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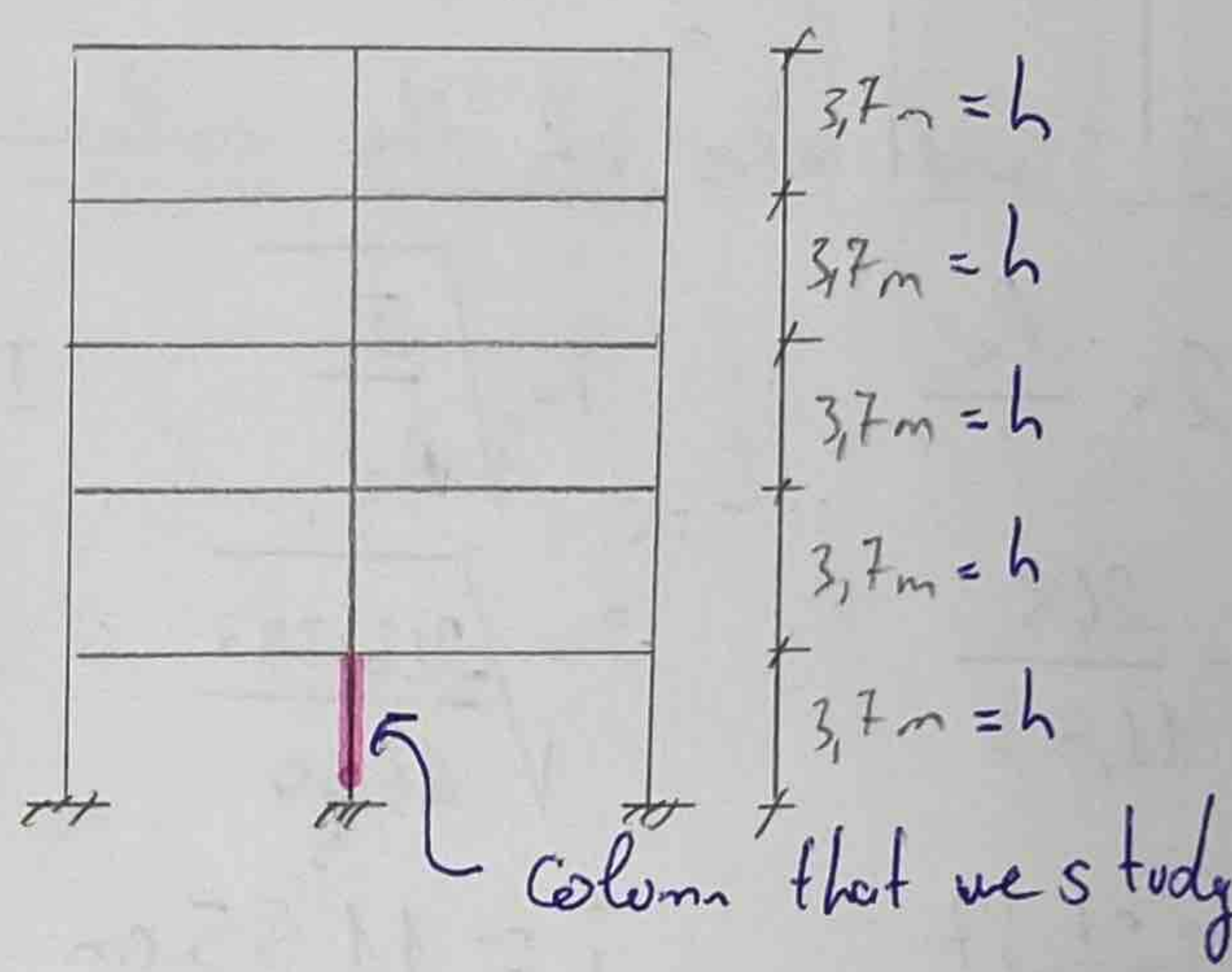
Task 4 : Design of the bottom floor column



1. Geometric imperfections

$$e_i = \Theta_0 \cdot \alpha_h \cdot \alpha_m \cdot \frac{l_0}{2}$$

$$e_i = \frac{1}{200} \cdot 1{,}13 \cdot 0{,}82 \cdot \frac{2{,}48}{2}$$

$$= \underline{0{,}0057 \ m}$$

$$\Theta_0 = \frac{1}{200}$$

$$\alpha_h = \frac{2}{\sqrt{h_c}} = \frac{2}{\sqrt{h - h_B}} = \frac{2}{\sqrt{3{,}7-0{,}6}} \simeq \underline{1{,}13}$$

$$\alpha_m = \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)}$$

$$\alpha_m = \sqrt{0{,}5 \cdot \left(1 + \frac{1}{3}\right)} \simeq \underline{0{,}82}$$

$$l_0 = 0{,}8\,h = 0{,}8 \times (3{,}7 - 0{,}6)$$

$$= \underline{2{,}48 \ m}$$

1.1. Additional moment due to geometry imperfection

From Idea StatiCa

$$M_{imp} = N_{Ed} \cdot e_i$$

Head of the column

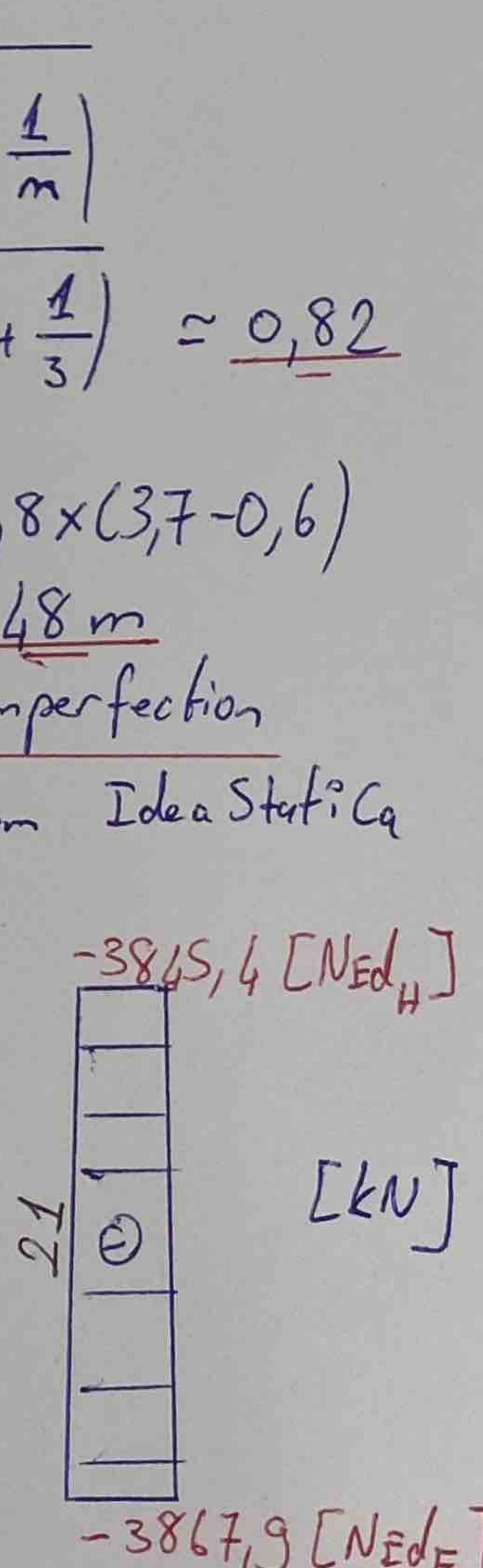
$$|M_{imp}| = N_{Ed_H} \cdot e_i$$

$$= 3845{,}4 \times 0{,}0057 \simeq \underline{21{,}92 \ kN.m}$$

Foot of the column

$$|M_{imp}| = N_{Ed_B} \cdot e_i$$

$$= 3867{,}9 \times 0{,}0057 \simeq \underline{22{,}05 \ kN.m}$$



Bottom column

	M [kN.m]	Head of the column	Foot of the column
	$\lvert M_{imp} \rvert$	21,92	22,05
Comb 1	M_{Ed}	0	0
	$M_{Ed,I}$	21,92	22,05
Comb 2	M_{Ed}	69,7	-39,3
	$M_{Ed,I}$	91,62	61,35

1.2. Slenderness of the column

$$\lambda = \frac{l_0}{i} \quad ; \quad i = \sqrt{\frac{I}{A_c}} \quad ; \quad I = \frac{1}{12} \cdot b_{col} \cdot h_{col}^3$$

$$\lambda = \frac{248}{11,55} \qquad i = \sqrt{\frac{213\,333}{1600}} \qquad = \frac{1}{12} \times 40 \times 40^3 \simeq 213\,333 \text{ cm}^4$$

$$\underline{\lambda \simeq 21,47} \qquad i \simeq 11,55 \text{ cm} \qquad A_c = 40 \times 40 = 1600 \text{ cm}^2$$

$$l_0 = 0,8\,h = 2,48 \text{ m}$$

1.2.1 Limiting slenderness

$$\lambda_{lim} = \frac{20\,ABC}{\sqrt{n}}$$

$$\lambda_{lim} = \frac{20 \times 0,7 \times 1,1 \times 0,7}{\sqrt{1,21}} \simeq \underline{9,80}$$

$A = 0,7$

$B = 1,1$

$$n = \frac{N_{Ed}}{A_c f_{cd}} = \frac{3867,9 \times 10^3}{0,4^2 \times 20 \times 10^6} \simeq 1,21$$

$C = 0,7$

$\lambda > \lambda_{lim}$ The column is slender

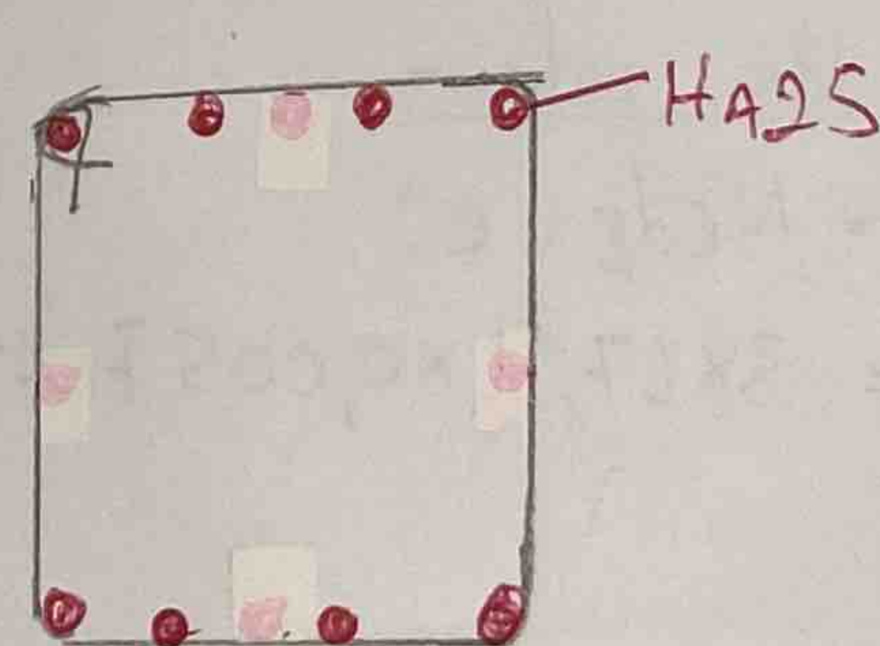
$(21,47) > (9,80)$

2 Design of reinforcement

$$A_{s,req,1} = \frac{N_{Ed} - 0,8\,A_c \cdot f_{cd}}{\sigma_s} = \frac{3867,9 \times 10^3 - (0,8 \times 0,4^2 \times 20 \times 10^6)}{400 \times 10^6} \times 10^4$$

$$\simeq 32,7 \text{ cm}^2$$

From table of steel bars 8 HA25 (39,27 cm^2) > $A_{s,rq\,1}$



2.1 Design of reinforcement 2nd method

$$\mu = \frac{M_{Ed,I}}{b_c \cdot h_c^2 \cdot f_{cd}} = \frac{91{,}62 \times 10^3}{0{,}4 \times 0{,}4^2 \times 20 \times 10^6} \simeq \underline{0{,}071}$$

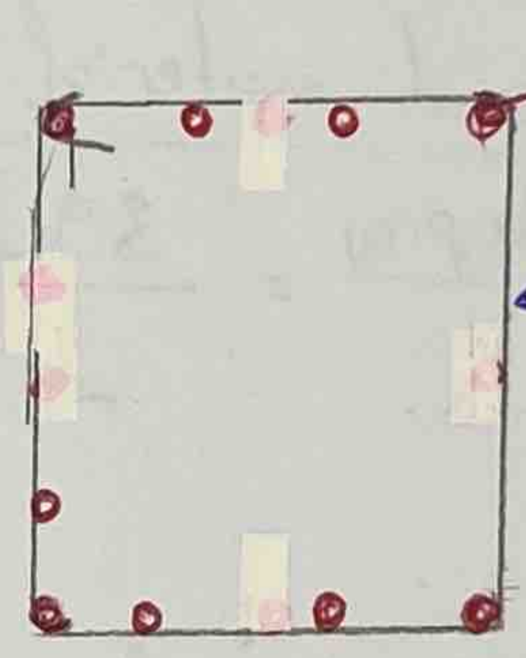
$$\nu = \frac{N_{Ed}}{b_c \cdot h_c \cdot f_{cd}} = \frac{3867{,}9 \times 10^3}{0{,}4 \times 0{,}4 \times 20 \times 10^6} \simeq \underline{1{,}209}$$

From chart $\underline{w = 0{,}4}$

$$A_{s,req2} = \frac{w \cdot A_c \cdot f_{cd}}{f_{yd}} = \frac{0{,}4 \times 0{,}4^2 \times 20}{435} \times 10^4 \simeq \underline{29{,}42\ cm^2}$$

$$A_{s,req} = \max(A_{sreq1} ; A_{sreq2})$$

$$\max(32{,}7 ; 29{,}42) = \underline{32{,}7\ cm^2}$$



Asprov: 8HA25 (39,27 cm²)

2.2. Check detailing rules

$$A_{s,min} = \max\left(0{,}1 \frac{N_{Ed}}{f_{yd}} ; 0{,}002 A_c\right)$$

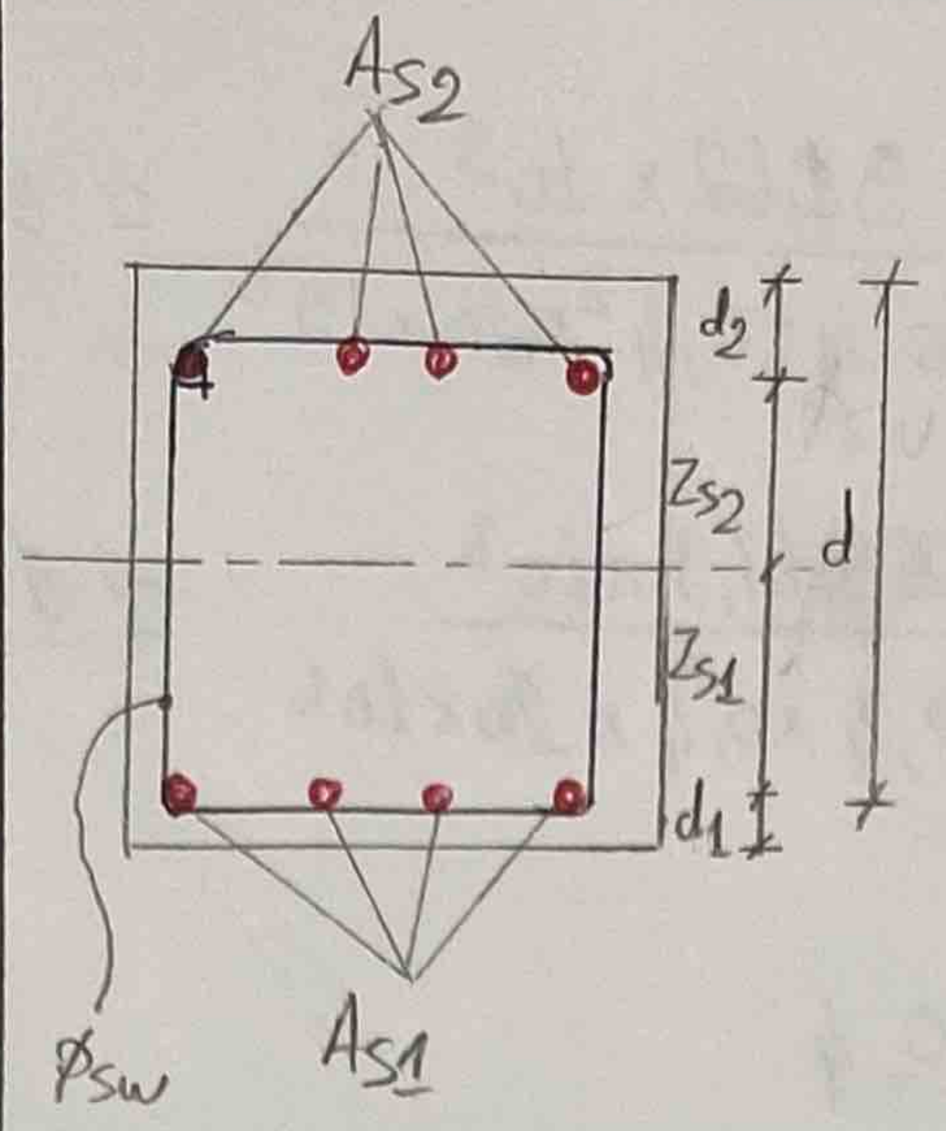
$$\max\left(0{,}1 \times \frac{3867{,}9 \times 10^3}{435 \times 10^6} \times 10^4 ; 0{,}002 \times 40^2\right)$$

$$\max(8{,}89\ cm^2 ; 3{,}2\ cm^2) = 8{,}89\ cm^2 < A_{s,prov}$$

CHECKED !

$A_{smax} = 0{,}04 A_c = 0{,}04 \times 40^2 = 64\ cm^2 > A_{s,prov}$ CHECKED.

3- Check of column - Interaction diagram



Dimensions of cross section

$b_{col} = h_{col} = 400$ mm

$c = 25$ mm (cover depth)

$\phi_{sw} = 6$ mm (hypothesis)

$\phi_s = 25$ mm

$d = h_{col} - c - \phi_{sw} - \frac{\phi_s}{2}$

$400 - 25 - 6 - \frac{25}{2} \approx \underline{356 \text{ mm}}$

$Z_{s1} = Z_{s2} = \frac{1}{2}(h_{col} - 2c - 2\phi_{sw} - \phi_s)$

$= \frac{1}{2}(400 - 2 \times 25 - 2 \times 6 - 25)$

$\approx \underline{156 \text{ mm}}$

$d_1 = d_2 = \frac{h_{col}}{2} - Z_{s1} = \frac{400}{2} - 156 = \underline{44 \text{ mm}}$

Cross section and material properties

$A_{s1} = A_{s2} = \frac{A_{s,prov}}{2} = \frac{39{,}27}{2} \simeq 19{,}63 \text{ cm}^2$

$f_{cd} = 20$ MPa

$A_c = 40^2 = 1600 \text{ cm}^2$

$f_{yd} = 435$ MPa

$\sigma_s = 400$ MPa

$\varepsilon_{cd} = 0{,}0035$ (limit strain of concrete)

$E_s = 200\,000$ MPa (Young's modulus of steel)

Point "0" (Pure compression)

$N_{Rd,0} = b_{col} \cdot h_{col} \cdot f_{cd} + A_{s1} \cdot \sigma_s + A_{s2} \cdot \sigma_s$

$= (400 \cdot 400 \cdot 20 + 1963 \cdot 400 + 1963 \cdot 400) \times 10^{-3} = \underline{4770 \text{ kN}}$

$M_{Rd,0} = (A_{s2} Z_{s2} - A_{s1} \cdot Z_{s1}) \cdot \sigma_s = \underline{0}$

Point 1 (strain in tensile reinforcement $\varepsilon_{s1} = 0$)

$N_{Rd,1} = F_c + F_{c2} = 0{,}8 \cdot b_{col} \cdot d \cdot f_{cd} + A_{s2} \cdot f_{yd}$

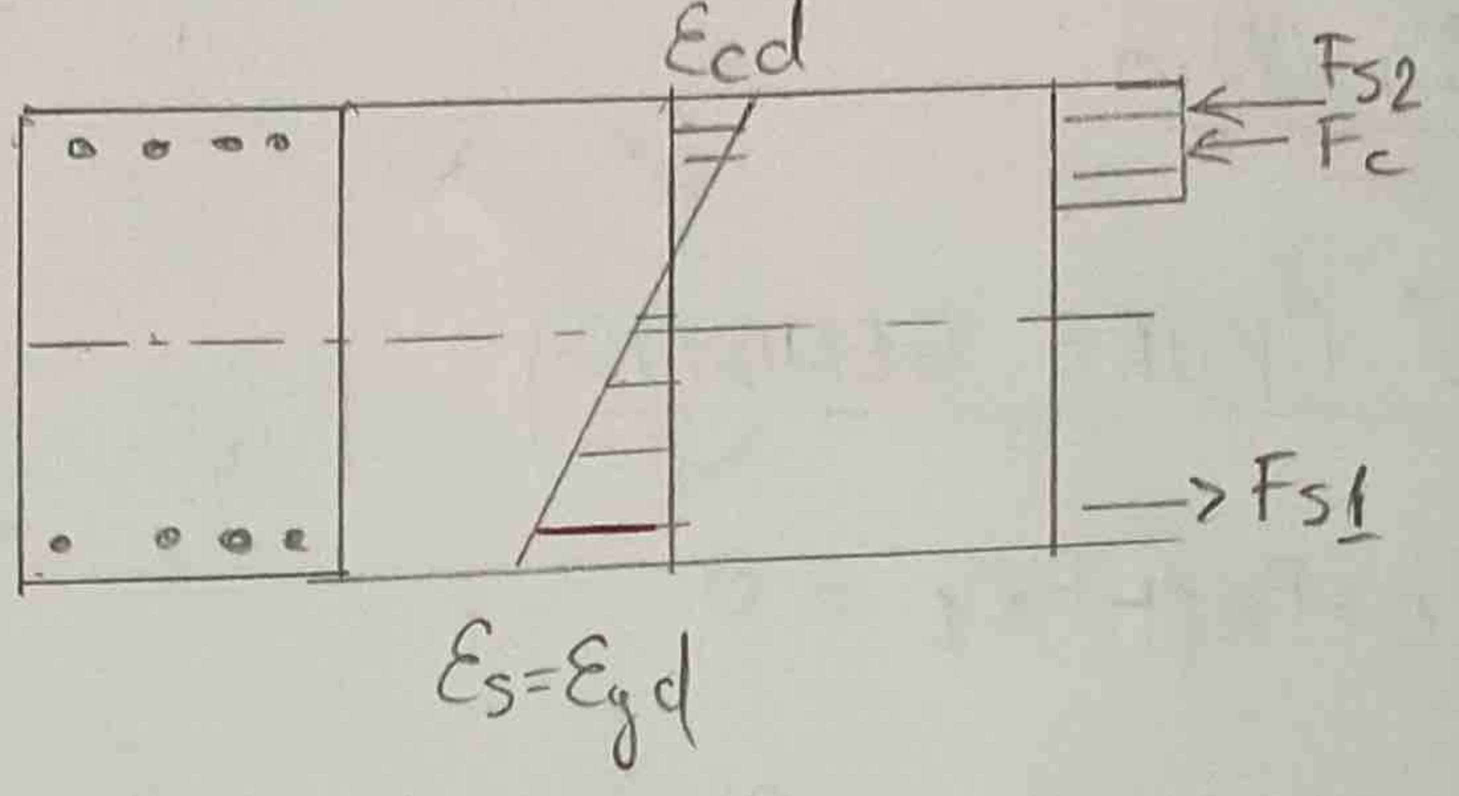
$= (0{,}8 \cdot 400 \cdot 356 \cdot 20 \cdot 1963 \cdot 435) \times 10^{-3} = \underline{3132 \text{ kN}}$

$$M_{Rd,1} = F_c \cdot z_c + F_{s2} \cdot z_{s2} = 0{,}8 \cdot b_{col} \cdot d \cdot f_{cd}\left(\frac{h}{2} - 0{,}4d\right) + A_{s2} \cdot f_{yd} \cdot z_{s2}$$

$$= \left[0{,}8 \cdot 0{,}4 \cdot 0{,}356 \cdot 20\times10^{6}\left(\frac{0{,}4}{2} - 0{,}4 \cdot 0{,}356\right) + 1963\times10^{-6} \cdot 435\times10^{6} \cdot 0{,}156\right] \times10^{-3}$$

$$\simeq \underline{264 \text{ kNm}}$$

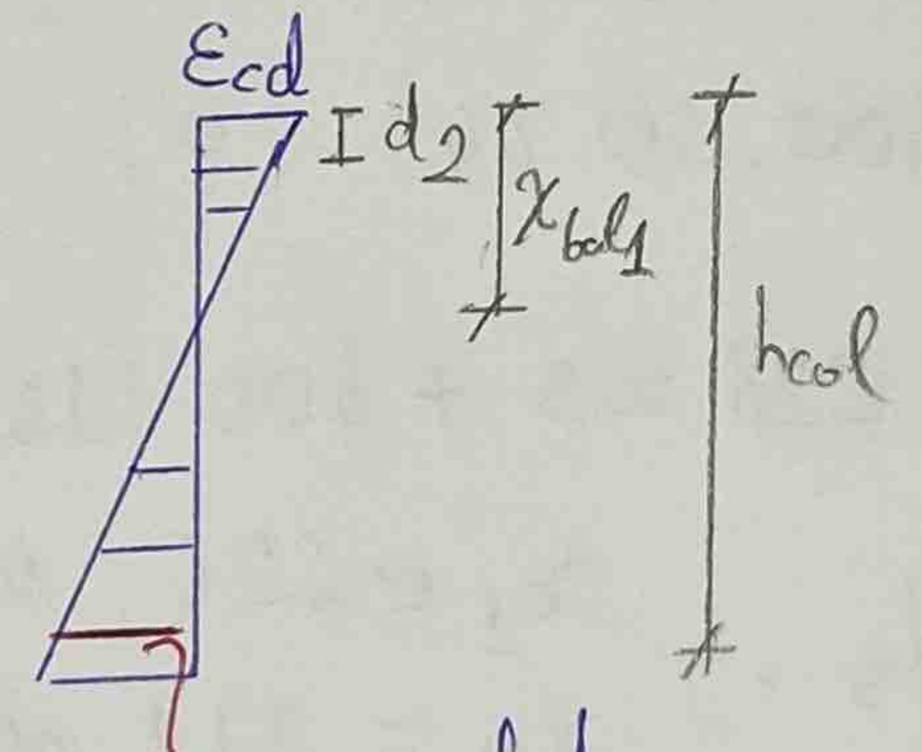
Point 2 (stress in tensile reinforcement is on yield limit ($\sigma_{s1} = f_{yd}$))



$$N_{Rd,2} = F_c + F_{s2} - F_{s1} = 0{,}8 \cdot b_{col} \cdot x_{bal,1} \cdot f_{cd} + A_{s2} \cdot \sigma_{s2} - A_{s1} \cdot f_{yd}$$

$$x_{bal,1} = \xi_{bal,1} \cdot d = \frac{700}{700 + f_{yd}} d = \frac{700}{700 + 435} \cdot 356$$

$$\simeq 219 \text{ mm}$$



$$\frac{\varepsilon_{cd}}{x_{bal,1}} = \frac{\varepsilon_{s2}}{x_{bal,1} - d_2}$$

$$\varepsilon_{s2} = \varepsilon_{cd}\left(1 - \frac{d_2}{x_{bal,1}}\right)$$

$$\varepsilon_{s1} = \varepsilon_{yd} = \frac{f_{yd}}{E_s}$$

$$= 0{,}0035\left(1 - \frac{44}{219}\right) \simeq 0{,}00280$$

$$\underline{2{,}8 \text{‰}}$$

$$\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{435}{200\,000} = 2{,}18\%$$

$\varepsilon_{s2} > \varepsilon_{yd}$ therefore $\underline{\sigma_{s2} = f_{yd}}$
(28‰) (2,18‰) $= 435$ MPa

$$N_{Rd,2} = F_c + F_{s2} - F_{s1} = 0{,}8 \cdot b_{col} \cdot x_{bal,1} \cdot f_{cd} + \overbrace{A_{s2} \cdot \overset{f_{yd}}{\sigma_{s2}} - A_{s1} \cdot f_{yd}}^{=0}$$

$$= (0{,}8 \times 400 \times 219 \times 20) \times 10^{-3} \simeq \underline{1401 \text{ kN}}$$

$$M_{Rd,2} = F_c \cdot z_c + F_{s2} \cdot z_{s2} + F_{s1} z_{s1}$$

$$= 0,8 \cdot b_{col} \cdot x_{bal,1} \cdot f_{cd} \left(\frac{h}{2} - 0,4\, x_{bal,1}\right) + A_{s2} \sigma_{s2} z_{s2} + A_{s1} \cdot f_{yd} \cdot z_{s1}$$

$$= \left[0,8 \cdot 0,4 \cdot 0,219 \cdot 20^{\times 10^6} (0,2 - 0,4 \cdot 0,219) + 1963 \cdot 435 \cdot 0,156 + 1963 \cdot 435 \cdot 0,156\right] \times 10^{-3}$$

$$\simeq \underline{424 \text{ kN.m}}$$

Point "3" (PURE BENDING)

$$N_{Rd,3} = F_C + F_{S2} - F_{S1} = 0$$

To find the value of σ_{s2} we can derive quadratic equation:

$$\sigma_{s2}^2 \cdot A_{s2} - \sigma_{s2}(A_{s1} \cdot f_{yd} + A_{s2} \cdot \varepsilon_{cd} \cdot E_s) + \varepsilon_{cd} \cdot E_s \cdot (A_{s1} \cdot f_{yd} - 0,8 b_{col} \cdot f_{cd} d_2) = 0$$

$$\sigma_s^2 \, 1963 - \sigma_{s2}(1963 \cdot 435 + 1963 \cdot 0,0035 \times 200\,000) + 0,0035 \cdot 200\,000 \times (1963 \cdot 435 - 0,8 \cdot 400 \cdot 20 \cdot 44) = 0$$

$$\sigma_{s2}^2 \, 1963 - \sigma_{s2} \cdot 2\,228\,005 + 400\,613\,500 = 0$$

We obtains 2 roots: $x_1 \simeq 224$ MPa, $x_2 \simeq 911$ MPa

The second root doesn't make sense because its bigger than fyd (435 MPa)

$$\underline{\sigma_{s2} = 224 \text{ MPa}}$$

Height of compressed part $x = \dfrac{A_{s1} \cdot f_{yd} - A_{s2} \cdot \sigma_{s2}}{0,8 \cdot b_{col} \cdot f_{cd}}$

$$x = \frac{1963 \times 435 - 1963 \cdot 224}{0,8 \cdot 400 \cdot 20}$$

$$\underline{x \simeq 64,72 \text{ mm}}$$

$M_{Rd,3} = F_c z_c + F_{s2} \cdot z_{s2} + F_{s1} \cdot z_{s1}$

$= 0{,}8 b_{col} \cdot x \cdot f_{cd} \left(\frac{h}{2} - 0{,}4 x \right) + A_{s2} \sigma_{s2} z_{s2} + A_{s1} \cdot f_{yd} \cdot z_{s1}$

$= 0{,}8 \cdot 0{,}4 \times 0{,}06672 \times 20 \times 10^6 \, (0{,}2 - 0{,}4 \times 0{,}06672) + (1963 \cdot 224 \cdot 0{,}156$

$+ 1963 \cdot 435 \cdot 0{,}156)$

$\simeq 273922 \; N \cdot m \approx \underline{273 \; kN \cdot m}$

Point "4" (strain in compressed reinforcement $\varepsilon_{s2} = 0$)

$N_{Rd,4} = F_{s1} = A_{s1} \cdot f_{yd} = (1963 \cdot 435) \times 10^{-3} \simeq \underline{853 \; kN}$

$M_{Rd,4} = F_{s1} \cdot z_{s1} = A_{s1} \cdot f_{yd} \cdot z_{s1} = 853 \times 0{,}156$

$\simeq \underline{133 \; kN \cdot m}$

Point "5" (pure tension)

$N_{Rd,5} = F_{s1} + F_{s2} = (A_{s1} + A_{s2}) \cdot f_{yd} = (2 \times 1963) \cdot 435 \times 10^{-3}$

$\simeq \underline{1707 \; kN}$

$M_{Rd,5} = F_{s1} \cdot z_{s1} - F_{s2} \cdot z_{s2} = \overbrace{(A_{s1} \cdot z_{s1} - A_{s2} \cdot z_{s2})}^{=0} \cdot f_{yd}$

$= 0$

Restriction of compressive resistance

Minimum eccentricity: $e_0 = \max \left(\frac{h_{col}}{30} ; 20 \, mm \right) = \max \left(\frac{400}{30} ; 20 \right)$

$(13{,}3 ; 20)$

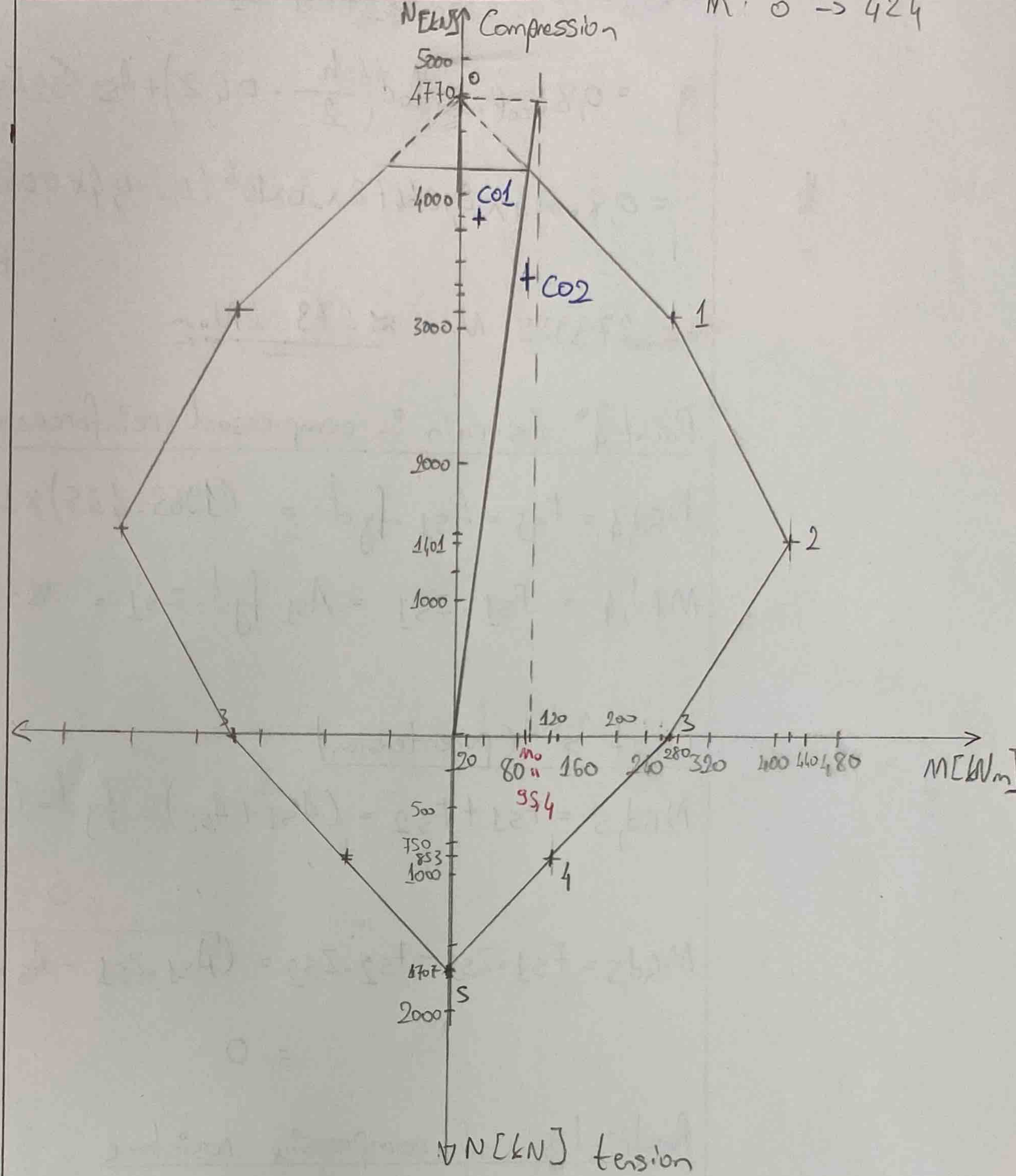
$= \underline{20 \; mm}$

Minimum bending moment: $M_0 = N_{Rd,0} \cdot e_0$

$= 4770 \times 0{,}02 = \underline{95{,}4 \; kN \cdot m}$

INTERACTION DIAGRAM

N 4770 -> 0
M: 0 -> 424



Load combinations	N_{Ed} [kN]	M_{Ed} [kN.m]
CO1	3867,9	22,05
CO2	3366,3	91,62

All the extreme combinations of internal forces lay inside interaction diagram

Column checked