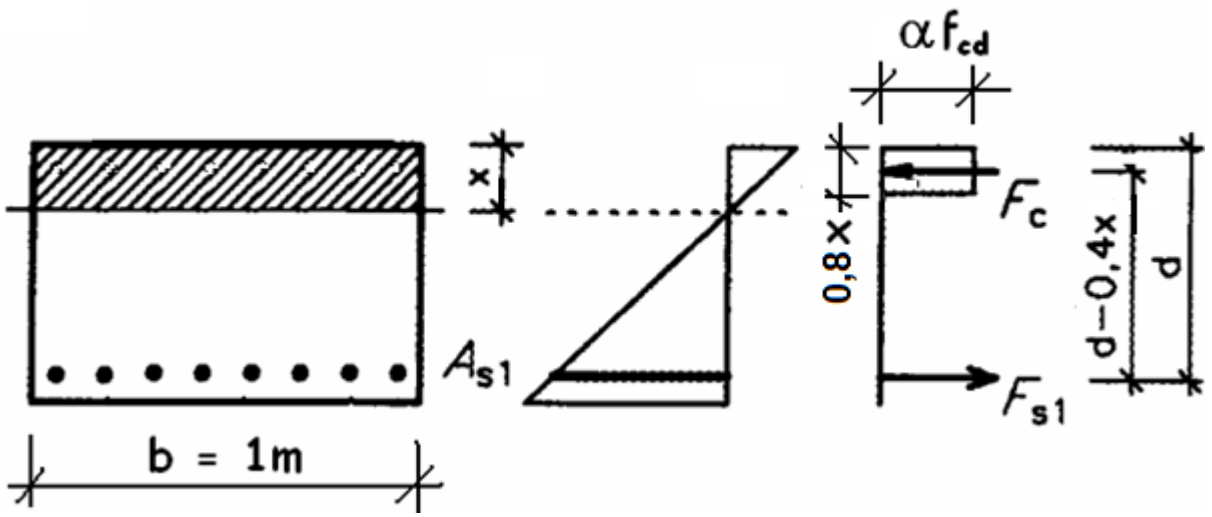
→ $f_{cd} = f_{ck}/1.5$

Actual value of lever arm of internal forces →

$$z = d - 0.4x$$

$$m_{Rd} = a_{s,prov} f_{yd} z$$

$$\boxed{m_{Rd} \geq m_{Ed}}$$



Detailing rules

- Relative height of compressed zone:

$$\xi = \frac{x}{d} \leq 0.45$$

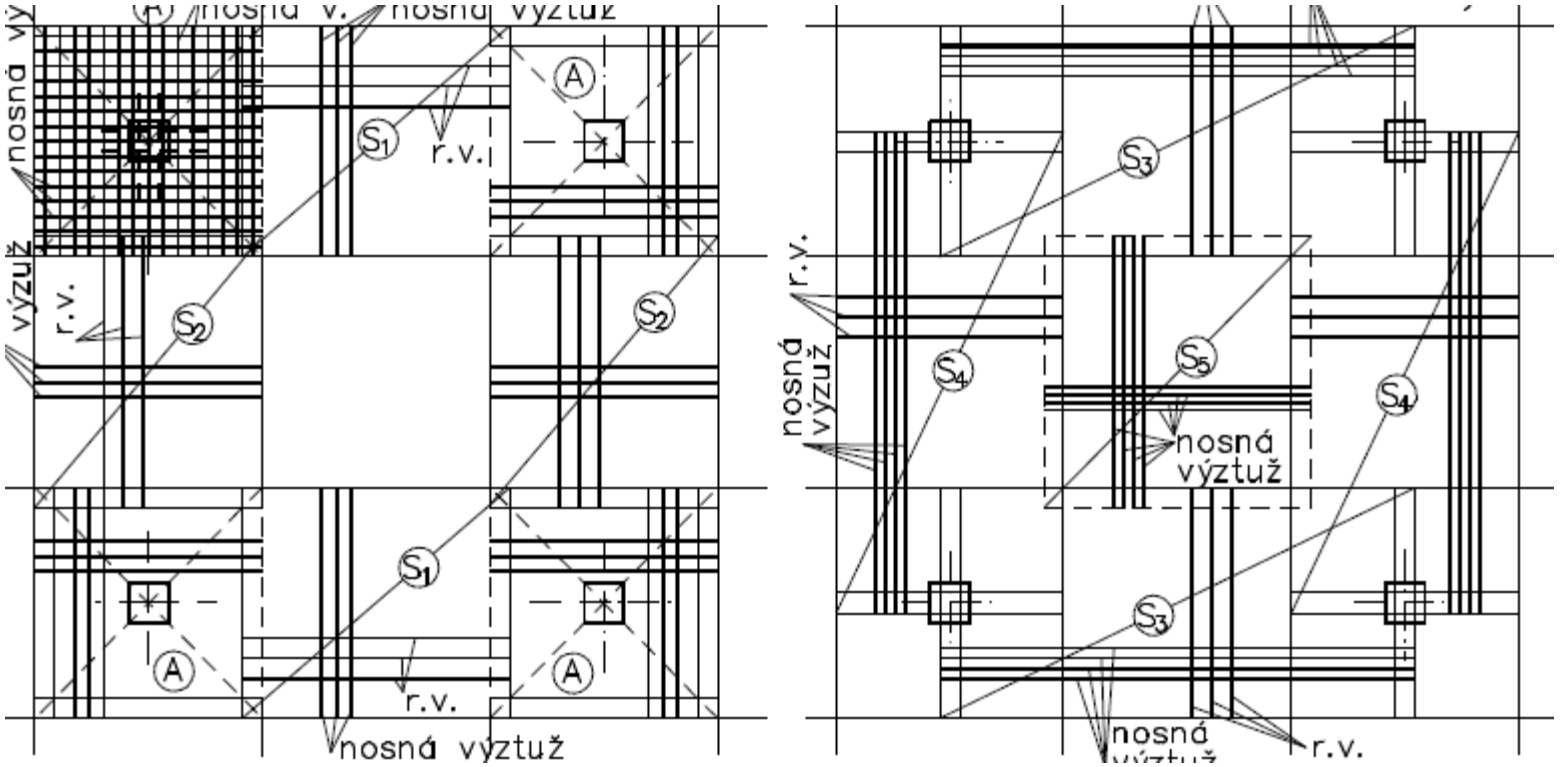
- Spacing of rebars:

$$s \leq \min\left(2h_s;\ 250\ \text{mm}\right)$$

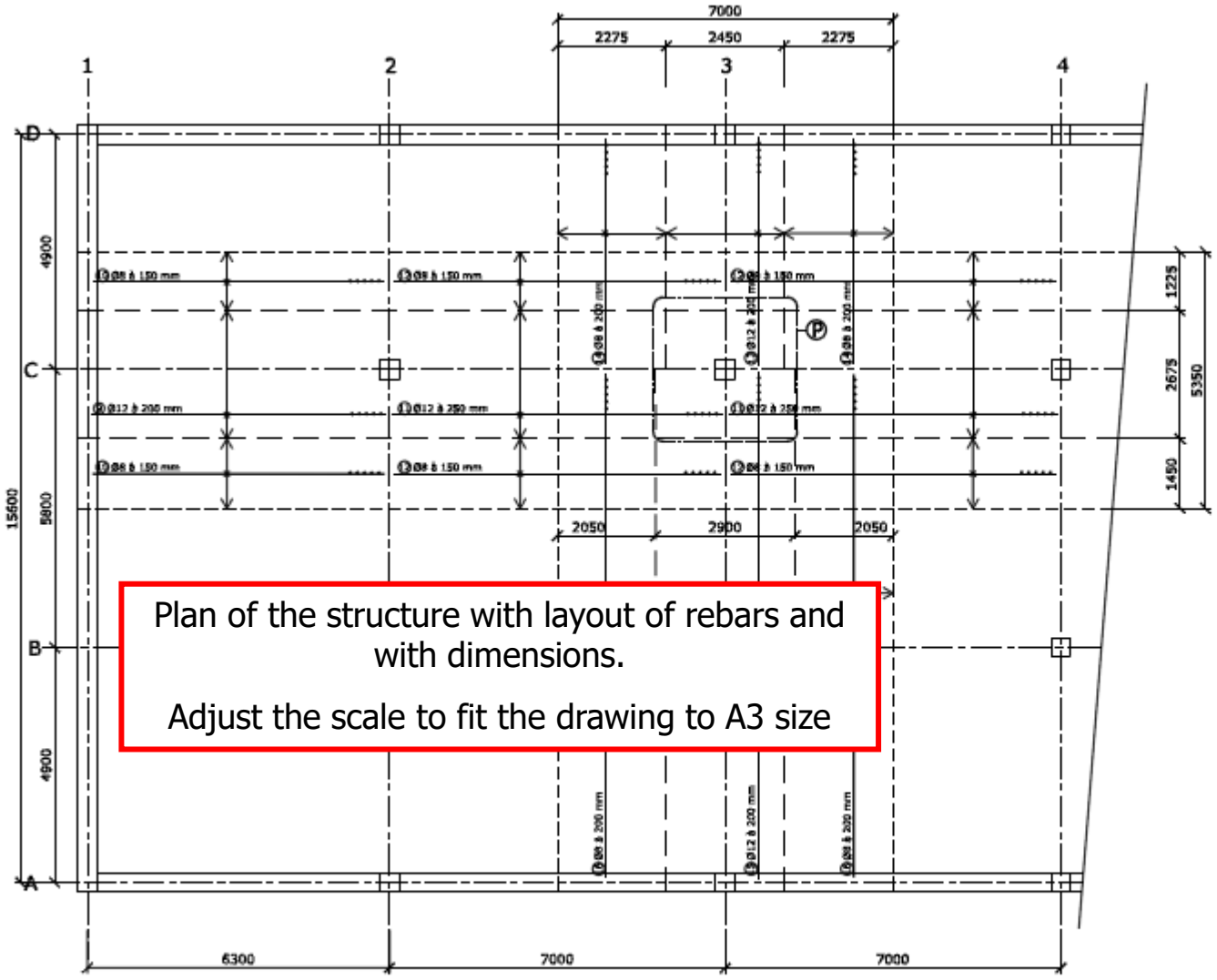
- Recommendation: Spacing not less than 100 mm
- RULE: It is always better to use higher number of smaller bars than lower number of bigger bars (deflections, stress distribution…)

Drawing of bending reinforcement

- 1 drawing for upper reinforcement
- 1 drawing for lower reinforcement
- Calculation of anchorage length – see 5th seminar

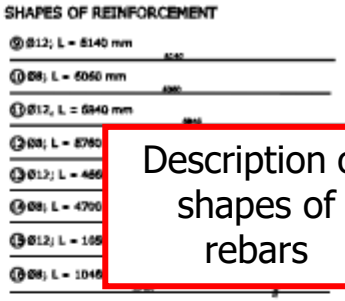


Drawing of bending reinforcement



Plan of the structure with layout of rebars and with dimensions.

Adjust the scale to fit the drawing to A3 size



Description of shapes of rebars

LIST OF REINFORCEMENT

Item no.	diam. [mm]	L [mm]	pcs	Length by profiles R8	Length by profiles R12
1	12	6140			
2	8	6060			
3	12	6840			
4	8	6780			
5	12				
6	8				
7	12				
8	8				
Total length [m]					
Weight per meter [kg]					
Total weight [kg]					
Total weight of reinforcement bars [kg]					

List of reinf.

NOTES:
- PUNCHING
  SEPARATE
- REINFORCEMENT BARS DIMENSIONS ARE
  ALLIGNED AT THE CENTER LINE

Notes

MATERIALS:
CONCRETE C
REINFORCEM

COVER DEPT
ANCHORAGE
ANCHORAGE

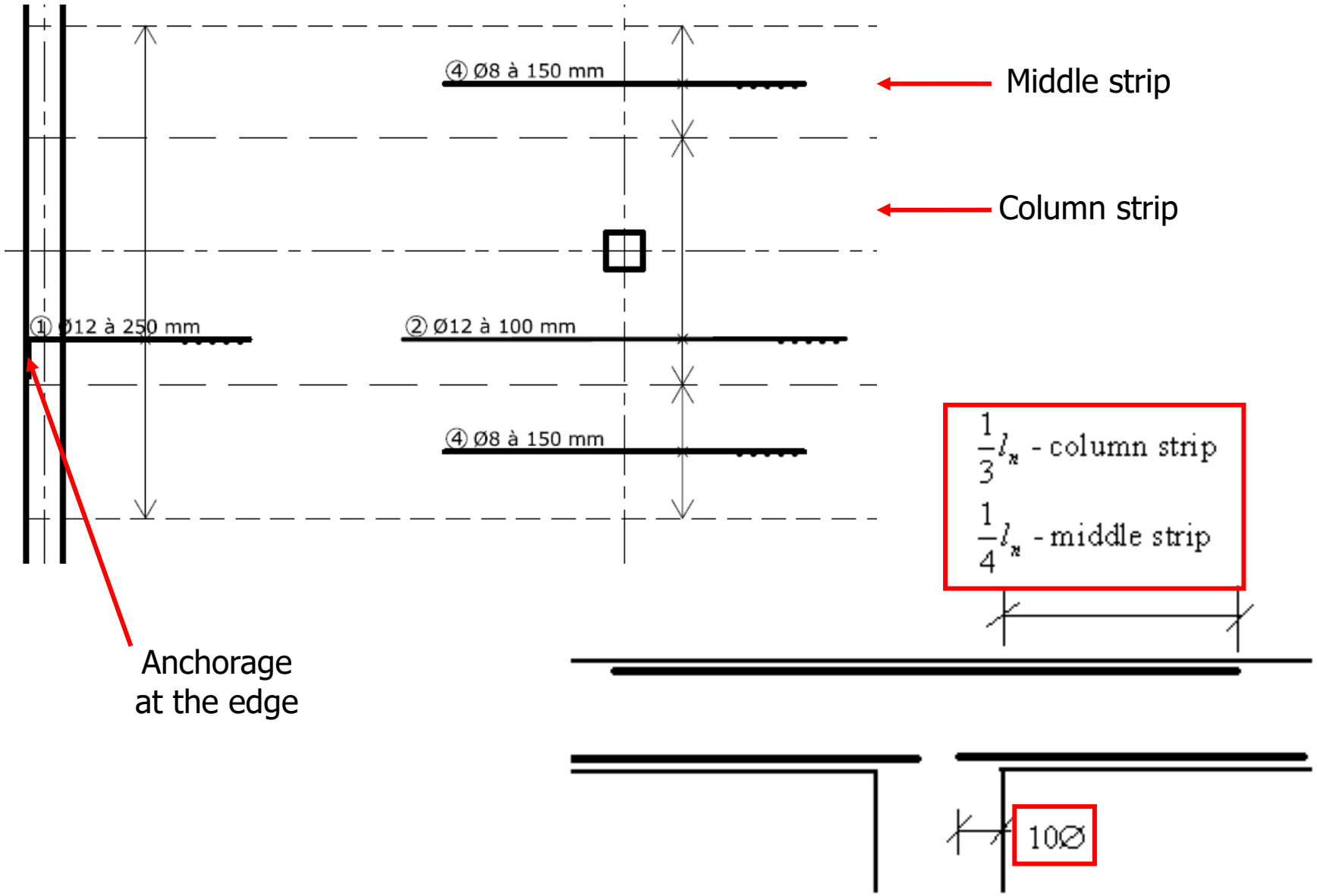
Cover depth, materials, anchorage lengths

SKETCH OF REINFORCEMENT DRAWING – LOWER LAYER		CST2
d by: JÁRA CIMRMAN	WT 1910/11	
Checked by: prof. KLOKNER	1:100	

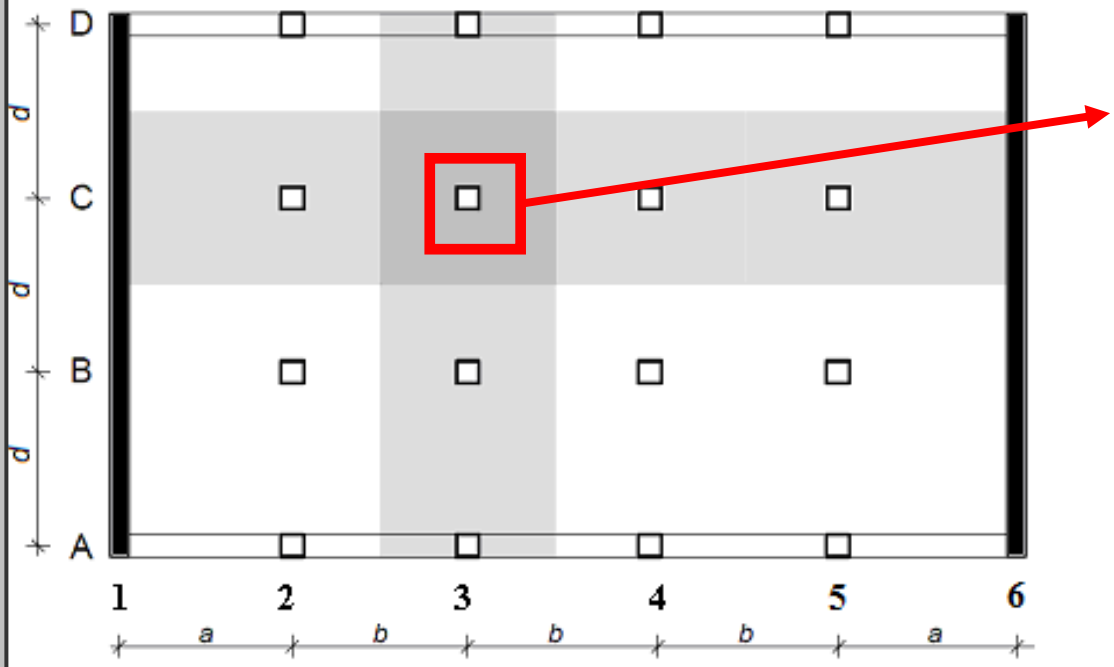
Drawing title

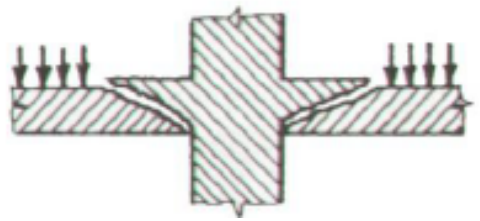
See example on my website

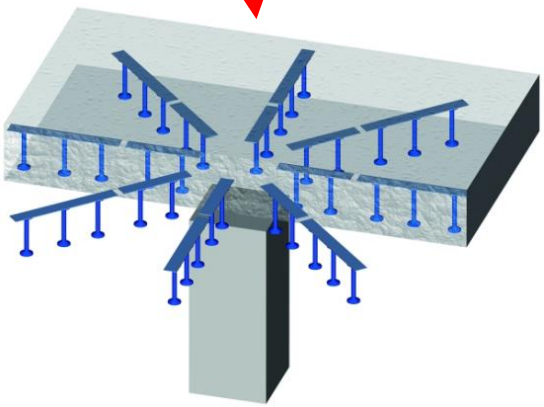
Layout of bending reinforcement



Punching in column C3







Double-headed studs
connected to a spacer bar

Detailed check of punching

- 3 steps
- *Is the resistance of compressed concrete in the face of the column sufficient?* Already checked in preliminary design.

$$v_{\mathrm{Ed,max}} \leq v_{\mathrm{Rd,max}}$$

- *Is the slab able to carry the load without reinforcement?*

$$v_{\mathrm{Ed,1}} \leq v_{\mathrm{Rd,c}}$$

- *Is the designed reinforcement sufficient?*

$$v_{\mathrm{Ed,1}} \leq v_{\mathrm{Rd,cs}}$$

Resistance without reinforcement

$$v_{Ed,1} = \frac{\beta V_{Ed}}{u_1 d} \le v_{Rd,c} = \max\left[C_{Rd,c} \cdot k \cdot \sqrt[3]{(100\rho_l \cdot f_{ck})};\quad 0,035\sqrt{k^3 f_{ck}} \right]$$

Stress in control perimeter u_1 - see preliminary design

Punching shear resistance of a slab <u>without</u> punching reinforcement

Reinforcement ratio for tensile reinforcement

Minimum value of $v_{Rd,c}$ called v_{min}

$$d = \frac{d_C + d_3}{2}$$

$$\rho_l = \sqrt{\rho_{lC} \cdot \rho_{l3}} \le 0,02$$

$$\rho_{lC} = \frac{a_{sC}}{1000 d_C},\ \rho_{l3} = \frac{a_{s3}}{1000 d_3}$$

If you used caps (steel flanged collars), use u_1 calculated with the cap

a_{sC} and a_{s3} are cross-sectional areas of upper reinforcement per 1 m in belt C and belt 3 (in the distance of $3d$ from perimeter u_0)

Remaining coefficients – see preliminary design (7th seminar)

$v_{Ed,1} \le v_{Rd,c}$ ☺ No punching reinforcement needed

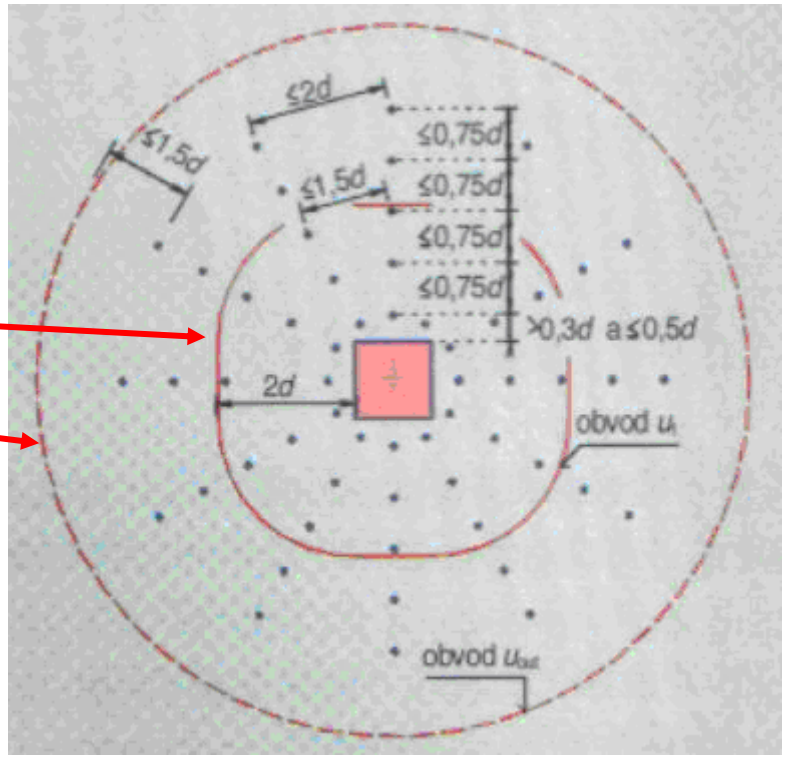
$v_{Ed,1} > v_{Rd,c}$ ☹ Design of punching reinforcement

Design of punching reinforcement

- Control perimeter where the reinforcement is not necessary ($v_{Ed,out} = v_{Rd,c}$) and its radius:

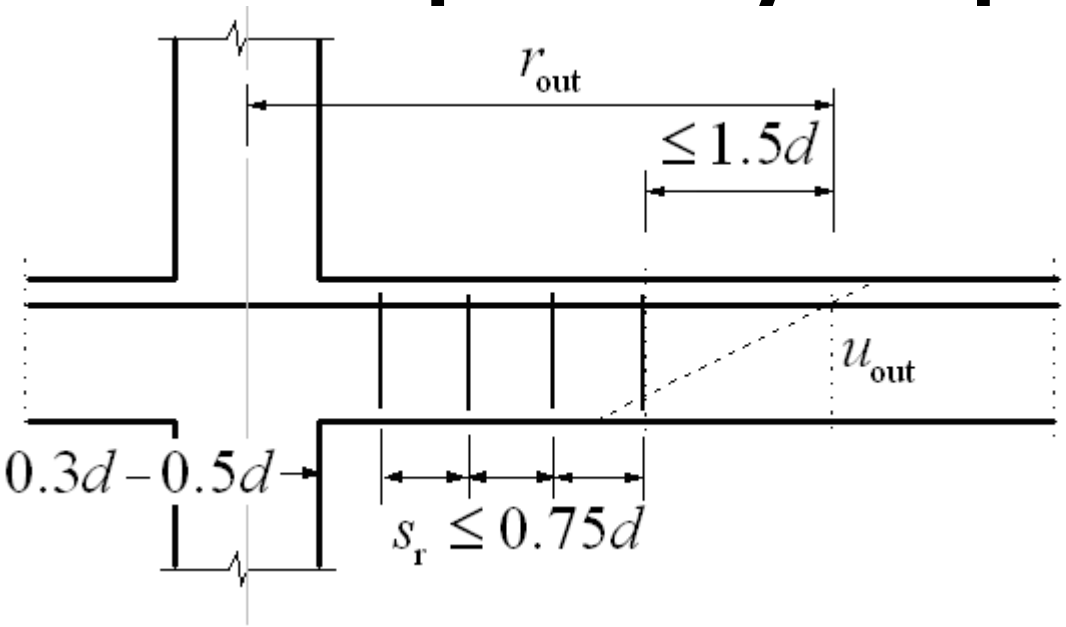
$$u_{out} = \frac{\beta \, V_{Ed}}{v_{Rd,c} d}$$

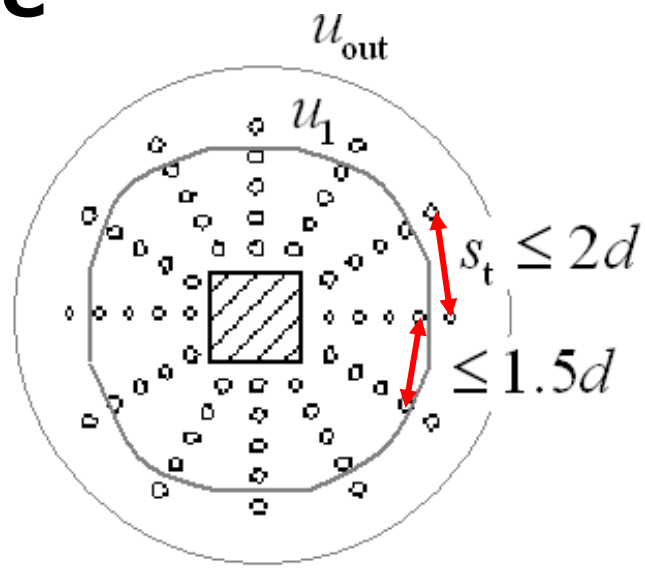
$$r_{out} = \frac{u_{out}}{2\pi}$$



Layout of punching reinforcement

- First stud (0.3–0.5)d behind the face of column
- Last stud not more than 1.5d from u_{out}
- Spacing of intermediate studs $s_r \leq 0.75d$
- Spacing of rails $s_t \leq 2d$
- Spacing of rails in perimeter u_1 less than 1.5d
- **See example on my webpage**





Design of punching reinforcement

- Number of rails („sun rays")

$$n \geq \max\left(\frac{2\pi(r_{out} - 1{,}5d)}{2d}; \frac{u_1}{1{,}5d}\right)$$

- Diameter of studs: 10 – 14 mm (up to 25 mm if necessary) => cross-sectional area of one stud $A_{sw,1}$
- Cross-sectional area of studs in one perimeter:

$$A_{sw} = n.A_{sw,1}$$

- If you used caps (steel flanged collars), use u_1 calculated with the cap

Check of punching reinforcement

$$v_{Ed,1} = \frac{\beta V_{Ed}}{u_1 d} \leq v_{Rd,cs}$$

Stress in control perimeter u_1

Punching shear resistance of a slab with punching reinforcement

$$v_{Rd,cs} = 0,75 v_{Rd,c} + 1,5 \frac{d}{s_r} A_{sw} f_{ywd,ef} \frac{1}{u_1 d} \sin\alpha \leq k_{max} v_{Rd,c}$$

Punching shear resistance of a slab without punching reinforcement

Spacing of studs

Effective design strength of punching reinforcement

Angle between studs and slab, 90°

$$f_{ywd,ef} = 250 + 0,25d \leq f_{ywd}$$

Design strength of reinforcement steel, ask the producer of studs or take 435 MPa

$v_{Ed,1} \leq v_{Rd,cs}$ ☺ Checked

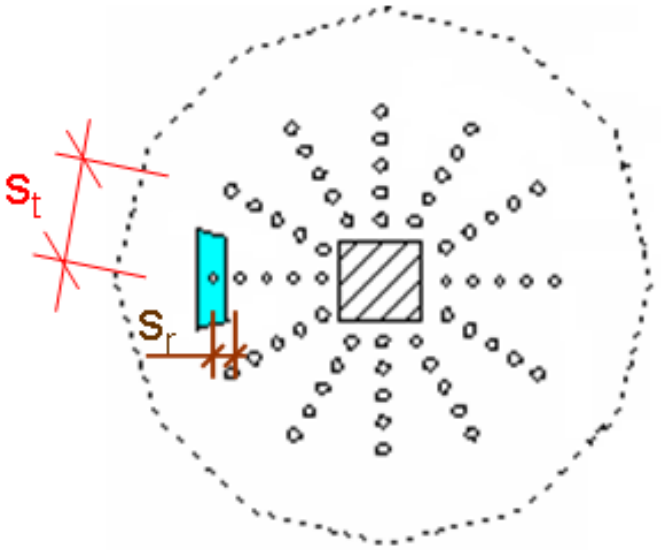
$v_{Ed,1} > v_{Rd,cs}$ ☹ Redesign: Decrease s_r or increase $A_{sw,1}$

Punching reinforcement ratio

$$\rho_{sw} = 1,5\frac{A_{sw,1}}{s_r s_t} \geq \rho_{sw,min} = 0,08\frac{\sqrt{f_{ck}}}{f_{yk}}$$

Punching reinforcement ratio of your slab

Minimum punching reinforcement ratio



$\rho_{sw} \geq \rho_{sw,min}$ ☺ Checked

$\rho_{sw} < \rho_{sw,min}$ ☹ Redesign: Decrease s_r or increase $A_{sw,1}$

Sketch of punching reinforcement

- Plan and section in scale 1:20
- See example on my website