A Rational Approach to Fire Resistance Analysis
of Reinforced Concrete Columns
Subjected to Uniaxial/Biaxial Bending and Axial Restraint

Truong Thang Nguyen, Kang Hai Tan
1. Introduction

1.1. Published Research Works

1. Mostly on columns subjected to axial load and uniaxial bending.
   An indirect approach using Bresler’s formula was proposed by Tan and Yao (2003) for biaxially-loaded columns under fires;
   EC2 for ambient design specifies an approximate approach;
   Fire provisions of current codes neglect biaxial bending;
2. Additional axial forces due to axial restraint were rarely considered;
3. Mostly conducted in standard fires. Based on failure time in fire tests to validate ultimate loads predicted by analytical/numerical methods;
4. Limited information of fire tests on biaxially-loaded columns have been used for verification.

\[
\frac{1}{P_n(T)} = \frac{1}{P_{nx0}(T)} + \frac{1}{P_{ny0}(T)} - \frac{1}{P_0(T)}
\]

Tan & Yao (2003)  
BS EN 1992-1-1:2004 (E)  
\( \frac{M_{Edx}}{M_{Rdz}} + \frac{M_{Edy}}{M_{Rdy}} \leq 1.0 \)

BS EN 1992-1-2:2004

1.2. Proposed Approach

1. Conduct temperature-dependent sectional analysis directly based on inclined neutral axes;
2. Additional thermal-induced forces are considered;
3. Any fire curve can be adopted. Based on both variation of acting loads and deteriorations of strength/stiffness to predict failure time;
4. Fire tests on either uniaxially- or biaxially-loaded columns under either standard or nonstandard fires are used for validation.
2. Principle of Analysis

Step 1. Temperature-time Curve

Step 2. Thermal Analysis

SAFIR software

Step 3. Sectional Analysis

Failure modes

ANALYTICAL MODEL

(a) Cross section
(b) Position of a neutral axis
(c) Scan for the n.a. position

EUROCODE Material Model

Thermal-induced Axial Force
3. Analytical Approach and Computer Program

### Strain Failure Criterion

<table>
<thead>
<tr>
<th>Concrete</th>
<th>$T_{cr}$ (°C)</th>
<th>$f_{ct}^0$ (N/mm²)</th>
<th>$f_{cte}^0$ (N/mm²)</th>
<th>$f_{ct}^0$ (N/mm²)</th>
<th>$f_{cte}^0$ (N/mm²)</th>
<th>$f_{ct}^0$ (N/mm²)</th>
<th>$f_{cte}^0$ (N/mm²)</th>
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<tbody>
<tr>
<td>39</td>
<td>2.0</td>
<td>28.0</td>
<td>3.0</td>
<td>0.600</td>
<td>0.600</td>
<td>0.600</td>
<td>0.600</td>
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<tr>
<td>50</td>
<td>6.0</td>
<td>32.5</td>
<td>6.0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>38.0</td>
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<td>27.0</td>
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<td>1.500</td>
<td>1.500</td>
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</tbody>
</table>

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**FRRC Model**

- **Load Resistance:**
  \[
  N_{rd} = \sum \alpha d_1^2 + \sum A_{sk} (f_{sk}^0 - \alpha) + \sum A_{ck} f_{ck}^0 \\
  M_{rd} = \sum \alpha d_1^2 z_1 + \sum A_{sk} (f_{sk}^0 - \alpha) z_1 + \sum A_{ck} f_{ck}^0 z_1 \\
  M_{rd} = \sum \alpha d_1^2 y_1 + \sum A_{sk} (f_{sk}^0 - \alpha) y_1 + \sum A_{ck} f_{ck}^0 y_1
  \]

- **Direct Approach:**
  If $N_{Ed} = N_{rd}$ then $M_{Ed} < M_{rd}$ and $M_{Ed} < M_{rd}$

- **Indirect Approach:**
  If $N_{Ed} = N_{rd}$ then $M_{Ed}^2 + M_{Ed}^2 < 1$

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**Subroutines of FRRC program**

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.P</td>
<td>Input for geometrical and material properties</td>
</tr>
<tr>
<td>2.AO</td>
<td>Determination of applied loads (first-order and second-order)</td>
</tr>
<tr>
<td>3.CA</td>
<td>Analysis of coordinate</td>
</tr>
<tr>
<td>4.TA</td>
<td>Determination of temperature</td>
</tr>
<tr>
<td>5.CM</td>
<td>Material model of concrete at elevated temperatures</td>
</tr>
<tr>
<td>6.RM</td>
<td>Material model of reinforcing steel at elevated temperatures</td>
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<tr>
<td>7.FC</td>
<td>Criteria for failure of material at elevated temperatures</td>
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<tr>
<td>8.CC</td>
<td>Computing concrete contribution to the load resistance</td>
</tr>
<tr>
<td>9.RC</td>
<td>Computing reinforcing steel contribution to the load resistance</td>
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<tr>
<td>10.FD</td>
<td>Description of failure</td>
</tr>
<tr>
<td>11.OP</td>
<td>Output for time, neutral aids and load resistance at failure</td>
</tr>
<tr>
<td>12.ID</td>
<td>Determination of interaction diagrams</td>
</tr>
</tbody>
</table>
4. Experimental Verification

| Test No. | H | B | d | H | B | d | H | B | d | H | B | d | H | B | d | H | B | d | H | B | d |
|----------|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| 1        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 2        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 3        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 4        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 5        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 6        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 7        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 8        | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |

**Table 1.** Experimental verification

**Figure 7.** Validation used Haas’s tests (1986)

**Figure 8.** Validation used NTU’s tests (2010)

- Biaxially-loaded column
- Specimen in Furnace
- Concrete Spalling
- Failure at Mid-Height Cross Section
6. Summary

6.1. Proposed Approach
1. Can be used for columns under either uniaxial bending ($\beta=0$ and $\beta=\pi/2$) or biaxial bending ($0<\beta<\pi/2$);
2. Considers additional thermal-induced axial forces due to axial restraint;
3. Incorporates any type of fire curve, time-dependent temperature at any point inside the column can be assessed;
4. Can directly predict failure time of the most possible failure mode and describe stress/strain distributions at failure.

6.2. Experimental Validation
1. Good agreement between the proposed approach and experimental data can be obtained;
2. Fire tests by Hass (1986): Only for uniaxially-loaded columns, no axial restraint, standard fire $\rightarrow (t^{FRRC}/t^{Test})_{mean}=0.947$;
   Fire tests in NTU (2010): For both uniaxially- and biaxially-loaded columns with axial restraint $\rightarrow (t^{FRRC}/t^{Test})_{mean}=1.099$;
3. Less conservative results of tests on axially-restrained columns may be due to transient strain / concrete spalling.

6.3. Future Works
1. Incorporate the effects of transient strain into the approach;
2. Conduct parametric study to propose a practical equation for the prediction of column fire resistance.

7. Acknowledgement

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