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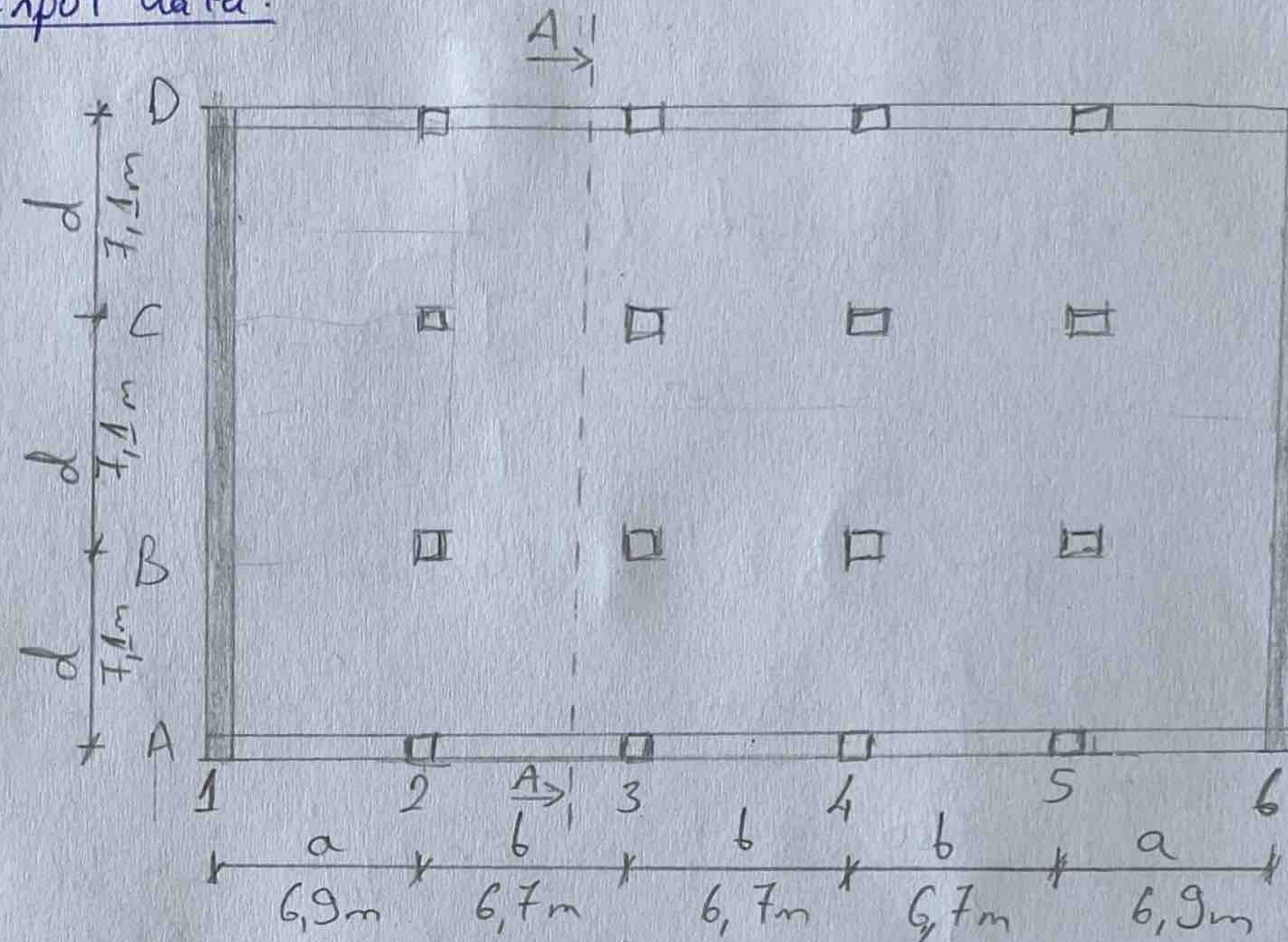
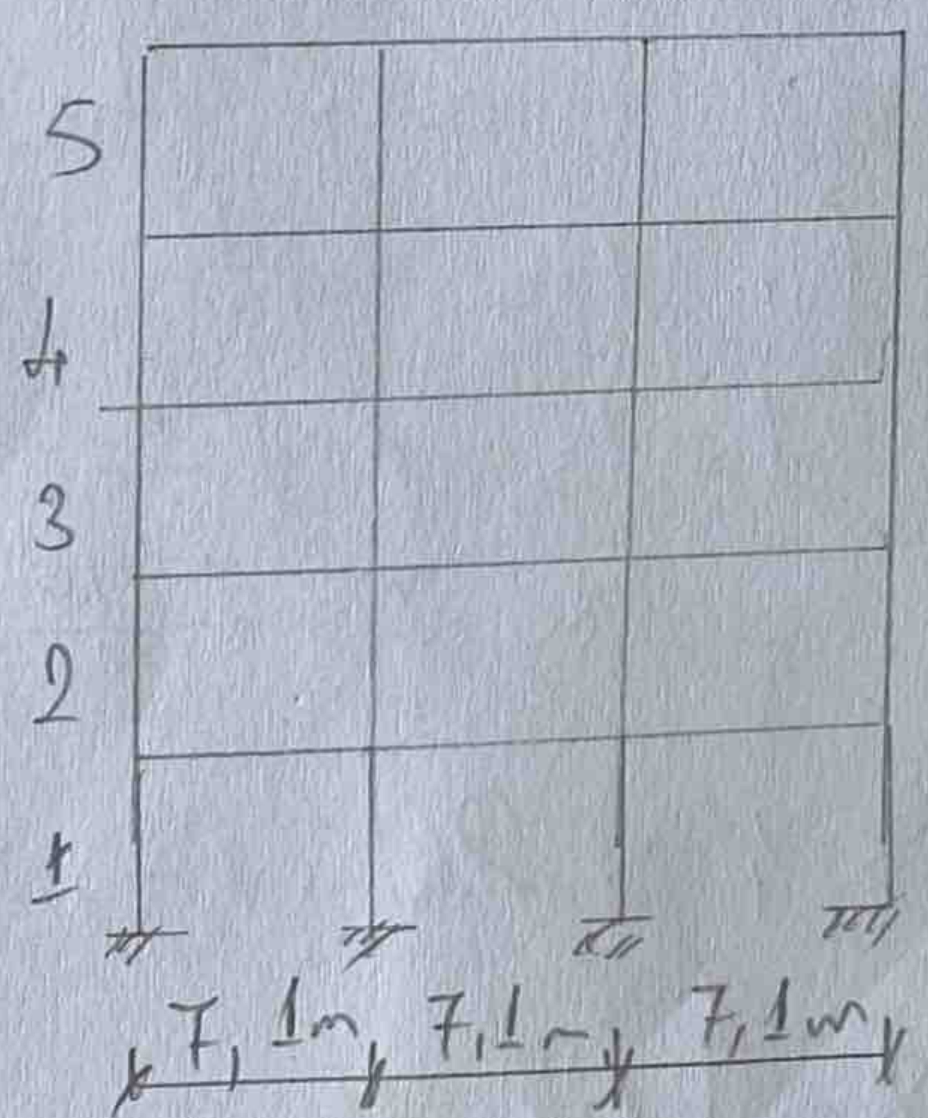
TASK 3: Two-way slab supported on columns

Input data:



Section A-A



Materials

Concrete : C 30/37
- XC2
- 50 years

Loads (typical floor):

$(g-g_0)_{floor,k} = 1,6\ kN/m^2$

$q_{floor,k} = 3,9\ kN/m^2$

1. Preliminary design of dimensions of load-bearing elements

1.1. Depth of the slab hs

• Emprical estimation: $h_s = \frac{1}{33} l_{n,max}$ | $l_{n,max}$ used because we don't know dimensions of column yet.

$= \frac{1}{33} \times 7,1$

$\simeq 0,215$ m

We choose 22 cm for depth of the slab as preliminary design.

• Effective depth d: $d = h_s - c - \frac{\phi}{2}$ | $c = 25$ mm (from task 1)

$d = 220 - 25 - \frac{10}{2}$

$d = 190$ mm

• Span/depth ratio (deflection control):

$\lambda = \frac{l}{d} \overset{?}{\leq} \lambda_{lim}$

$\lambda = \frac{7100}{190} \simeq 37,4$

$\lambda_{lim} = K_{c1} \cdot K_{c2} \cdot K_{c3} \cdot \lambda_{d,tab}$, with $K_{c1} = 1$, $K_{c2} = 1$, $K_{c3} = 1,2$

$\lambda_{d,table}$ → table (C30/37 and $\rho = 0,5\%$)

$\lambda_{d,table} = 24,6$

$\lambda_{lim} = 1 \cdot 1 \cdot 1,2 \cdot 24,6 = 29,52$

$\lambda > \lambda_{lim}$, we should increase hs:

Let's try 28 cm for the depth of the slab

• effective depth: $d = h_s - c - \frac{\phi}{2}$

$= 280 - 25 - \frac{10}{2} = 250$ mm

$\lambda = \frac{l}{d} = \frac{7100}{250} = 28,4 < \lambda_{lim}$ (29,52) checked ✓

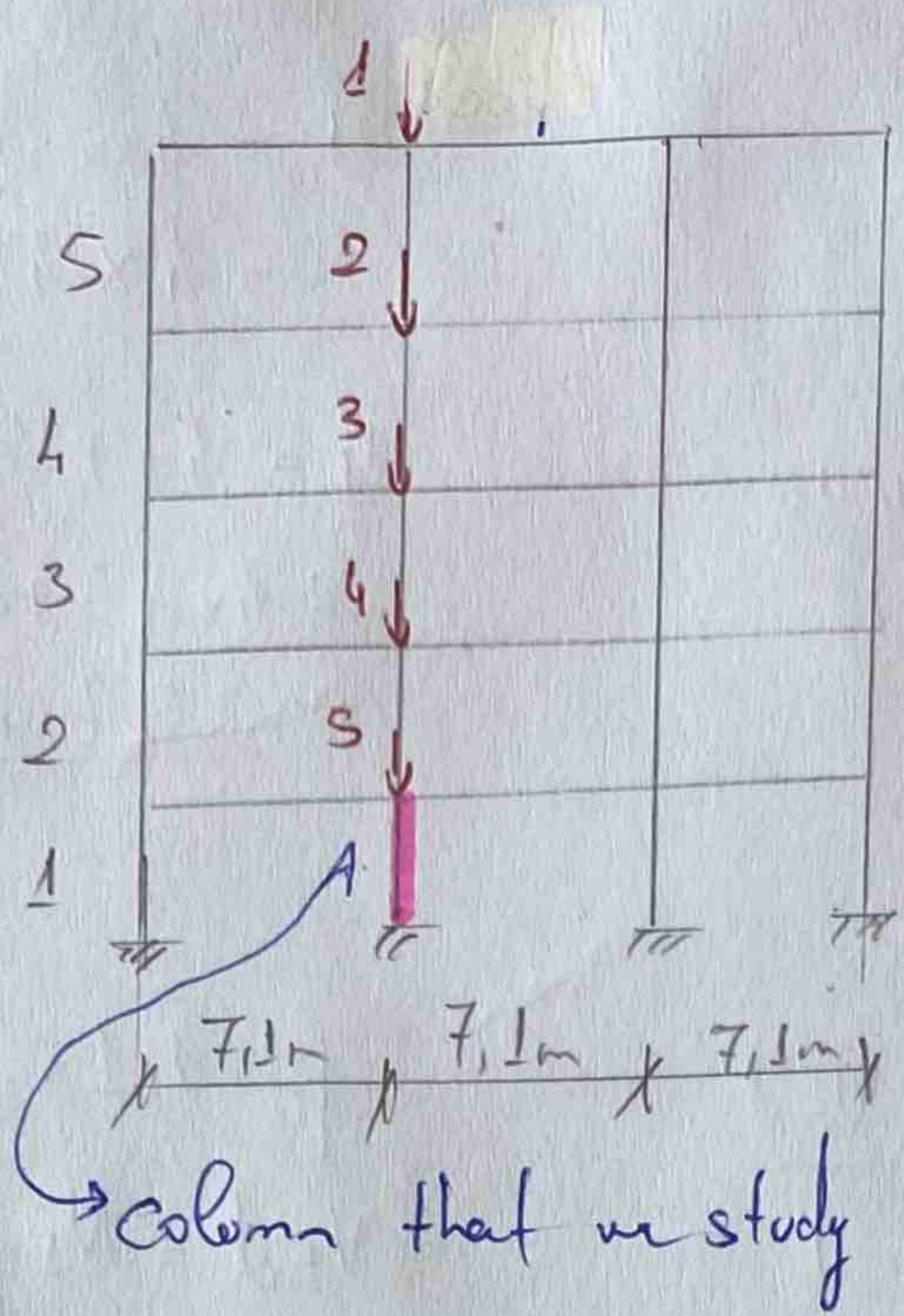
Therefore: $h_s = 28$ cm

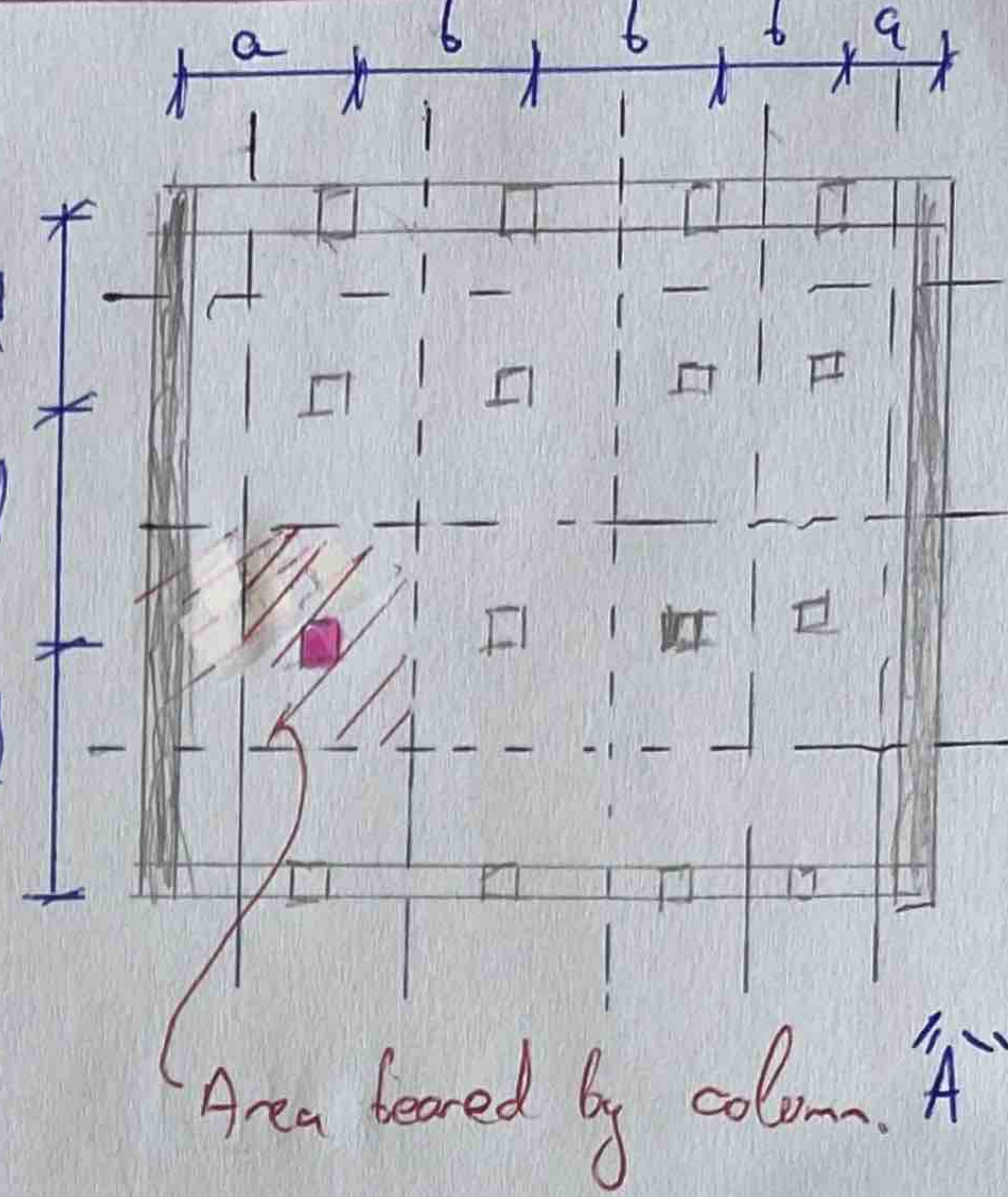
Total load of the slab

Type	Name	f_k [kN/m²]	γ_F	f_d [kN/m²]
Permanent	$(g - g_0)_{floor,k}$	1,6	1,35	2,16
	Self-weight of the slab	25x0,28 = 7	1,35	9,45
Variables	$q_{floor,k}$	3,9	1,5	5,85

$f_d = 2,16 + 9,45 + 5,85 = 17,46$ kN/m²

1.2 Preliminary design of the most loaded column.





$$A = \left(\frac{a}{2} + \frac{b}{2}\right) \times d = \left(\frac{6,9}{2} + \frac{6,7}{2}\right) \times 7,1 = 48,28 \ m^2$$

Loads from typical floor

g_1 = self-weight of the slab = $A \times h_s \times$ Concrete

$= 48,28 \times 0,28 \times 25 \simeq 338 \ kN$

$q = q_{floor,k} \times A = 3,9 \times 48,28 \simeq 188 \ kN$

$g_2 = (g - g_0)_{floor,k} \times A = 1,6 \times 48,28 \simeq 77 \ kN$

Design at ULS

$N_{Ed} = 1,35 \times \Sigma G + 1,5 \Sigma Q$

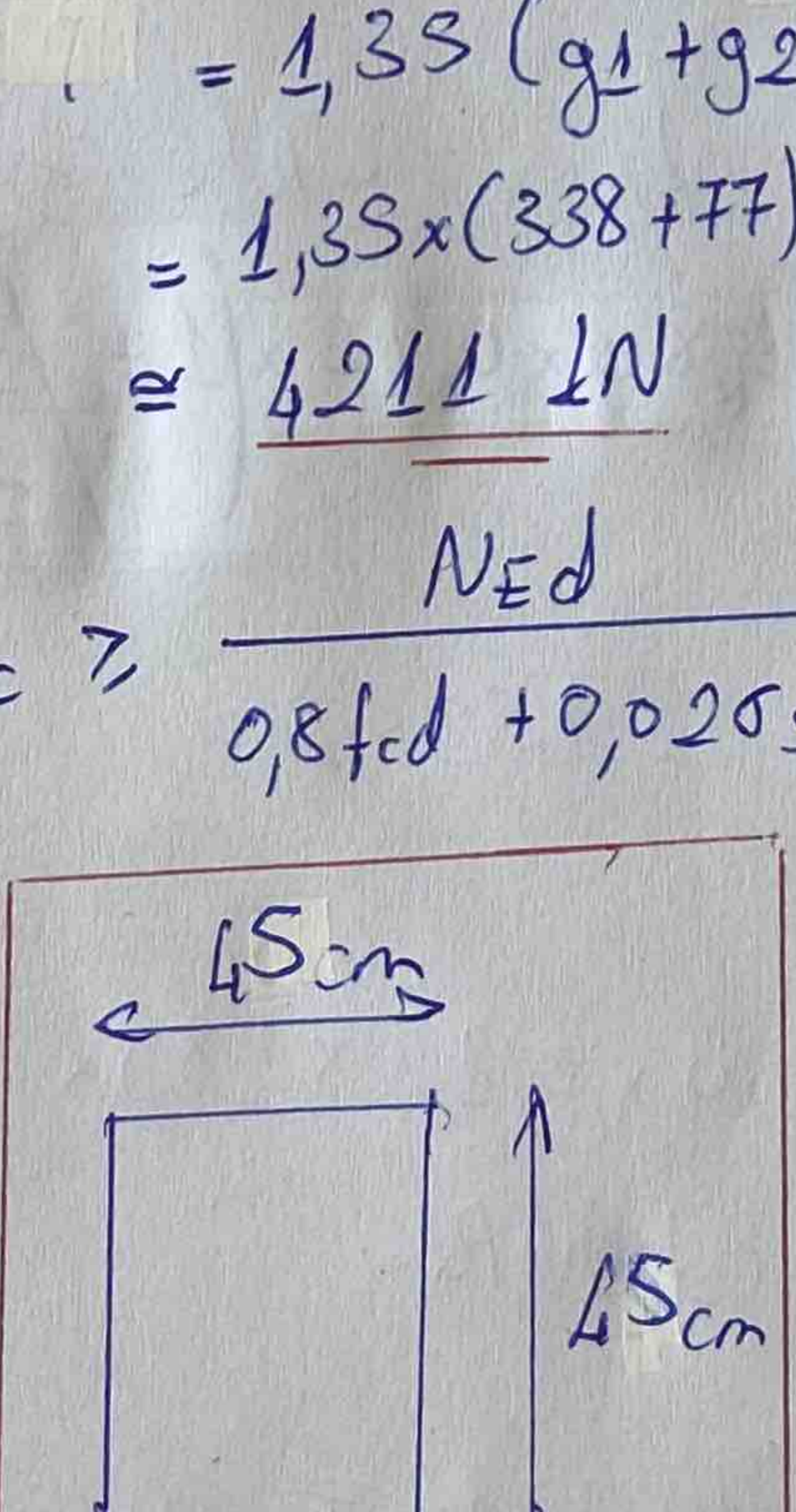
$= 1,35 \, (g_1 + g_2) \times n + 1,5 \times q \times n$ with n = number of floors

$= 1,35 \times (338 + 77) \times 5 + 1,5 \times 188 \times 5$

$\simeq$ **4211 kN**

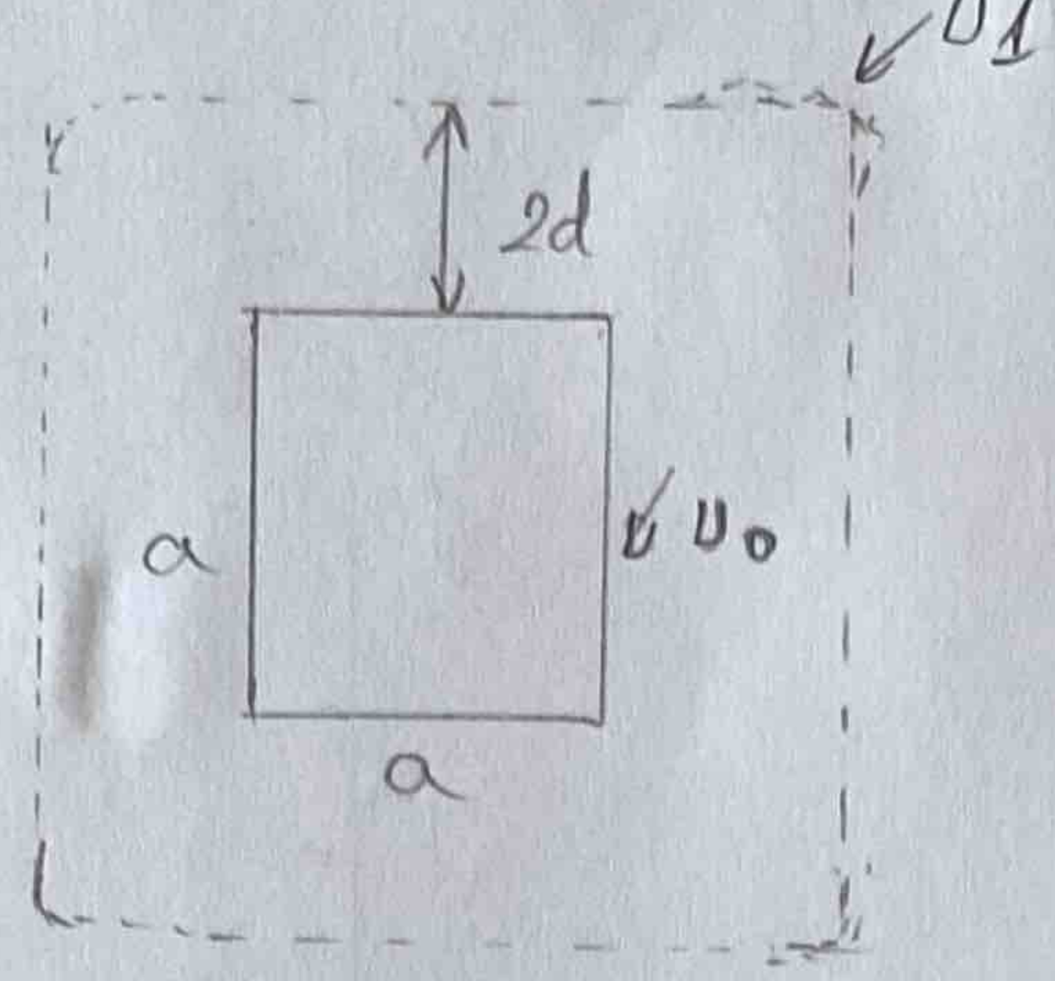
$$A_c \geq \frac{N_{Ed}}{0,8 f_{cd} + 0,02 \sigma_s} = \frac{4211 \times 10^3}{(0,8 \times 20 \times 10^6) + (0,02 \times 400 \times 10^6)}$$

$\simeq 0,175 \ m^2$



$A_c = 0,20 \ m^2 > A_{c\,req} = 0,175 \ m^2$

1.3. Preliminary check of punching



$u_0 = 4a$
$= 4 \times 45 = \underline{180}$ cm

$u_1 = 4a + 2\pi \cdot 2d$
$= 4 \times 45 + 2\pi \cdot 25$
$\simeq \underline{337}$ cm

* $v_{Ed,0} = \dfrac{\beta \cdot V_{Ed}}{u_0 \cdot d}$

$= \dfrac{1{,}15 \times 842 \times 10^3}{1800 \times 250}$

$\simeq 2{,}15$ MPa

$\beta = 1{,}15$
$V_{Ed} = N_{Ed\,1floor}$
$= 1{,}35g + 1{,}5q$
$= 1{,}35 \times (338+77) + 1{,}5 \times 188$
$\simeq \underline{842}$ kN

• $v_{Rd,max} = 0{,}4 \nu \cdot f_{cd}$
$= 0{,}4 \times 0{,}528 \times 20$
$= 4{,}22$ MPa

$f_{cd} = 20$ MPa
$\nu = 0{,}6\left(1 - \dfrac{f_{ck}}{250}\right)$
$0{,}6\left(1 - \dfrac{30}{250}\right) = 0{,}528$

$v_{Ed,0} < v_{Rd,max}$ checked ✓

* $v_{Ed,1} = \dfrac{\beta \cdot V_{Ed}}{u_1 \cdot d} = \dfrac{1{,}15 \times 842 \times 10^3}{3370 \times 250} \simeq 1{,}15$ MPa

* $k_{max} \cdot v_{Rd,c} = k_{max} \cdot C_{Rd,c} \cdot k \cdot \sqrt[3]{100\, \rho_1 \cdot f_{ck}}$

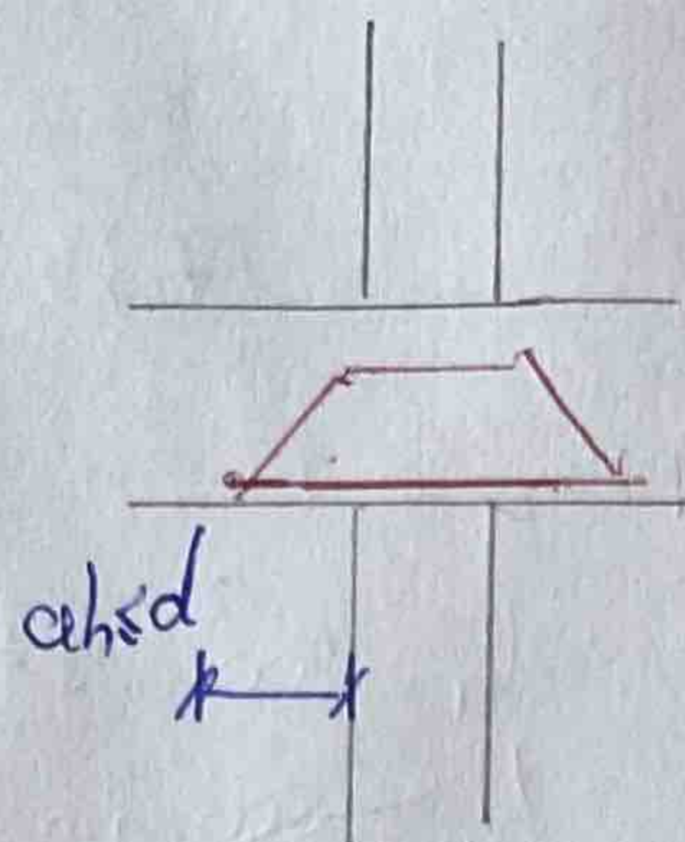
$k = 1 + \sqrt{\dfrac{200}{d}} \leq 2$

$= 1 + \sqrt{\dfrac{200}{250}} = 1{,}89 < 2$

* $k_{max} \cdot v_{Rd,c} = 1{,}80 \times 0{,}12 \times 1{,}89 \times \sqrt[3]{100 \times 0{,}005 \times 30}$
$= 1{,}01$ MPa

$v_{Ed,1} > k_{max} \cdot v_{Rd,c}$ NOT CHECKED !

1.4 Design columns with caps



$a_h = d = 25\ cm$

$U_0 = 4a + 8a_h$
$= 4 \times 45 + 8 \times 25 = 380\ cm$

$U_1 = 4a + 8a_h + 2\pi \cdot 2d$
$= 380 + 2\pi \cdot 2 \times 25 = 537\ cm$

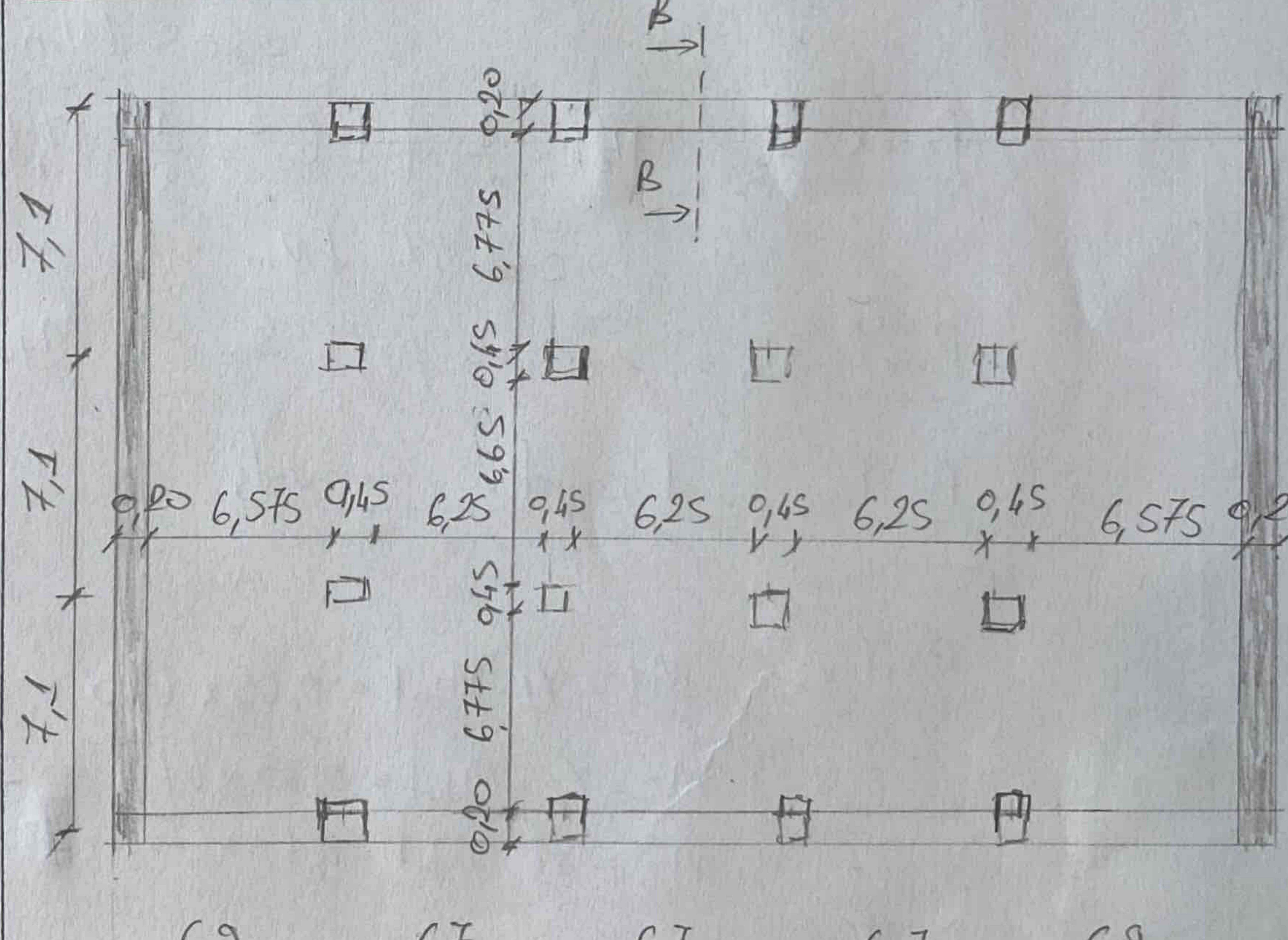
$v_{Ed,0} = \frac{\beta \cdot V_{Ed}}{U_0 \cdot d} = \frac{1,15 \times 842 \times 10^3}{3800 \times 250} = 1,01\ MPa < v_{Rd,max}\ (4,22\ MPa)$

Checked!

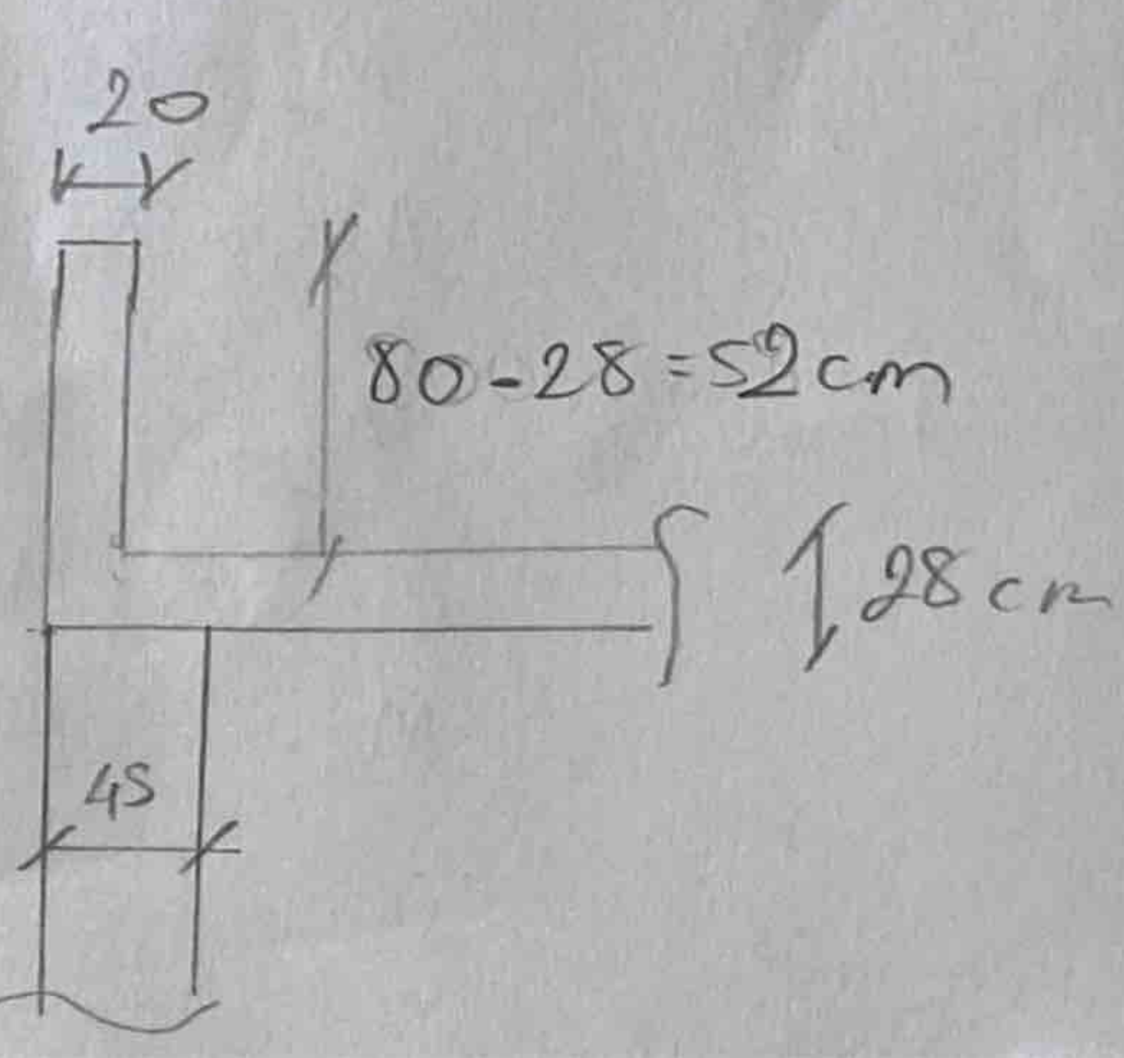
$v_{Ed,1} = \frac{\beta \cdot V_{Ed}}{U_1 \cdot d} = \frac{1,15 \times 842 \times 10^3}{5370 \times 250} = 0,72\ MPa < k_{max} \cdot v_{Rd,c}\ (1,01\ MPa)$

Checked!

2. Sketch of the structure

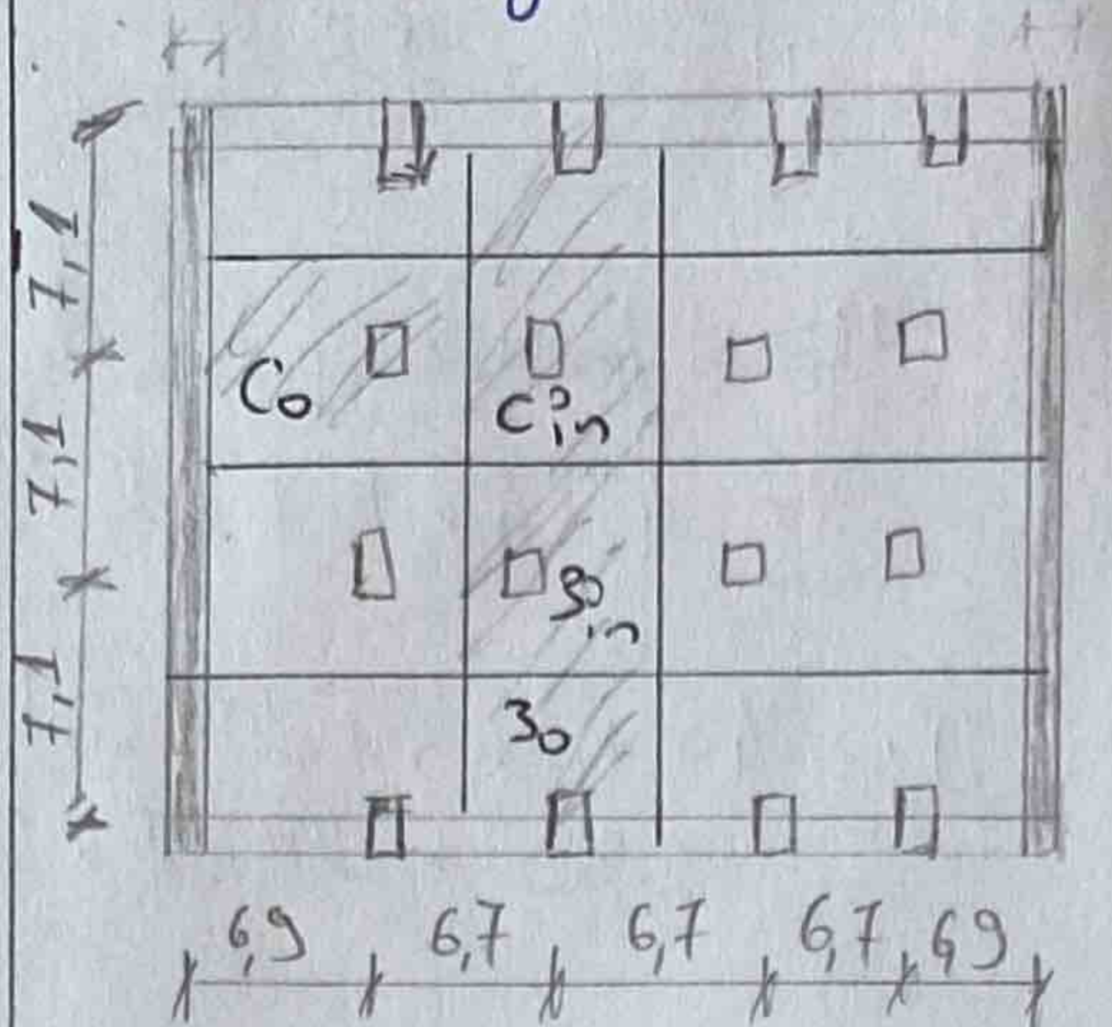


Section B-B



3. Structural analysis of the slab

3.1. Bending moments in strips C and 3 using direct design method



$h_S = 280$ mm

$f_d = 17,46$ kN/m²

$a = 45$ cm (dimension of column)

$b_T = 200$ mm - width of the beam

$h_T = 800$ mm - height of the beam

TOTAL MOMENTS (o = outer ; in = inner)

Panel C_o: $M_{tot} = \frac{1}{8} \cdot f \cdot b \cdot l_n^2 = \frac{1}{8} \cdot 17,46 \cdot 7,1 \cdot \left(6,9 - \frac{0,2}{2} - \frac{0,45}{2}\right)^2$

$\simeq 670$ kNm

Panel C_{in}: $M_{tot} = \frac{1}{8} \cdot f b \cdot l_n^2 = \frac{1}{8} \cdot 17,46 \times 7,1 \times \left(6,7 - \frac{0,45}{2} - \frac{0,45}{2}\right)^2$

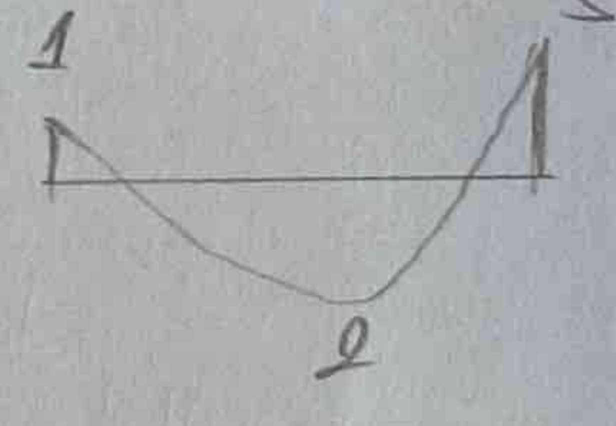
$\simeq 605$ kNm

Panel 3_o: $M_{tot} = \frac{1}{8} \cdot f \cdot b \cdot l_n^2 = \frac{1}{8} \cdot 17,46 \times 6,7 \times \left(7,1 - \frac{0,45}{2} - \frac{0,45}{2}\right)^2$

$\simeq 646$ kNm

Panel 3_{in} = same as panel 3_o => $M_{tot} = 646$ kNm

Positive and negative moments



Panel C_o: $M_1 = \gamma_1 \cdot M_{tot} = 0,65 \times 670 = 435,5$ kNm

$M_2 = \gamma_2 \cdot M_{tot} = 0,35 \times 670 = 234,5$ kNm

$M_3 = \gamma_3 \cdot M_{tot} = 0,65 \times 670 = 435,5$ kNm

Panel C_{in}: $M_1 = \gamma_1 \cdot M_{tot} = 0,65 \times 605 = 393,25$ kNm

$M_2 = \gamma_2 \cdot M_{tot} = 0,35 \times 605 = 211,75$ kNm

$M_3 = \gamma_3 \cdot M_{tot} = 0,65 \times 605 = 393,25$ kNm

Panel 3_o: $M_1 = \gamma_1 \cdot M_{tot} = 0,3 \cdot 646 = 193,8$ kNm

$M_2 = \gamma_2 \cdot M_{tot} = 0,5 \times 646 = 323$ kNm

$M_3 = \gamma_3 \, M_{tot} = 0,7 \times 646 = 452,2$ kNm

Panel 3_{in}: $M_1 = \gamma_1 \cdot M_{tot} = 0,65 \times 646 = 419,9$ kNm

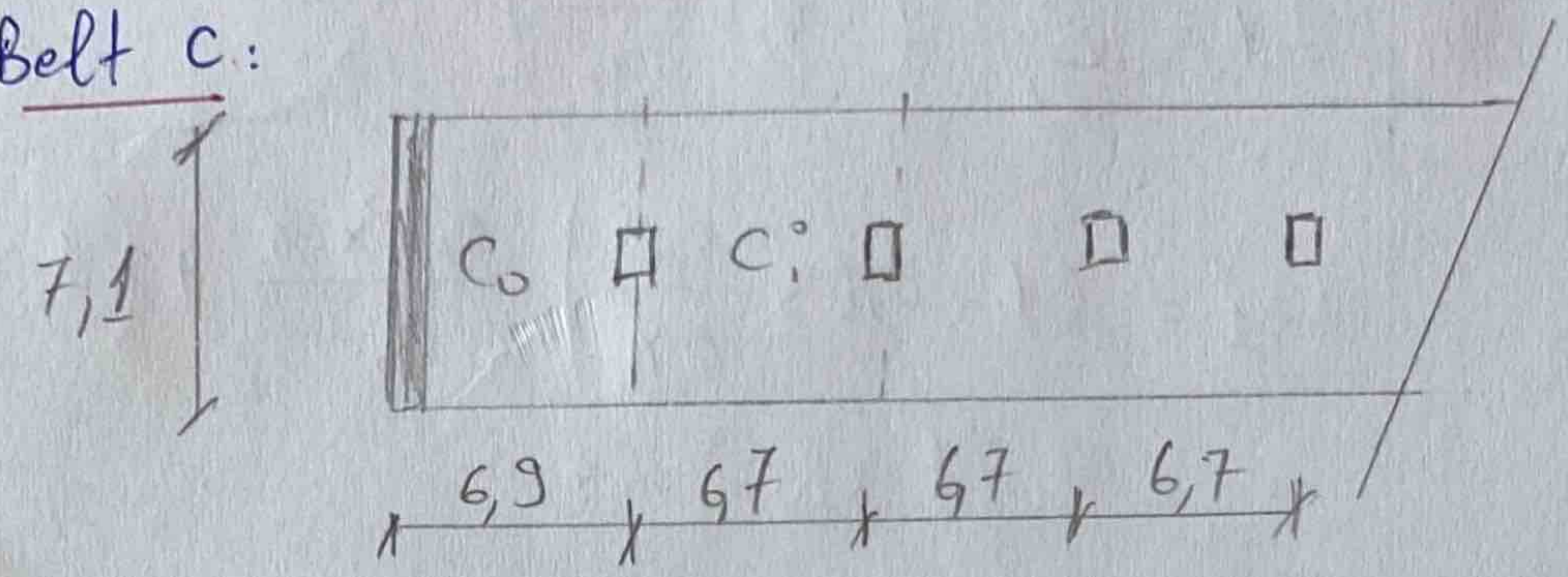
$M_2 = \gamma_2 \cdot M_{tot} = 0,35 \times 646 = 226,1$ kNm

$M_3 = \gamma_3 \cdot M_{tot} = 0,65 \times 646 = 419,9$ kNm

Moments in colum and middle strips

Dividing belts into strips

Belt C:



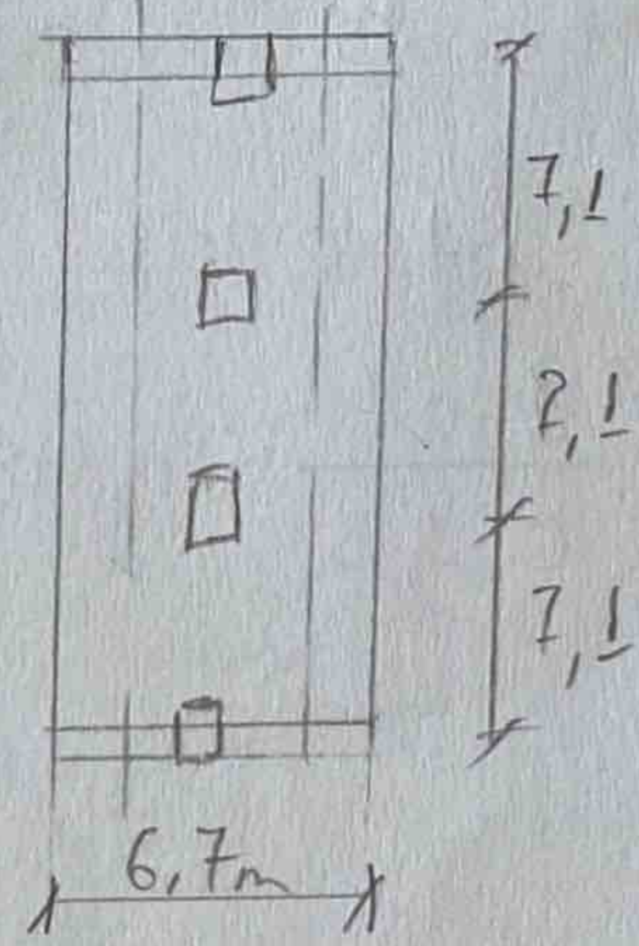
Panel C_0: colomn strip : $\frac{6,9}{4} + \frac{6,9}{4} = 3,45$m

middle strip: $7,1 - 3,45 = 3,65$m

Panel C_{in} : column strip : $\frac{6,7}{4} + \frac{6,7}{4} = 3,35$m

middle strip : $7,1 - 3,35 = 3,75$m.

Belt 3:

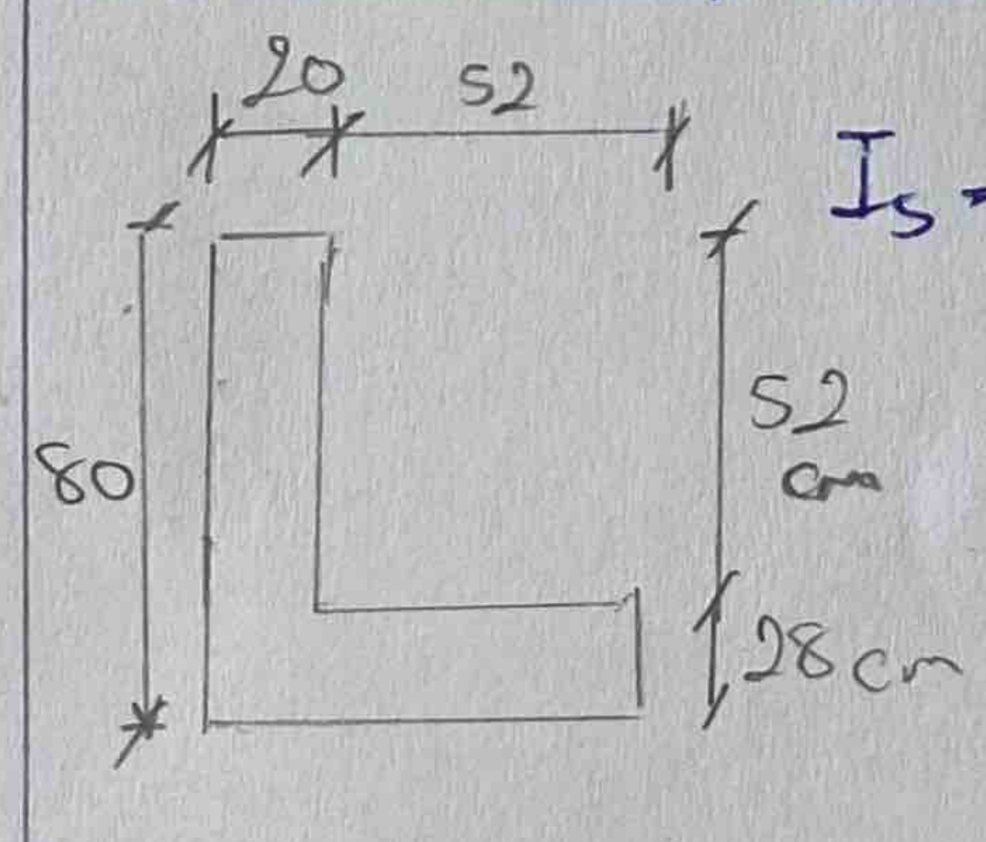


Panel 3_0: $\frac{6,7}{4} + \frac{6,7}{4} = 3,35$m column strip.

Panel 3_{in}: $\frac{6,7}{4} + \frac{6,7}{4} = 3,35$m

middle strip = $6,7 - 3,35 = 3,35$m

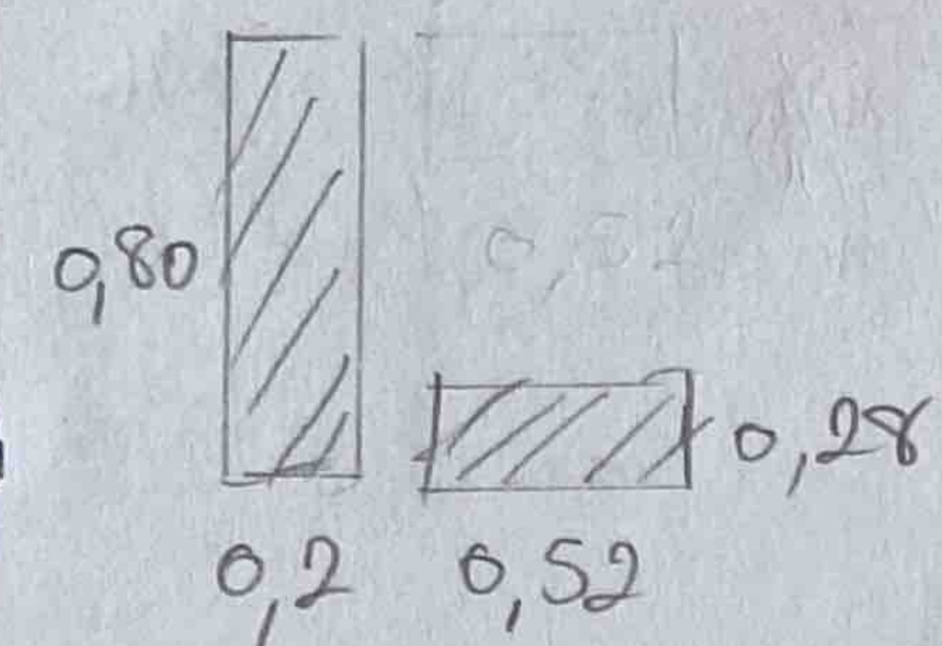
Calculation of β_t edge beam



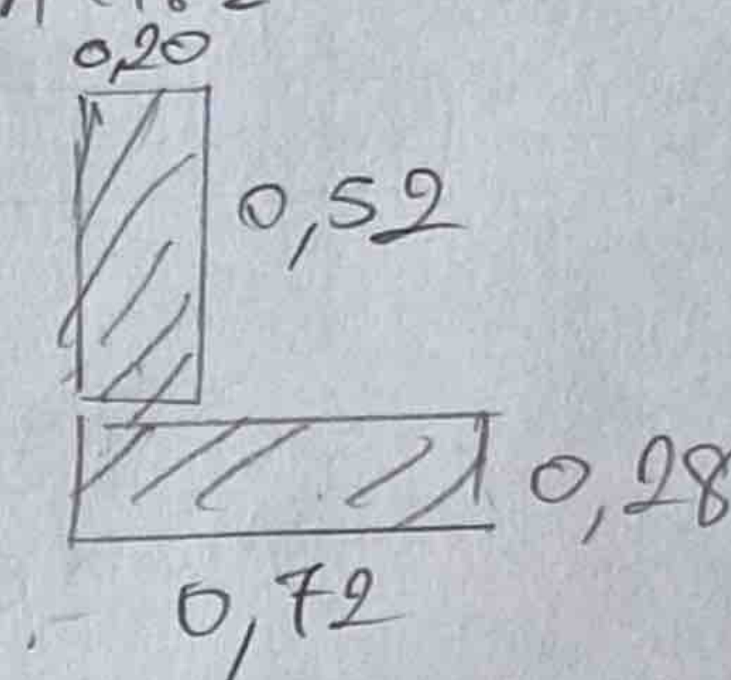
$I_s = \frac{1}{12} \cdot b \cdot h_s^3 = \frac{1}{12} \times 6,7 \times 0,28^3$

$12,26 \times 10^{-3}$ m^4

Alt. 1



Alt. 2



$$I_t = \sum_{i=1}^{n} \left(1 - 0{,}63 \frac{t_i}{a_i}\right) \cdot \frac{t_i^3 \cdot a_i}{3}$$

$$I_{t1} = \left(1 - 0{,}63 \cdot \frac{0{,}2}{0{,}8}\right) \cdot \frac{0{,}2^3 \times 0{,}8}{3} + \left(1 - 0{,}63 \cdot \frac{0{,}28}{0{,}52}\right) \cdot \frac{0{,}28^3 \cdot 0{,}52}{3}$$

$$\simeq 4{,}31 \times 10^{-3} \; m^4$$

$$I_{t2} = \left(1 - 0{,}63 \cdot \frac{0{,}2}{0{,}52}\right) \cdot \frac{0{,}2^3 \cdot 0{,}52}{3} + \left(1 - 0{,}63 \cdot \frac{0{,}28}{0{,}72}\right) \cdot \frac{0{,}28^3 \cdot 0{,}72}{3}$$

$$\simeq \boxed{5{,}03 \times 10^{-3} \; m^4}$$

$$\beta_t = \frac{I_t}{2 I_s} = \frac{5{,}03 \times 10^{-3}}{2 \times 12{,}26 \times 10^{-3}} = \underline{0{,}205}$$

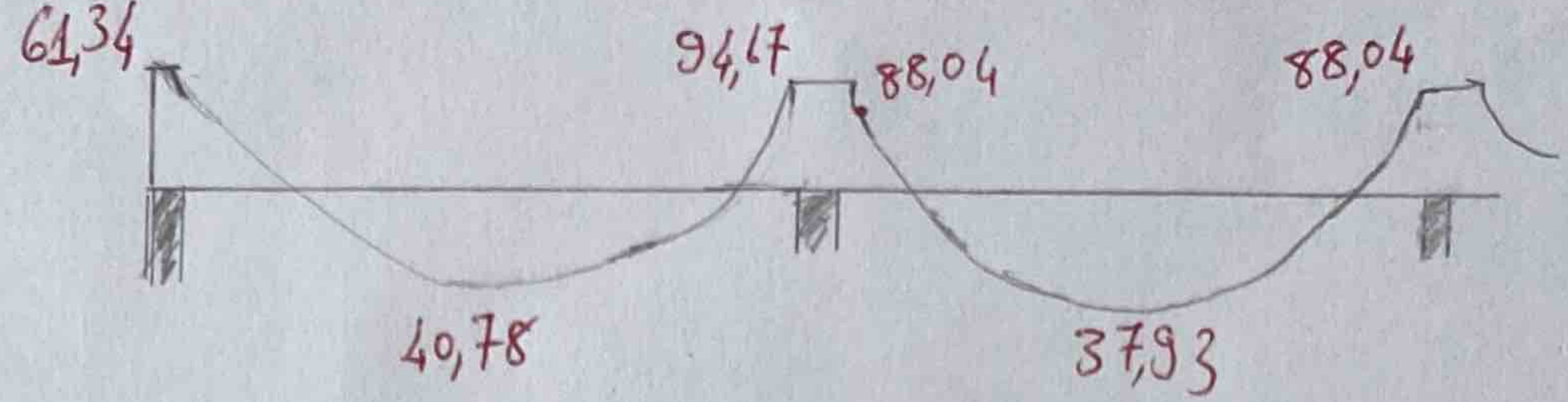
Calculation of moments in column and middle strips

Value of w for $\beta_t = 0{,}205$:

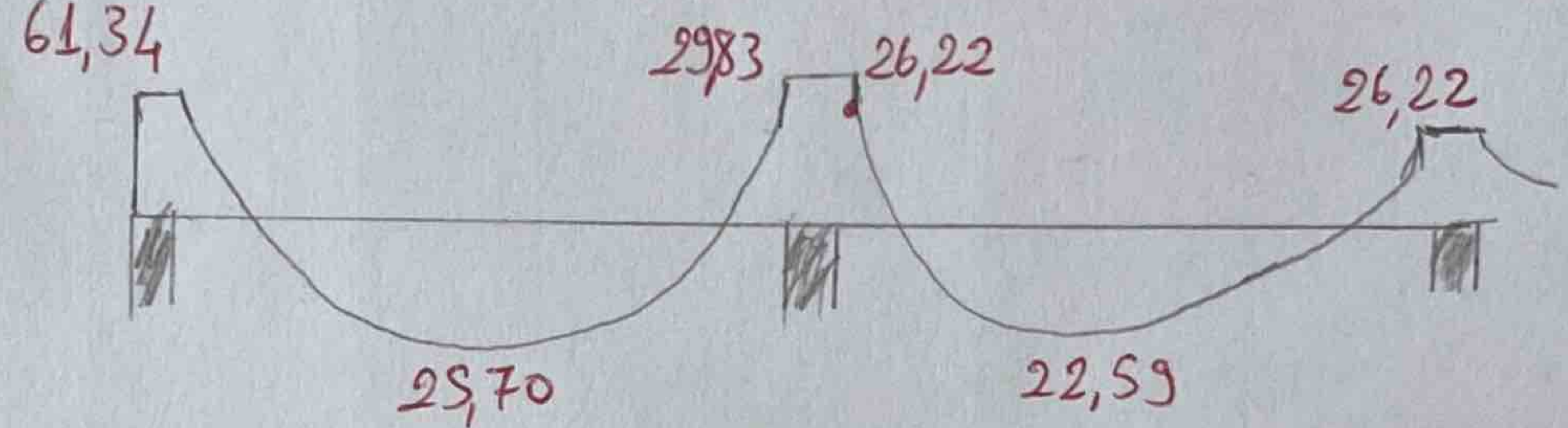
$$w_{\beta t} = 1 + \left(\frac{1 - 0{,}75}{0 - 2{,}5} \times 0{,}205\right) \simeq \underline{0{,}979}$$

3.2. Moment Curves [kNm/m]

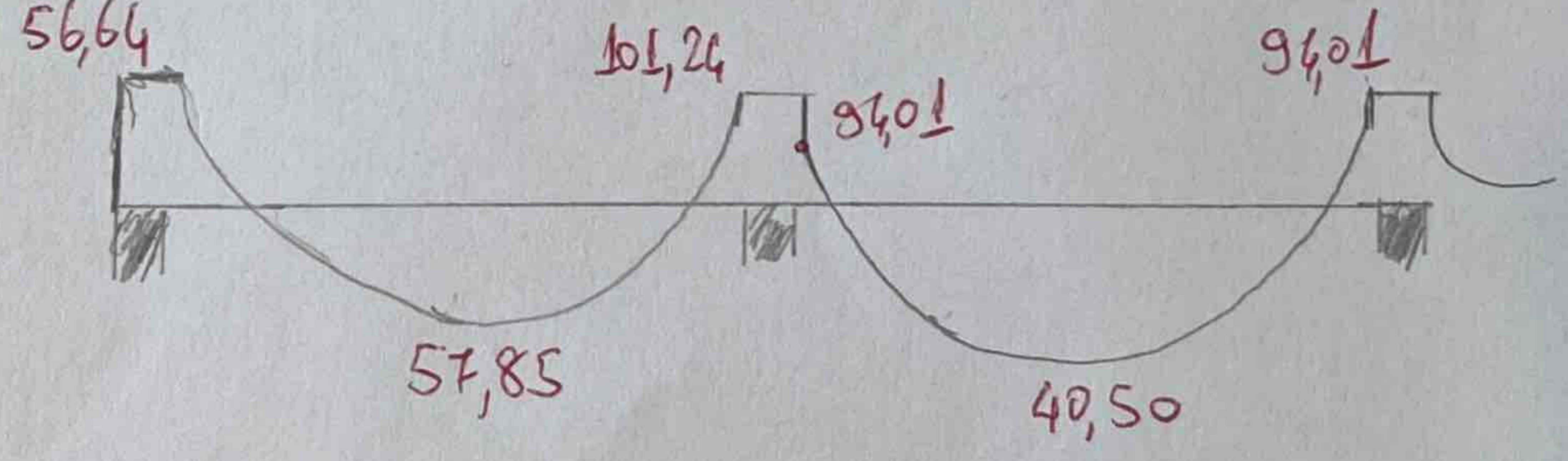
C-column strip



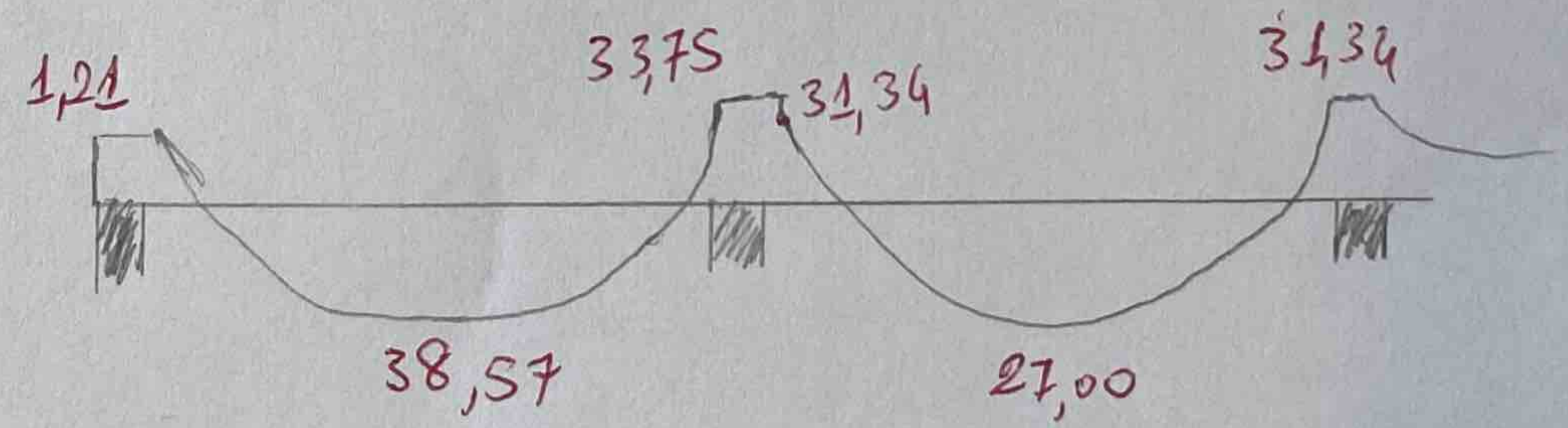
C-middle strip



3-column strip



3-middle strip



Moment in column and middle strips							
Panel	Cross-section	Positive/negative moment M_i (kNm)	Strip	ω	Moment in column/middle strip M_i (kNm)	Width of the strip s_j (m)	Moment per 1m of the slab mj (kNm/m)
C_o	1 (left support)	435.5	no division	1.00	435.50	7.10	61.34
	2 (midspan)	234.5	Column	0.60	140.70	3.45	40.78
			Middle		93.80	3.65	25.70
	3 (right support)	435.5	Column	0.75	326.63	3.45	94.67
			Middle		108.88	3.65	29.83
C_{in}	1 (left support)	393.25	Column	0.75	294.94	3.35	88.04
			Middle		98.31	3.75	26.22
	2 (midspan)	211.75	Column	0.60	127.05	3.35	37.93
			Middle		84.70	3.75	22.59
	3 (right support)	393.25	Column	0.75	294.94	3.35	88.04
			Middle		98.31	3.75	26.22
3_o	1 (left support)	193.8	Column	0.979	189.73	3.35	56.64
			Middle		4.07	3.35	1.21
	2 (midspan)	323	Column	0.60	193.80	3.35	57.85
			Middle		129.20	3.35	38.57
	3 (right support)	452.2	Column	0.75	339.15	3.35	101.24
			Middle		113.05	3.35	33.75
3_{in}	1 (left support)	419.9	Column	0.75	314.93	3.35	94.01
			Middle		104.98	3.35	31.34
	2 (midspan)	226.1	Column	0.60	135.66	3.35	40.50
			Middle		90.44	3.35	27.00
	3 (right support)	419.9	Column	0.75	314.93	3.35	94.01
			Middle		104.98	3.35	31.34