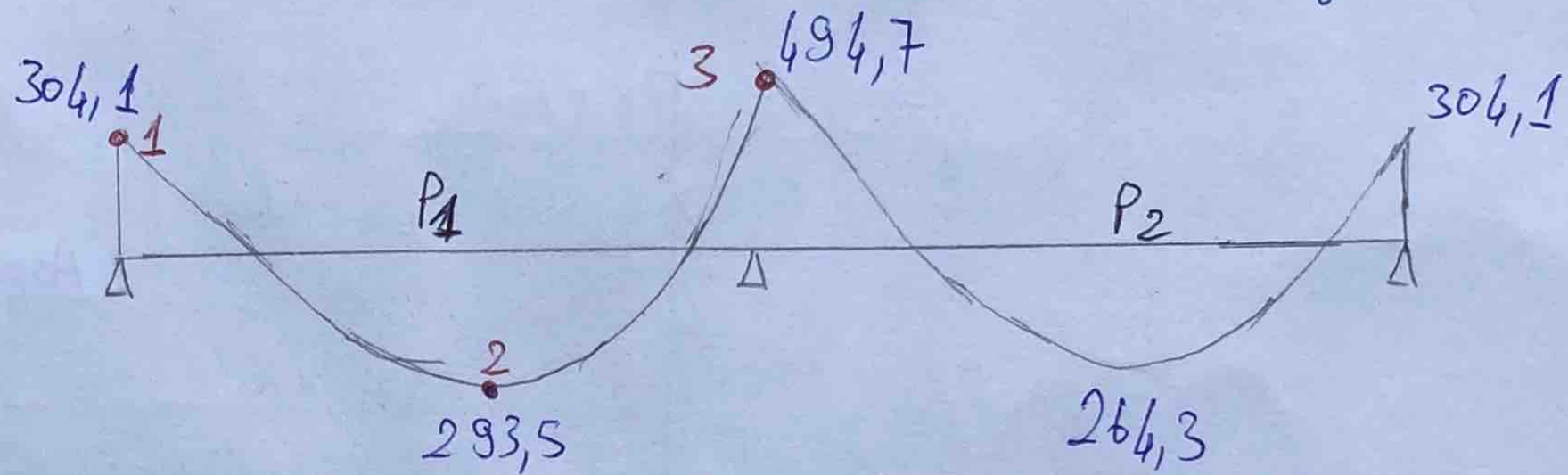
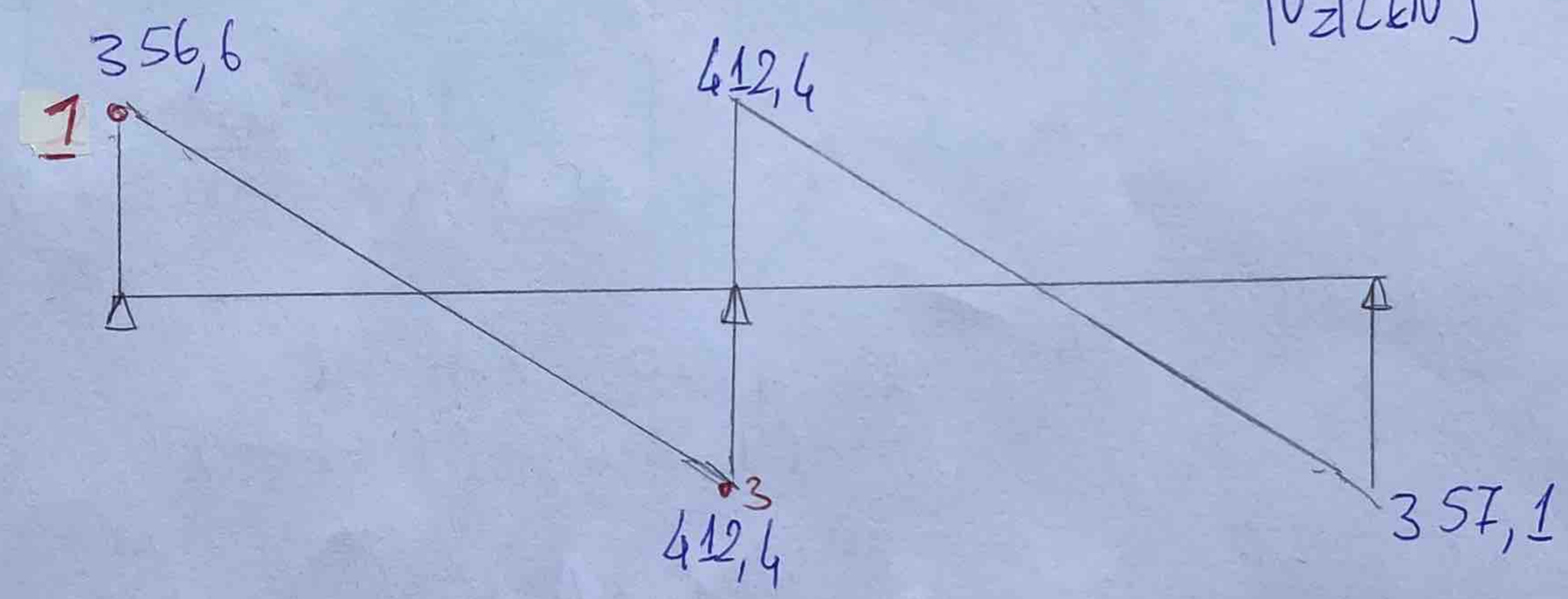
Task 3: Design of reinforcement

Internal forces from "Envelope"

$|M_y|$ [kN.m]



$|V_z|$ [kN]



1. Bending reinforcement P1

$$|M_{Ed,red1}| = |M_{Ed,FEM1}| - |V_{Ed,FEM1}| \frac{b_{sup1}}{2}$$

$$= 304,1 - 356,6 \times \frac{0,4}{2} \simeq \underline{232,78\ kN.m}$$

$$|M_{Ed,red2}| = |M_{Ed,FEM2}| = \underline{293,5\ kN.m}$$

$$|M_{Ed,red3}| = |M_{Ed,FEM3}| - |V_{Ed,FEM}| \cdot \frac{b_{sup3}}{2}$$

$$= 494,7 - 412,4 \times \frac{0,4}{2} = \underline{412,22\ kN.m}.$$

$$A_{s,rq}\,d_1 = \frac{M_{Ed1}}{z \cdot f_{gd}} = \frac{M_{Ed1}}{0{,}9\,d_B \cdot f_{gd}}$$

$$d_B = h_B - \frac{\phi}{2} - \phi_{sw} - c$$

$$= 600 - \frac{16}{2} - 8 - 25 = 559\ mm$$

$$f_{gd} = \frac{f_{gk}}{\gamma_s} = \frac{500}{1{,}15} \simeq 435\ MPa$$

$$-A_{s,rq}\,d_1 = \left(\frac{232{,}78 \times 10^3}{0{,}9 \times 0{,}559 \times 435 \times 10^6}\right) \times 1$$

$$-A_{s,rq}\,d_2 = \frac{M_{Ed2}}{0{,}9\,d_B \cdot f_{gd}} = \left(\frac{293{,}5 \times 10^3}{0{,}9 \times 0{,}559 \times 435 \times}\right.$$

$$-A_{s,rq}\,d_3 = \frac{M_{Ed3}}{0{,}9 \cdot d_B \cdot f_{gd}} = \left(\frac{412{,}22 \times 10^3}{0{,}9 \times 0{,}559 \times 435 \times 10}\right.$$

$-A_{s,prov\,1}$: $4\phi 20$ $(12{,}57\ cm^2)$ $>$ $A_{s,rq\,d1}$ $(10{,}66\ cm^2)$

$A_{s,prov\,2}$: $4\phi 18 + 2\phi 16$ $(10{,}18) + (4{,}02)$ $>$ $A_{s\,rq\,d2}$ $(13{,}41\ cm^2)$

$-A_{s\,prov\,3}$: $4\phi 20 + 4\phi 16$ $(12{,}57\ cm^2)$ $(8{,}04\ cm^2)$ $>$ $A_{s\,rq\,d3}$ $(18{,}84$

1.1 Check of the design

$$-x_1 = \frac{A_{s,prov_1} \cdot f_{gd}}{0{,}8 \cdot b_B \cdot f_{cd}} = \frac{12{,}57 \times 10^{-4} \times 435}{0{,}8 \times 0{,}30 \times 20}$$

$$z = d_B - 0{,}4x$$

$$= 55{,}9 - 0{,}4 \cdot 11{,}4 = \underline{51{,}34\ cm}$$

$$-M_{Rd_1} = A_{s,prov_1} \cdot f_{gd} \cdot z$$

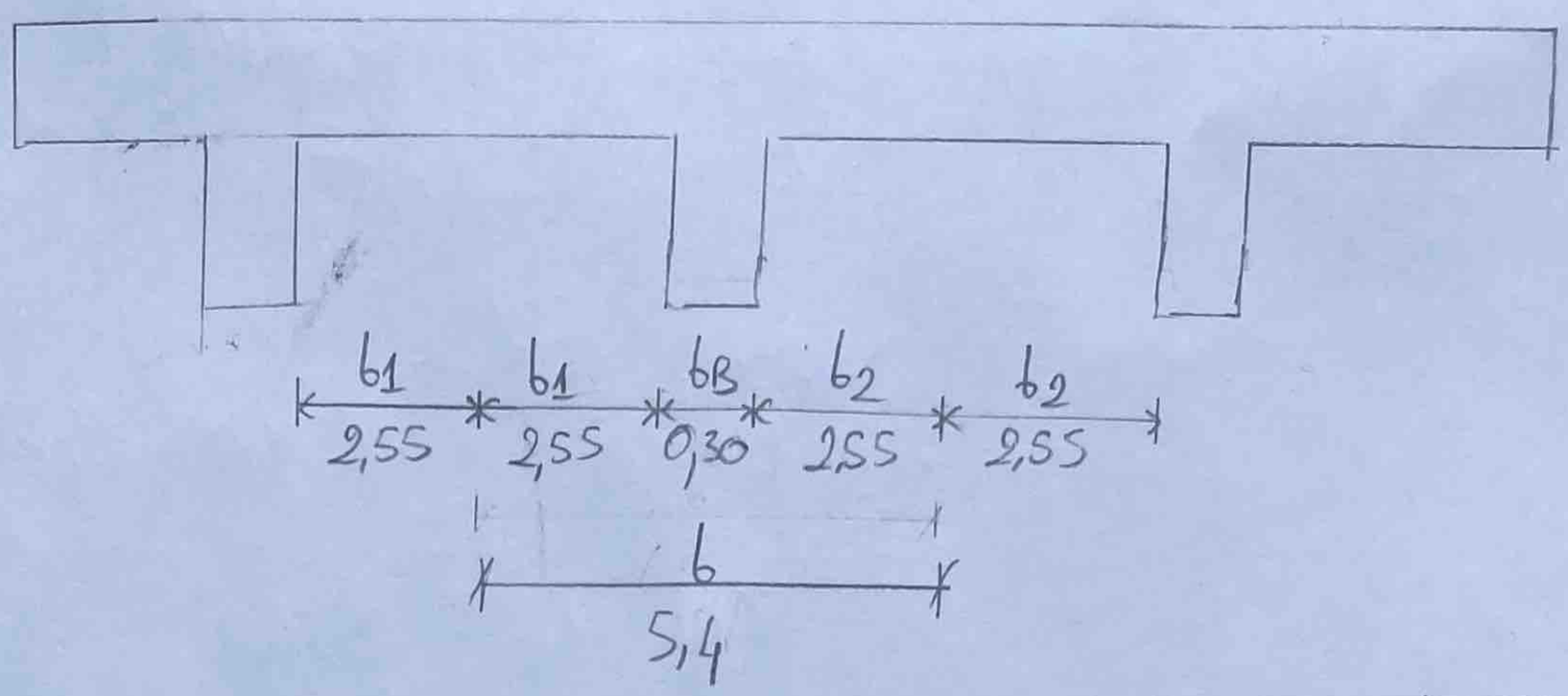
$$= (12{,}57 \times 10^{-4} \cdot 435 \times 10^6 \times 0{,}513$$

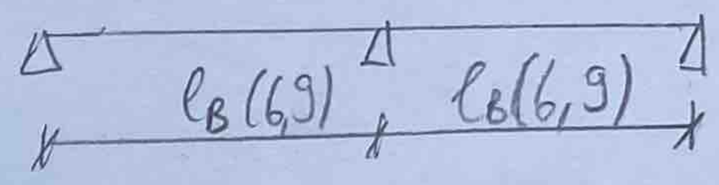
$M_{Ed\,red_1} < M_{Rd\,1}$

$(232{,}78\ kN.m) < (280{,}7\ kN.m)$

We don't usually combine different bars in one group of rebars. It is also impractical to use so many different diameters in one beam.
I would use just 20 mm bars.
1) 4 bars in cross-section 1, all in one layer
2) 5 bars in cross-section 2, all in one layer
3) 6 bars in cross-section 3. In this case, it is very close to the required area, but there will be some allowance in the check, you will see. Also, six bars will not fit in one layer due to minimum spacing requirements.
You have two possibilities - either widen the beam to 350 mm and fit all bars in one layer, or keep 300 mm width and divide the bars in two layers.
I would use the second one, but it will have some consequences - see the next comment.

For the checks, you should recalculate the dB using the actual designed diameter of bars. Also, if you have bars in two layers (possibly in cross-section 3), then dB should be taken to the centroid of both layers, i.e. it would be dB = h - c - stirrup diameter - bar diameter.





$b_{eff} = \Sigma b_{eff,i} + b_B \leq b$

$b_{eff,1} = b_{eff,2} = 0,2\, b_i + 0,1\, l_0 \leq 0,2\, l_0$

$0,2 \times 2,55 + 0,1 \times 0,85 \times 6,9 \leq 0,2 \times 0,85 \times 6,9$

$1,0965 \leq 1,173$ Checked ✓

$b_{eff\,1/2} < b_{1/2}$ Checked ✓

$(1,0965) < (2,25)\,m$

$b_{eff} = (2 \times 1,0965) + 0,30 \leq 5,4$

$= 2,49 < 5,4$ checked ✓

$\boxed{b_{eff} = 2,49\ m}$

$-x_2 = \dfrac{A_{s,prov_2} \cdot f_{yd}}{0,8 \cdot b_{eff} \cdot f_{cd}} = \dfrac{14,20 \times 10^{-4} \times 435}{0,8 \times 2,49 \times 20} \simeq 0,0155$

$= 1,5\ cm$

$z = d_B - 0,4x$

$= 55,9 - 0,4 \cdot 1,5 = \underline{55,3\ cm}$

$M_{Rd,2} = A_{s,prov_2} \cdot f_{yd} \cdot z$

$= (14,20 \times 10^{-4} \times 435 \times 10^6 \times 0,553) \times 10^{-3} \simeq \underline{341,6\ kN.m}$

$\boxed{M_{Edred,2} < M_{Rd,2}\quad (293,5\ kN.m < 341,6\ kN.m)}$

$$-x_3 = \frac{A_{sprov_3} \cdot f_{yd}}{0,8 \cdot 6_B \cdot f_{cd}} = \frac{20,61 \times 10^{-4} \times 435}{0,8 \times 0,30 \times 20} \simeq 0,1867$$

$\underline{18,7 \text{ cm}}$

$$z = d_B - 0,4 x_C = 55,9 - 0,4 \cdot 18,7$$
$$\simeq \underline{48,4 \text{ cm}}$$

$$M_{Rd,3} = A_{sprov3} \cdot f_{yd} \cdot z$$
$$= (20,61 \times 10^{-4} \times 435 \times 10^6 \cdot 0,484) \times 10^{-3} \simeq 434 \text{ kN.m}$$

$M_{Edred_3} < M_{Rd,3}$
(412,22) (436 kN.m)

1.3 Detailing rules

$$\xi_1 = \frac{x_1}{d_B} \leq \min\left(\xi_{bd,L} = \frac{700}{700 + f_{yd}} ; 0,45\right)$$

$$= \frac{11,4}{55,9} \overset{?}{\leq} 0,45 \Leftrightarrow 0,20 < 0,45 \quad \text{Checked!}$$

$$\xi_2 = \frac{x_2}{d_B} \overset{?}{\leq} 0,45 \Leftrightarrow \frac{1,5}{55,9} \overset{?}{\leq} 0,45 \Leftrightarrow 0,03 < 0,45$$
Checked!

$$\xi_3 = \frac{x_3}{d_B} \overset{?}{\leq} 0,45 \Leftrightarrow \frac{18,7}{55,9} \overset{?}{\leq} 0,45 \Leftrightarrow 0,33 < 0,45 \quad \text{Checked}$$

$$A_{smin} = \max\left(0,26 \cdot \frac{f_{ctm}}{f_{yk}} 6_B \cdot d_B ; 0,0013 6_B \cdot d_B\right) \times 10^4$$

$$= \max\left(0,26 \cdot \frac{2,9}{500} \cdot 0,30 \cdot 0,559 ; 0,0013 \cdot 0,30 \cdot 0,559\right)$$

$$= \max\left(2,53 \text{ cm}^2 ; 2,18 \text{ cm}^2\right) = \underline{2,53 \text{ cm}^2}$$

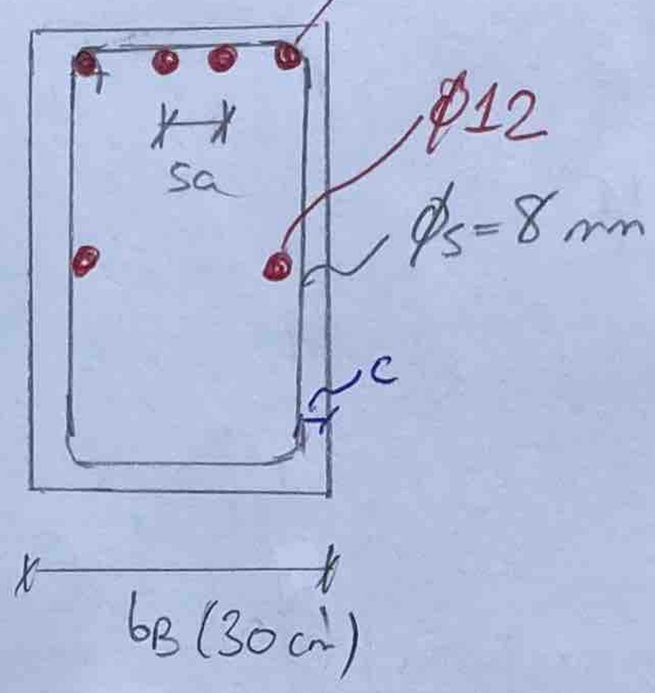
$A_{sprov1} = 12,57 \text{ cm}^2$
$A_{sprov2} = 16,20 \text{ cm}^2$
$A_{sprov3} = 20,61 \text{ cm}^2$
$\Big\} > A_{smin} (2,53 \text{ cm}^2)$

$A_{smax} = 0,04 \cdot b_B \cdot d_B \times 10^4$

$= 0,04 \times 0,30 \times 0,559 \times 10^4 = 67,08\ cm^2$

$A_{sprov1} = 12,57\ cm^2$
$A_{sprov2} = 14,20\ cm^2$
$A_{sprov3} = 20,61\ cm^2$
} $<$ A_{smax} (67,08 cm²).

Section 1



$$s_a = \frac{b_B - 2c - 2\phi_{st} - \phi}{n-1}$$

$$= \frac{300 - 2\times25 - 2\times8 - 20}{4-1}$$

$$\simeq \underline{71\ mm}$$

$s_c = s_a - \phi$

$= 71 - 20 = \underline{51\ mm}$

$s_{a,max} = \min(2h_B ; 250\ mm)$

$= \min(2\times600 ; 250mm)$

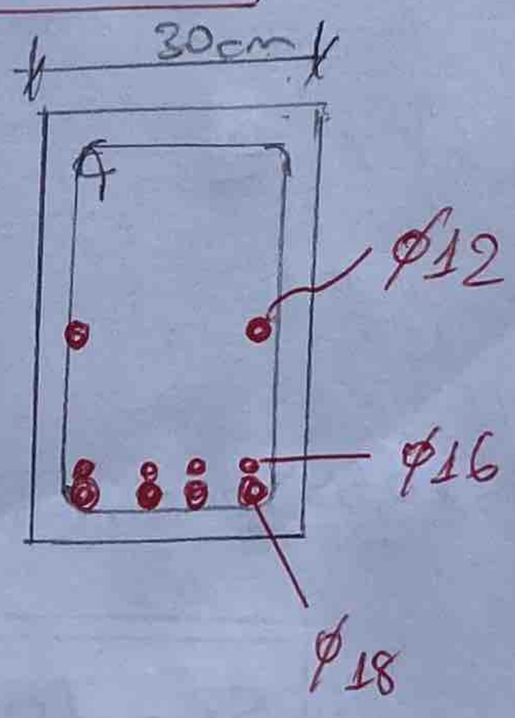
$= \underline{250\ mm} > 71\ mm$ checked!

$s_{c,min} = \max(20\ mm ; 1,2\phi)$

$= \max(20mm ; 1,2\times20)$

$= \underline{24\ mm} < 51\ mm$ checked!

Section 2



$$s_a = \frac{b_B - 2c - 2\phi_s + \phi_{18}}{4-1}$$

$$= \frac{300 - 2\times25 - 2\times8 - 18}{4-1}$$

$= \underline{72\ mm} < s_{a,max}(250\ mm)$ checked!

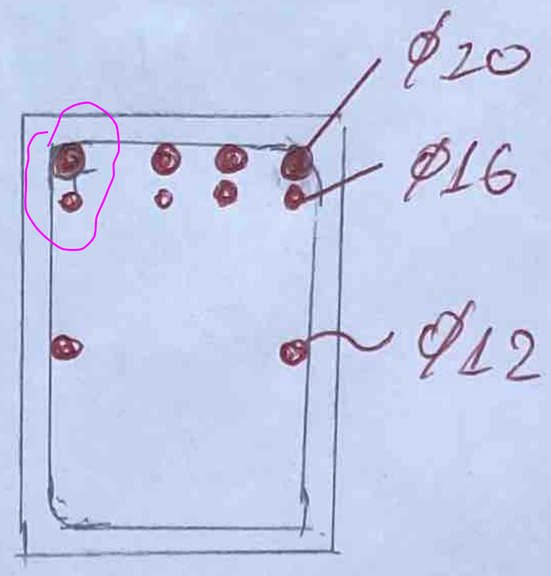
$s_c = s_a - \phi_{18}$

$= 72 - 18 = \underline{54\ mm} > s_{c,min}(21,6mm$ Checked!

$s_{c,min} = \max(20mm ; 1,2\phi_{18})$

$= \max(20mm ; 1,2\times18) = \underline{21,6mm}$

Section 3



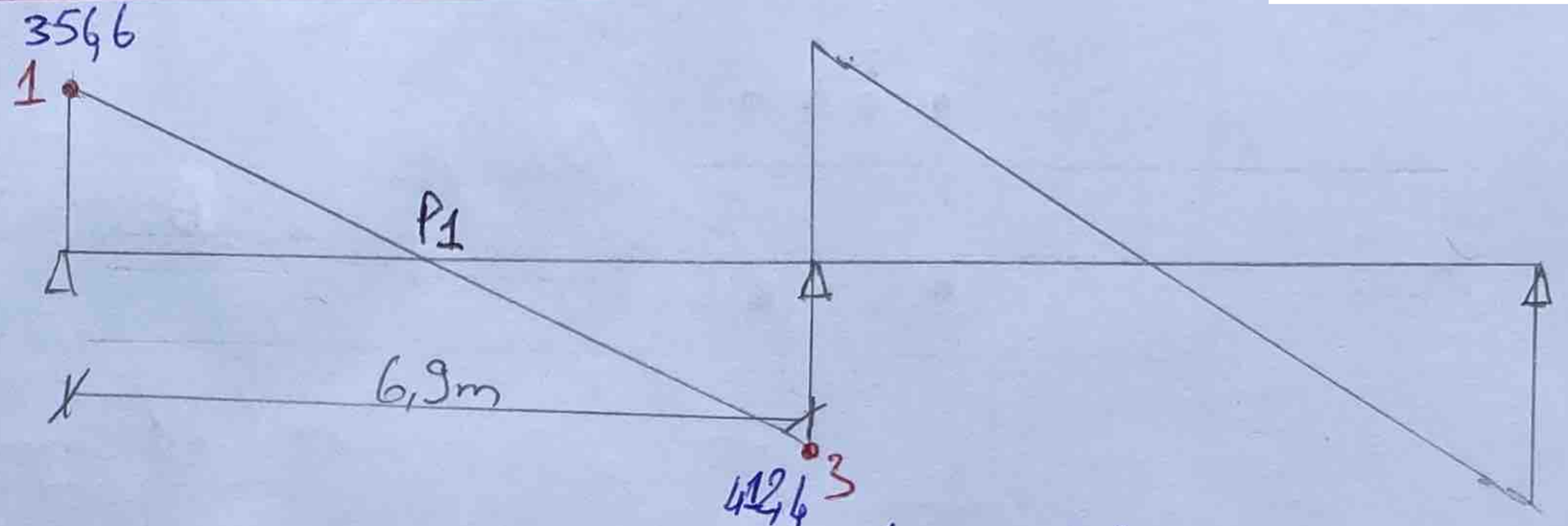
It's
// Seef

Samax
250mm

If you have bars in two layers, then the two bars laying above each other create a so-called "bundle". To calulate sc,min, you have to use the effective diameter of the bundle instead of actual diameter of the bars. You have to use the formula below. nb is the number of bars in the bundle, i.e. 2 in your case.

$$\phi_n = \phi\sqrt{n_b}$$

2. Shear reinforcement



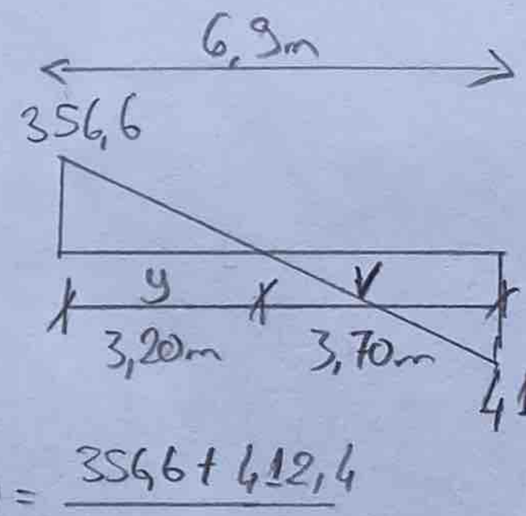
$$-|V_{Ed_3}| = |V_{Ed,max,FEM_3}| \frac{V-\left(\frac{b_{sup}}{2}+d_B\right)}{V}$$

$$-|V_{Ed,3}| = 412,4 \times \frac{3,70-\left(\frac{0,40}{2}+0,559\right)}{3,70}$$

$$V_{Ed,3} \simeq 327,8 \text{ kN}$$

$$-s_1 \leq \frac{A_{sw}\cdot f_{yd}}{V_{Ed,3}}\Delta l$$

$$\begin{cases} A_{sw} = \dfrac{n\pi\phi_{sw}^2}{4} = \dfrac{2\times\pi\times 8^2}{4} \simeq 100 \text{ mm}^2 \\ \Delta l = z\times\cot g\,\Theta = 48,4\times 1,5 = 72,6 \text{ cm} \end{cases}$$



$$q = \frac{356,6+412,4}{6,9}$$

$$\simeq 111,45 \text{ kN}$$

$$y = \frac{356,6}{111,45} \simeq 3,20 \text{ m}$$

$$V = 6,9-3,2 = 3,70 \text{ m}$$

$$s_1 \leq \left(\frac{100\times 435}{327,8\times 10^3}\times 0,726\right)\times 10^2$$

$$s_1 \leq 9,6 \text{ cm}$$

$$-s_1 \leq 0,75 d_B = 0,75\times 55,9 \simeq 41,9 \text{ cm}$$

$$s_1 \leq 400 \text{ mm}$$

$$s_1 \geq 100 \text{ mm}$$

The rule of $s_1 \geq 100$ mm isn't satisfied. Therefore, we have to change the diameter of stirrups.

with $\phi_{sw} = 10$ mm

$$A_{sw} = \frac{n \cdot \pi \cdot \phi_{sw}^2}{4} = \frac{2 \cdot \pi \cdot 10^2}{4} \simeq 157 \text{ mm}^2$$

$$s_1 \leq \frac{A_{sw} \cdot f_{yd}}{V_{Ed,3}} \cdot \Delta l = \left(\frac{157 \times 435}{327{,}8 \times 10^3} \times 0{,}726 \right) \times 10^2$$

$s_1 \leq 15{,}1$ cm

$$\begin{cases} s_1 \leq 40 \text{ cm} \\ s_1 \leq 0{,}75 d_B = 41{,}9 \text{ cm} \\ s_1 \geq 10 \text{ cm} \end{cases}$$ checked!

$\boxed{s_1 = 15 \text{ cm}}$

2.1 Check shear resistance

$$- V_{Rd,sw,1} = \frac{A_{sw} \cdot f_{yd}}{s_1} \Delta l \overset{?}{\geq} V_{Ed,3}$$

$$V_{Rd,sw1} = \left(\frac{157 \times 435}{0{,}15} \times 0{,}726 \right) \times 10^{-3} \simeq \underline{330{,}5 \text{ kN}}$$

$$V_{Ed,3} < V_{Rd,sw1}$$
$$(327{,}8 \text{ kN}) < (330{,}5 \text{ kN})$$

2.2. Check shear reinforcement ratio

$$\rho_{sw\,min} = \frac{0{,}08 \times \sqrt{f_{ck}}}{f_{yk}} = \frac{0{,}08 \cdot \sqrt{30}}{500} = 8{,}7 \times 10^{-4}$$

$$\rho_{s\,max} = \frac{0{,}5 \nu \cdot f_{cd}}{f_{yd}} \quad \text{with} \quad \nu = 0{,}6 \left(1 - \frac{f_{ck}}{250}\right)$$

$$= \frac{0{,}5 \times 0{,}528 \times 20}{435} \qquad = 0{,}6 \left(1 - \frac{30}{250}\right) = 0{,}528$$

$$= 1{,}2 \times 10^{-2}$$

$$\rho_{sw_1} = \frac{A_{sw}}{b_B s_1} = \frac{1{,}57}{30 \times 15} = 3{,}4 \times 10^{-3}$$

$\rho_{s\,max} > \rho_{sw_1} > \rho_{sw\,min}$ checked!

2.3 Stirrups in the middle part (s_{max})

$s_{max} \leq \min(0,7 d_B ; 600\,mm)$

$\leq \min(0,75 \cdot 55,8 ; 40)$

$s_{max} \leq 41,9\ cm$

$\boxed{s_{max} = 40\ cm}$

2.4 Check shear reinforcement ratio

$$\rho_{sw,min} = \frac{0,08\sqrt{f_{ck}}}{f_{yk}} = 8,7 \times 10^{-4}$$

$\rho_{s,max} = 1,2 \times 10^{-2}$

$$\rho_{sw,2} = \frac{A_{sw}}{b_B s_{max}} = \frac{1,57}{30 \times 60} \simeq 1,3 \times 10^{-3}$$

$\rho_{sw,min} < \rho_{sw2} < \rho_{smax}$ checked!

- Shear force for which smax is sufficient

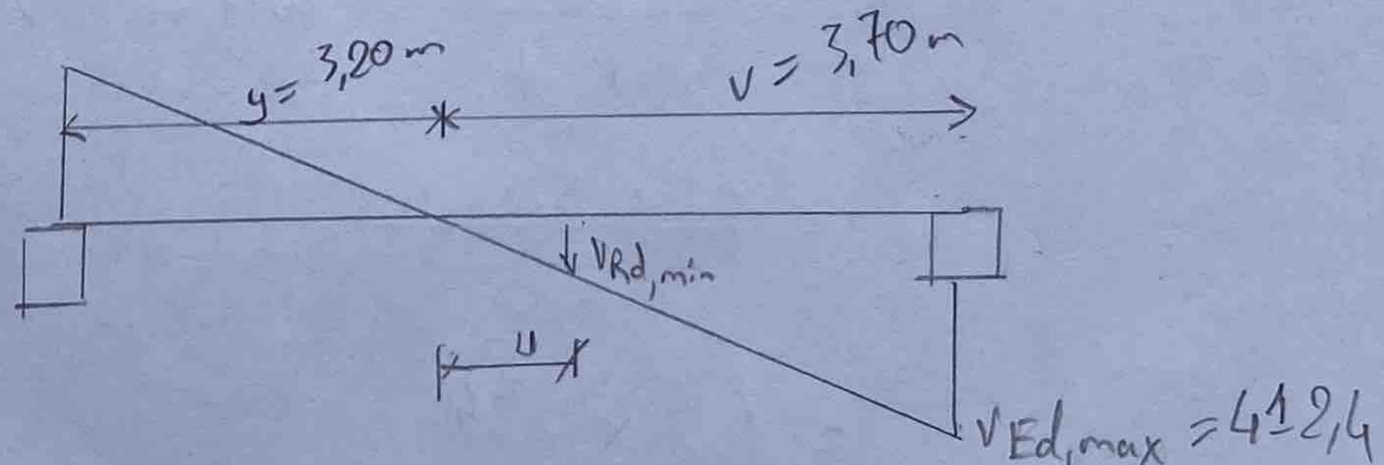
$$V_{Rd,min} = \frac{A_{sw} \cdot f_{yd}}{s_{max}} \Delta l \quad \text{with } \Delta l = z \cdot \cot g\theta$$

$$= 55,3 \times 1,5$$

$$\simeq 83\ cm$$

$$= \left(\frac{157 \times 435}{0,4} \times 0,83\right) \times 10^{-3}$$

$\simeq 161,7\ kN$



$$\frac{V_{Rd,mid}}{U} = \frac{V_{Ed,max}}{V} \Rightarrow U = \frac{V_{Rd,mid} \cdot V}{V_{Ed,max}} = \frac{161,7 \cdot 3,70}{412,4}$$

$\simeq 1,27\ m$

- Length of the area reinforced by stirrups with spacing s_{max}:

$W = U + \Delta l$

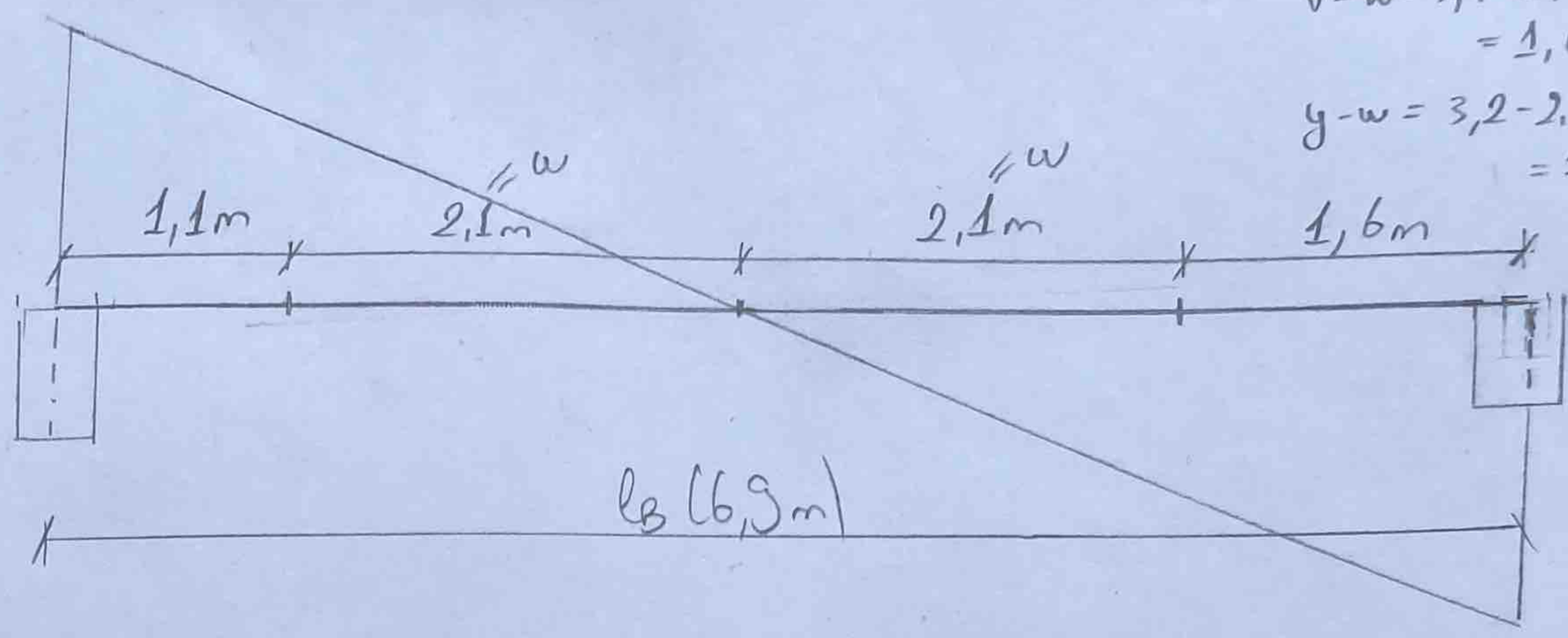
$= 1,27 + 0,83 = 2,1\ m$

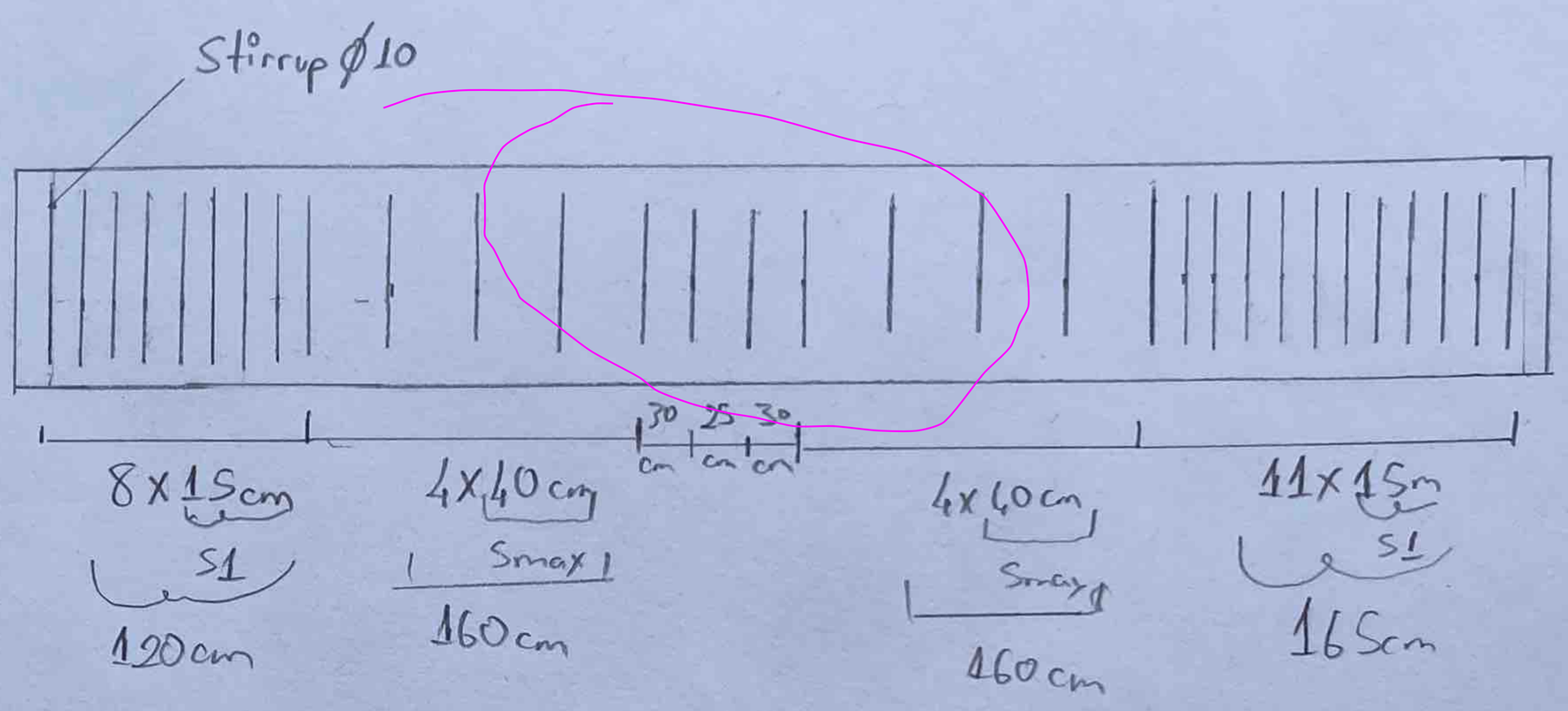
Scale: 1/50

w = 2,1 m

v - w = 3,7 - 2,1
= 1,6 m

y - w = 3,2 - 2,1
= 1,1 m





Here I would start with 400 mm spacing in the middle.
Then you will recieve some smaller gaps on the ends,
but it does not matter.
The way you designed is theoretically fine, but impractical
for the workers who prepare the reinforcement.