1C8
Advanced design of steel structures

prepared by

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List of lessons

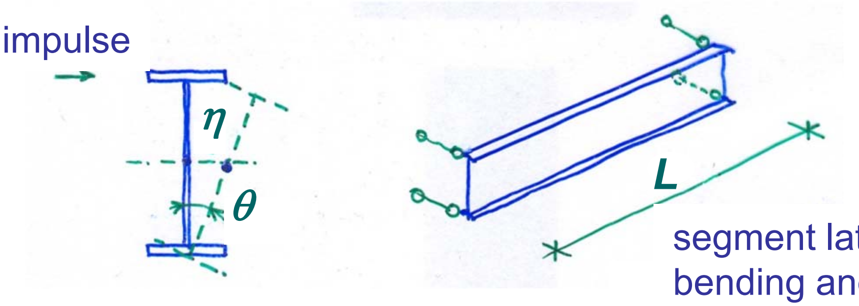
1) Lateral-torsional instability of beams.
2) Buckling of plates.
3) Thin-walled steel members.
4) Torsion of members.
5) Fatigue of steel structures.
6) Composite steel and concrete structures.
7) Tall buildings.
8) Industrial halls.
9) Large-span structures.
10) Masts, towers, chimneys.
11) Tanks and pipelines.
12) Technological structures.
13) Reserve.

1. Lateral-torsional buckling

- **Introduction (stability and strength).**
- **Critical moment.**
- **Resistance of the actual beam.**
- **Interaction of moment and axial force.**
- **Eurocode approach.**

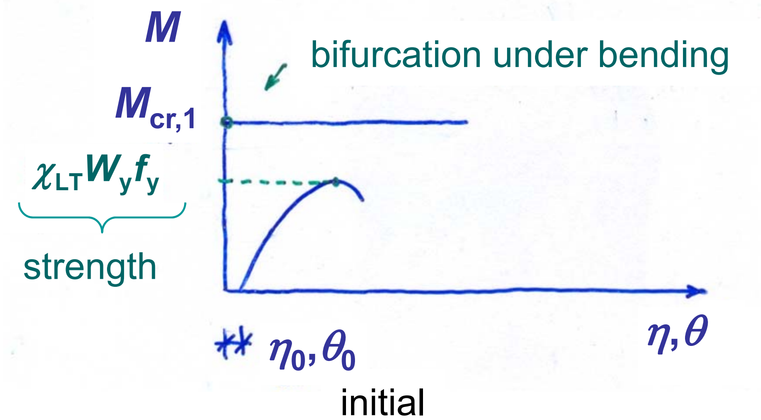
Introduction

Stability of ideal (straight) beam under bending



segment laterally supported in bending and torsion ➡ M_{cr}

Strength of real beam (with imperfections η_0, θ_0)

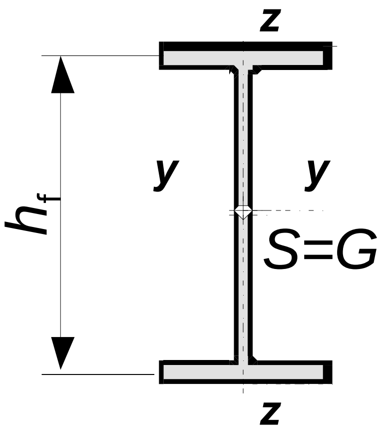


➡ $$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$

reduction factor χ_{LT} depends on: $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$

Critical moment

Stability of ideal beam under bending (determination of M_{cr})



"Basic beam" - with y-y axis of symmetry
(simply supported in bending and torsion, loaded only by *M*)

Two equations of equilibrium (for lateral and torsional buckling) may be unified into one equation:

$$EI_w \frac{d^4\theta}{dx^4} - GI_t \frac{d^2\theta}{dx^2} + \frac{M^2}{EI_z}\theta = 0$$

The first non-trivial solution gives $M = M_{cr}$:

$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}} = \mu_{cr}\frac{\pi\sqrt{EI_z GI_t}}{L}$$

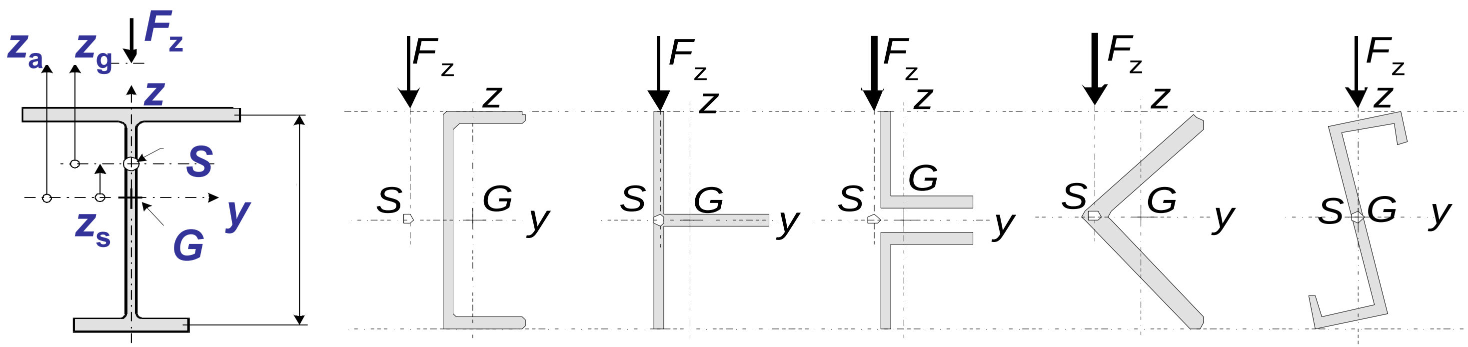
where $\mu_{cr} = \sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}} = \sqrt{1+\kappa_{wt}^2}$ $\qquad \kappa_{wt} = \frac{\pi}{L}\sqrt{\frac{EI_w}{GI_t}}$

Critical moment

Generally (EN 1993-1-1) for beams with cross-sections according to picture:

$$\mu_{cr} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}^2+\left(C_2\zeta_g - C_3\zeta_j\right)^2} - \left(C_2\zeta_g - C_3\zeta_j\right)\right]$$

$$\kappa_{wt} = \frac{\pi}{k_w L_{LT}}\sqrt{\frac{EI_w}{GI_t}} \qquad \zeta_g = \frac{\pi\, z_g}{k_z\, L_{LT}}\sqrt{\frac{EI_z}{GI_t}} \qquad \zeta_j = \frac{\pi\, z_j}{k_z\, L_{LT}}\sqrt{\frac{EI_z}{GI_t}}$$



symmetry about z-z

symmetry about k y-y, loading through shear centre

C_1 represents mainly shape of bending moment,

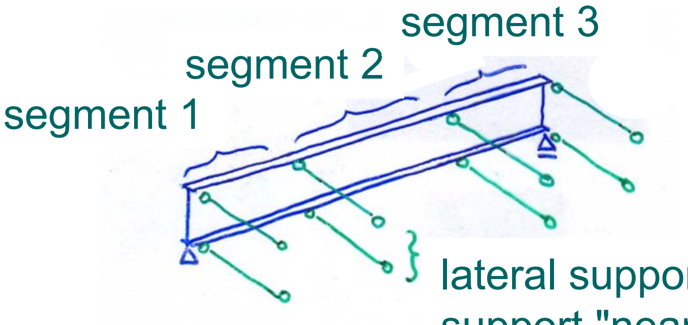
C_2 comes in useful **only** if loading is not applied in shear centre,

C_3 comes in useful **only** for cross-sections non-symmetrical about y-y.

Critical moment

Procedure to determine M_{cr}:

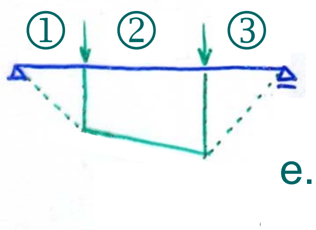
1. Divide the beam into segments of lengths L_{LT} according to lateral support:



e.g. segment 1: $(L_{LT,1})$

lateral support (bracing) in bending and torsion (lateral support "near" compression flange is sufficient)

2. Define shape of moment in the segment: → from table factor $C_1 \geq 1$:



e.g. segment 2: a) usually linear distribution

b) almost never

(because loading here creates continuous support)

Critical moment

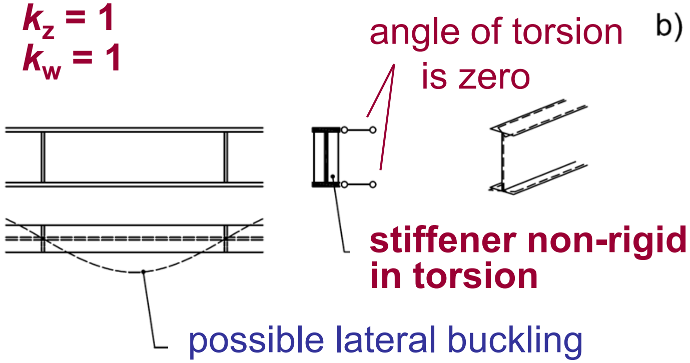
3. Determine support of segment ends:

(actually ratio of "effective length")

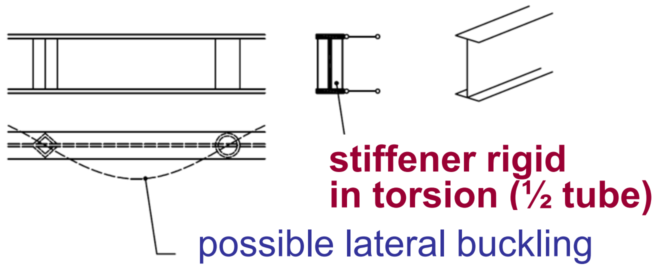
$k_z = 1$ (pins for lateral bending)
$k_w = 1$ (free warping of cross section)

Other cases of *k* :

a) **$k_z = 1$**
$k_w = 1$



b) **$k_z = 1$**
$k_w = 0{,}5$



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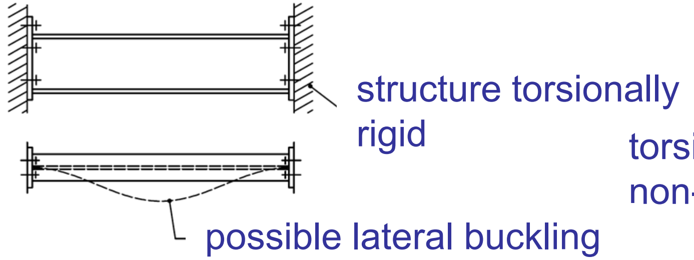
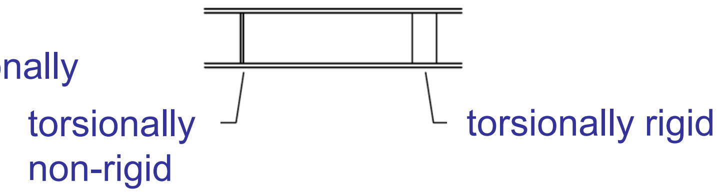
c) $k_z = 0{,}7$, $k_w = 1$ } **conservatively (theor. $k_z = k_w = 0{,}5$)**



d) $k_z = 1$, $k_w = 0{,}7$ **(conservatively 1)**



Cantilever: - only if free end is not laterally and torsionally supported (otherwise concerning M_{cr} this case is not a cantilever but normal beam segment),

- for cantilever with free end: $k_z = k_w = 2$.

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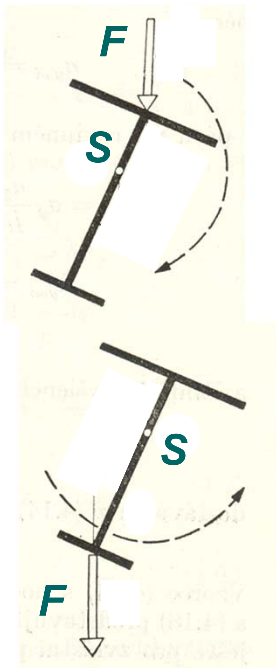
4. Formula for M_{cr} depends also on position of loading with respect to shear centre (z_g):

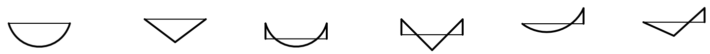
Come in useful for lateral loading (loading by end moments is considered in shear centre).



- lateral loading acting **to shear centre *S* (z_g > 0) is destabilizing:** it increases the torsional moment

- lateral loading acting **from shear centre *S* (z_g < 0) is stabilizing:** it decreases the torsional moment

Factor C_2 for moment shape *M*: (valid for I cross-section)



M_{el}	0,46	0,55	1,56	1,63	0,88	1,15
M_{pl} (plast. hinges)			0,98	1,63	0,70	1,08

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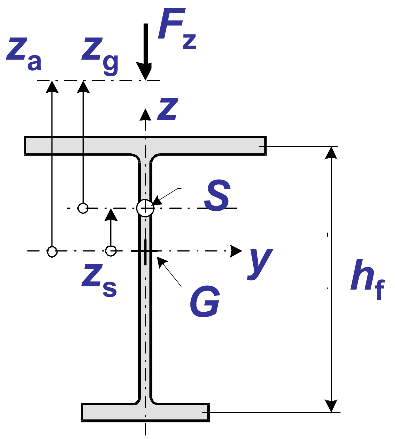
5. Cross-sections non-symmetrical about y-y



For I cross section with unequal flanges:

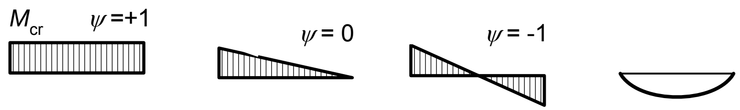
warping constant $$I_w = (1 - \psi_f^2) I_z (h_s/2)^2$$

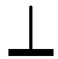
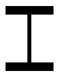
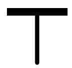
parameter of asymmetry $$\psi_f = \frac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$$

second mom. of area of compress. and tens. flange about z-z

$$z_j = z_s - \frac{0{,}5}{I_y} \int_A (y^2 + z^2)\, z\, dA \cong 0{,}45 \psi_f h_f$$

Factor C_3 greatly depends on ψ_f and moment shape (below for $k_z = k_w = 1$):



		$\psi = +1$	$\psi = 0$	$\psi = -1$	
	ψ_f = -1	1,00	1,47	2,00	0,93
	ψ_f = 0	1,00	1,00	0,00	0,53
	ψ_f = 1	1,00	1,00	-2,00	0,38

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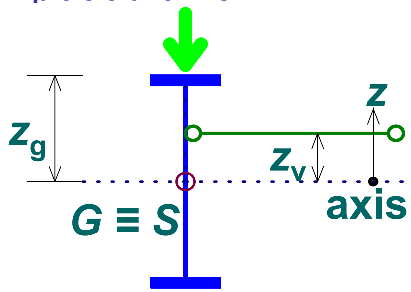
Cross sections with imposed axis of rotation

M_{cr} is affected by position of imposed axis of rotation (M_{cr} is always greater, holding is favourable)

For a simple beam with doubly symmetric cross section and general imposed axis:

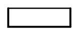


$$M_{cr} = \frac{\left[E\,I_w + E\,I_z z_v^2\right]\left(\dfrac{\pi}{k_w L_{LT}}\right)^2 + G\,I_t}{-\beta_1 z_v + \beta_2\left(z_g - z_v\right)}$$

For suck loading applied at tension flange:

$$M_{cr} = \frac{\left[E\,I_w + E\,I_z\left(\dfrac{h}{2}\right)^2\right]\left(\dfrac{\pi}{k_w L_{LT}}\right)^2 + G\,I_t}{\dfrac{h}{2}\beta_1}$$

coefficients β for shape of M:

	β_1	β_2
	2,00	0,00
	0,93	0,81
	0,60	0,81

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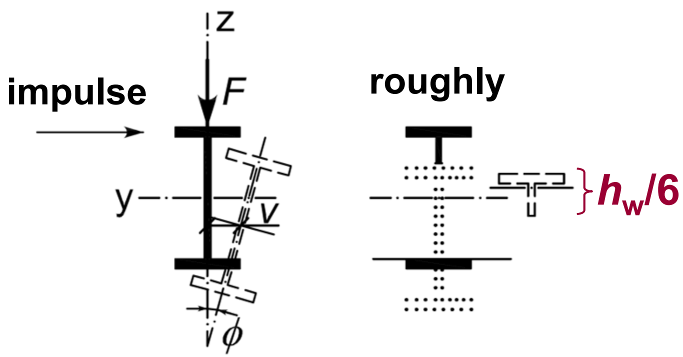
Approximate approach for lateral-torsional buckling



In buildings, the reduction factor for lateral buckling corresponding to "equivalent compression flange" (defined as flange with 1/3 of compression web) may be taken instead:

$$\bar{\lambda}_f = \frac{L_{LT}}{i_{f,z}\lambda_1}$$

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93{,}9\varepsilon$$

Note: According to Eurocode the reduction factor χ is taken from curve c, but for cross sections with web slenderness $h/t_w \leq 44\varepsilon$ from curve d. The factor due to conservatism may be increased by 10%.

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Practical case of a continuous beam (or a rafter of a frame):

The top beam flange is usually laterally supported by cladding (or decking). Instead of calculating stability to imposed axis a conservative approach may be used, considering loading at girder shear centre and neglecting destabilizing load location (C2 = 0):

M_{el} $C_1 = 2{,}23$

M_{pl} $C_1 = 1{,}21$

M_{el} $C_1 = 2{,}58$

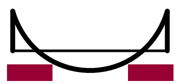
M_{pl} $C_1 = 1{,}23$

according to M distribution (for different *M* may be used graphs by A. Bureau)

Note:
For suck stabilizing effect should be applied (C2, $z_g < 0$).

It is desirable to secure laterally the dangerous zones of bottom free compression flanges against instability:

- **by bracing in the level of bottom flanges,**
- **or by diagonals (sufficiently strong) between bottom flange and crossing beams (e.g. purlins).**



Length L_{LT} then corresponds to the distance of supports).

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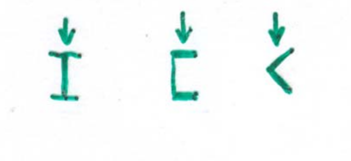
Beams that do not lose lateral stability:

1. **Hollow cross sections**
   Reason: high $I_t \Rightarrow$ high M_{cr}

2. **Girders bent about their minor axis**
   Reason: high $I_t \Rightarrow$ high M_{cr}

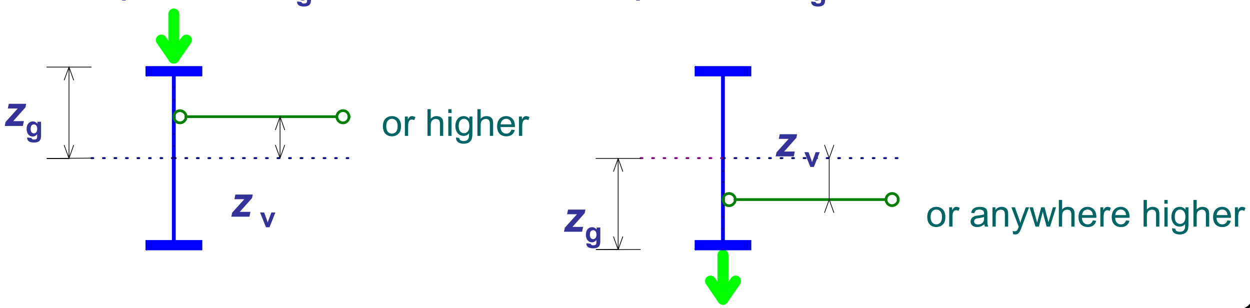
3. **Short segment ($\bar{\lambda}_{LT} \leq 0{,}4$) - all cross sections, e.g**
   Reason: $\chi_{LT} \approx 1$



4. **Full lateral restraint: "near" to the compression flange is sufficient (≈ approx. within h/4)**

compression flange loaded
$z_v \geq 0{,}47\ z_g$

tension flange loaded
$z_v \leq 0{,}47\ z_g$



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Resistance of the actual beam ($M_{b,Rd}$)

Similarly as for compression struts: actual strength $M_{b,Rd}$ < M_{cr} (due to imperfections)

e.g. DIN: $$M_{b,Rd} = M_{pl,Rd}\left[1 + \left(\bar{\lambda}_{LT}\right)^{2n}\right]^{1/n}$$ n = 2,0 (rolled)
= 2,5 (welded)

Eurocode EN 1993:

The procedure is the same as for columns: acc. to $\bar{\lambda}_{LT}$ is determined χ_{LT} with respect to shape of the cross section (see next slide).
Note: For a direct 2. order analysis the imperfections e_{0d} are available.

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$ ... W_y is section modulus acc. to cross section class

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \begin{cases} \chi_{LT} \le 1{,}0 \\ \chi_{LT} \le \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases}$$

$$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}^2\right]$$

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Resistance of the actual beam ($M_{b,Rd}$)

Similarly as for compression struts: actual strength $M_{b,Rd} < M_{cr}$ (due to imperfections)

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For common rolled and welded cross sections: $\bar{\lambda}_{LT,0} = 0{,}4$ $\beta = 0{,}75$
For non-constant *M* the factor may be reduced to $\chi_{LT,mod}$ (see Eurocode).

SUSCOS SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Resistance of the actual beam ($M_{b,Rd}$)

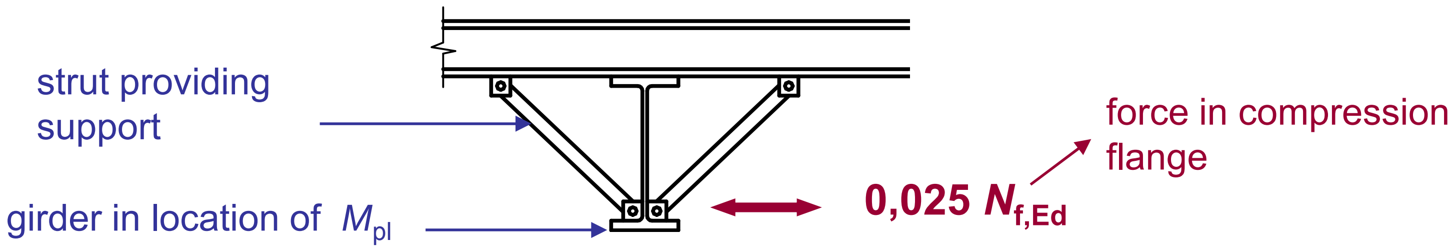
For common rolled and welded cross sections: $\bar{\lambda}_{LT,0} = 0,4$ $\beta = 0,75$
For non-constant *M* the factor may be reduced to $\chi_{LT,mod}$ (see Eurocode).

Choice of buckling curve:

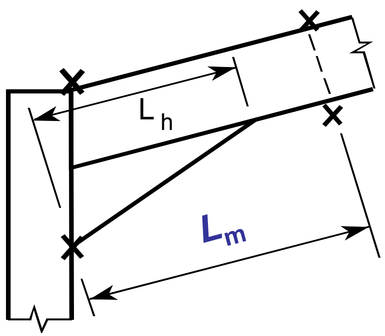
rolled I sections	shallow (rigid cross section)	h/b ≤ 2 (up to IPE300, HE600B)	b
	high	h/b > 2	c
welded I sections (greater residual stresses due to welding)		h/b ≤ 2	c
		h/b > 2	d

Resistance of the actual beam ($M_{b,Rd}$)

In plastic analysis (considering redistribution of moments and "rotated" plastic hinges) lateral torsional buckling in hinges must be prevented and designed for 2,5 % $N_{f,Ed}$:



Complicated structures (e.g. haunched girders may be verified using "stable length" L_m (in which χ_{LT} = 1) - formulas see Eurocode.



Interaction *M* + *N* ("beam columns")

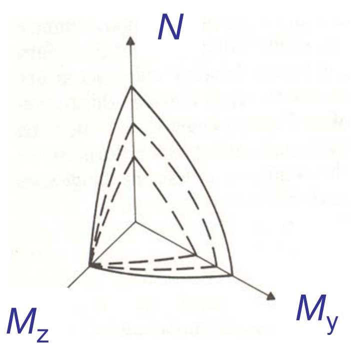
Always must be verified simple compression and bending in the most stressed cross section – see common non-linear relations.

In stability interaction two simultaneous formulas should be considered:

for class 4 only

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$



Usual case $M + N_y$:

$$\frac{N_{Ed}}{\chi_y N_{Rd}} + k_{yy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rd}} \le 1$$

$$\frac{N_{Ed}}{\chi_z N_{Rd}} + k_{zy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rd}} \le 1$$

factors $k_{yy} \le 1{,}8$; $k_{zy} \le 1{,}4$

(for relations see EN 1993-1-1, Annex B)

Note: historical unsuitable linear relationship (without 2nd order factors k_{yy}, k_{zy})

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{y,Ed}}{M_{b,Rd}} \le 1$$

Interaction *M* + *N* ("beam columns")

Complementary note:

Generally FEM may be used (complicated structures, non-uniform members etc.) to analyse **lateral and lateral torsional buckling**.
First analyse the structure linearly, second critical loading. Then determine:

$\alpha_{ult,k}$ - minimum load amplifier of design loading to reach characteristic resistance (without lateral and lateral-torsional buckling);

$\alpha_{cr,op}$ - minimum load amplifier of design loading to reach elastic critical loading (for lateral or lateral torsional buckling).

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \Rightarrow \chi_{op} = \min(\chi, \chi_{LT})$$

Resulting relationship:

$$\frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{M_{y,Rk}/\gamma_{M1}} \le \chi_{op}$$

SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Assessment

- Ideal and actual beam – differences.
- Procedure for determining of critical moment.
- Destabilizing and stabilizing loading.
- Approximate approach for lateral torsional buckling.
- Resistance of actual beam.
- Interaction *M*+*N* according to Eurocode.

Notes to users of the lecture

- This session requires about 90 minutes of lecturing.
- Within the lecturing, design of beams subjected to lateral torsional buckling is described. Calculation of critical moment under general loading and entry data is shown. Finally resistance of actual beam and design under interaction of moment and axial force in accordance with Eurocode 3 is presented.
- Further readings on the relevant documents from website of www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Keywords for the lecture:

  lateral torsional instability, critical moment, ideal beam, real beam, destabilizing loading, imposed axis, beam resistance, stability interaction.

Notes for lecturers

- Subject: Lateral torsional buckling of beams.
- Lecture duration: 90 minutes.
- Keywords: lateral torsional instability, critical moment, ideal beam, real beam, destabilizing loading, imposed axis, beam resistance, stability interaction.
- Aspects to be discussed: Ideal beam and real beam with imperfections. Stability and strength. Critical moment and factors which influence its determination. Eurocode approach.
- After the lecturing, calculation of critical moments under various conditions or relevant software should be practised.
- Further reading: relevant documents www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Preparation for tutorial exercise: see examples prepared for the course.