Composite Frames under both Vertical and Horizontal Loading

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ABSTRACT

Composite frames with semi-rigid joints are studied using numerical analysis methods, in order that effects of semi-rigidity of connections are introduced into global analysis. Differing from plastic approach and non-linear collapse analysis, characteristics of joints are employed into linear elastic analysis, by introducing elastic element end releases.

Semi-rigidity of joints is possible to be employed for moment redistribution of composite frames, leading to a suitable coherence between hogging and sagging bending capacity of composite beams, while effects of concrete cracking is limited but unneglectable. Critical load factor, together with moment redistribution and deflection are analysed within this paper. Suggestion on joint stiffness selection and using of FEM programme are provided.

Keywords: Semi-rigid composite joints, composite frame, critical load factor, moment redistribution, cracking analysis

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Composite Frames under both Vertical and Horizontal Loading

1. INTRODUCTION

Moment connections are widely used in steel and composite frames. The semi-rigidity of steel connections has been acknowledged since steel was used as structure materials, and research on its effects on frames have been widely reported in the last decades. As for composite frames, full-rigid full-strength connections seem to be impossible due to their complexity, or to be unnecessary due to their high expense.

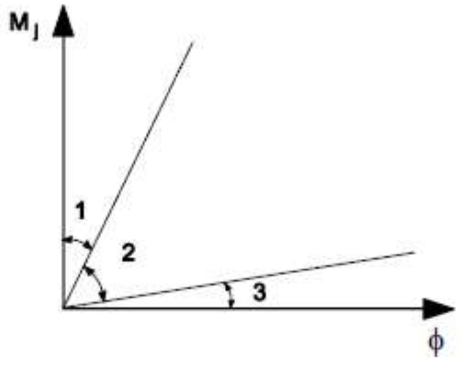
This paper, starting from simplified plastic analysis methods by Nethercot (Nethercot & Stylianidis, 2008) and frame instability research by Demonceau (Demonceau, 2008), tries to introduce semi-rigidity joints into global analysis of composite frames, based on numerical analysis using a commercial software package. The study highlights the possibility of global analysis of semi-rigid frames, by means of application of end elastic releases on beam ends. Limitations of finite element analysis programmes are also introduced.

1.1. Joints classification

As recognised, steel and/or composite joints have some degree of rigidity or deformability, which varies in accordance with the applied loading (Simões da Silva, 2008). Composite joints, as well as steel joints, are classified, in Eurocode4 (EN1994-1-1, 2009) and in Eurocode3-1-8 (EN1993-1-8, 2005), into three categories either by rotational stiffness or by resistance to bending moments. The categories are:

- for stiffness: rigid, semi-rigid, and nominally pinned;
- for strength: full-strength, partial-strength, and nominally pinned.

The three stiffness classes, which are relevant to elastic global analysis, are shown in Figure 1.1. Comparing with a 'rigid' joint, the flexibility of a connection leads to probably significant redistribution of elastic moments in a frame (Johnson, 2004), related with the stiffness of the joints.



Zone 1: rigid, if $S_{j,ini} \geq k_b EI_b / L_b$

where:

k_b = 8 for frames where the bracing system reduces the horizontal displacement by at least 80 %

k_b = 25 for other frames, provided that in every storey $K_b/K_c \geq 0,1$ *)

Zone 2: semi-rigid

All joints in zone 2 should be classified as semi-rigid. Joints in zones 1 or 3 may optionally also be treated as semi-rigid.

Zone 3: nominally pinned, if $S_{j,ini} \leq 0,5 EI_b / L_b$

*) For frames where $K_b/K_c < 0,1$ the joints should be classified as semi-rigid.

Key:

K_b is the mean value of I_b/L_b for all the beams at the top of that storey;
K_c is the mean value of I_c/L_c for all the columns in that storey;
I_b is the second moment of area of a beam;
I_c is the second moment of area of a column;
L_b is the span of a beam (centre-to-centre of columns);
L_c is the storey height of a column.

Figure 1.1: Classification of joints by stiffness (Figure 5.4 in EN1993-1-8)

When the rotational behaviour is considered, the structural properties of a joint assumed in design include bending resistance, rotational stiffness and rotation capacity, noted by $M_{j,Rd}$, S_j (or initial stiffness $S_{j,ini}$) and Φ_{Cd} respectively.

1.2. Semi-rigid joints in Composite frames

Semi-rigid joints are almost unavoidable in composite frames due to the composite effects of steel and concrete. Pinned joints might be recommended for braced frame, but the longitudinal reinforcement in the slab, responsible for control of cracking, will lead to partly resistance for composite beams connected to an internal column. As for rigid, full-strength connections, extended end plates with stiffeners will probably needed, as for steel joints. Even through the joints and beam sections are strengthened by additional haunches, the column web zone may yield some degree of flexibility, not considering the expense caused by complexity.

Under this criterion, semi-rigid joints or semi-rigid frames are widely studied during the last decades. Nethercot (Nethercot D. , 1995) (Nethercot & Stylianidis, 2008) recommended simplified plastic design methods with necessary verification of required rotation capacity of beams and of joints. This method provides an intermediate solution between the traditional “simple construction” and "continuous construction", resulting in semi-continuous construction or "partially restrained (PR) construction". Within this plastic method, the rotation capacity, instead of rotation stiffness of joints, governs the design of frames.

This plastic analysis assumes a "plastic mechanism", with the hinges firstly form at the joints. In other words, the column should be strong to avoid failure from strength or instability. As steel or even composite columns involved, local and/or global instability should be considered. Critical load or critical load factor is mostly employed in this field.

For some load case or combination, critical load factor α_{cr} is calculated with eigenvalue methods, based on elastic theory. The materials, elements and all other components involved into the structure are assumed to be idealized elastic. From strength side, a plastic load factor α_{pl} is used to foresee the ultimate load of structures. Since loading capacity of frames is related with their elastic and strength characteristic, both factors, α_{cr} and $\alpha_{pl,}$ contribute to the ultimate loading capacity. The "Merchant-Rankine approach" and a new "Eurocode" approach (Demonceau, 2008) employ the two factor into an equation to foresee the static capacity of structures.

1.3. Frames and joints studied

This paper is intended to study property of composite frames, mostly of composite beams with semi-rigid connections. Effects of cracking, and that of joint rigidity, are studied.

The study starts with a fixed beam, where the cracking property of composite beam is recognized. This composite beam is then put into a rigid frame under load cases, in order that the effects of stiffness ratio between column and beam are studied. In the following parts, rotational springs are introduced, trying to approach suitable global analysis methods for semi-rigid frames.

The characteristics of joints studied in this paper are cited from technical report of a European research project. The end plate composite joints are classified as semi-rigid in most frames.

The studied composite beams, which are directly end-supported or connected with columns in frames, are assumed to be laterally restrained in span, as a result, lateral buckling is not possible to appear.

A series of one-span one-storey steel and composite frames are employed, with parametric factors, namely span, height, and joint stiffness, Critical load factor of frame, moment of critical section, joint rotation and deflection are studied. The dominated analysis method is linear elastic analysis, while rotation stiffness of joints is realised with linear elastic end releases. (Dlubal, 2013a)

2. COMPOSTIE BEAMS UNDER VERTICAL LOADINGS

A fixed composite beam is studied here, to highlight the effects of concrete cracking and/or of joint rotation. As simple composite one is more useful for one-span beam, this case rarely happens in practice, but forms principle of plastic design.

Self-weight of structure elements is deactivated for highlighting the effects of external loading.

2.1. Quasi-plastic approach

For an idealized fixed beam under vertical loading, such as self-weigh or imposed loading, the hogging moments are larger than sagging moments in span, which gives us an assumption that the moments at the supports, due to increasing loading, will reach the yielding stage, and results plastic hinges there. This is proposed by Nethercot for plastic design of semi-continuous frames.

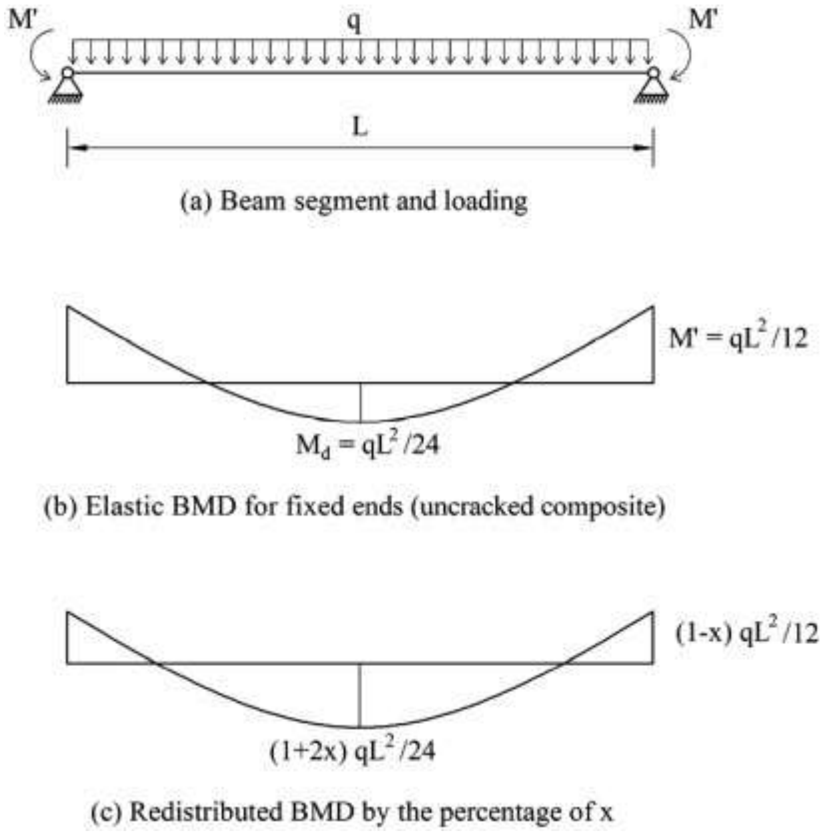


Figure 2.1 Moments redistribution and quasi-plastic design (Nethercot & Stylianidis, 2008, Fig2)

Since the hogging bending resistance (M_{pl-}) of a composite beam is normally less than its sagging bending capacity (M_{pl+}), it is reasonable to assume the plastic hinges appears firstly at hogging zone, namely the fixed ends. Put it further, when the bending resistance of joints (M_j) is even less, the hinges are more probably to form before M_{pl-} is reached.

Following this assumption, moments redistribution is developed after M_j (when less than M_{pl-}) is reached at the joints, until the maximal sagging moment at span reaches at M_{pl+}, or at a reasonable percentage of M_{pl+}. The level of span moment, which is linked to the connection rotation capacity (Φ_j), is recommended to be 0.80~0.95 times of M_{pl+}, as the required rotation capacity is easily met (Nethercot & Stylianidis, 2008).

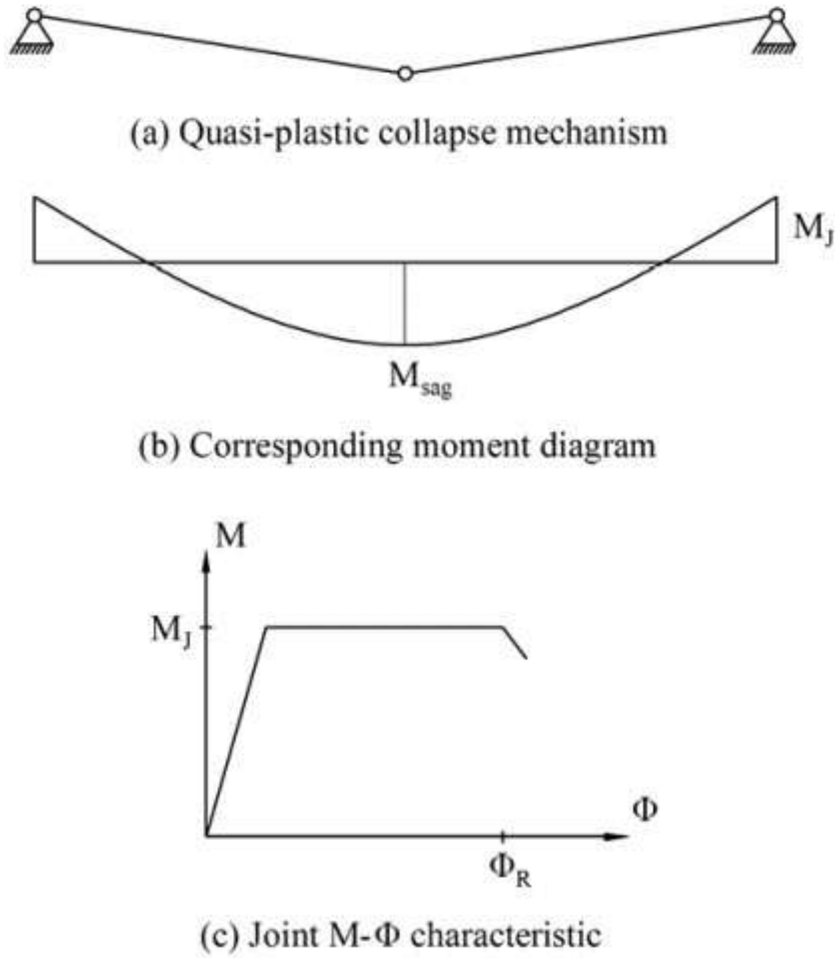


Figure 2.2 Quasi-plastic design methods (Nethercot & Stylianidis, 2008, Fig3)

2.2. Cracked region of composite beams

Nethercot's method is based on plastic theory; therefore the cracking length of beams is out of discussion, since its contribution is considered in plastic resistance of components. As elastic analysis is concerned, the concrete cracking of composite beam, when under hogging moments, comes into consideration. Within the cracked region, where the reduced stiffness E_aI_2 of cracked section is applied, the extreme fibre tensile stress in the concrete exceeds twice the strength f_{ctm} or f_{lctm} of concrete slab. The hogging moment at which the tensile stress at the extreme fibre of concrete slab equals to $2f_{ctm}$ is defined as "cracking moments" in this paper.

The envelope of the internal forces and moments, determined by an irritation process beginning at an initial cracked length by an "un-cracked analysis" with flexural stiffness E_aI_1 of the un-cracked sections, is response to a characteristic combination. This distribution of stiffness is used for both ultimate and serviceability limits states for the considered design load combination (EN1994-1-1, 2009). As the amplitude of load case may be referred as either limit state in this study, the length of cracked region should be corrected for real design analysis.

The studied composite beam is showed as in Figure 2.3. The beam is composited by IPE300 steel with 110mm concrete slab, spaced by 50mm sheeting. A steel beam with the same profile IPE300 is calculated for comparison.

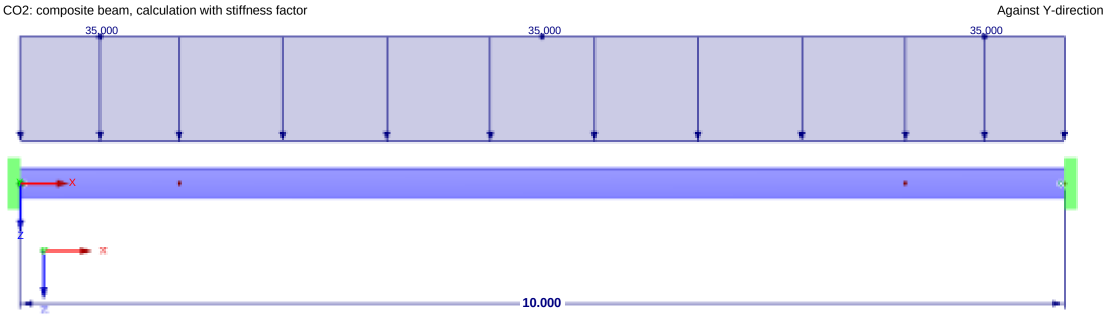


Figure 2.3 Model of Beam with End Releases

The sectional bending resistance of steel and composite beams is listed in Table 2-2 .

Table 2-1 Plastic Moments of Beam Section

Material	Profile	M_{pl}^- (kNm)	M_{pl}^+ (kNm)	$[q_{ED}]$ plastic (kN/m)	$[q_{ED}]$ elastic (kN/m)
Steel	IPE300	147.67	147.67	23.63	17.72
composite	IPE300+110slab	198.78	332.33	42.49	29.86

After irritations, the cracked length of composite beam is set as 1.521m under the applied loading level. Moments at the fixed ends (233.04kNm) is above the section bending resistance (198.78kNm), and will lead to further moments redistribution assuming idealized fixed by supports.

If the length of the hogging moment zone (Com_2) is used instead, the moments at fixed ends (233.00kNm) reduce slightly as a result of larger cracking length ratio. The errors seem to be acceptable if the cracking moments of beams are unavailable at the concept design stage, due to uncertainty of effective slab width and/or of reinforcement. The results (Span L=10m) are listed in Table 2-2 .

Table 2-2 Moments and Displacement of Elastic Beams

Beam	I_1 or I_2 (cm4)	$EI_{\_com}$ /$EI_{\_a}$	Cracked length (X_M_{cr}) (m)	Mom. at fixed end (kNm)	M_mid (kNm)	Dis. of mid-span (mm)	Dis. With 100GA$_z$ (mm)	Length of hogging Moments ($x_{\_M=0}$) (m)
Steel (I_a)	8356	1.000	n.a.	291.67	145.83	54.68	51.97	2.113
Uncr.Com.(I_1)	30145.86	3.608	2.041	291.67	145.83	17.13	14.42	2.113
Cra.Com.(I_1 and I_2)	30145.86 11175.75	3.608 1.337	1.521	233.04	204.46	26.35	23.64	1.582
Com_2			1.582	233.00	204.50	26.36	23.65	1.582
Dis. Due to shear							2.71	

As showed in Table 2-2 , the cracking length ratio is about 0.15, with a redistribution of end moment at 20%. While the cracking length ratio here (15%) is comparable with that in simplified method (15%) (EN1994-1-1, 2009) 5.4.2.3, the rotation capacity of frames is normally able to provide further moments redistribution, as recommended in EN1994-1-1, Table 5.1, for both un-cracked and cracked analysis.

2.3. Stiffness reduction of cracked beams

When used in frame, the stiffness of a cracked beam comes into consideration, related to internal forces distribution and deformation, especially when simplified method is approached.

2.3.1. Effects of Shear deflection

To determine the bending stiffness of the cracked beams, the displacement of mid-span points is used as the beam and loading are symmetry. Since the composite beam is calculated as an assumed steel element with multiplier factor modifying the flexural stiffness, deflection of beams due to shear forces will be the same as the bare steel beam, if no multiplier factor is applied to its shear stiffness. This accuracy of deflection calculation of steel elements makes unfortunately the comparing of the bending stiffness of beams uneasy. Since the shear deformation of composite beams, within influence of the concrete slab, un-cracked or cracked, is assumed to reduce, and since the interaction of shear and bending effects is out of our study, the deflection due to shear force is probably excluded.

The shear stiffness is multiplied by a factor of 100 in order to gain the displacement due to bending only. The displacement without and with this configuration are listed in the above Table 2-2 and Table 2-3 .

2.3.2. Stiffness of cracked beam

Calculated from the bending displacement of the beams, the stiffness of the cracked beam reduced to 61.6% of the un-cracked one. Considering the relatively high load level, it is possibly concluded that stiffness of cracked composite beams in frames will no less than 60% of EI_1.

Table 2-3 Equivalent Stiffness of Cracked Composite Beam

Beam	I_1 or I_2 (cm4)	Dis. of mid-span (mm)	Dis. With $100GA_z$ (mm)	Dis/Dis_un-cracked (bending)	I_{eq} of the cracked composite beam(cm4)
Un-cracked Composite (I_1)	30145.86	17.13	14.42	1.000	
Cracked Composite (I_1 and I_2)	30145.86 11175.75	26.35	23.64	1.639	18557
Equivalent Beam (I_{eq})	22557.82	n.a.	19.27		n.a.

Ammerman and Leon provided in 1995 an effective equation for equivalent bending stiffness of a composite with cracked regions (WangJ.F. & LiG.Q., 2008a),

$$I_{eq}=0.6I_1+0.4I_2 \qquad \textbf{Equation 2-1}$$

which is recommended by the AISC design guide (Leon, Hoffman, & Staeger, 1996). The recommended equivalent bending stiffness is about 75% of that of un-cracking beam (EI_1), which seems suitable for frames, as less restrain on beam ends is expected than fixed supports. This is further explained in section 3.2.1.

Cracking of the concrete slab causes moment redistribution as well as reduction of element stiffness. An important problem for the equivalent beam method is its failure to perform the reduction of beam end moments. An end moment equation or chart is necessary to determine the initial end moments for frame analysis.

3. COMPOSITE FRAMES WITH RIGID JOINTS

As showed above, the cracking phenomenon of composite beams results in reduction of end moments and bending stiffness. This will cause moment redistribution in frames, comparing with a beam with constant stiffness.

The beam is composited by IPE300 steel with 110mm concrete slab as described above, while the column is partly encased HEB260. Characteristics of the elements are list in Table 3-1.

Table 3-1 Characteristic of Element Sections

Element	Section	M_{pl} (kNm)	$(EI)_{eff}/E_a$ (cm^4)	Multiplier
Beam (M+)	IPE300+110slab	332.33	30145.86	3.608
Beam (M-)	IPE300+4D12	198.78	11175.75	1.337
Column	HEA260,encased		10481	1.003
Beam	IPE300	147.67	8356	1.000
Column	HEA260		10450	1.000

The above mentioned beams are used in one-storey one-span frame with the same amplitude of vertical loading. The steel column in the steel frame is HEA260, while the composite column in composite frames is a partially encased one with the same profile. As recommended in EN1994, a calibration factor (K_0=0.9) is applied, together with a correction factor ($K_{e,II}$=0.5) for concrete with long-term effects (EN1994-1-1, 2009). As a result, the stiffness of composite column has a slight increase (0.3%), while that of the beams increase significantly, as showed in the above section.

3.1. Cracked region of composite beams

The method in Eurocode 4 for cracking of composite beams is described in 2.2. The supports in a frame are less stiff than above assumed fixed end support, which leads to further reduction of the end moments and as a response, the cracked region will shorten so that the cracked beams are stiffer under the same level of loading.

A composite beam in one-storey one-span frame is studied in this part.

3.1.1. Composite beam in frame under vertical loading

Due to increased stiffness ratio of beam to column, un-cracked composite frame behaviours differently as the bare steel frame. The moments and the deflections (considering share deformations or not) drop dramatically while the shear force stays at the same level.

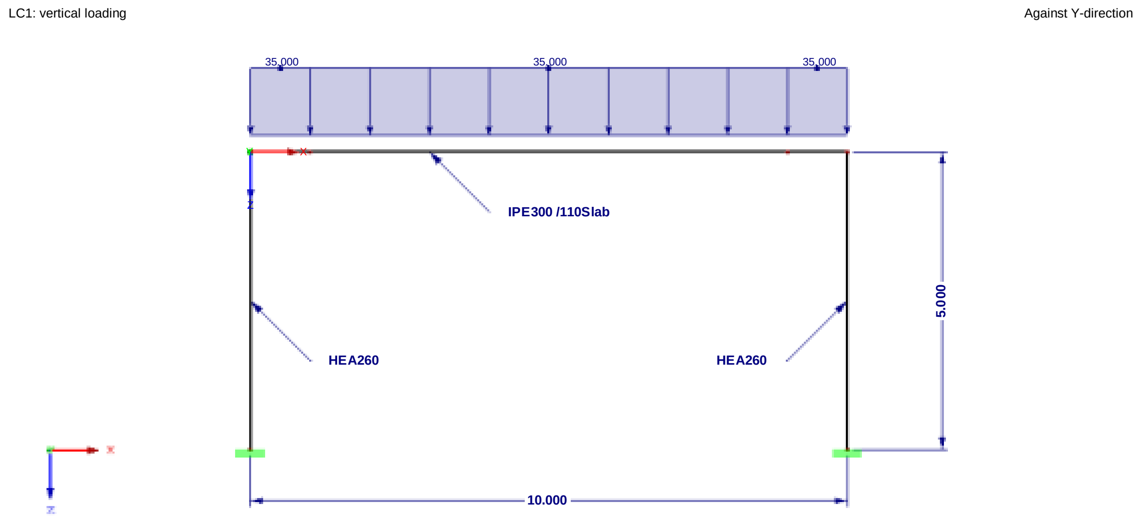


Figure 3.1 Frame under Vertical Loading

Table 3-2 Moments of Composite Beam in Rigid Frame (Span L=10m)

Frame type	I_1 or I_2 ($10^{-8}m^4$)	EI_com /EI_a	Cracked length (X_M$_{cr}$) (m)	Mom. at beam end (kNm)	M_mid (kNm)	Dis.of mid-span (mm)	Length of hogging Moments (x_M=0) (m)	I_b/I_c
Steel (I_a)	8356	1.000	n.a.	240.48	197.02	91.4	1.645	0.800
Uncr.Comp. (I_1)	30145.86	3.608	1.002	165.17	272.33	42.5	1.055	2.876
Crack. Comp. (I_1 & I_2)	30145.86 11175.75	3.608 1.337	0.907	151.71	285.79	44.7	0.959	n.a.
Equ. Beam(I_{eq})	22558	2.700	1.150	185.41	252.09	50.5	1.205	2.152

M_{pl}^{-}=198.78kNm, M_{pl}^{+}=332.33kNm

Effects of concrete cracking included, the end moments of beam drop further at about 9% with a reduced cracked stiffness at hogging moment region, and lead to a lager deflection.

Equivalent stiffness beam gives a safe estimate of deflection by 47.8mm (without shear deflection), a difference of 13.8% from 44.7mm.

Within a frame, the moments of beam are not exceed plastic resistance, both negative and positive, respectively. It should be mentioned that the sagging moments will firstly reach at bending resistance (1.16<1.31), due to the long span of beam and relative stiffness of column.

Under vertical loading, the moments at ends of beams will reduce as well as the stiffness of the beam reduces, leading to a reduced column end moments.

3.1.2. Composite beam in a frame under horizontal loading

The cracked length of composite beam is set as the same as that under the above described vertical loading, as horizontal loading is definitely combined at least with gravity loading. For this assuming loading case, the internal forces, mostly end moments of beam, will compared with those in a bare steel frame.

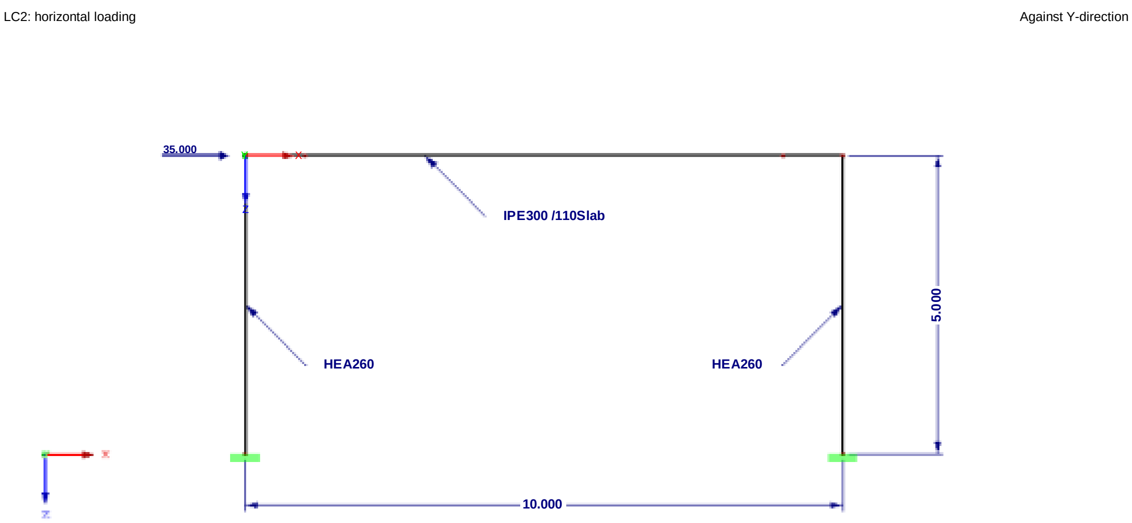


Figure 3.2 Frame under Horizontal Loading (LC2)

The applied horizontal force at the left node of the floor is 35kN, 10% of the total vertical loading.

Table 3-3 Moments of beam due to horizontal loading

Frame type	I_1 or I_2 (cm^4)	EI_{com}/EI_a	Mom. at beam end (kNm)	Dis.of floor (Hor.) (mm)	Cracked length (m)	I_b/I_c	M_{bottom} of Column (kNm)	V_{column} (kN)
Steel (I_a)	8356	1.000	30.81	16.4	n.a	0.800	56.98	17.56
Uncr.Comp. (I_1)	30145.86	3.608	39.16	11.7	n.a	2.876	48.78	17.59
Crack. Comp. (I_1 & I_2)	30145.86 11175.75	3.608 1.337	36.27	13.3	0.907	n.a.	51.63	17.58
Equ. Beam(I_{eq})	22558	2.700	37.82	12.5	n.a.	2.152	50.08	17.58
Equ/cracked			1.043	0.940			0.970	

The end moments of beam reduce when the cracked stiffness of beam is applied, with an increasing moments at the column bottom. With the equivalent beam, the results vary slightly from the cracked beam method, with a difference less than 6%.

Under horizontal loading, the restrain of columns by beams is reduced with the introduction of the rotational spring. The moments of the columns will increase. The moments of beams differ from the rigid frame.

3.1.3. Composite beam in a frame under vertical and horizontal loading

The assumed horizontal loading is combined with the above disscused vertical laoding. Frames are treated with linear elastic analysis.

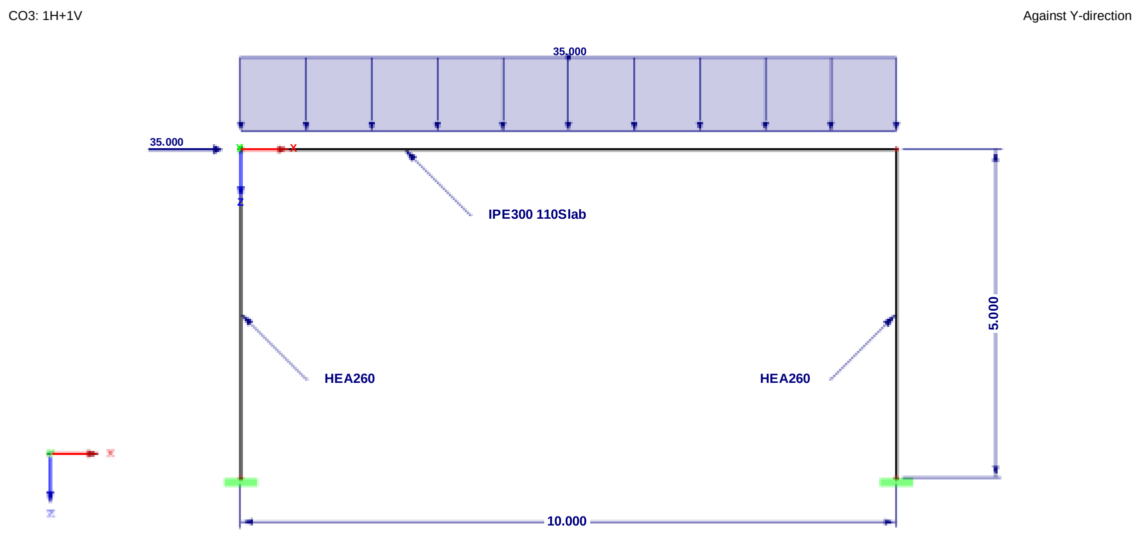


Figure 3.3 Frame under vertical and horizontal loading

The main results are listed in Table 3-4 .

Table 3-4 Moments of beam due to vertical and horizontal loading

Frame type	I_1 or I_2 (cm^4)	$EI_{\_com}/EI_{\_a}$	Cracked length (m)	Mom. at left end (kNm)	Mom. at right end (kNm)	Mom. at span (max) (kNm)	Dis.of floor (Hor.) (mm)	Ver.Dis.of beam (mm)	$V_{\_column1}$ (kN)	$V_{\_column2}$ (kN)
Steel (I_a)	8356	1.000	n.a	209.66	271.16	197.09	2.6	91.7	53.02	88.02
Uncr.Comp. (I_1)	30145.86	3.608	n.a	126.01	204.00	272.49	1.2	42.6	30.89	65.89

Crack. Comp. (I_1 & I_2)	30145.86 11175.75	3.608 1.337	0.692 1.104	115.05	187.12	287.16	0.6	44.7	26.80	61.80
Crack. 2 L_cr_Ver	30145.86 11175.75	3.608 1.337	0.907	115.44	187.71	285.93	0.6	44.7	26.95	61.95
Equ. Beam(I_{eq})	22558	2.700	n.a.	147.58	222.96	252.23	1.6	50.5	36.84	71.84

M_{pl}^{-}=198.78kNm, M_{pl}^{+}=332.33kNm

The equivalent beam methods seems to be unbelievable, mostly because its results for vertical loading. If corrected initial end moments of cracked beams, which are mostly provided by human "hand-calculation" and following input, are used, the redistribution of moments between beams and columns would follow a "right" way. Until this correction is included in the some commercial software, this method would stay in discussion for its unpractical usage.

The cracked lengths differ when considering the unequal bending moments at both ends induced by horizontal loading. Fortunately this difference makes slight effects on stiffness and internal forces of frame, if linear elastic analysis is permitted for frames in discussion. The "errors" are less than 1% in this example.

3.2. Effects of cracked length ratio of beam

It would be important for us to determine the cracked length considering the applied load combinations on a resistance system. As these lengths differs for varied combination, it will be practical to set the lengths of one load (combination) case as the most performed value for other combinations.

3.2.1. Stiffness of cracked beam

Based on the above consideration, the effects of cracked length ratio is studied, and the results are showed as in Table 3-5. The resistance capacity of structures is excluded here, as an increases loading (p=50kN/m) is introduced for comparison. The bending stiffness of steel elements is multiplied by a modifier to simulate composite ones, while the section area and shear stiffness are left as the initial value.

Table 3-5 Effects of length of cracked regions (I_2/I_1=0.371)

			Beam ends fixed I_2/I_1=0.3707 (P=35kN/m)	Beam in Frame I_2/I_1 =0.37	Beam ends fixed I_2/I_1=0.3707 (P=50kN/m)	Beam in Frame I_2/I_1 =0.3707 I_{column}/I_1=0.3477

					07, I_{column}/I_1=0.3477 (P=35kN/m)				(P=50kN/m)	
No.	Cr.ed length ratio	EI_{eq}/EI_1	M_1 (kNm)	v_1 (mm)	M_1 (kNm)	v_1 (mm)	M_1 (kNm)	v_1 (mm)	M_1 (kNm)	v_1 (mm)
0	0	1.000	291.67	17.1	165.17	42.6	416.67	24.5	235.96	60.8
1	0.02	0.809	274.21	20.5	159.69	43.6	391.73	29.2	228.13	62.3
2	0.04	0.716	260.85	22.8	155.80	44.2	372.64	32.5	222.57	63.1
3	0.06	0.667	250.82	24.3	153.29	44.5	358.31	34.7	218.99	63.6
4	0.08	0.637	243.50	25.3	151.98	44.6	347.86	36.2	217.11	63.9
5	0.1	0.621	238.42	25.9	151.70	44.7	340.59	37.0	216.71	63.8
6	0.12	0.613	235.18	26.2	152.31	44.6	335.96	37.4	217.58	63.8
7	0.14	0.610	233.46	26.3	153.69	44.7	333.51	37.6	219.56	63.8
8	0.16	0.608	233.00	26.4	155.73	44.7	332.86	37.7	222.47	63.9
9	0.18	0.610	233.58	26.3	158.34	45.0	333.69	37.6	226.19	64.2
10	0.2	0.610	235.03	26.3	161.42	45.3	335.75	37.6	230.6	64.7
11	0.22	0.608	237.18	26.4	164.90	45.8	338.82	37.7	235.57	65.5
12	0.24	0.603	239.90	26.6	168.72	46.6	342.71	37.9	241.02	66.6
13	0.26	0.598	243.08	26.8	172.80	47.6	347.25	38.3	246.86	67.9
14	0.28	0.588	246.62	27.2	177.11	48.7	352.32	38.9	253.01	69.6
A	0.5	0.370	291.67	41.6	227.19	76.4	416.67	59.4	324.56	109.1

Equivalent stiffness of equivalent beam is calculated from bending deflection of fixed beams, excluding shear effects of steel section. The calculation method by Ammerman and Leon (WangJ.F. & LiG.Q., 2008a) leads to a larger stiffness (75%), comparing with 61%~67% here.

It seems that a range of cracking length exists, with which the stiffness of beam is possibly considered to be stable. A small shift of that range, with less shortened cracking length, results in comparably accurate moment distribution of frame, as both end moments and deflection of beams are accurate for engineering usage. The cracked lengths (0.692, 0.907 and 1.104 in frame) of studied beam stay in this region (0.06~0.14 times of span). Supposing these be common for other frames, the cracked length under vertical loading is recommenced for analysis model.

3.2.2. End moments of cracked beam

Restrains by frame members are less stiff than fixed support, which induces reduced end moments of beams. This redistribution results a shortened cracking region of composite beams, making effects of cracking less significant in frame than in a continuous beam, similar as in a fixed beam. The reduction of end moments of cracked beam is about 10% in study, while that of a fixed beam is about 20%, as showed in section 2.2.

As horizontal loading is considered, the columns are required to resistance the induced shear forces. Since the top moments reduce, balanced with the end moments of beams, the bottom moments increase as a result of reduced stiffness, by introduction of cracked stiffness.

4. COMPOSITE FRAMES WITH SEMI-RIGID JOINTS

Before semi-rigid joints are widely studied, pinned connections are recommended in braced frames for their easy fabrication and low expense. Their little flexural strength leads to a simple global analysis and quick design, besides its full usage of the ultimate sagging bending capacity. Internal forces and moments are statically determinate, which excludes the effects of cracking, creep or shrinkage of concrete.

Similarly, rigid connections are preferred in practice thanks to its simplicity of global analysis. As a result, full-strength rigid connections are needed, providing the connection part, by strength and stiffness, is stronger enough than nearby elements connected. Eurocode classifies joints by stiffness with varied criterion for braced and un-braced frames.

Between the regarded pinned and rigid classes there is a large gap which is filled by semi-rigid connections, neglecting the reality that each connection has some degree of stiffness and bending strength. EN1994 allows designer neglect effects of behaviour of joints, if its effects on distribution of internal forces and moments, and on the overall deformations of the structure, are not significant. Even through, rigid joints in composite frames are criticized for its complexity and high expense.

Recent research on joints is questioning the traditional analysis with idealized rigid or pinned joints. The behaviour of the joints needs to be taken into account in those structures with semi-continuous joints (EN1994-1-1, 2009). Development of finite elements analysis methods provides possibility to account effects of flexibility of joints.

4.1. Research on semi-rigid frames

Semi-rigid connections are almost unavoidable in composite frames due to the composite effects of steel and concrete. Pinned joints might be recommended for braced frame, but the longitudinal reinforcement in the slab, responsible for control of cracking, will lead to partly resistance for composite beams connected to an internal column. As for rigid, full-strength connections, extended end plates with stiffeners will probably needed, as for steel joints. Even through the joints and beam sections are strengthened by additional haunches, the column web zone may yield some degree of flexibility, not considering the expense caused by complexity.

With this consideration, semi-rigid joints or semi-rigid frames are widely studied during the last decades. Nethercot (Nethercot D. , 1995) (Nethercot & Stylianidis, 2008) recommended simplified plastic design methods for braced frames, or non-sway frames under vertical loading. As for global instability analysis, Demonceau proposed an approach, which is comparable with the column verification method in Eurocode. In his approach, critical load factor α_{cr} and plastic load factor α_{pl} are used in a formula to foresee a reduction factor χ. By multiplying χ with $\alpha_{pl,}$ the ultimate load factor is expected (Demonceau, 2008).

4.2. Bending stiffness of joints

Knowledge of the connection's moment-rotation (M-Φ) characteristic is vital for estimating the effects of joint performance on the frame behaviour. A general form of curve by Zandonini (1989) is given in Figure 4.1, (Nethercot D. , 1995) which highlights the non-linearity of joint's performance.

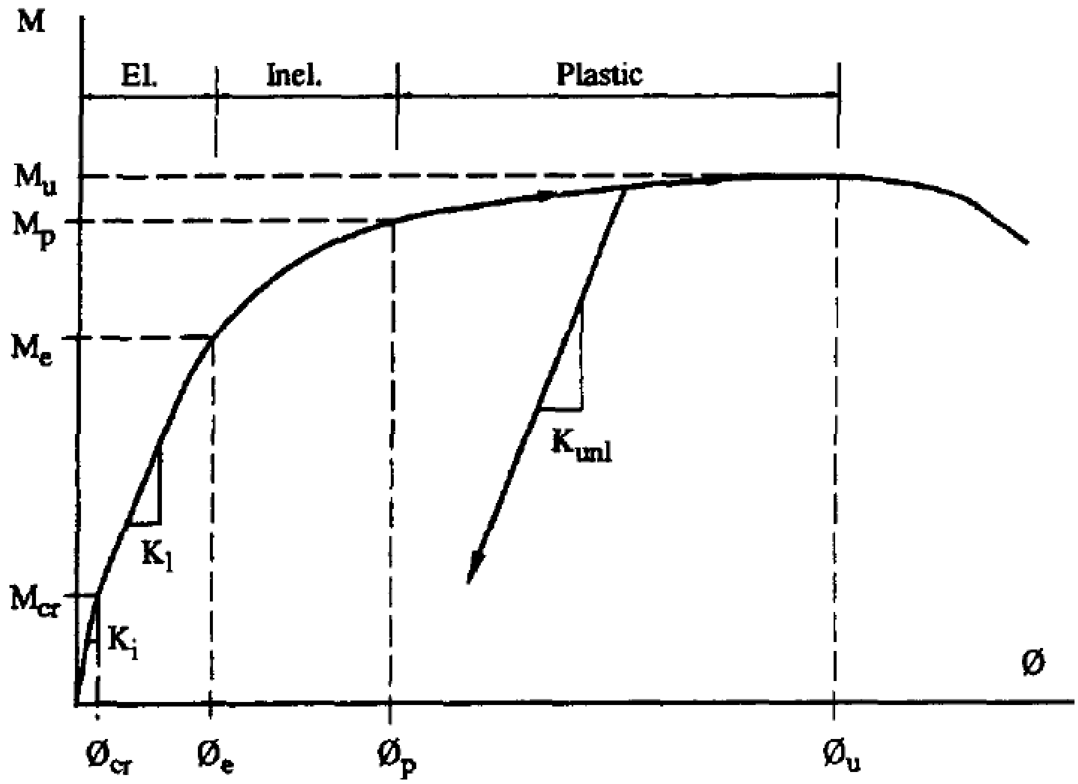


Figure 4.1 Most general form of moment-rotation characteristic (Nethercot D. , 1995)

As numerical analysis of frames concerned, equations for the complete M-θ curves have been proposed, from databases based on tests. Beside the simplified bilinear elastic-plastic curve, power function models and Richard-Abbott model have been processed, but are criticised for their uncertain and unreliable variables. (Tamboli, 1999)

A detailed nonlinear finite element analysis of connection has been used to develop M-θ curves. Component method approach is proposed by Eurocodes, where each deformation mechanism of joint components involved is activated in analysis. While this study focuses on global analysis, the CM approach is briefly introduced in following section.

4.2.1. Components Method

Components Method is recommended in EN1993-1-8, to determine the joints' properties in frames. Basic components and their properties are identified, while a joint is modelled as an assembly of these components. Applying resistance of components with cooperation conditions, the main structural properties, namely moment resistance $M_{j,Rd}$, rotational stiffness S_j (or initial stiffness $S_{j,ini}$)and rotation capacity Φ_{Cd}, are to determined.

By numerical analysis with finite element software package, the M-θ curve of joints is possibly to be simulated, when properties of each component are activated (Eldemerdash, Abu-Lebdeh, & Al Nasra, 2012). As for end-plate connection under hogging moments, components involved mostly are listed in Table 4-1. (Demonceau, 2008)

Table 4-1 Components to be considered for end-plate connection under hogging moments (Demonceau, 2008)

Zones	Components
Compression zone	Column web in compression
	Beam flange and web in compression
Tension zone	Column web in tension
	Column flange in bending
	Bolts in tension
	End-plate in bending

	Beam web in tension Slab rebars in tension
Shear zone	Column web panel in shear

Aside from commercial software package, spring model is widely used, where the components are represented by spring, with their strength and stiffness either in tension or in compression or in shear, and their interactions are defined in numerical analysis. By assembling the springs, relationship of loading and response is analysed and M-θ curve of studied connections is possible obtained. (Drozd, 2012)

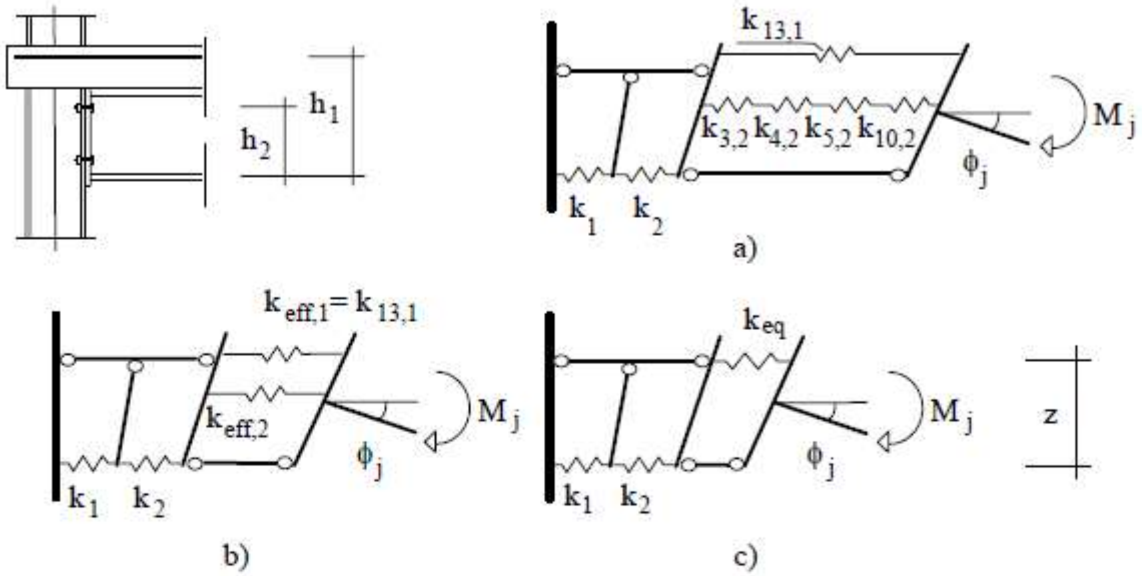


Figure 4.2 Example of a spring model for a composite end-plate connection (Demonceau, 2008)

4.2.2. Bending stiffness at unloading stage

Another comment on joint's property is its different bending stiffness when loading and unloading, which might influence response of frame under loading process. Chen declaimed impacts of stiffness difference on frames under wind loading (Chen & Atsuta, 1997) (Chen, 1999).

As showed in Figure 4.3, if the bending the joints in the opposite direction, reduction of moment of joint will be almost linearly, for example the joints at the side facing wind. The stiffness when unloading would be initial stiffness when loading, while that of the other side will be tangent, which is less with moment amplitude. The gap might be significant depending on the amplitude caused by vertical loading. The joints might lose its stiffness and behaviour like a pin at ultimate (or analytically plastic) bending capacity, while others obtain a high stiffness as moments drop. This phenomenon will probably appear on frames with relatively heavy gravity loading, where the ultimate capacity is tended.

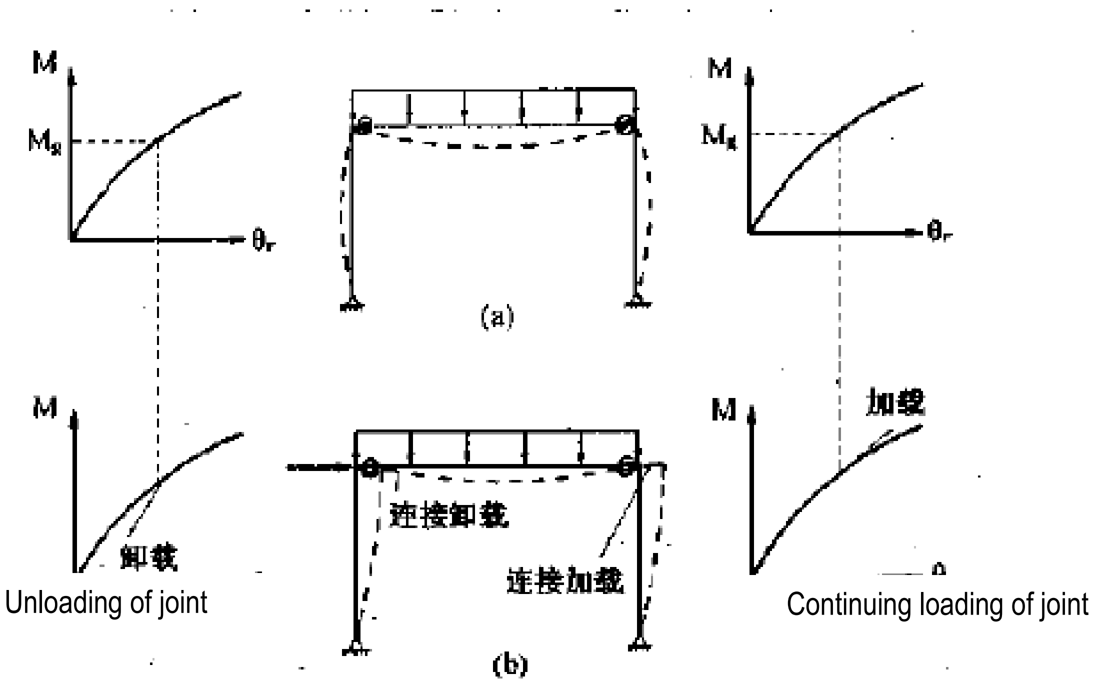


Figure 4.3 Characteristic of joints when loading or unloading

4.2.3. Composite joints studied

One of the considered composite joints is showed in the following Figure 4.4. Other joints used in this study are same or similar steel profile with varied arrangement of bolts.

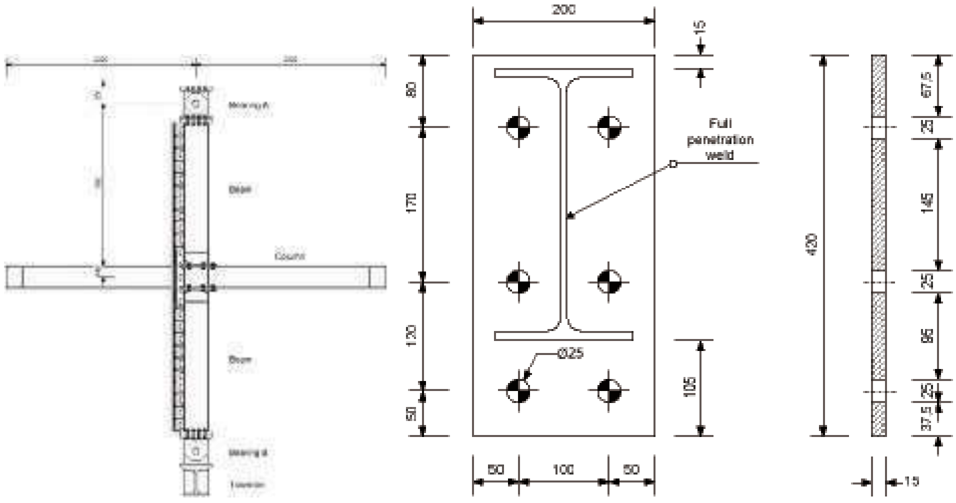


Figure 4.4 Detailing of composite joints

Main characteristic of joints used in this paper are listed in Table 4-2, which is extracted from a European research report (EUR21913) (Bitar, et al., 2006).

Table 4-2 Characteristic of composite joints

	Joint1 (type4, external,hogging, Liège)	Joint2 (Type8,internal, Aachen)	Joint3(type11, internal,Aachen)	Joint4(CM, Liège)
Beam	IPE300+110slab	IPE300+110slab	IPE300+110slab	IPE300+110slab
Column	HEB260	HEB280	HEB280	HEB280
End plate	420x200x15	420x200x15	340x180x12	340x180x12
Bolts	6M24	6M24	4M20	4M20
$S_{j,ini}$ (kNm/rad)	65000	30849	17850	13681
M_j (kNm)	201.58	182.7	142.5	117.40
Φ_u (rad)	0.031	0.048	0.10	n.a.(computation)

4.3. Internal forces and moments of beams in semi-rigid frames

End moments of beams with rotation spring at both ends might be calculated with the following Equation 4-1

, where M_{FA} and M_{FB} are end moments of beams with idealized fixed supports, while S_{beam}, S_{kA} and S_{kB} are stiffness of beam and of rotation springs respectively. (Wang, Li, & Li, 2003)

$$M_A = \frac{M_{FA} + 6\alpha_B M_{FB}}{1 + 4\alpha_A + 4\alpha_B + 12\alpha_A\alpha_B}$$

$$M_B = \frac{M_{FB} + 6\alpha_A M_{FA}}{1 + 4\alpha_A + 4\alpha_B + 12\alpha_A\alpha_B}$$

$$S_{beam} = E_{beam} I_{beam} / L_{beam}$$

$$\alpha_A = S_{beam} / S_{kA}$$

$$\alpha_B = S_{eam} / S_{kB}$$

Equation 4-1

These equations might be used for steel frame, followed by global analysis using equivalent beam stiffness methods. For composite beams the uncertainty of cracking length makes it less useful as the beam stiffness is in question.

4.3.1. Moments of semi-rigid frames under vertical loading

A composite frame with rotation spring at beam to column connections is studied, under vertical loading only. Element profile is the same as described above. Numerical analysis using programme gives results as showed in Table 4-4.

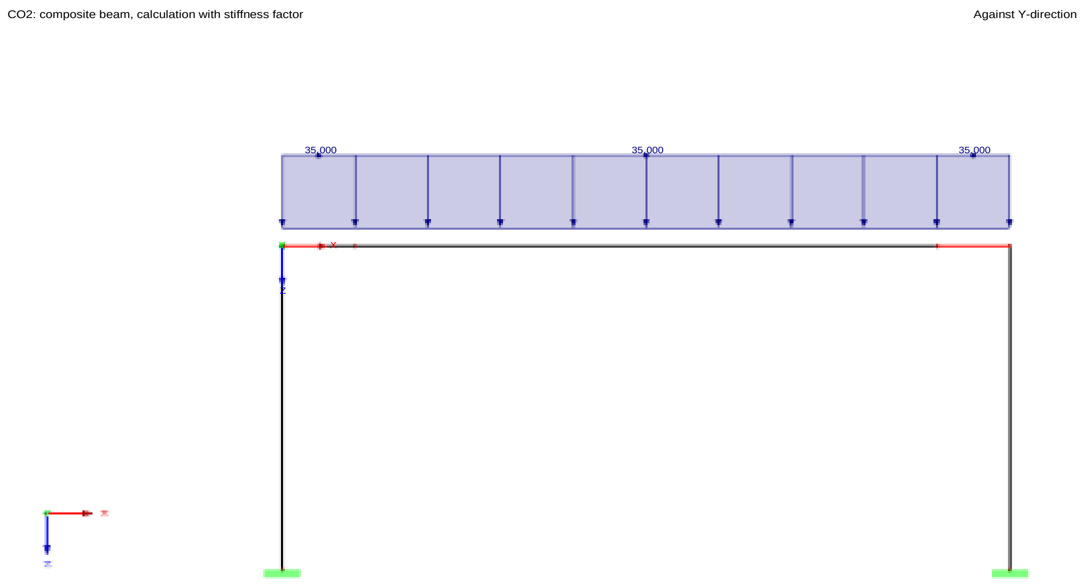


Figure 4.5 Composite frame with semi-rigid connection

Table 4-3 Moments of beams in semi-rigid frames

Frame	$S_{j,ini}/S_{beam}$ $=2S_j/S_{beam}$	M_j (kNm)	Mom. at beam ends (kNm)	M_mid (kNm)	Dis.of mid-span (mm)	Length of cracked region ($M=M_{cr}$) (m)	Length of hogging moment region (M=0) (m)
Un-cracked_Com	∞		165.17	272.33	42.6	1.583	1.645
Cracked_Com	∞		151.70	285.80	44.7	0.907	1.055
Spr_32500	35.02	201.58	127.61	309.89	49.7	0.742	0.792
Spr_15425	16.62	182.70	108.19	329.31	53.6	0.614	0.662
Spr_8925	9.62	142.50	89.10	348.40	57.5	0.491	0.538
Spr_6840	7.38	117.40	78.93	358.57	59.5	0.427	0.473

$S_{beam}=(EI)_{beam}/L_{beam}=E_a*1855.7$, $M_{pl+}=332.33$kNm,$M_{pl-}=198.78$kNm

The cracked stiffness of the beam is assumed to be $E_a*18557cm^4$, as calculated in 2.3.2(page 9). The stiffness ration is listed in Table 4-4. Shear force effects on steel beam is significant. This should be reduced for composite beam. But there is nothing said about this in the modified steel structures method.

Rotation stiffness of joints will reduce the stiffness of related beam, but a simple reduction factor multiplied with the stiffness of beam will not function. Beside the reduction of beam stiffness, the end moments of the beams decrease as a result of increased rotation at the connection part. The reduction of stiffness ratio between beam and columns leads to a further reduction of moments redistributed on connected column.

Even though beam cracked stiffness is uncertain, depending on length of cracked region under specialized loading level, it is possible to have a relative stiffness ration of joints to beams, using the equivalent beam stiffness.

As showed in Table 4-4, even though the bending resistance of joints might be able to bear the moments reduced by joint's rotation (Spr_8925, $M_j=142.50>M_{ed}=89.10$), the beam may tend to reach its bending capacity M_{ed} (Spr_8925, $M_u=332.33<M_{ed}=348.40$) before plastic hinges form at the beam ends, and possibly produce collapse or at least large deformation. To avoid this type of collapse, Nethercot (Nethercot & Stylianidis, 2008) suggested to have a M_{ed}/M_{pl} (or M_d/M_p) ratio at 0.80~0.95, i.e. to have a safety supplement when plastic design is applied.

Considering a plastic design for this case (Spr_8925), from M_j=142.50kN/m and M_{total}=437.5kNm, the maximal sagging moment is expected to be M_{ed} (or M_d) =295.0kNm, which is 0.88 times M_{pl+} (or M_p). The simplified methods, based on the assumption that plastic hinges firstly form at beam ends, might lead to an un-conservative result for joints with lower rotation stiffness. In this case, the rotation capacity of the composite beam at span should be checked. It should be mentioned that the discussed joints might be classed as rigid in a braced frame as $S_{j,ini}/(EI_b/L_b)$=9.62, which is larger than k_b=8, if Eurocode is applied. (EN1994-1-1, 2009)

4.3.2. Internal forces and moments of semi-frames under horizontal loading

In this part of calculation, cracked length of beams is set as under above mentioned vertical loading, as reasoned in 3.1.2. The stiffness of joints is set firstly at the same value of secant stiffness as used for vertical loading, then at initial and secant stiffness respectively according to 4.2.2,. Results are showed in Table 4-4, and Table 4-5 respectively.

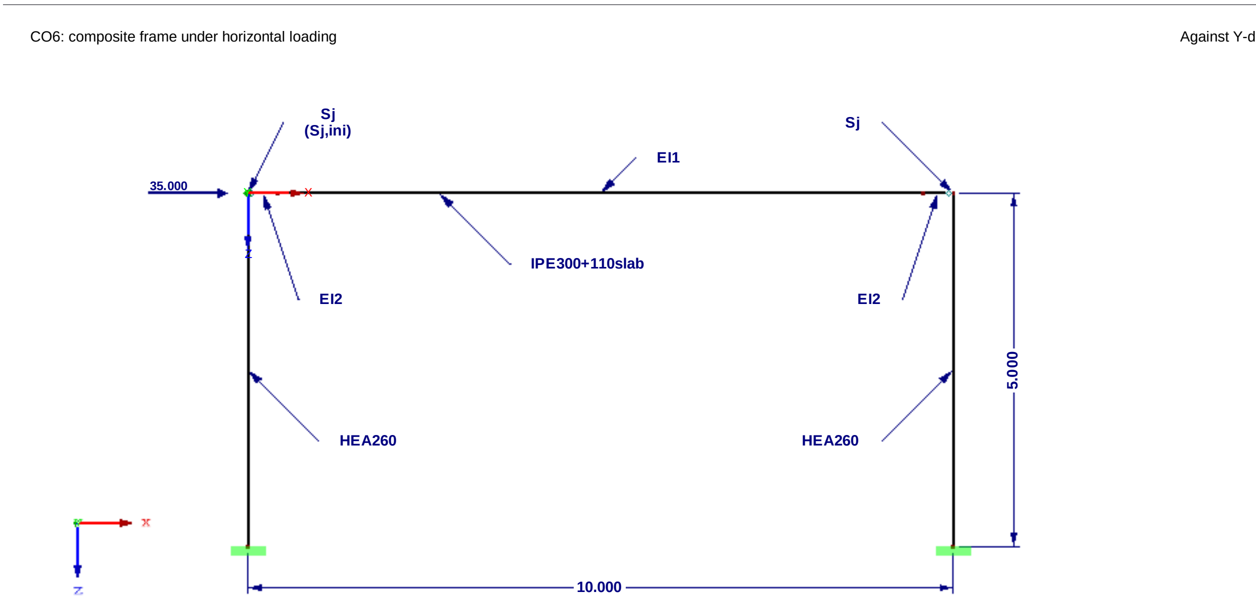


Figure 4.6 Semi-rigid frames under horizontal loading

As showed in Table 4-4, end moments of beam are almost the same, and both columns behaviour similar, as a symmetrical frame under counter-symmetrical loading. The slight difference is due to effects of compression of beam.

Table 4-4 Internal forces of semi-rigid frames (S_j)

Frame type	Cracked length (m)	Mom. at beam end_1 (kNm)	Mom. at beam end_2 (kNm)	M_bottom of Column1 (kNm)	M_bottom of Column2 (kNm)	Dis.of floor (Hor.) (mm)	V_column1 (kN)	V_column2 (kN)
Uncr.Comp. (I_1)	n.a	39.16		48.78		11.7	17.59	
Equ. Beam(I_{eq})	n.a.	37.82		50.08		12.5	17.58	
Crack. Comp. (I_1 & I_2) I=18557	0.907	36.27		51.63		13.3	17.58	
Spr_32500	0.742	32.95	32.71	54.93	54.41	15.2	17.58	17.42
Spr_15425	0.614	29.84	29.63	58.01	57.52	17.0	17.57	17.43
Spr_8925	0.491	26.32	26.15	61.50	61.02	19.0	17.57	17.43
Spr_6840	0.427	24.22	24.07	63.59	63.12	20.1	17.56	17.44

On the contrary, the end moments of beams differ considering stiffness hardening at the so called "unloading" joint. The stiffer joint and its connected column are willing to bear more external effects with their higher resistance capacity.

Table 4-5 Internal forces of semi-rigid frames ($S_{j,ini}$ &S_j)

Frame type	Cracked length (m)	Mom. at beam end (kNm)	Mom.at right end (kNm)	M_bottom of Column1 (kNm)	M_bottom of Column2 (kNm)	Dis.of floor (Hor.) (mm)	V_column1 (kN)	V_column2 (kN
Uncr.Comp. (I_1)	n.a	39.16		48.78		11.7	17.59	
Equ. Beam(I_{eq})	n.a.	37.82		50.08		12.5	17.58	
Crack. Comp. (I_1 & I_2) I=18557	0.907	36.27		51.63		13.3	17.58	
Spr_32500	0.742	35.27	32.18	54.70	52.85	14.7	17.99	17.01
Spr_15425	0.614	33.98	28.79	57.53	54.70	16.0	18.30	16.70

Spr_8925	0.491	32.09	25.13	60.72	57.06	17.6	18.56	16.44
Spr_6840	0.427	30.74	23.02	62.63	58.62	18.6	18.67	16.33

It seems neglecting the stiffness hardening will lead to a conservative result, as the structure is more slender. (Eldemerdash, Abu-Lebdeh, & Al Nasra, 2012)

4.4. Critical load factor α_{cr}

Above results of linear elastic analysis, or first order analysis, neglected effects of deformation. Under loading, deformation of elements and structures from their initial geometry, might lead to redistribution of internal forces and thus to further deformation. These impacts should be included if the structure, normally a slender or relative slender one, is sensitive to deformation. In other words, if the increase of the relevant internal forces or moments or any other change of structural behaviour caused by deformations cannot be neglected, and a so-called second order analysis should be employed.

For steel and/or composite structures, if the increase of relevant forces or moments caused by the deformations given by first-order analysis is less than 10%, effects of loading induced deformation might be neglected, thus results from a first order analysis are regarded accurate enough for engineering usage. Structures extended that limitation should be treated with second order analysis. (EN1993-1-1, 2005) (EN1994-1-1, 2009)

4.4.1. Critical load analysis

Critical load theory dated back to Euler for his famous column formula, and has been expanded to include any member, component, or structural system with significant portions in compression. First-order elastic critical analysis is traditionally an eigenvalue problem, with assumed buckled mode shapes.

Even though elastic critical loads, as upper bound values, are for most buildings considerably higher than the inelastic limit points, and have limited practical value for frame design, they have proved useful in determining amplification factors to approximate second-order effects (Ziemian, 2010). A critical load factor is also possible obtained with second order analysis, using modern analysis software. Values to estimate structure behaviour are showed in Figure 4.7.

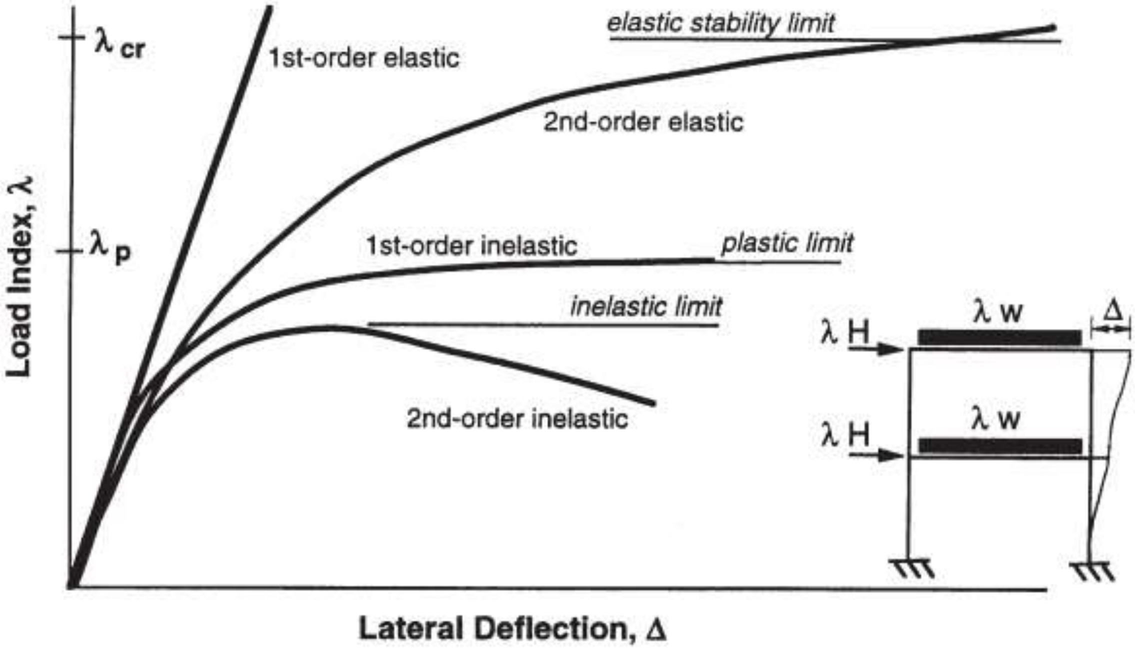


Figure 4.7 Schematic comparison of load-deflection behaviour ((Ziemian, 2010) Figure 16.1)

The reason of the higher value of critical load factor than plastic or inelastic one is its idealized elastic assumption. All the components are assumed to be idealized elastic, without resistance limitation. The assumption should be coherently applied when software is used in analysis.

4.4.2. Critical load factor of semi-rigid frames

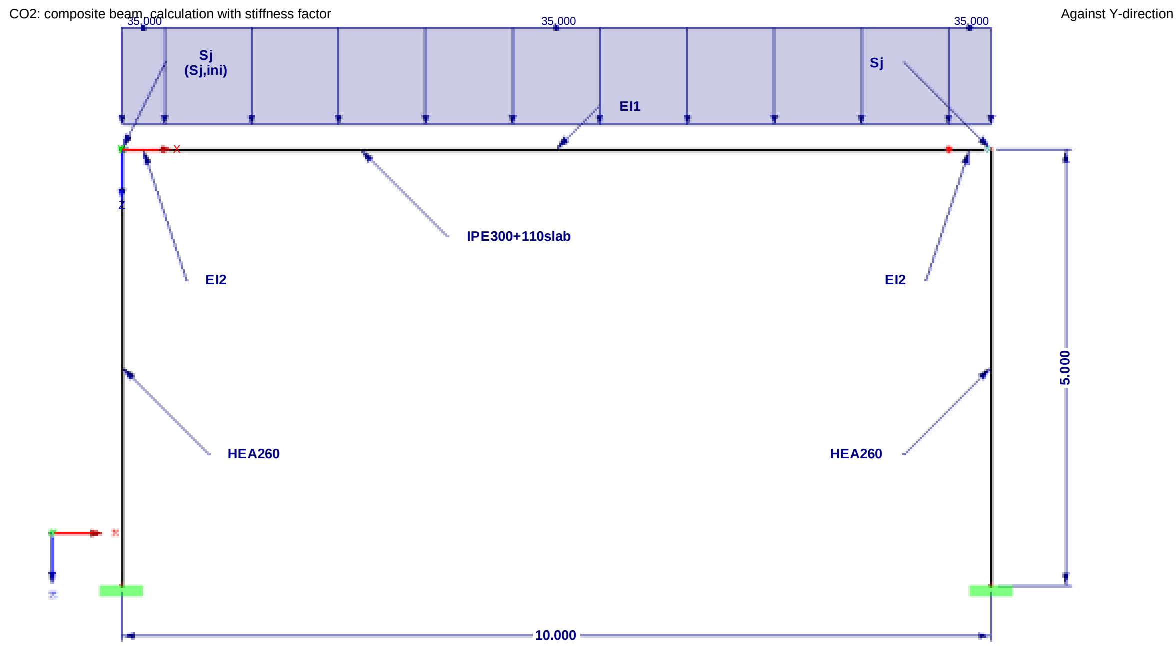


Figure 4.8 Load case for critical load factor calculation

The critical load factors are calculated based on two methods. RSBUCK, an add-on module of RSTAB software, is an eigenvalue analysis tool for frameworks to determine the critical load factors and buckling shapes. RSTAB also provides an alternative method, using a step-by-step nonlinear calculation method, within second order analysis (Dlubal, 2013). The later procedure determines only the lowest eigenvalue of structures, considering all nonlinear elements involved. The results are listed in Table 4-6.

Table 4-6 Critical load factors of semi-rigid frames (S_j)

Frame type	Crkd length (m)	M_j (kNm)	α_{cr} (RSBUCK) LD_V	α_{cr} (RSBUCK) LD_V+H	α_{cr} (2nd order) id.ela. joints LD_V	α_{cr} (2nd order) id.ela. joints LD_V+H	α_{cr} (2nd order) M_j induced LD_V or V+H
Uncr.Comp. (I_1)	n.a	n.a.	25.79	25.37	37.20	36.80	n.a..
Equ. Beam(I_{eq})	n.a.	n.a.	25.65	25.22	34.60	35.40	n.a..
Crack. Comp. (I_1 & I_2) I=18557	0.907	n.a.	25.88	25.43	32.90	32.90	n.a.
Spr_32500	0.742	201.58	23.93	23.64	29.00	28.60	12.20
Spr_15425	0.614	182.70	22.26	22.07	25.90	25.10	12.20
Spr_8925	0.491	142.50	20.55	20.45	23.00	22.10	12.20

Spr_6840	0.427	117.40	19.63	19.55	21.50	20.70	12.20
pinned	n.a	0.00	12.20	12.20	12.20	12.20	12.20

4.4.2.1 Bending capacity of joints in second order approach

As showed in Table 4-6, second order analysis approach gives similar value, when the stiffness and bending capacity of joints are considered. The bending resistance of joints is assumed to be reached and plastic hinges are formed when a certain load level is applied. After that the joints work similarly as pinned support, which leads to a critical load almost the same as a pinned frame. In other words, the contribution of joint stiffness is hidden by its bending capacity.

Since the nonlinearity of joints is activated in second order calculation, the rotation capacity of joints is verified to be the weakest property of structure with increased loading. Until this configuration is considered in programme, non-linear analysis might fail in critical analysis. To solve this problem, the joints should be set as idealised elastic (Demonceau, 2008), which means the bending resistance of joints should be infinitive.

4.4.2.2 Idealized stiffness of joints in second order approach

Since critical load analysis should be based on idealized elastic properties of structure, of all its elements or components, properties of joints should defined as idealized elastic to have a reasonable analysis. That is, the joints are defined as a rotation spring with an infinite bending capacity. With this configuration, contribution of joint stiffness is possible to be evaluated, as showed in Table 4-6. (columns with id.ela.joints) With decrease of joint stiffness, critical load factor of frames reduced, as expected.

4.4.2.3 Eigenvalue analysis approach

Differing from the second order approach, RSBUCK determines buckling shapes as well as critical load factors with an eigenvalue approach. It seems the limitation of bending capacity is not activated in this module, as critical load factors are related with joints stiffness, even though the gaps between values from two approaches are quite large.

It is mentioned in the Manual of RSTAB8.0 and RSBUCK (April 2013) that, “minor difference between the critical load factors from RSTAB and RSBUCK cannot be completely excluded”, as results of two different methods. Un-similarity of calculation parameters, such as favourable effect due to tension forces and stiffness modifications, might lead to a significant difference. Unfortunately, the difference might be significant for some frames, ranging from 0% for pinned frame to 29.4% for rigid one, as showed in this example.

4.4.3. Study on Critical load factor of semi-rigid steel frames

To identify effects of semi-rigid joints, steel frames are used here. Steel frames with same profile are calculated, without stiffness modifiers and introduction of crack region. The properties of composite joints are used here. Results are listed in Table 4-7 and Table 4-8

Similar as the composite frames, critical load factors of semi-rigid frames with limited rotation capacity of joints (M_j) are same as a pinned frame, using the second order analysis method. Differing from the RSBUCK eigenvalue approach, RSTAB determines the critical load factor based on a nonlinear calculation method while all nonlinear elements, including failing members or supports are considered. The end release will work as a pin when its bending capacity is reached under a relatively lower loading level, which leads to similar critical load as a pinned beam. Comparing with an infinite rotation spring, the M_j seems to be too small to have any effect on critical load factor.

The second phenomenon is the results of both approaches for idealized rotation springs. The results of second-order analysis increase with the joint stiffness, large as 55.2% for rigid frame. As explained in software package, the RSBUCK eigenvalue analysis is based on the load case definition in the main programme, which is defined and used for RSTAB analysis. The internal forces and deflection of elements are introduced into RSBUCK as foundation of buckling shapes. There is unfortunately no clue to identify the result difference of the two methods; even if the manuals declaim "minor difference" is expected.

Variations are made in frame elements, trying to find the effects of both approaches. Steel frames are used, to minimise influence of stiffness modifiers. The gaps reduce with a smaller span, having same steel profile of beam and columns. Slender frames (such as long spans) are mostly labelled with different values, while frames with two spans and/or two storeys have similar values, considering both calculation methods.

Table 4-7 Critical load factor of steel frames (L=10m)

Frame type	$S_{j,ini}/S_b$ $=2S_jL/EI_b$	M_j (kNm)	α_{cr} (2nd order) id.ela. joints LD_V	α_{cr} (2nd order) id.ela. joints LD_V+H	α_{cr} (2nd order) id.ela. joints LD_2V+H	α_{cr} (RSBUCK) LD_V	α_{cr} (RSBUCK) LD_V+H	α_{cr} (RSBUCK) LD_2V+H	α_{cr} (2nd order) M_j induced LD_V
rigid	∞	n.a.	16.80	15.40	14.40	26.08	25.58	25.10	16.80
Spr_32500	37.04	201.58	16.00	14.50	13.50	24.24	23.93	23.63	12.30
Spr_15425	17.58	182.70	15.50	13.90	12.90	22.63	22.43	22.24	12.30
Spr_8925	10.17	142.50	15.30	13.70	12.50	20.95	20.83	20.72	12.30
Spr_6840	7.80	117.40	15.40	13.70	12.50	20.02	19.93	19.86	12.30
Spr_3420	3.90	58.70	16.50	17.50	17.20	17.56	17.54	17.53	12.30
pinned	0	0.00	12.30	12.30	12.30	12.38	12.40	12.43	12.30

Table 4-8 Critical load factor of steel frames (L=7m)

Frame type	$S_{j,ini}/S_b$ $=2S_jL/EI_b$	α_{cr} (2nd order) id.ela. joints LD_V	α_{cr} (2nd order) id.ela. joints LD_V+H	α_{cr} (2nd order) id.ela. joints LD_2V+H	α_{cr} (RSBUCK) LD_V	α_{cr} (RSBUCK) LD_V+H	α_{cr} (RSBUCK) LD_2V+H
rigid	∞	43.50	45.70	47.80	44.14	43.59	43.05
Spr_32500	37.04	38.90	40.40	39.50	39.20	38.89	38.61
Spr_15425	17.58	35.20	35.50	34.10	35.42	35.24	35.09
Spr_8925	10.17	31.80	31.10	29.90	31.89	31.80	31.74
Spr_6840	7.80	30.00	29.10	28.00	30.09	30.04	30.01
Spr_3420	3.90	25.60	24.50	23.80	25.70	25.70	25.73
pinned	0	17.60	17.60	17.60	17.69	17.71	17.78

5 PARAMETRIC STUDY

Parametric study on critical load factor and moment redistribution, of steel and composite frame, is performed in this part. The variables include rotation stiffness of joints, together with span and height of frame. The calculation is carried out with FEM programme RSTAB 8.0, element end releases are used to simulate function of rotation characteristics of joints. (Dlubal, 2013) (Dlubal, 2013a)

5.1 Steel frames

To eliminate effects of concrete cracking, results of a series of steel frames are presented in this section. The principal frames are showed in the following figure.

Against Y-

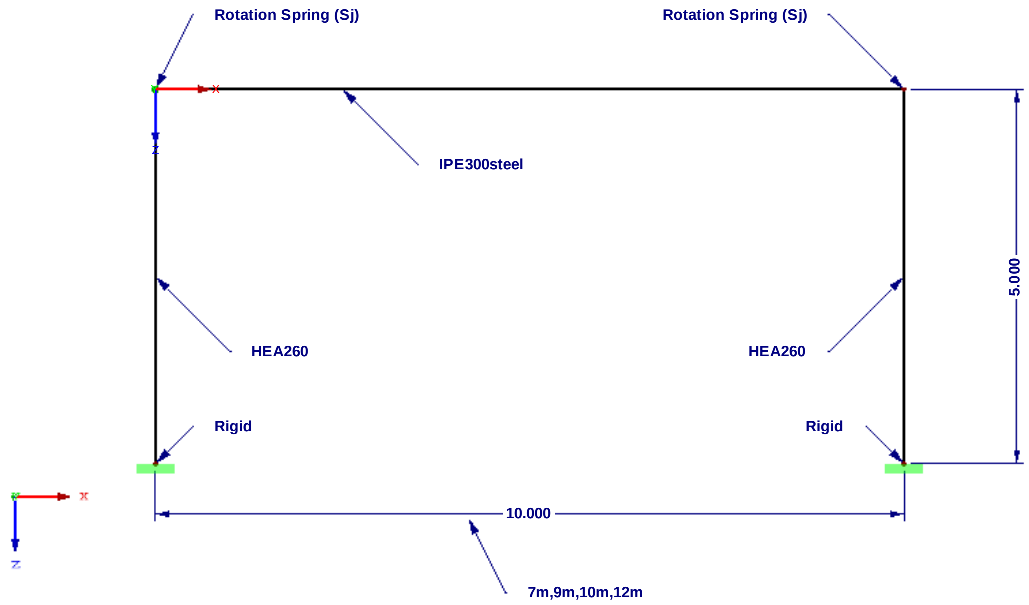


Figure 5.1 Steel frames studied

5.1.1 Critical load factors

A series of steel frames are studied, with varied span, height, and joint rotation stiffness. Critical load factors are calculated with eigenvalue analysis approach in RSBUCK module. Results are showed in Table 5-1.

It should be mentioned that there is difference from eigenvalue analysis and a second-order stability approach. The reason might be out-plane element buckling or strength capacity of cross-section, as any limited capacity with non-linearity of any part of the frame will lead to buckling performance of the whole frame. Different values of critical load factors are curved in following figure.

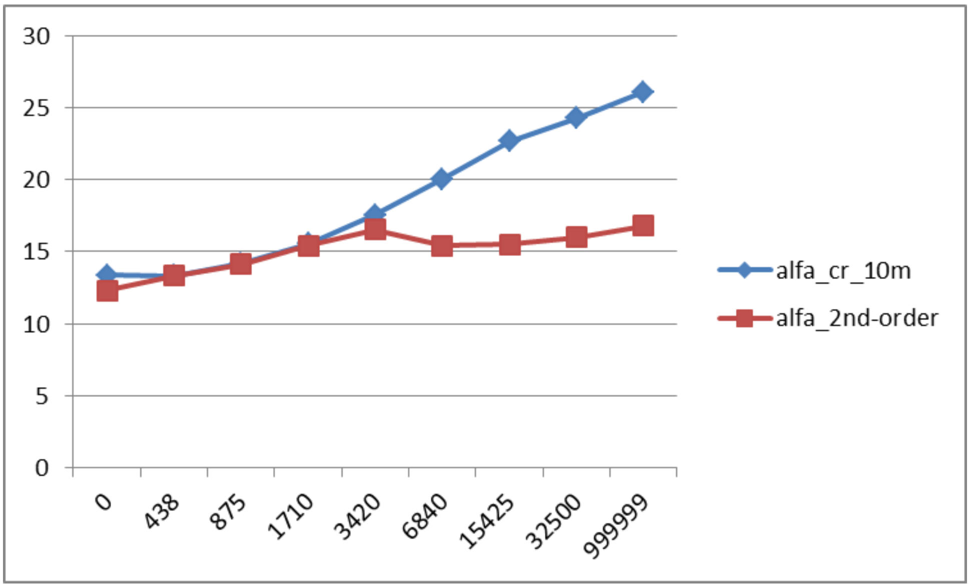


Figure 5.2 α_{cr} Value according to different approaches

As showed in the following figures, both rigid and semi-rigid frames lose part of its capacity for critical load, as span increase. The main reason is the increased axial force of columns, while less restrain is provided by large span beams.

Table 5-1 Critical load factor of steel frames (variable span)

S pan (m)													0				2				5			
h eigh(m)		.5		.5		.5		.5		.5		.5		.5		.5		.5		.5		.5		.5
0 (pinned)	7.76	9.99	4.38	0.2	6.97	1.42	7.41	4.43	0.98	6.66	3.54	1.22	8.88	4.99	2.19	0.10	5.73	2.50	0.16	.42	2.59	0.00	.13	.73
7 5	2.17	3.93	7.95	3.45	0.07	4.20	9.92	6.72	3.35	8.79	5.47	2.98	1.00	6.89	3.91	1.67	7.47	4.05	1.57	.70	3.92	1.20	.22	.73
1 710	5.80	7.17	0.85	6.08	2.56	6.41	1.91	8.52	5.19	0.43	6.95	4.32	2.61	8.34	5.21	2.85	8.73	5.18	2.59	0.64	3.80	2.10	.97	.42
3 420	1.90	2.55	5.63	0.37	6.58	9.97	5.09	1.37	8.04	2.97	9.22	6.37	5.03	0.51	7.16	4.62	0.47	6.80	4.07	1.98	0.75	2.35	0.91	.32
6 840	0.46	0.01	2.19	6.18	1.90	4.64	9.22	5.05	1.53	6.09	2.02	8.88	7.86	3.09	9.5	6.73	2.21	8.51	5.68	3.48	0.04	1.41	1.73	0.16
1 5425	1.82	9.73	0.58	3.47	8.38	0.29	4.17	9.40	5.31	9.54	5.13	1.67	0.68	5.76	1.97	8.97	3.52	0.00	7.19	4.93	0.18	1.14	2.24	0.80
3 2500	0.64	7.15	6.86	8.83	2.96	4.30	7.65	2.43	7.67	1.77	7.15	3.48	2.28	7.38	3.5	0.38	4.01	0.73	8.01	5.75	0.47	1.72	2.39	1.07
(999999) Rigid	3.27	7.58	5.52	6.07	8.92	9.53	2.17	6.33	0.30	4.36	9.57	5.68	3.84	9.13	5.24	2.03	4.18	1.34	8.81	6.62	0.89	2.12	2.40	1.26

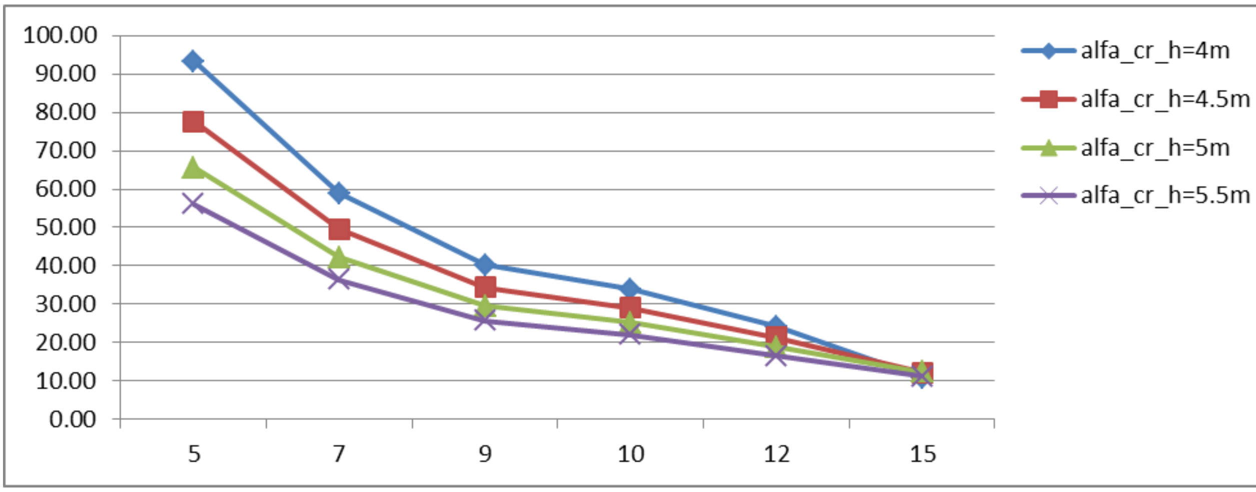
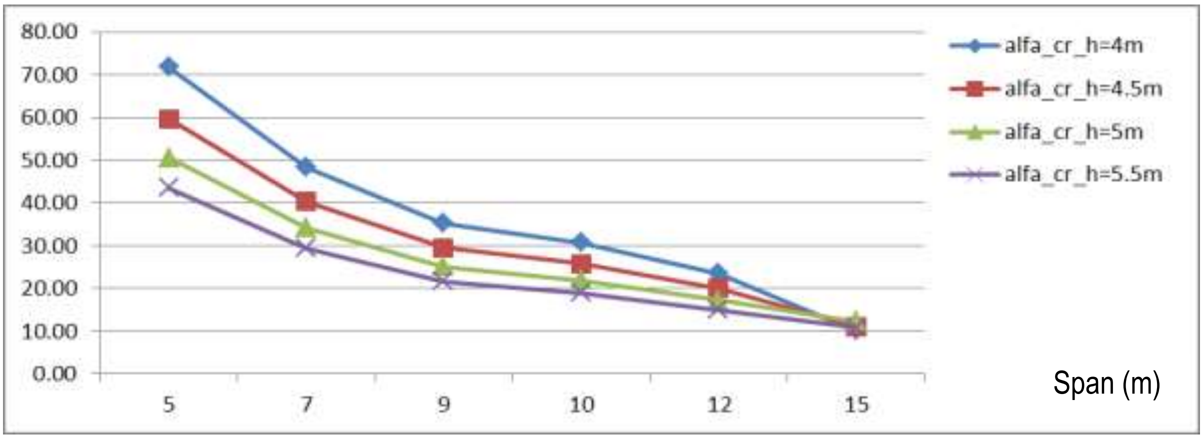
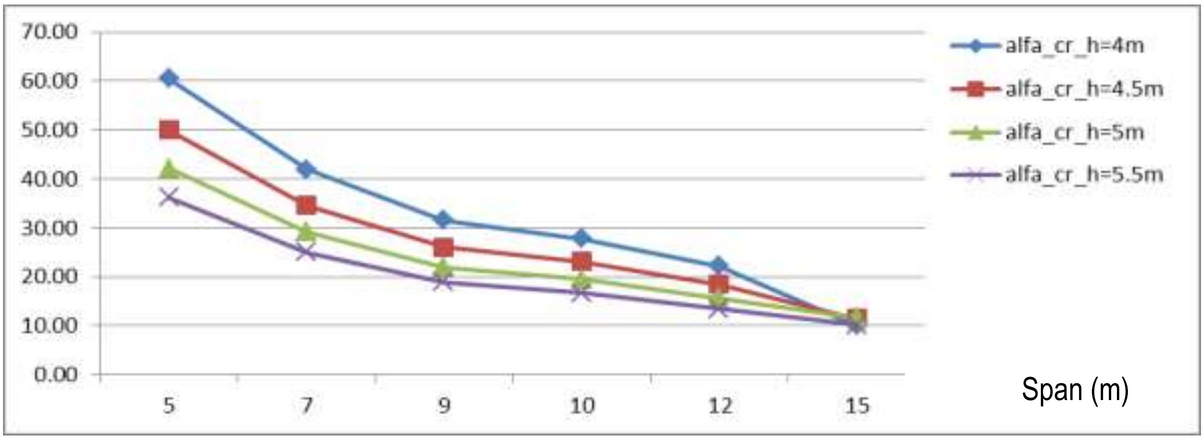


Figure 5.3 Critical Load factors vs. Span



(Rigid for span=15m, Rotation stiffness S_j=15425kNm/rad)

Figure 5.4 Critical load factors (Semi-rigid steel frames)



(Rotation stiffness S_j=15425kNm/rad)

Figure 5.5 Critical load factors (Semi-rigid steel frames)

With reduction of the rotation stiffness, critical load of frames reduced. It is mentioned that α_{cr} might reduce 15%~25%, when S_j/S_{beam} is equal to 25, which is considered as rigid joints to EN 1994 and EN1993.

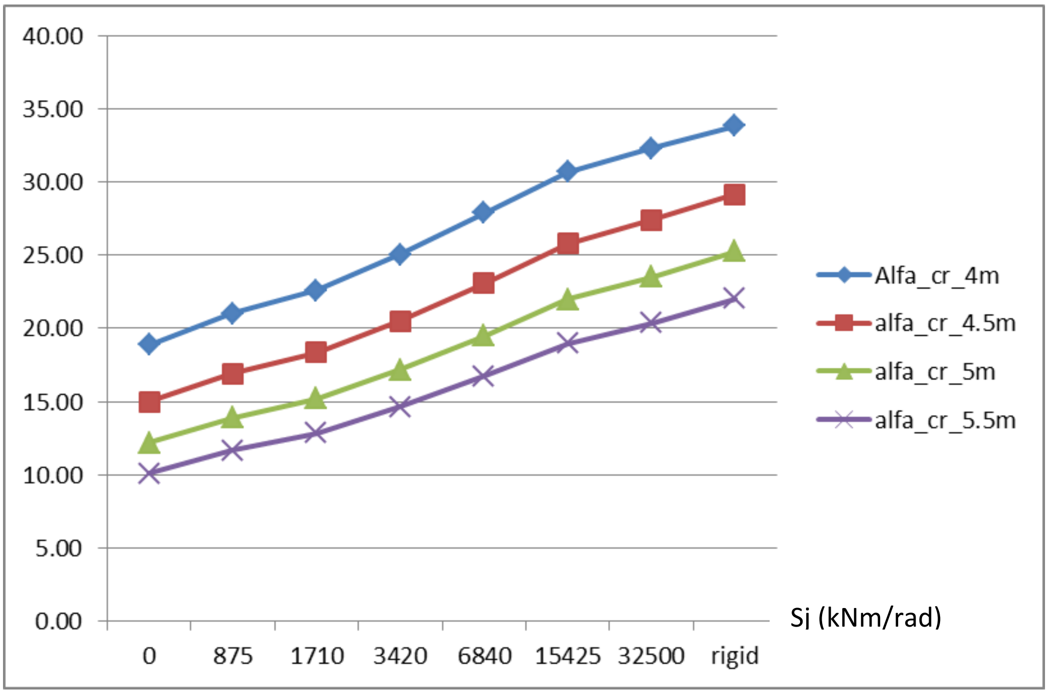


Figure 5.6 Critical load factor vs. Joints stiffness

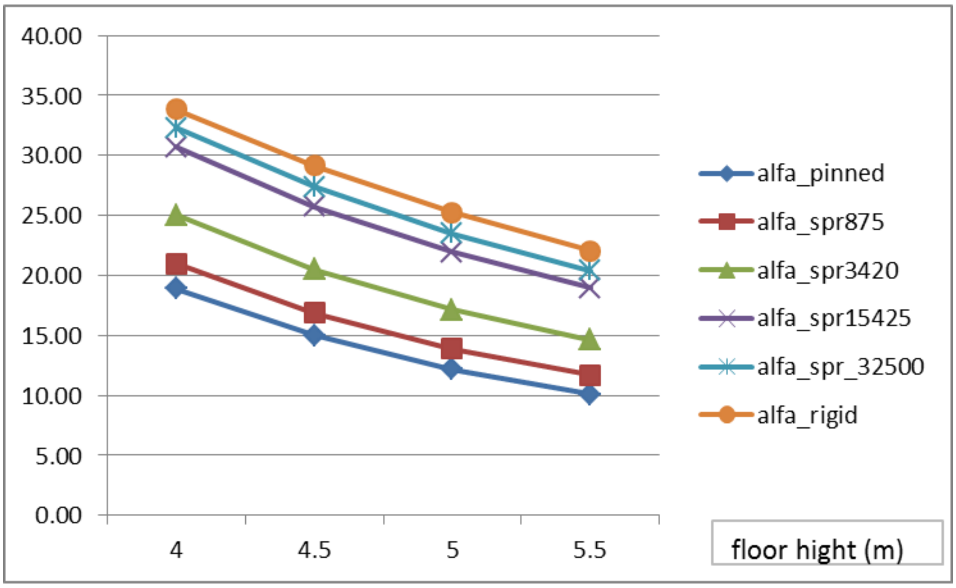


Figure 5.7 Critical load factor vs. column height

5.1.2 Moments at joints

5.1.2.1 Effects of Joint stiffness

The frames studied are the same as for critical load analysis. Vertical loading is employed. Some results are listed in Table 5-2.

Table 5-2 Moment of joint in a 1-spam 1-storey frame

pan (m)	Sj spring (kNm/rad)	joint classification	ight (m)	2 M3 (kNm)	M5 (Load_V+H) (kNm)	a lfa_ RSBUCK	Alfa _2order	ngle ϕ _3- ϕ _2 (mrad)
0	pi nned	pinn ed			0	1 3.38	12.3	3.1
0	43 8	pinn ed		1.66	35. 47	1 3.33	13.3	2.3
0	87 5	semi -rigid		5.96	62. 74	1 4.16	14.1	4
0	17 10	semi -rigid		9.64	100 .60	1 5.52	15.4	2.5
0	34 20	semi -rigid		30.94	147 .10	1 7.56	16.5	8.3
0	68 40	semi -rigid		70.13	191 .33	2 0.02	15.4	4.8
0	15 425	semi -rigid		04.13	229 .78	2 2.63	15.5	3.2
0	32 500	rigid		22.77	250 .89	2 4.24	16	.8
0	rig id	rigid		42.79	273 .59	2 6.08	16.8	

The results are curved below, where M2 is end moment of beam under vertical loading, and M5 is the maximum end moment of beam under vertical and horizontal loading. Rotation angle of joints are computed from rotation deformation of column and beam at joint.

For a joint stiffness at $0.25S_{beam}$ (where $S_{j,ini}/S_{beam}$=0.5) , which could be considered as pinned, the joint moment is 31.66kNm and 35.47kNm, about 8% of mid-span moment of pinned beams.

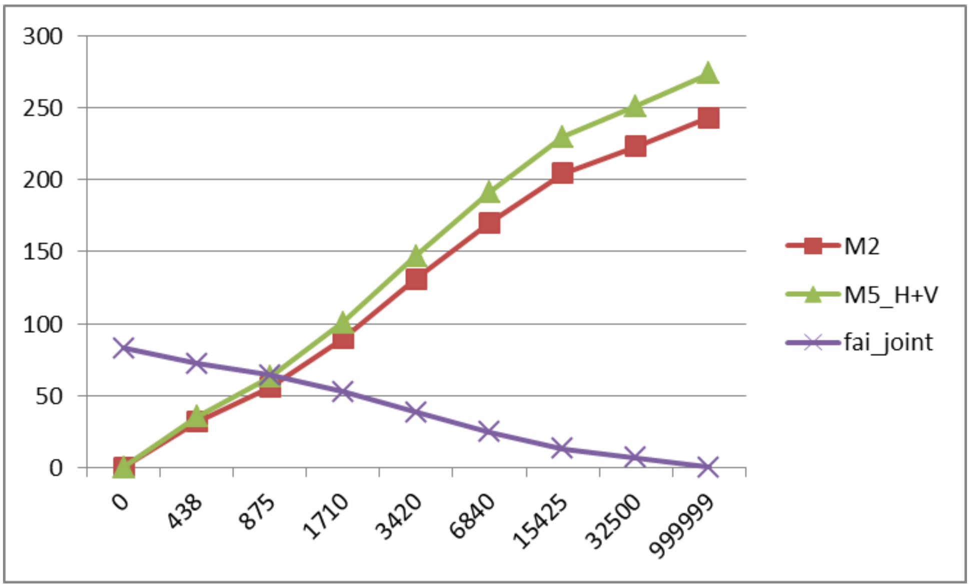


Figure 5.8 Moment at joint and Joint rotation

5.1.2.2 Effects of joints stiffness with varied spans

End moments of beam reduce when end releases are employed, as showed in the following figures. With certain components arrangement of connections, the characteristics of joint might be considered similar. When used in frames with different spans, moment reduction induced seems to be larger in smaller span frames.

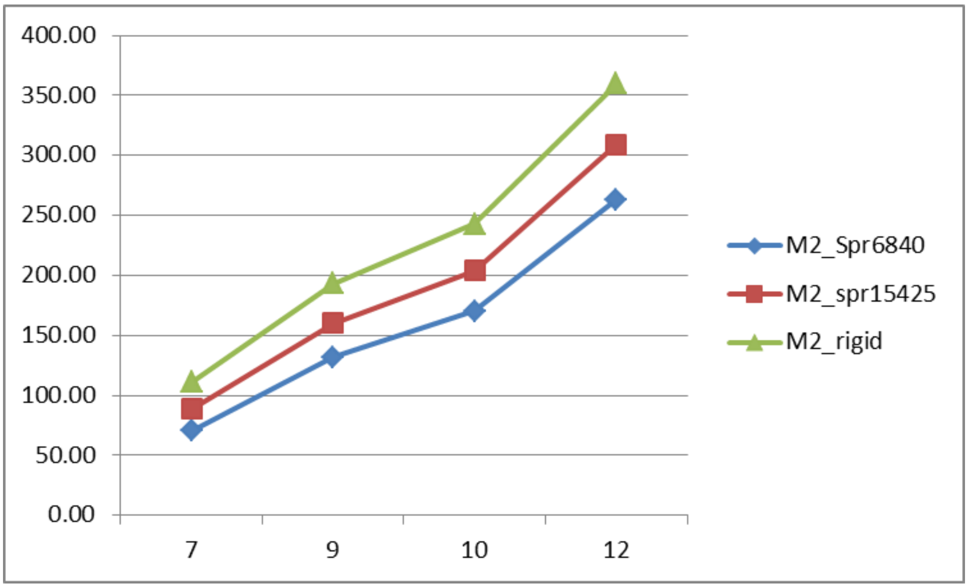


Figure 5.9 Moment reduction with Semi-rigid joints

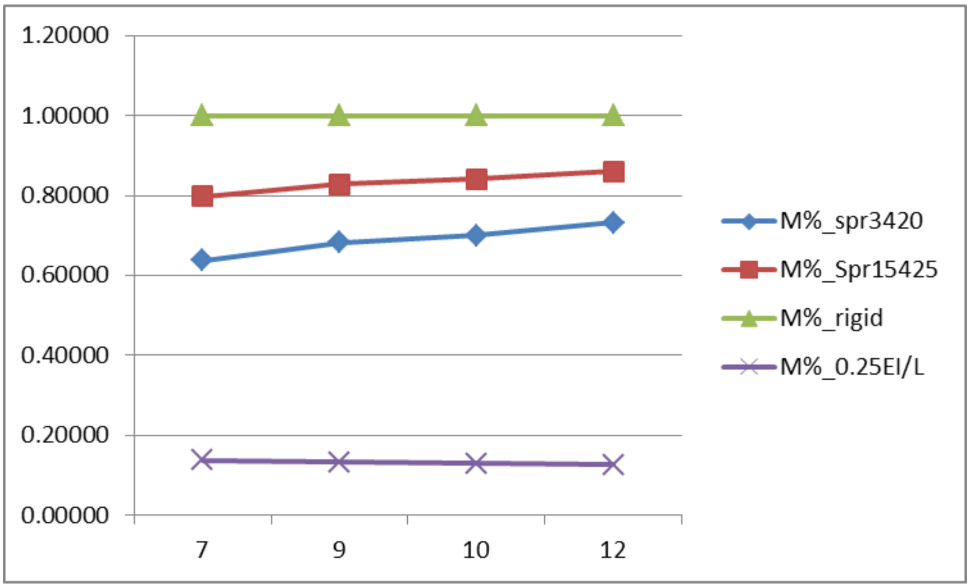


Figure 5.10 Moment Redistribution with semi-rigid joints

Moment redistribution will be proportionally, if the stiffness ratio is constant, as showed for $0.25S_j/S_{beam}$ in the below Figure 5.10 and Table 5-3.

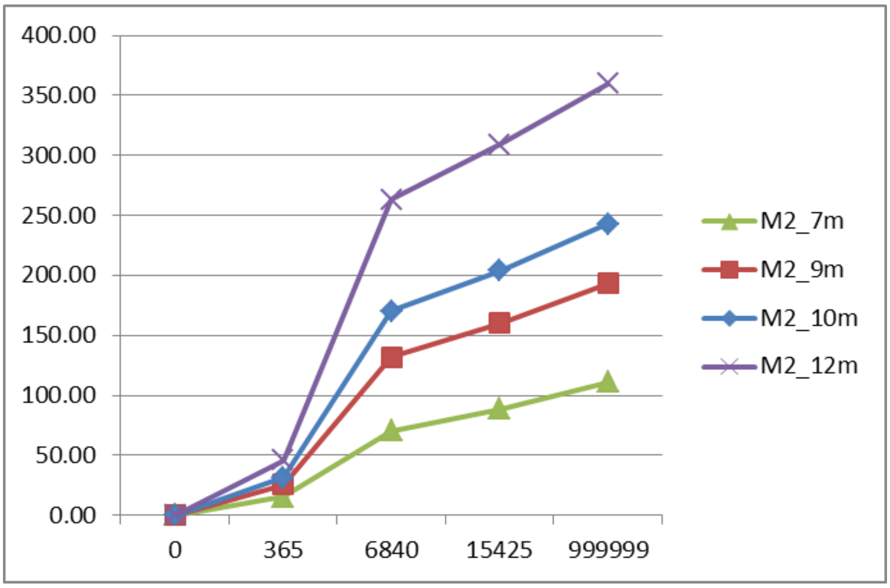


Figure 5.11 Moment of joint in frames

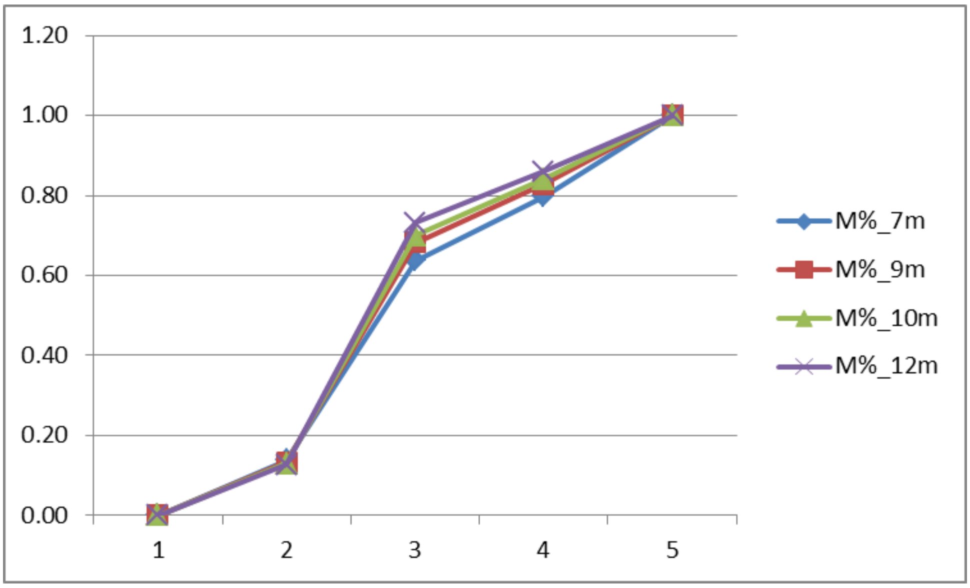


Figure 5.12 Moment percentage comparing with rigid frames

Table 5-3 End Moment and Percentage

Span (m)	**7**		**9**		**10**		**12**	
Stiffness (kNm)	**M (kNm)**	**%**	**M (kNm)**	**%**	**M (kNm)**	**%**	**M (kNm)**	**%**
pinned	0.00	.00	0.00	.00	0.00	.00	0.00	.00
365							45.75	.127
438					31.66	.130		
487			25.59	.133				
626	15.37	.138						
875					55.96	.231		
1710					89.64	.369		
3420					130.94	.539		
6840	70.75	.637	131.72	.682	170.13	.701	263.23	.732
15425	88.66	.798	160.03	.829	204.13	.841	309.37	.860
32500					222.77	.918		

rigid	111.05	.000	193.09	.000	242.79	.000	359.60	.000

5.1.3 Stiffness of beam

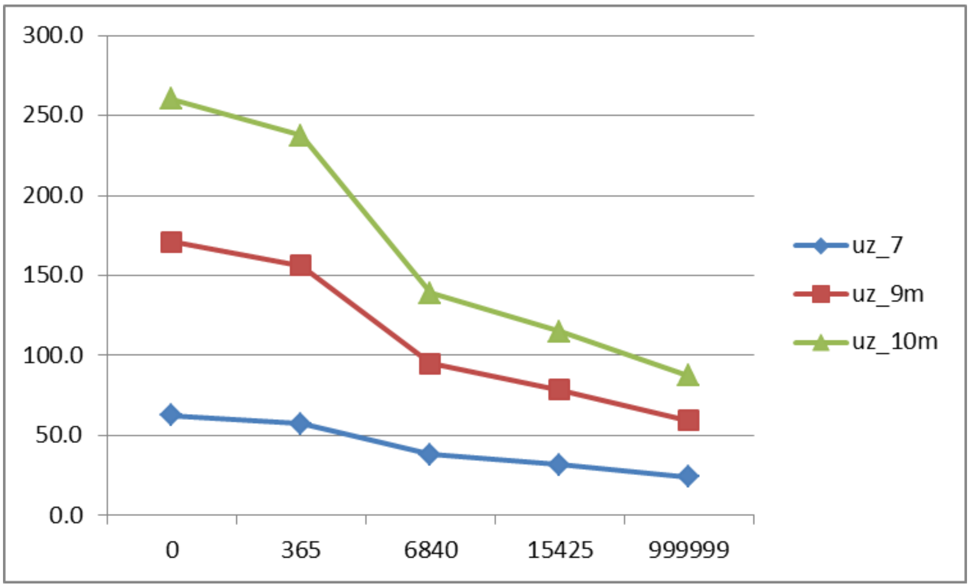


Figure 5.13 Deflection of beams (mm)

5.1.4 Rotation of joints

Rotation capacity of joints is less important for rigid frames, as ductility is provided most by respective beams. For pinned frame the joints are normally designed for free rotation, which might be unreal for most case. For a beam at 12m span under studied loading, the required rotation of joints is larger than 140 mrad, which should be properly provided with sufficient shear capacity.

The rotation angle at stiffness of 365kNm/rad is calculated with $0.25S_{beanm}$, representing joint classification boundary in Eurocode. The used spring values are set according to beam span, namely 365, 438, 487, and 626, for frame with span at 12m, 10m, 9m, and 7m respectively. It should be mentioned that required joint rotation of semi-rigid frame might be less than 40mrad, if respective stiffness is provided.

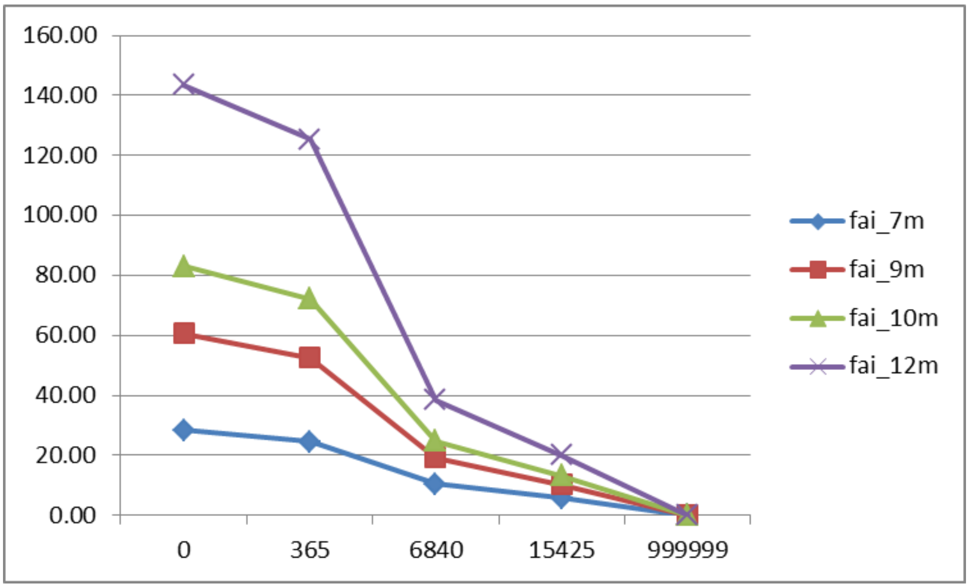
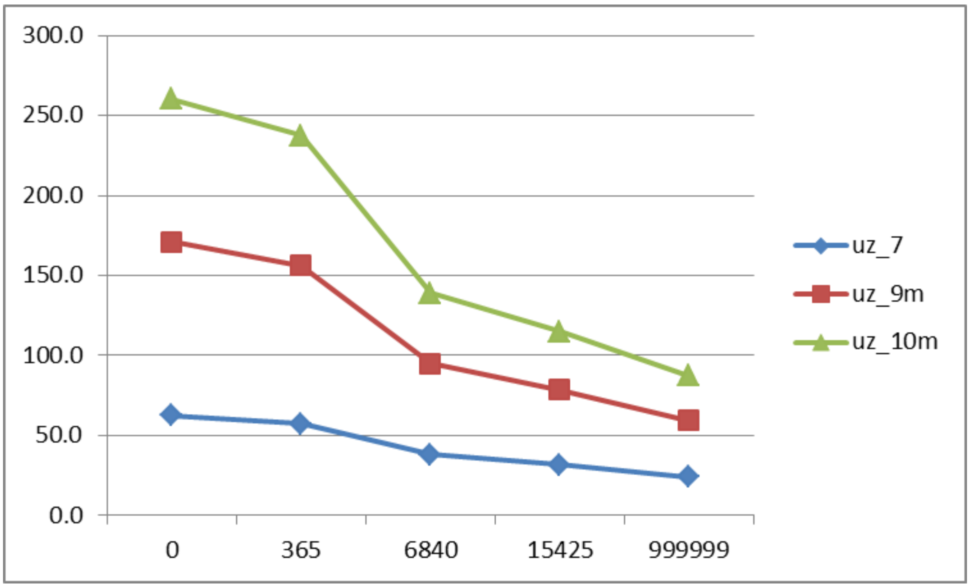


Figure 5.14 Rotation of joints (h=5m, unit: mrad)

Following this consideration, stiffness ratio (S_j/S_{beam}) might not less than 4 for span up to 10m, and 10 for larger span. Further research should be introduced for more complicated frames.

Table 5-4 Joint Rotation Requirement under vertical loading

Span (m)	Sj spring (kNm/rad)	Sj /Sbeam	joint classification	Angle ϕ_3-ϕ_2 (mrad)
10	3420	3.897969	semi-rigid	38.3
7	6840	5.457157	semi-rigid	10.4
9	6840	7.016344	semi-rigid	19.3
10	6840	7.795938	semi-rigid	24.8
12	6840	9.355125	semi-rigid	38.5
7	15425	12.30653	semi-rigid	5.7
9	15425	15.82268	semi-rigid	10.3



5.1.5 Stiffness of frame

With the reduction of joint rotation stiffness, the frame becomes slender. This is illustrated in the following figure. Column rotation deflection at the joint decreases as joint moments reduces, with a smaller critical load factor.

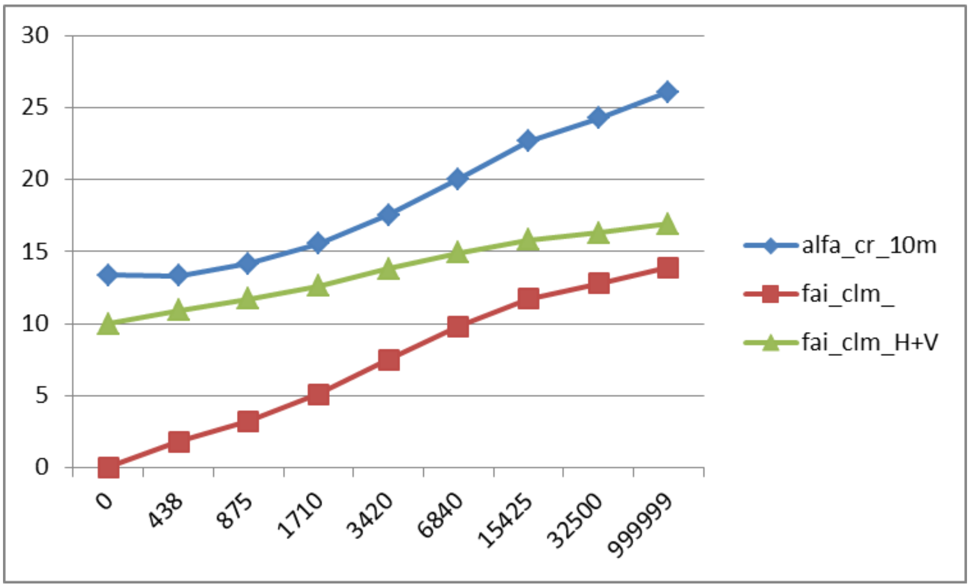


Figure 5.15 α_{cr} and Rotation angle of Column

This is also showed by the following figures, which display the horizontal displacement at the left node of floor. The varied span has less influence as columns are the main elements to resistant horizontal forces.

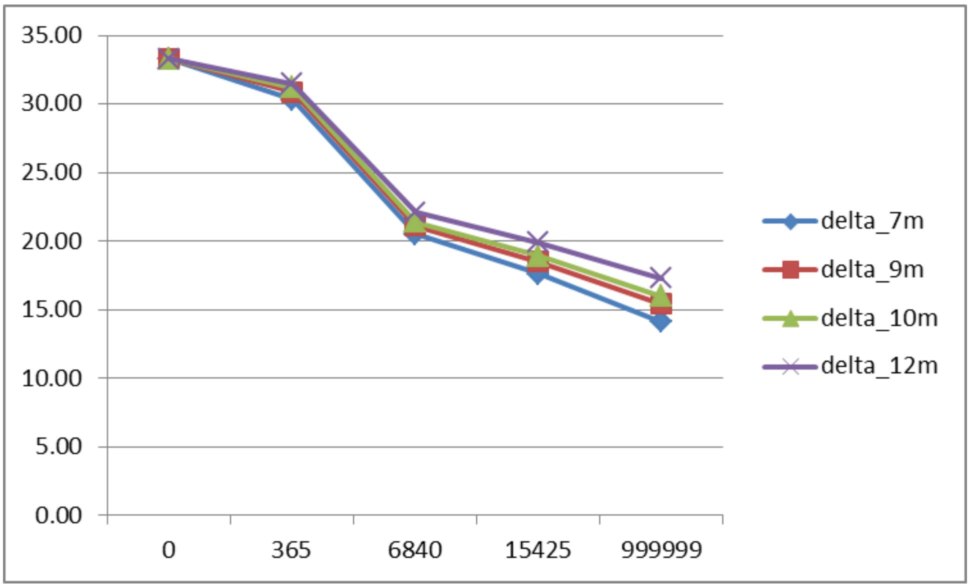


Figure 5.16 Horizontal displacement of frame (h=5m unit: mm)

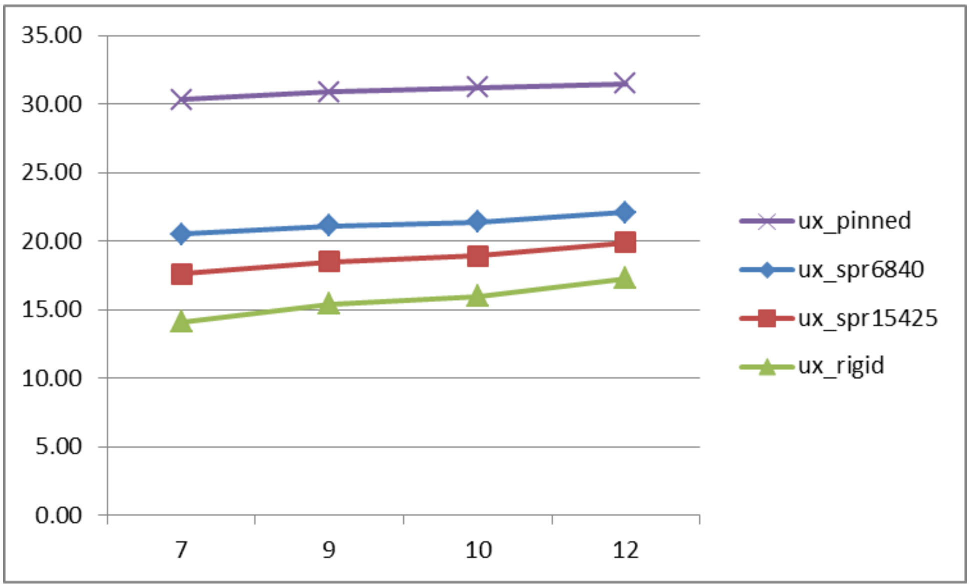


Figure 5.17 Horizontal displacement of frame (h=5m, unit: mm)

5.2 Composite frames

A series of one-span one-storey frames are studied, using RSTAB8.0 programme. The setting of spans and heights is showed as in Figure 5.18 . A load case with only vertical loading is used, for study of cracking length, moment distribution, beam stiffness, joint rotation, and critical load analysis. Additional load case is set when horizontal node force is combined with the above vertical loading. Cracking lengths of beams are set as result of vertical loading, as proposed in Part 2. The loading is showed in Figure 5.19.

Against

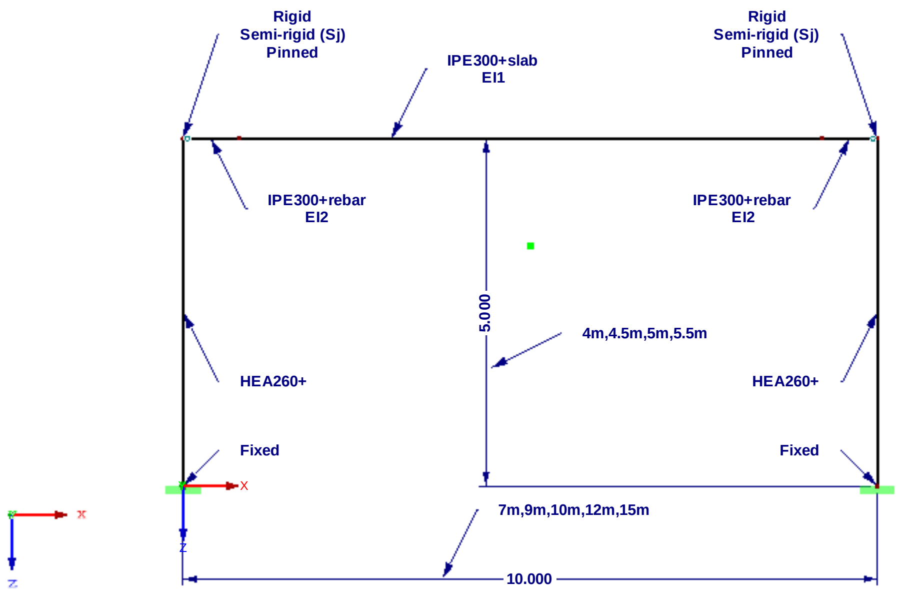


Figure 5.18 Composite frame studied

Cracking length of beams is calculated iteratively, while the moment and cracking length coverage on perfect coherence for almost all case but one. The exception is a 7-meter span at 5-meter height frame, with a joint stiffness of 1000kNm/rad. The respective S_j/S_{beam} is 0.221, which is considered as pinned according to EN1994-1-1. The cracking length used in study is the conservative one with no cracking, which leads to some degree of incoherence, as showed in following explaination.

Table 5-5 Example of divergent iteration

irritation order	initial x_crk	End Moment	x cracked length @M=Mcrk=7.351	Frame data
1	.099	7.08	-0.0022	L=7m M_crk=7.35kNm q=35kN/m R1=122.5kN
2	.000	22.62	0.1269	
3	.127	7.08	-0.0022	

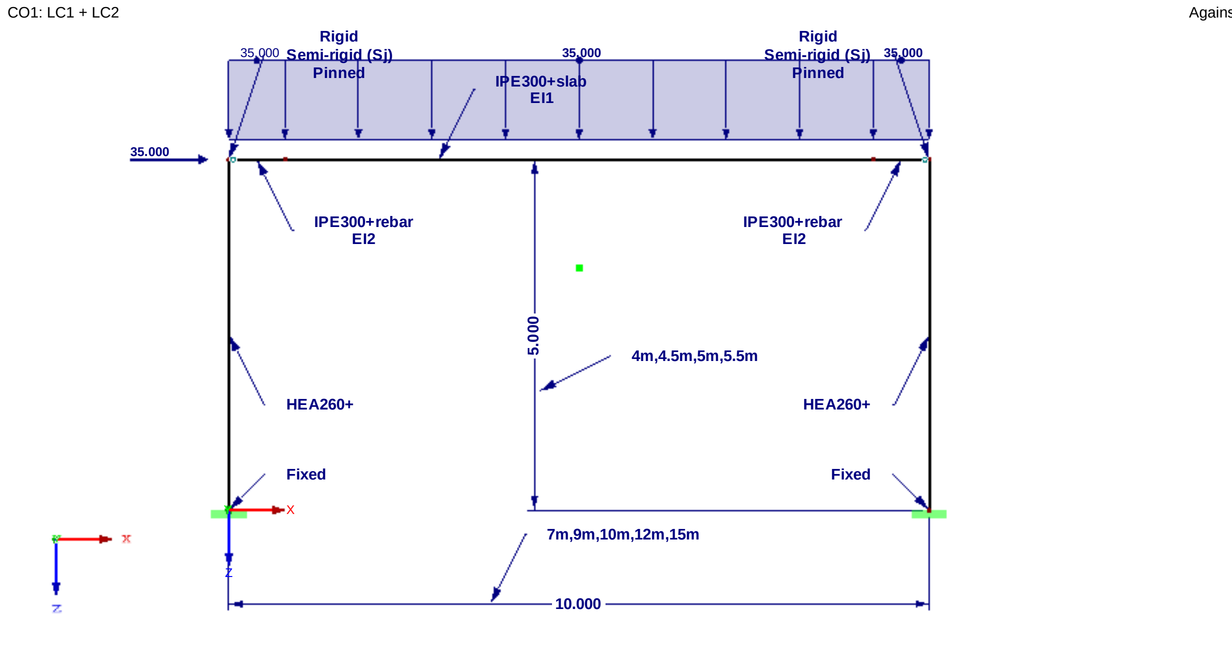


Figure 5.19 Load case: Horizontal and vertical Loading

The moments at critical sections are collected, while rotations of joints at the column end and beam end are provided to account for joint rotation angle under respective loading case. Vertical displacements of mid-span or maximal at span are recorded, as well as the horizontal displacement of floor (loaded node) for respective loads.

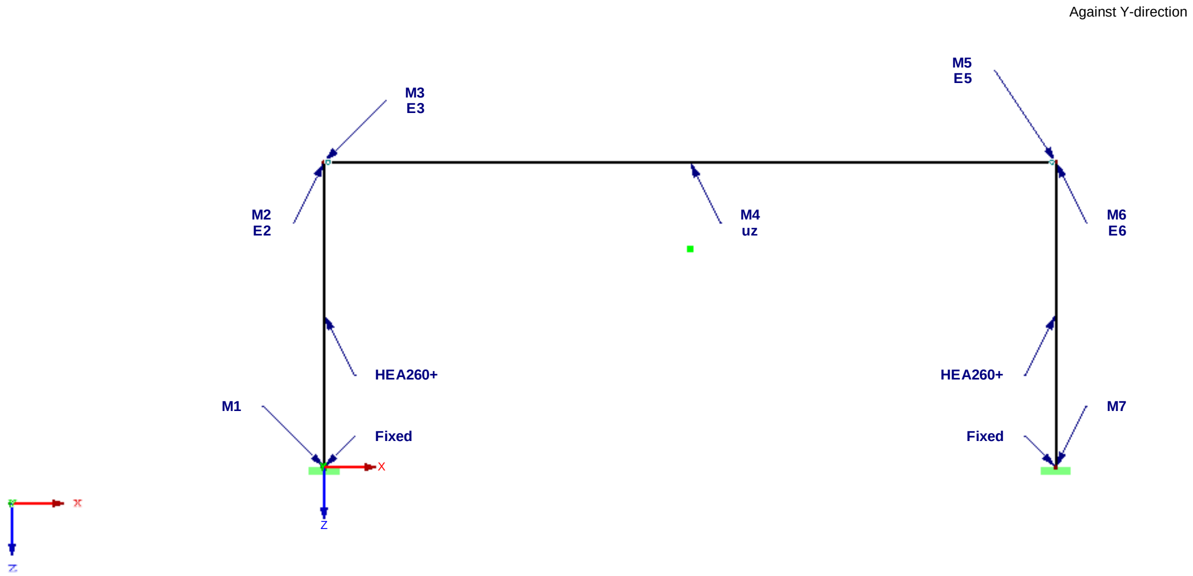


Figure 5.20 Critical Sections and reference displacements

5.2.1 Critical load factors

As showed in following figures, the critical load factor will reduce toward that of a pinned frame when the stiffness of joints decreases. Rotation stiffness at value of 999999 is the result of idealized rigid frame, if not further explained in the following parts.

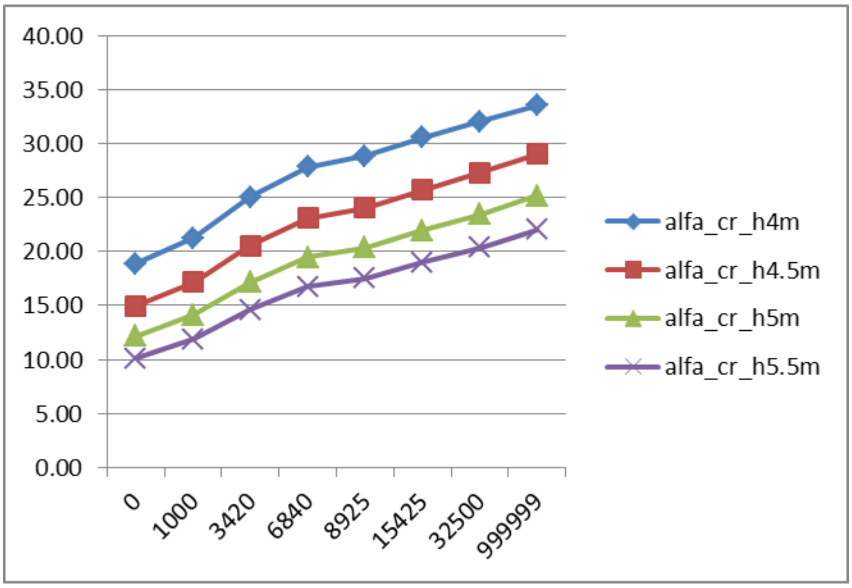


Figure 5.21 α_{cr}-S_j curve 1 (Span =10m)

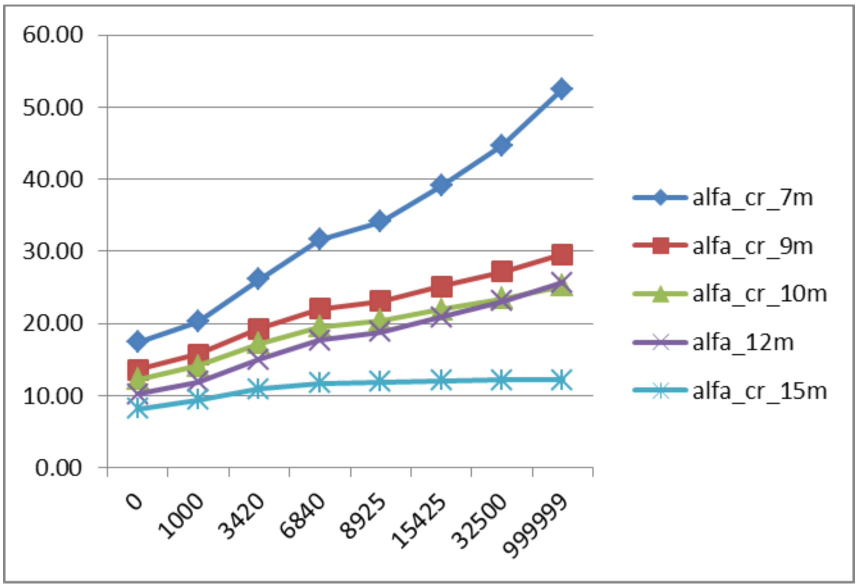


Figure 5.22 α_{cr}-S_j curve 2 (h=5m)

The curves are not parallel at spans, especially whit a large span, where the beam stiffness is more sensitive to the joint stiffness.

Critical load decrease whit increased height of frame, as influence of slender frame, especially impact of slenderer columns.

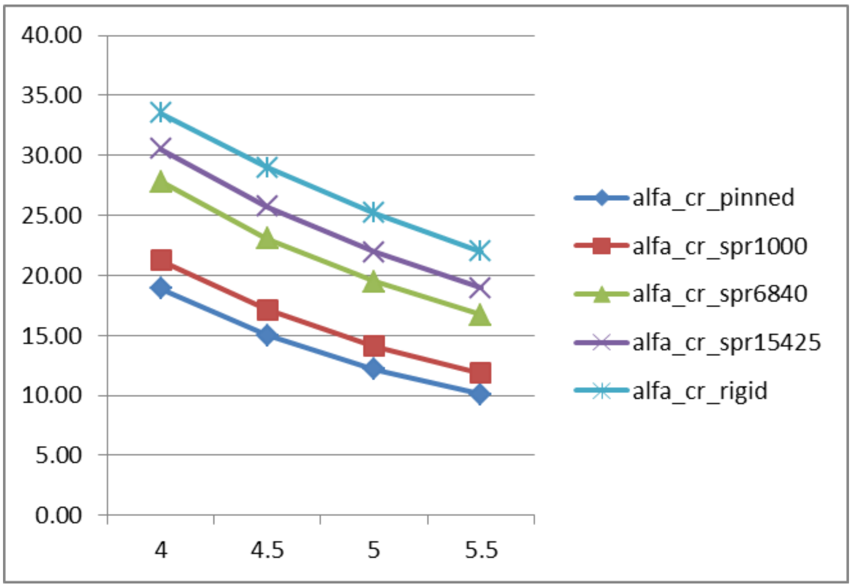


Figure 5.23 α_{cr}-h_i curve

5.2.2 Moments at joints

Moments of beam end are equal to that of respective end of column, which is determined by balance of moments, no matter stiffness of joints. Apart from rigid joints, rigidity of joints leads to moment redistribution of respective beams. The percentage is related with the joint and beam stiffness ratio, but not linearly.

The increased value of moment within 7m-span frame is results of zero cracking, as mentioned at the beginning of this part. If value of beam with cracking length used, curve will be coherent with that of other spans.

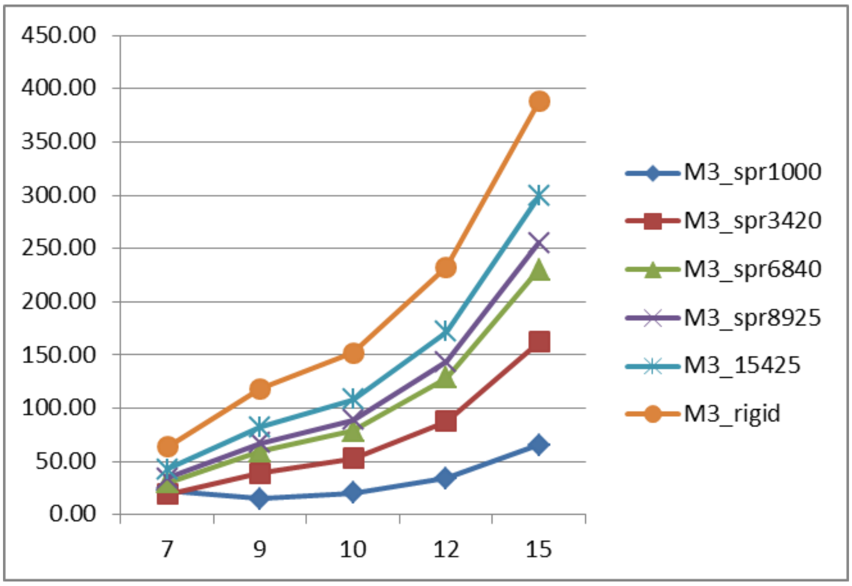


Figure 5.24 Moment of beam ends under vertical loading (h=5m)

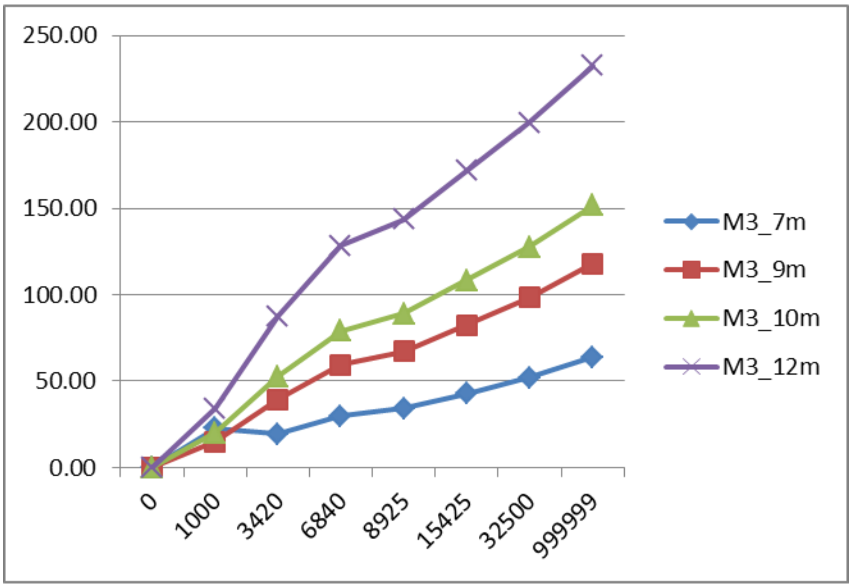


Figure 5.25 Moment redistribution as result of joints stiffness (h=5m)

Varity of frame height effects slightly the moments at joints, as bending stiffness of column changes slightly.

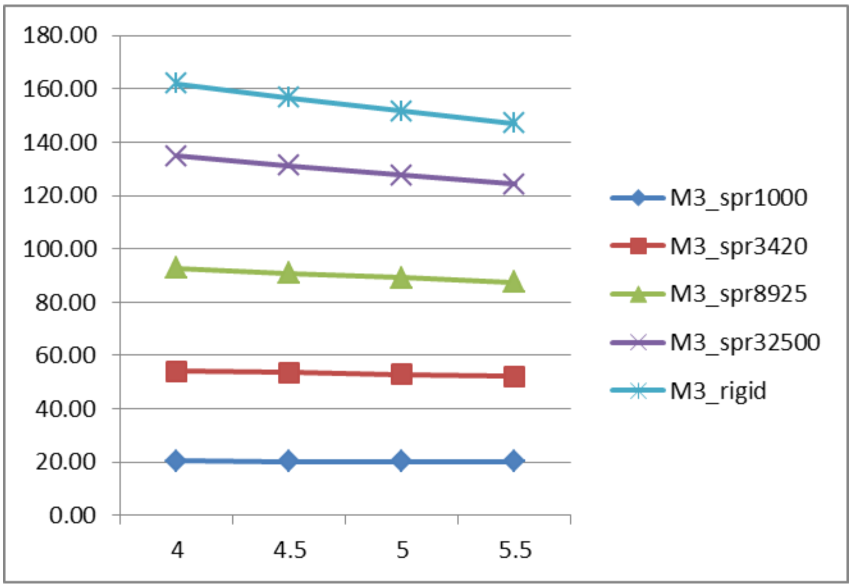


Figure 5.26 Moment redistribution (Span=10m)

5.2.3 Cracking length of beam

Cracking length extends with increasing rigidity of joints up to rigid joints. Cracking length seems might to be proportion to beam span with other constant condition. This is more obvious when the cracking ratio, namely the ratio of cracking length to span, is curved with joint stiffness, considering relative stiffness is not applied.

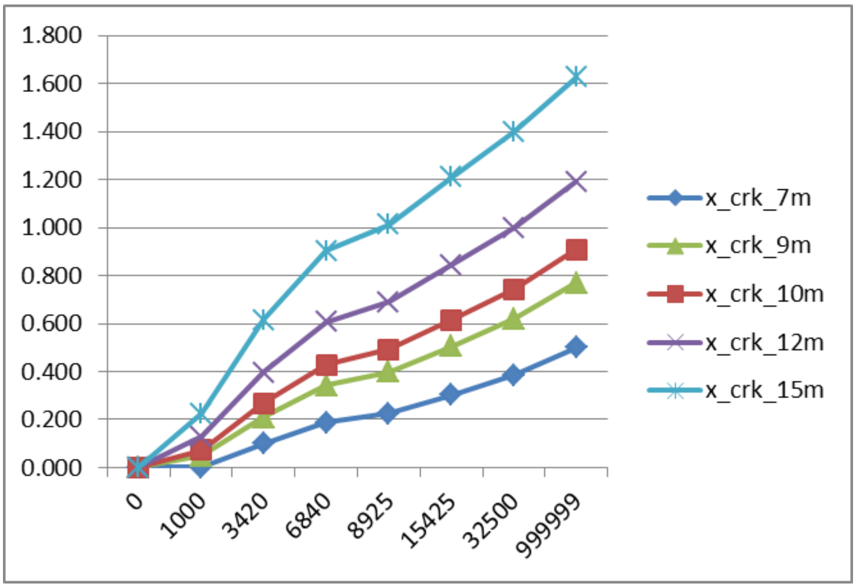


Figure 5.27 Cracking length of composite beam (h=5m)

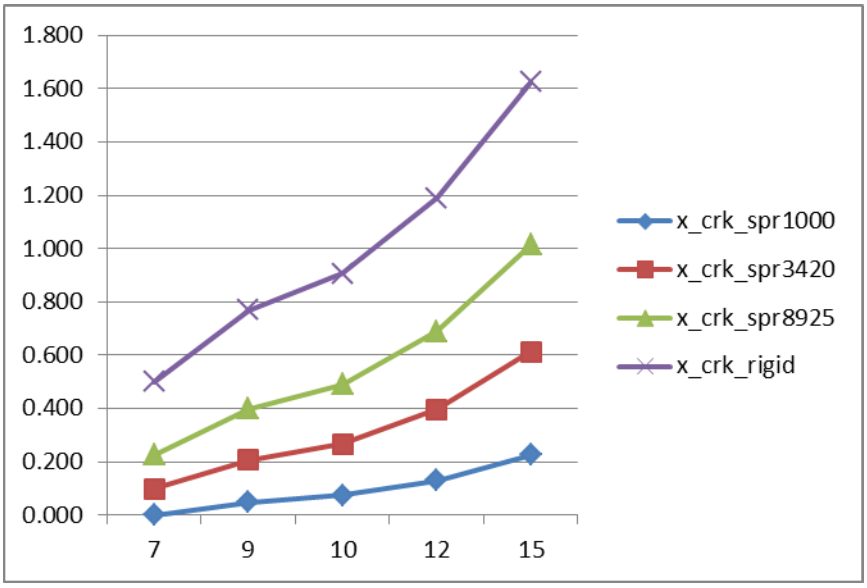


Figure 5.28 Cracking length vs. span

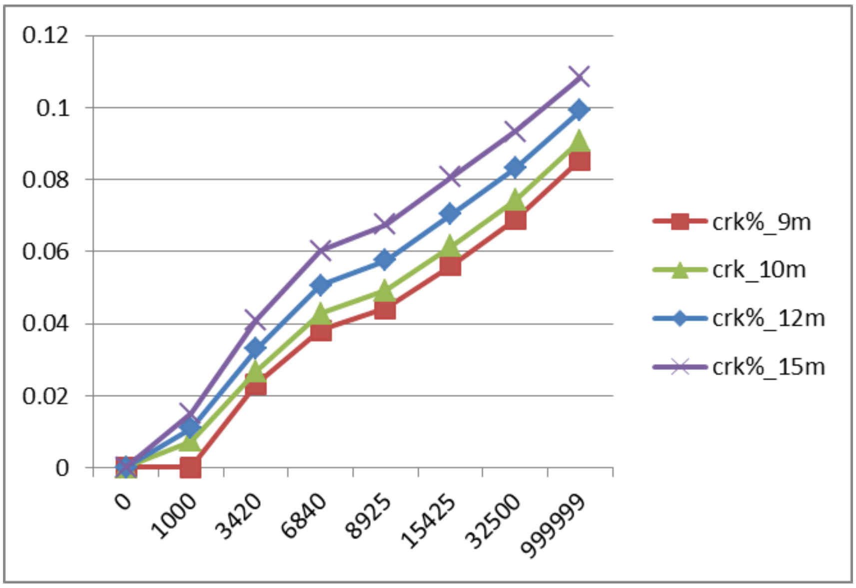


Figure 5.29 cracking ratio of composite beams

As showed in the following figure, cracking length is less effected by column height as the column bending stiffness varies slightly in this study.

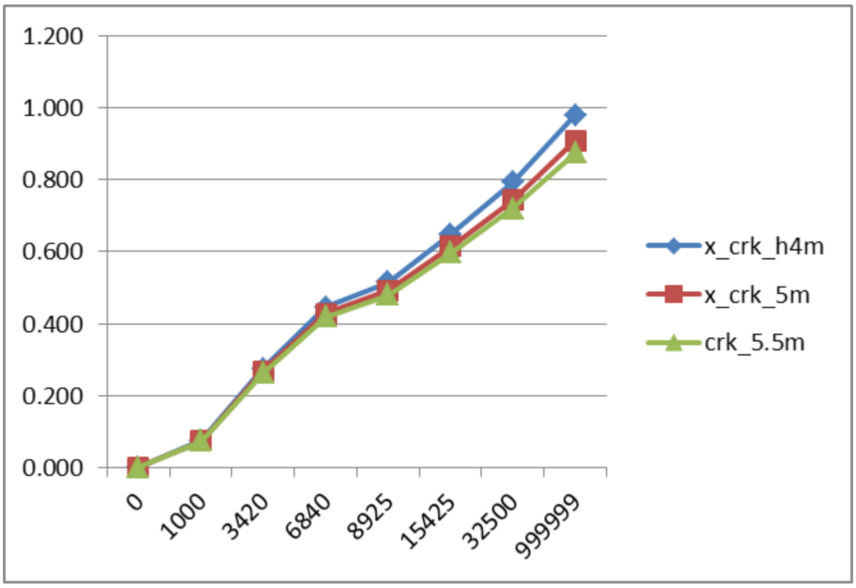


Figure 5.30 Cracking length of composite beam (span=10m)

5.2.4 Stiffness of beam

Stiffness of beam decreases when respective joints lose their rigidity. The displacement increases largely within slender beams, as a result of large span. If relative displacement is applied, that is, the ratio of displacement within semi-rigid beam to that of rigid beam, the curve seems to converge. The relation seems to be controllable unless the joint is dominantly pinned, as indicated in a 7-span beam, or the beam is too slender to perform service limit situation, which is not presented here. The beams with span of 12m and 15m have produced unfavourable displacements here, as a result of overloading, comparing to real design.

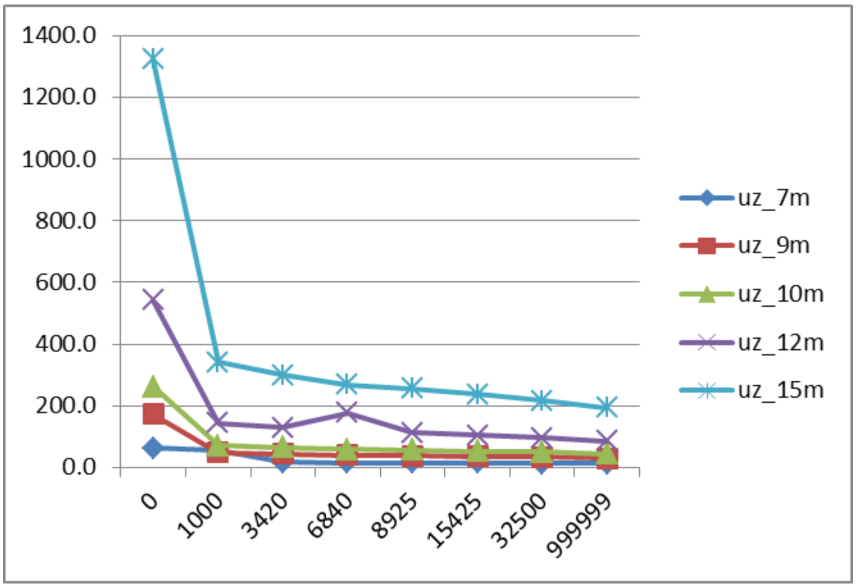


Figure 5.31 displacement of beam at mid-span (h=5m)

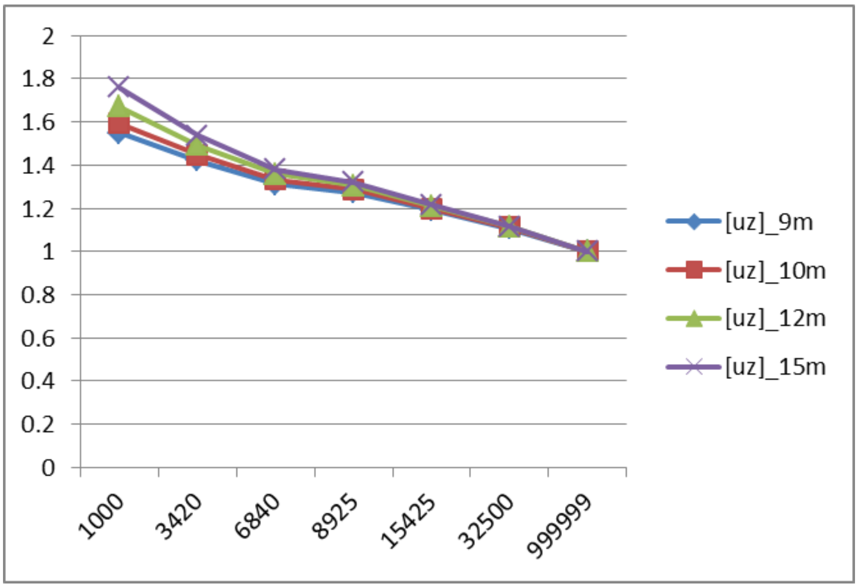


Figure 5.32 Displacement ratio vs. rotation stiffness of joint

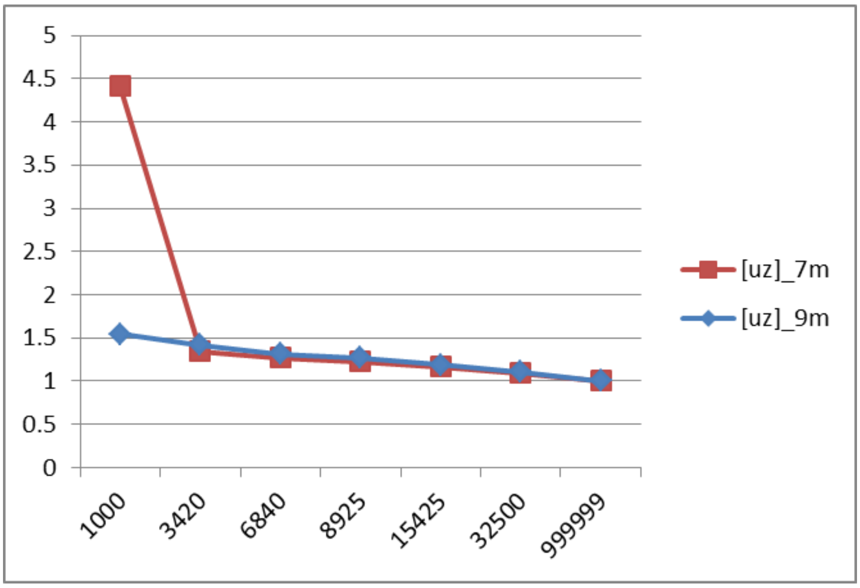


Figure 5.33 Lose of beam rigidity with soft joint (h=5m, span=7m)

5.2.5 Rotation of joints

To perform moment redistribution as discussed above, available joint rotation should not be less than value from elastic analysis. As showed in above parts, most joints are possible to have a rotation capacity of 31mrad, which extends to 100mrad when the joint stiffness reduced to 8425kNm/rad (initial stiffness of joint will be 17850kNm/rad) (Bitar, et al., 2006).

Table 5-6 Required Joint rotation (mrad)

pan (m)	EI /L (kNm)	Sj spring (kNm/rad)	Sj /Sbeam	joint classification	ngle $\phi\_3 - \phi\_2$ (mrad)
0	6331.174	3420	1.08	semi-rigid	5.8
2	5275.978	6840	2.593	semi-rigid	8.7
5	4220.783	15425	7.309	semi-rigid	9.4
0	6331.174	1000	0.316	pinned	0.4
	9044.534	1000	0.221	pinned	2.6
2	5275.978	3420	1.296	semi-rigid	5.7
5	4220.783	8925	4.229	semi-rigid	8.5
5	4220.783	6840	3.241	semi-rigid	3.6
2	5275.978	1000	0.379	pinned	4.4
5	4220.783	3420	1.621	semi-rigid	7.4
5	4220.783	1000	0.474	pinned	5.7

For span up to 12m, the required joint rotation will be less than 35mrad, with a secant stiffness of 1000kNm/rad, which is considered as pinned as stiffness ratio (0.38) is less than 0.5. Larger span requires an increased rotation capacity of joints. A 15m-span frame demands joint rotation at 65.7mrad, which is not presented in the following figure. The required bending moment in span is beyond section bending capacity, which involves a larger height of section, normally a larger steel profile. Fortunately such a span is not common for multi-storey buildings.

The incoherence of 7m-span frame is caused by divergence of cracking length, as discussed above. Pinned frames require a relatively large rotation capacity, as effect of cracking is not valid for moment redistribution. The un-cracked stiffness of composite beams is applied in such case.

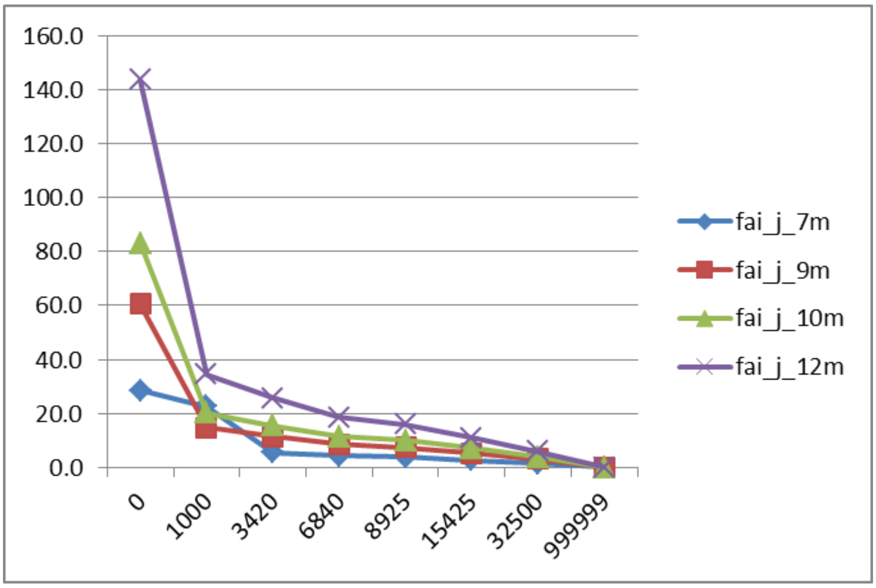


Figure 5.34 Rotation angle of joints (h=5m)

Frames with stiff joints perform a small joint rotation, which is coherent with the rigidity of joint and its less ductility. The required rotation ability increases with an extended span, on which attention should be drawn, especially when a soft joint is applied.

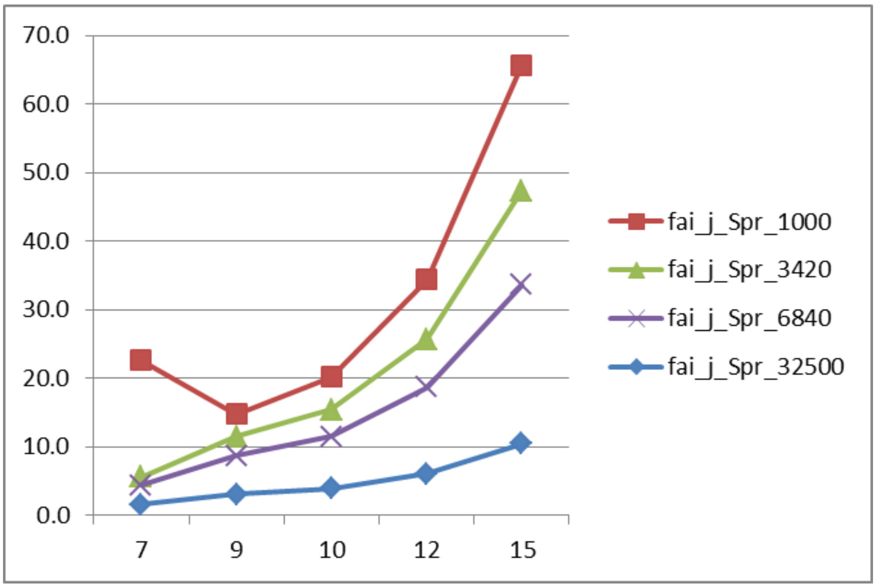


Figure 5.35 Rotation angle of joints vs. span (h=5m)

Frame height plays little role here, as stiffness of column has limited variation with a column height range from 4m to 5.5m.

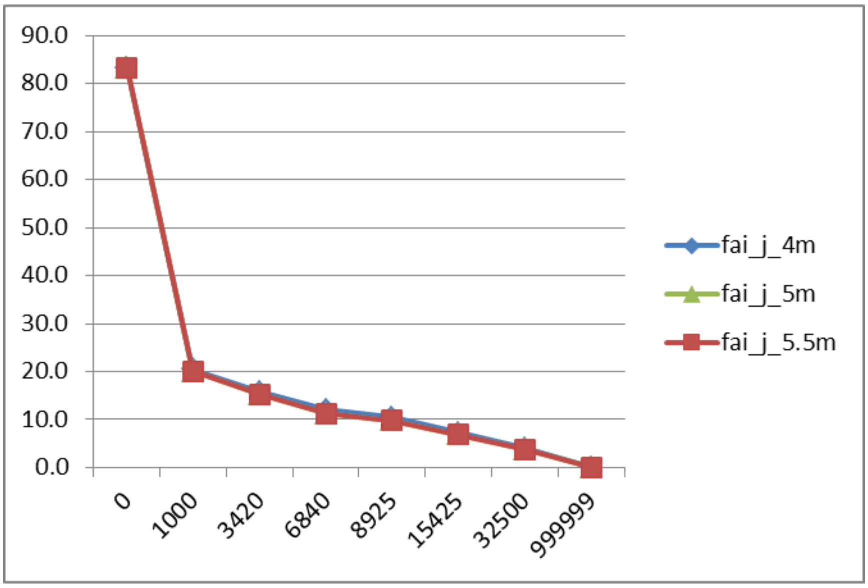


Figure 5.36 Rotation angle of joints (span=10m)

Reduction of joint stiffness is possible to be used for moment redistribution, which fits the bending capacity of composite beams. Even within reduced joint moments, rotation of joints will tend to increase, which requires a larger joint rotation capacity. As showed in Table 5-7 and Figure 5.37, when stiffness ratio approaches 10, required joint rotation is about 3.9mrad. Even when a practical pinned connection with stiffness ratio at 0.3, required rotation of beam is about 20, less than available value from test.

It should be mentioned that, required rotation capacity might increase significantly with large span. Nethercot (Nethercot & Stylianidis, 2008) deduced a value large than 120mrad for a 16m span beam, using quasi-plastic design approach.

Table 5-7 Moment and Rotation of joint in frame

pan L (m)	S_j spring (kNm/rad)	$_j/S_{beam}$ (S_jL/EI_1)	joint classification	ight (m)	racked length (m)	M oment _joint (kNm)	req uired ϕ _j (mrad)
0	rig id		rigid		.907	1 51.71	0.0
0	32 500	0.27	semi -rigid		.742	1 27.61	3.9
0	15 425	.873	semi -rigid		.614	1 08.19	7.1
0	89 25	.819	semi -rigid		.491	8 9.10	10.0
	68		semi			7	11.5

0	40	.161	-rigid		.427	8.93	
0	34 20	.08	semi -rigid		.267	5 2.84	15.4
0	10 00	.316	pinn ed		.074	2 0.19	20.2
0	pi nned		pinn ed		.000	0 .00	83.1

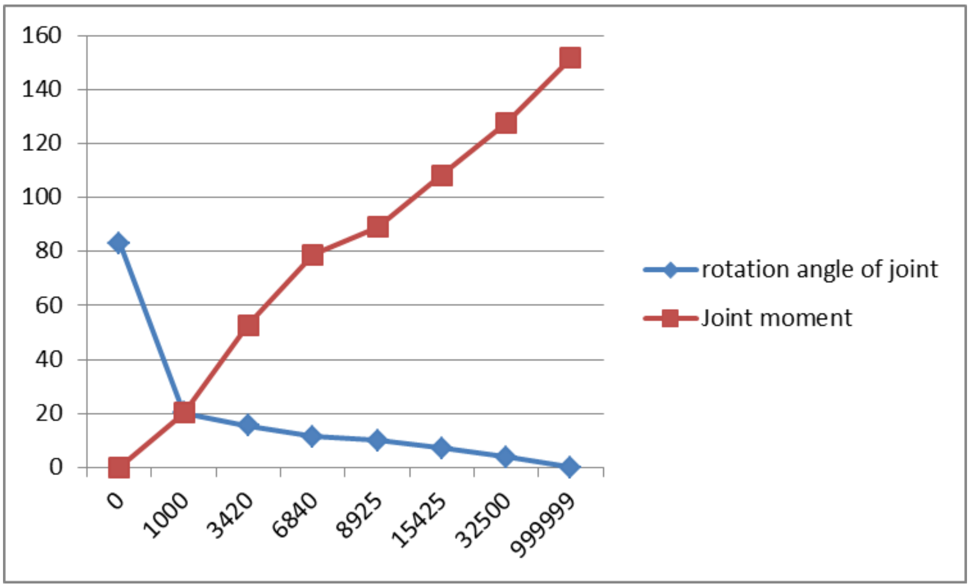


Figure 5.37 Joint moment and joint rotation (span=10m, h=5m, q=35kN/m)

5.3 Comparison of steel and composite frames

Critical load factors are employed here, to obtain ideas about global stability of composite structures. The specimen studied here are some of before discussed frames, with varied span and 5m height. Results are listed in Table 5-8.

Table 5-8 Critical load factor of steel and composite frames

Ty pe	Sj \span	7	9	1 0	1 2
ste el	pinn ed	1 7.69	1 3.76	1 3.38	1 0.32
ste el	0.25 S_beam	1 9.60	1 4.92	1 4.92	1 0.98
ste el	6840	3 0.09	2 2.63	2 0.02	1 6.06
ste el	1542 5	3 5.42	2 5.94	2 2.63	1 7.64
ste	rigid	4	3	2	1

el		4.14	0.68	6.08	9.27
co mposite	pinn ed	1 7.41	1 3.54	1 2.19	1 0.16
co mposite	1000	2 0.24	1 5.71	1 4.12	1 1.90
co mposite	6840	3 1.64	2 2.02	1 9.49	1 7.68
co mposite	1542 5	3 9.04	2 5.12	2 1.95	2 0.88
co mposite	3250 0	4 4.71	2 7.13	2 3.46	2 3.03
co mposite	rigid	5 2.49	2 9.54	2 5.19	2 5.63

Part of the result is curved in Figure 5.38. Simple frames perform similarity considering buckling load, as a result of similar strength capacity of column. Stiffness strengthening from composite beams leads to an increase of critical load factor for rigid frames.

Results of joints with a rotation stiffness of 15425kNm/rad are performed as those of semi-rigid frames. Composite frames seem to have a slight higher buckling load if same stiffness ratio are applied.

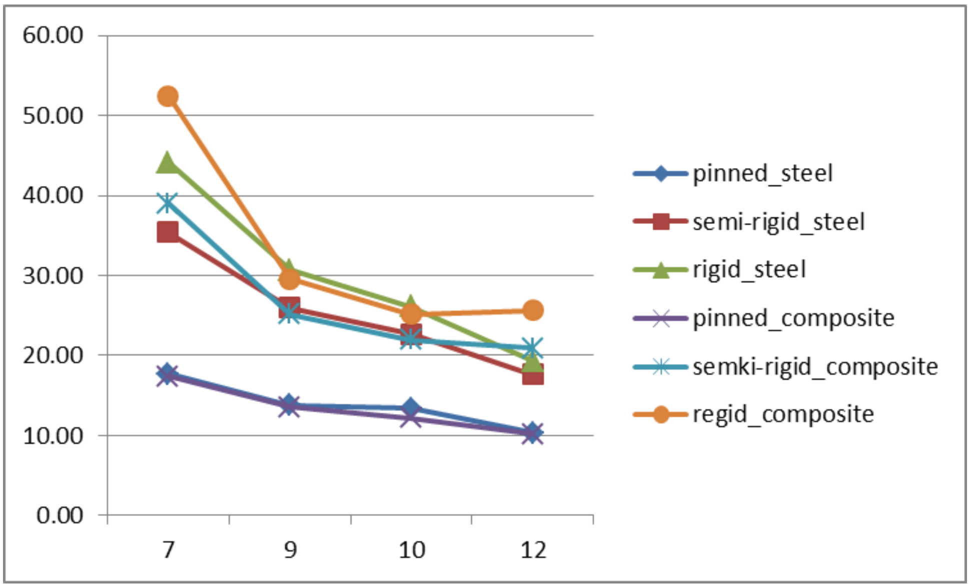


Figure 5.38 Critical load factors of steel and composite frames

6. COCLUSION

When semi-rigid joints are introduced into frames, the principle consideration might be how to set an appropriate degree of rotation stiffness and bending capacity of joints. As discussed above, effects of joints on frames may be significant, which should be included within global analysis.

6.1. Concrete cracking

Effects of cracking region should be considered in global analysis by iterative method, where convergence is reached in most case. Divergence is encountered during this study, which happens with a tiny stiffness of joint and a large one of beam. Cracking length is recommended to count for its effects, differing from un-cracking stiffness used for pinned beam, even when the joints are almost similar as a simple one.

Exactness of cracking length makes slight difference of moment distribution in frame, as the stiffness of beam varies slightly. It is recommended that the cracked length under characteristic combination of vertical loading be used for modelling; as a result, a determined structure is set for further analysis. With this simplification, modification factors and analysis methods are possible to be defined for each load case, and results combination is available considering load history.

Considering effects of concrete cracking, end moments of beams is expected to reduce about 10% in rigid frames. This moment redistribution is limited within semi-rigid frame, without consiferation of plastic characteristics of composite beams.

6.2. Semi-rigid joints

By introducing end releases at beam ends, effects of rotation stiffness of joints are possible to be simulated. Suitable moment redistribution, which will benefit usage of composite beams, is possible to be obtained by employing certain rigidity of joints, while the critical load of frame is expected to reduce.

Stiffness ratio of joints and beams is used for joint classification, but stiffness of composite beam in frame is hard to determined, as cracking length is related with level and distribution of loading. Un-cracked stiffness of composite beams is used in this study, as calculation, based on value of joint rigidity, is not influenced by stiffness ratio. With introducing of joint elements into modelling, classification of joints is less important, comparing traditional extreme setting as either rigid or simple joints.

Critical load analysis is possibly processed using either eigenvalue analysis or step-by-step nonlinear second-order approach. Eigenvalue analysis method is recommended as buckling shapes are provided to further determine buckling length for element verification. Second order approach might be used when nonlinearity of components should be activated for consideration. Programme used in this study might provide a lower value, especially within long span frames, when the later method is employed. Furthermore, results from that show discontinuity with varied parameters. In this situation, eigenvalue analysis is recommended in practice.

End moments of beams slide from results of rigid frame to those of simple one, when reduction of rigidity of joints performs. This desirable moment redistribution might benefit composite structures if coherence of rigidity and strength capacity of joints is proofed to be provided. Complicity of analysis process hinders practical application of semi-rigid semi-strength connection in building industry, thus it is expected that combination of software package of joint design and global analysis will bring semi-rigid frame into practice. A high degree of joint rigidity is recommended in sway frame, considering global stability and required joint rotation.

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