PART 5-4: Beam with bending and compression loads

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1 TASK

This example deals with a beam subjected to a uniform load, which causes a bending moment, and an axial load. Stability phenomena have to be considered. The beam is part of an office building. A hollow encasement of gypsum is chosen for fire protection. Due to a concrete slab the beam is exposed to fire on three sides. There is no shear-connection between the beam and the slab. The required standard fire resistance class for the beam is R 90.

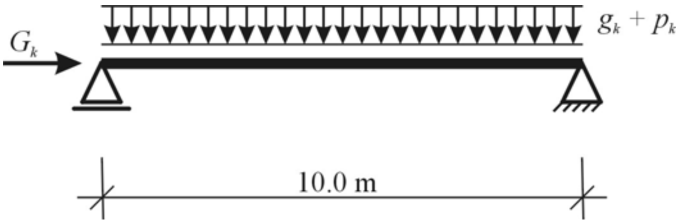

Figure 1. Static system

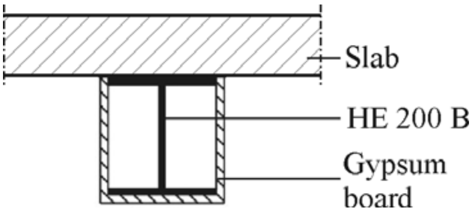

Figure 2. Beam Cross-section

Material properties:

Beam:

Profile: rolled section HE 200 B
Steel grade: S 235
Cross-section class: 1
Yield stress: f_y = 235 N/mm²
Elastic modulus: E = 210,000 N/mm²
Shear modulus: G = 81,000 N/mm²
Cross-sectional area: A_a = 7810 mm²
Moment of inertia: I_z = 2000 cm^4

Torsion constant: I_t = 59.3 cm^4
Warping constant: I_w = 171,100 cm^6
Section moduli: $W_{el,y}$ = 570 cm^2
$W_{pl,y}$ = 642.5 cm^3

Encasement:
Material: gypsum
Thickness: d_p = 20 mm (hollow encasement)
Thermal conductivity: λ_p = 0.2 W/(m·K)
Specific heat: c_p = 1700 J/(kg·K)
Density: ρ_p = 945 kg/m^3

Loads:
Permanent Loads: G_k = 96.3 kN
g_k = 1.5 kN/m
Variable Loads: p_k = 1.5 kN/m

2 FIRE RESISTANCE OF BEAM WITH BENDING AND COMPRESSION LOADS

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

The combination of mechanical actions during fire exposure shall be calculated as an accidental situation:

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

The combination factor for office buildings is $\psi_{2,1} = 0.3$. The design loads under high temperature conditions are:

$$N_{fi,d} = 96.3 \text{ kN}$$

$$M_{fi,d} = \left[1.5 + 0.3 \cdot 1.5\right] \cdot \frac{10.0^2}{8} = 24.38 \text{ kNm}$$

2.2 *Calculation of steel temperatures*

EN 1993-1-2

The steel temperature is given by the Euro-Nomogram (ECCS No.89). Therefore the section factor A_p/V is needed. For a hollow encased member exposed to fire on three sides, the section factor is:

$$\frac{A_p}{V} = \frac{2 \cdot h + b}{A_a} = \frac{2 \cdot 20.0 + 20.0}{78.1} \cdot 10^2 = 77 \text{ m}^{-1}$$

Section 4.2.5.2

With

$$\frac{A_p}{V} \cdot \frac{\lambda_p}{d_p} = 77 \cdot \frac{0.2}{0.02} = 770 \ \frac{\text{W}}{\text{m}^3 \cdot \text{K}},$$

ECCS No.89

the critical temperature arises to:

$$\Rightarrow \quad \theta_{a,max,90} \approx 540 \text{ °C}$$

2.3 *Verification in the temperature domain*

EN 1993-1-2

Due to section 4.2.4 (2) of EN 1993-1-2 the verification in the temperature domain may not be accomplished, because of stability problems of the beam.

Section 4.2.4

2.4 *Verification in the strength domain*

Members with a Class 1 cross-section should be analysed for the problem of flexural buckling and of lateral torsional buckling.

2.4.1 *Flexural buckling*

The verification for flexural buckling is:

$$\frac{N_{fi,d}}{\chi_{\min,fi}\cdot A\cdot k_{y,\theta}\cdot f_y/\gamma_{M,fi}}+\frac{k_y\cdot M_{y,fi,d}}{W_{pl,y}\cdot k_{y,\theta}\cdot f_y/\gamma_{M,fi}}\le 1$$

Section 4.2.3.5

The reduction factor $\chi_{min,fi}$ is the minimum of the two reduction factors for flexural buckling $\chi_{y,fi}$ and $\chi_{z,fi}$. The non-dimensional slenderness for the temperature θ_a is needed for the calculation of these reduction factors.

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures have to be determined.

EN 1993-1-1

$$\bar{\lambda}_y=\frac{L_{cr}}{i_y\cdot\lambda_a}=\frac{1000}{8.54\cdot 93.9}=1.25$$

Section 6.3.1.3

$$\bar{\lambda}_z=\frac{L_{cr}}{i_z\cdot\lambda_a}=\frac{1000}{5.07\cdot 93.9}=2.10$$

The needed reduction factors $k_{y,\theta}$ and $k_{E,\theta}$ are given in EN 1993-1-2 Table 3.1:

EN 1993-1-2

$\Rightarrow \quad k_{y,\theta}=0.656$

Section 3.2.1

$k_{E,\theta}=0.484$

With the reduction factors, the non-dimensional slenderness in the fire situation can be determined:

$$\bar{\lambda}_{y,\theta}=\bar{\lambda}_y\sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}=1.25\sqrt{\frac{0.656}{0.484}}=1.46$$

Section 4.2.3.2

$$\bar{\lambda}_{z,\theta}=\bar{\lambda}_z\sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}=2.1\sqrt{\frac{0.656}{0.484}}=2.44$$

With

$$\alpha=0.65\cdot\sqrt{235/f_y}=0.65\cdot\sqrt{235/235}=0.65$$

and

$$\varphi_{y,\theta}=\frac{1}{2}\cdot\left(1+\alpha\cdot\bar{\lambda}_{y,\theta}+\bar{\lambda}_{y,\theta}^{\,2}\right)=\frac{1}{2}\cdot\left(1+0.65\cdot 1.46+1.46^2\right)=2.04\,,$$

$$\varphi_{z,\theta}=\frac{1}{2}\cdot\left(1+\alpha\cdot\bar{\lambda}_{z,\theta}+\bar{\lambda}_{z,\theta}^{\,2}\right)=\frac{1}{2}\cdot\left(1+0.65\cdot 2.44+2.44^2\right)=4.27$$

the reduction factors $\chi_{y,fi}$ and $\chi_{z,fi}$ can be calculated:

$$\chi_{y,fi} = \frac{1}{\varphi_{y,\theta} + \sqrt{\varphi_{y,\theta}{}^2 - \bar{\lambda}_{y,\theta}{}^2}} = \frac{1}{2.04 + \sqrt{2.04^2 - 1.46^2}} = 0.29$$

$$\chi_{z,fi} = \frac{1}{\varphi_{z,\theta} + \sqrt{\varphi_{z,\theta}{}^2 - \bar{\lambda}_{z,\theta}{}^2}} = \frac{1}{4.27 + \sqrt{4.27^2 - 2.44^2}} = 0.13$$

Verification:

Section 4.2.3.5

$$\frac{96.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5} + \frac{1.50 \cdot 2438}{642.5 \cdot 0.656 \cdot 23.5} = 0.98 < 1 \quad \checkmark$$

where:

$$\begin{aligned} \mu_y &= \left(1.2 \cdot \beta_{M,y} - 3\right) \cdot \bar{\lambda}_{y,\theta} + 0.44 \cdot \beta_{M,y} - 0.29 \\ &= \left(1.2 \cdot 1.3 - 3\right) \cdot 1.46 + 0.44 \cdot 1.3 - 0.29 \\ &= -1.82 \end{aligned}$$

$$k_y = 1 - \frac{\mu_y \cdot N_{fi,d}}{\chi_{y,fi} \cdot A_a \cdot k_{y,\Theta} \cdot f_y / \gamma_{m,fi}} = 1 - \frac{-1.82 \cdot 96.3}{0.29 \cdot 78.1 \cdot 0{,}656 \cdot 23.5/1.0} = 1.50$$

2.4.2 *Lateral torsional buckling*

The second verification deals with the problem of lateral torsional buckling.

$$\frac{N_{fi,d}}{\chi_{z,fi} \cdot A \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} + \frac{k_{LT} \cdot M_{y,fi,d}}{\chi_{LT,fi} \cdot W_{pl,y} \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} \leq 1$$

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures has to be determined.

EN 1993-1-1

Section 6.3.2.2

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} \cdot f_y}{M_{cr}}} = \sqrt{\frac{642.5 \cdot 23.5}{14{,}203.5}} = 1.03$$

where:

DIN 18800-2

$$\begin{aligned} M_{cr} &= \zeta \cdot \frac{\pi^2 \cdot E \cdot I_z}{l^2} \cdot \left(\sqrt{c^2 + 0.25 \cdot z_p^2} + 0.5 \cdot z_p\right) \\ &= 1.12 \cdot \frac{\pi^2 \cdot 21{,}000 \cdot 2000}{\left(1.0 \cdot 1000\right)^2} \cdot \left(\sqrt{1241.9 + 0{,}25 \cdot \left(\frac{20}{2}\right)^2} - 0.5 \cdot \frac{20}{2}\right) \\ &= 14{,}203.5 \text{ kNcm} \end{aligned}$$

with: $$c^2 = \frac{I_\omega + 0.039 \cdot l^2 \cdot I_T}{I_z} = \frac{171{,}100 + 0.039 \cdot 1000^2 \cdot 59.3}{2000} = 1241.9$$

During fire exposure, the non-dimensional slenderness changes to:

EN 1993-1-2

Section 4.2.3.3

$$\bar{\lambda}_{LT,\theta} = \bar{\lambda}_{LT} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 1.03 \cdot \sqrt{\frac{0.656}{0.484}} = 1.20$$

with

$$\phi_{LT,\theta} = \frac{1}{2}\cdot\left(1+\alpha\cdot\bar{\lambda}_{LT,\theta}+\bar{\lambda}_{LT,\theta}{}^{2}\right) = \frac{1}{2}\cdot\left(1+0.65\cdot 1.20+1.20^{2}\right) = 1.61,$$

the reduction factor $\chi_{LT,fi}$ is calculated to:

$$\chi_{LT,fi} = \frac{1}{\phi_{LT,\theta}+\sqrt{\phi_{LT,\theta}{}^{2}-\bar{\lambda}_{LT,\theta}{}^{2}}} = \frac{1}{1.61+\sqrt{1.61^{2}-1.20^{2}}} = 0.37$$

Verification:

Section 4.2.3.5

$$\frac{96.3}{0.13\cdot 78.1\cdot 0.656\cdot 23.5/1.0}+\frac{0.80\cdot 2438}{0.37\cdot 642.5\cdot 0.656\cdot 23.5/1.0}$$

$$= 0.62+0.53 = 1.15 \leq 1 \quad ↯$$

where:

$$k_{LT} = 1-\frac{\mu_{LT}\cdot N_{fi,d}}{\chi_{z,fi}\cdot A\cdot k_{y,\theta}\cdot\dfrac{f_y}{\gamma_{M,fi}}} = 1-\frac{0.33\cdot 96.3}{0.13\cdot 78.1\cdot 0.656\cdot\dfrac{23.5}{1.0}} = 0.80$$

$$\begin{aligned}\mu_{LT} &= 0.15\cdot\bar{\lambda}_{z,\theta}\cdot\beta_{M,LT}-0.15 < 0.9\\ &= 0.15\cdot 2.44\cdot 1.3-0.15\\ &= 0.33 < 0.9\end{aligned}$$

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Literatur for MCr (for example: Steel Construction Manual)

QUALITY RECORD	WP5		
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