ELASTIC BUCKLING OF STEEL COLUMNS UNDER THERMAL GRADIENT

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A. [Introduction]

¬ Members on the perimeter of a building sustain a clear thermal gradient over their cross-section

¬ Mechanical properties of the material depend on the imposed temperature field according to Eurocode 3

¬ Thermal gradient causes the shift of the elastic neutral axis and, as a result, the generation of initial eccentricity

¬ Thermal gradient causes varying thermal elongation over the cross-section leading to column bowing

¬ The deflection of the column is amplified due to second-order effects

B. [Scope]

¬ Analytical treatment of steel pin-ended columns under thermal gradient

¬ Two different approaches:
+ Effect of thermal gradient on the shift of the elastic neutral axis with bowing being omitted
+ Combined effect of thermal gradient and bowing

¬ An IPE300 European cross-section will be used for the study of the above effects for several lengths in order to obtain the reduction of the maximum elastic axial load due to instability

¬ A linear temperature gradient is imposed across the y-y axis. The flexural buckling of the minor axis is not within the context of the present work since the influence of the thermal gradient is of main interest

C. [Material]

¬ Eurocode 3 – Part 1.2 proposes reduction factors for the description of the stress – strain relationships according to the imposed temperatures

¬ The shape of these functions is linear – elliptical – linear, but, for the sake of simplicity, bilinear laws are adopted

¬ Hence,

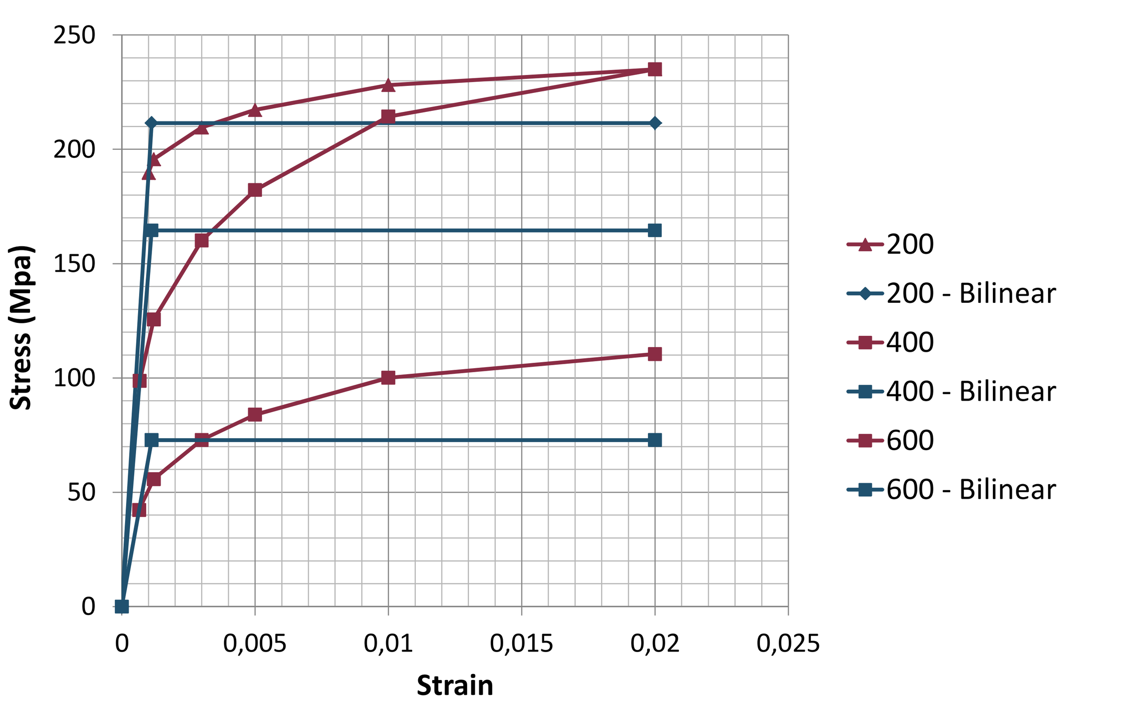
$$\frac{E_\vartheta}{E_{20}} = \frac{f_{y,\vartheta}}{f_{y,20}} = 2{,}347\sin\left(0{,}5275T + 2{,}6\right) + 0{,}193\sin\left(7{,}803T - 1{,}438\right)$$

where

$T = \vartheta/1000$ — θ is the applied temperature in °C

$f_{y,\vartheta}$, $f_{y,20}$ — are the yield strength at the applied and ambient temperature respectively and

E_ϑ, E_{20} — are the modulus of elasticity at the applied and ambient temperature respectively



D. [Equivalent section]

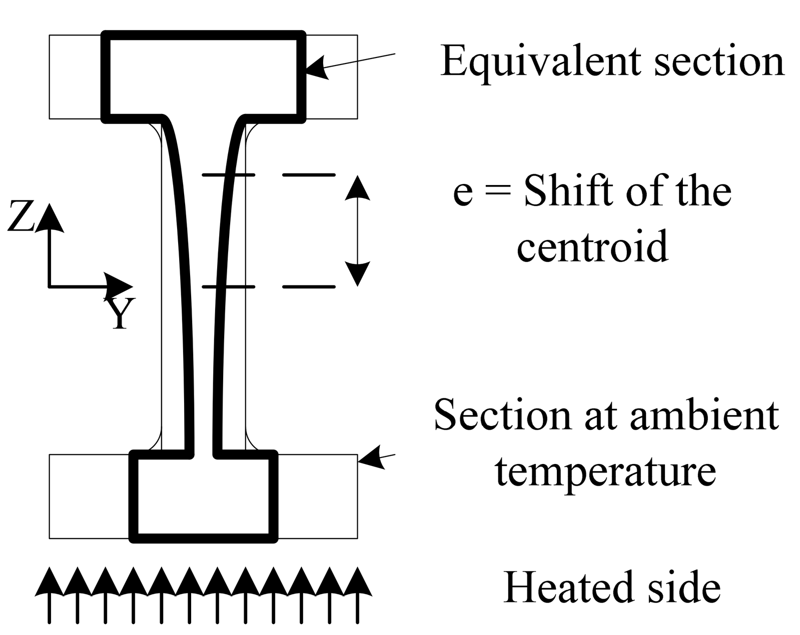
The reduction of the modulus of elasticity is dependent on the temperature of the material. That produces an arbitrary field of modulii along the cross-section. To overcome this obstacle and apply a constant E_{20}, the thickness of the web can be scaled according to the imposed temperature at each point of the cross-section:

$$z_g = \frac{\int_A E(\theta) z\, dA}{\int_A E(\theta)\, dA} = \frac{\int_0^H \int_0^{Beq} z\, dy\, dz}{\int_0^H \int_0^{Beq} dy\, dz}$$

where

$B_{eq} = (E(\vartheta(z))/E_{20})B(z)$ — the width of the equivalent section at a given z coordinate

$\vartheta = \Delta\vartheta\,(z/H) + \vartheta_{min}$ — the reference temperature at distance z from the extreme fiber



E. [1st approach]

On the assumption of the absence of thermal expansion effects, the column behaves like a beam-column. The differential equation is given:

$$P(e + w(x)) = -EI_{eq} w''(x)$$

where

e — is the distance between the mid-height of the cross section and the geometrical centroid of the equivalent section
P — is the axial force
$w(x)$ — is the deflection curve
E — is the modulus of elasticity at ambient temperature
I_{eq} — is the moment of inertia of the equivalent cross-section

The maximum deflection of the column occurs at the middle of the column. At that position, the initial yield criterion is applicable. Following an iterative procedure, one can thus determine the maximum allowable mean stress:

$$\sigma_{y,\theta\max} = k_{E,\max} P\left(\frac{1}{A_{eq}} + \frac{e}{I_{eq}} c \sec\left(\frac{l}{2}\sqrt{\frac{P}{EI_{eq}}}\right)\right)$$

where

c — is the distance from the centroid to the extreme fiber
$k_{E,max}$ — is the reduction coefficient of the modulus of elasticity for the maximum imposed temperature
$\sigma_{y,\vartheta max}$ — is the yield stress for the maximum imposed temperature
A_{eq} — is the area of the equivalent cross-section

'The eccentricity that arises from the shift of the centroid cannot be studied independently of thermal expansion effects'

F. [2nd approach]

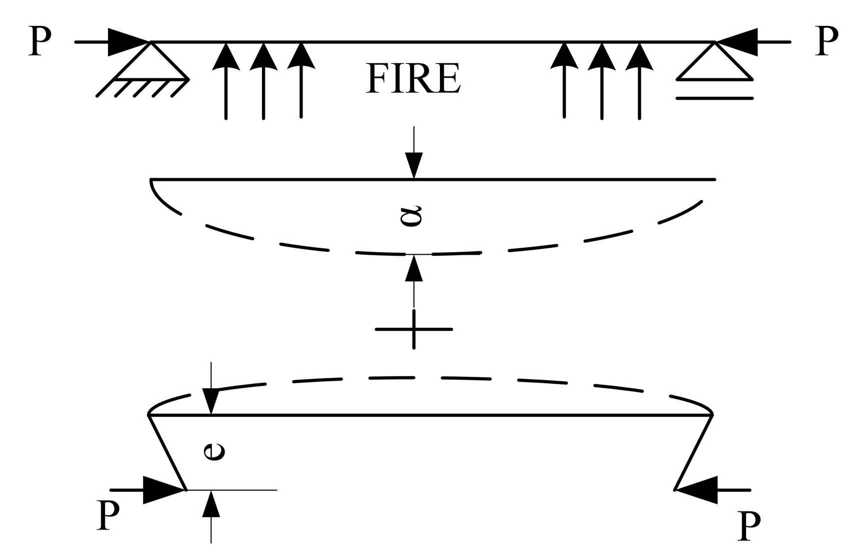
Assume the thermal bowing effect on perfect columns to be analogous to initial imperfection that exists on real columns. Against this phenomenon, acts the shift of the centroid. The edge moments, resulting from the slope of the thermal gradient, counteract the bowing of the member. The net effect reads:

$$w(x) = U_{bow} - U_{ecc} + e \Rightarrow$$

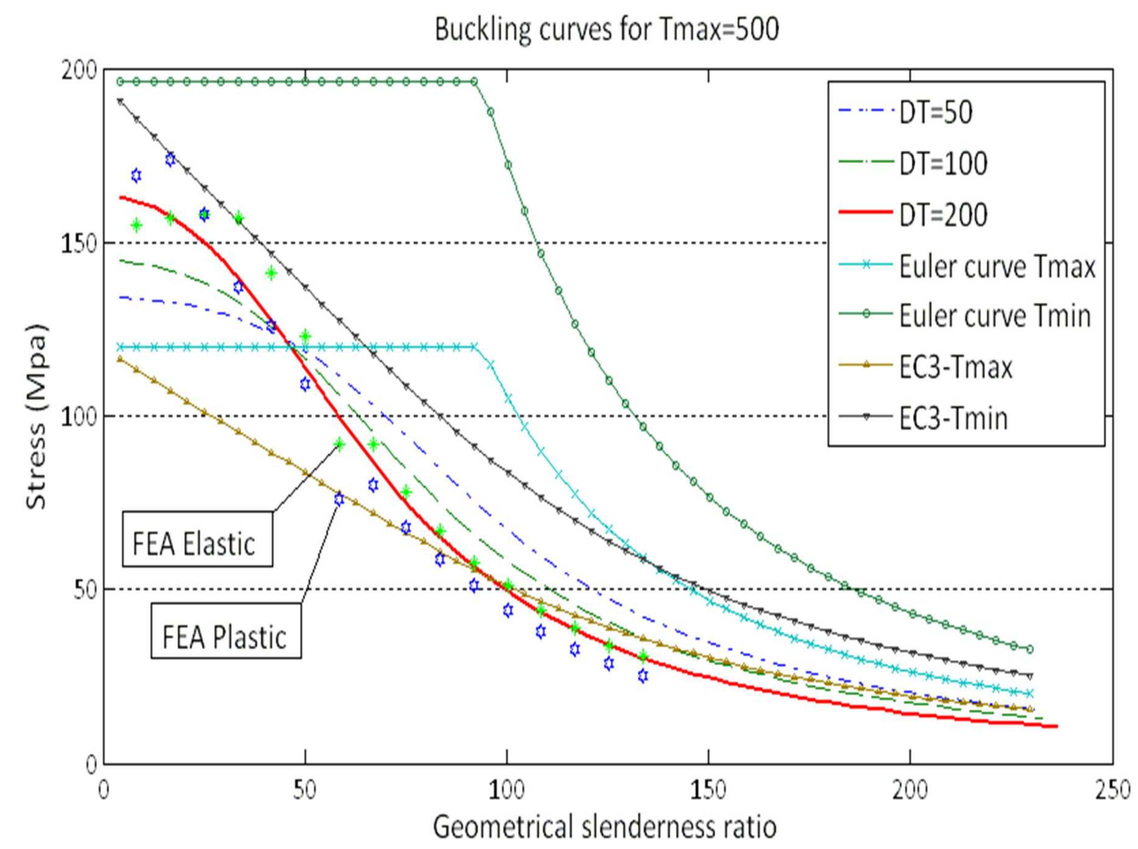
$$w(x) = \frac{a_{\Delta\vartheta}}{1 - P/P_{cr,eq}} \sin\left(\frac{\pi x}{l}\right) - \frac{M_0}{2EI_{eq}} x(l-x) + e$$

where

$a_{\Delta\vartheta}$ — is the maximum deflection at the middle due to thermal bowing
P — is the applied force on the mid-height of the cross section
$P_{cr,eq}$ — is the Euler buckling load of the equivalent cross section
M_0 — is the edge-moment due to the shift of the centroid



Based on the concept of maximum allowable stress at the middle of the column:



The analysis of the behaviour of the simply-supported steel column under the combined effect was validated with the general purpose finite element package ABAQUS. For the description of the material behaviour, both the mentioned bilinear laws and the true laws, that include the elastoplastic regions as given by Eurocode, are applied.

G. [Conclusions]

¬ The basic equation of beam-column with initial curvature can be used with the proper manipulation for the analytical treatment of steel columns under thermal gradient

¬ There is good agreement between the analytical solution and the finite element analysis

¬ The equation will be checked for various types of steel cross-sections and thermal cases

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