4 STRUCTURAL MODELLING

Joint behaviour has a significant effect on the response of the structural frame and must be included in both the global analysis and design. The types of joint modelling with respect to their stiffness and resistance are summarised in Table 4.1. In the case of elastic global frame analysis, only the stiffness properties (the initial stiffness for the Serviceability Limit State and stability calculations and the secant stiffness for the Ultimate Limit State calculations) of the joint are relevant for the joint modelling. In the case of rigid-plastic analysis, the principal joint features are its resistance, and its rotational capacity both of which need to be checked. In all other cases, both the stiffness and the resistance properties should be included in the joint model. These different models are illustrated in Table 4.2 and Figure 4.1. For most applications, separate modelling of the connection and the web panel behaviour is not convenient, but may be useful in some cases.

Table 4.1 Types of joint modelling

<table>
<thead>
<tr>
<th>STIFFNESS</th>
<th>RESISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>Full-strength</td>
</tr>
<tr>
<td>Semi-rigid</td>
<td>Partial-strength</td>
</tr>
<tr>
<td>Pinned</td>
<td>Pinned</td>
</tr>
</tbody>
</table>

- **Full-strength**
  - **Rigid**
  - **Semi-rigid**
  - **Pinned**

- **Partial-strength**
  - **Rigid**
  - **Semi-rigid**
  - **Pinned**

- **Pinned**
  - **Semi-rigid**
  - **Simple**

![Diagram](image)

*Figure 4.1 Design joint properties based on the type of global analysis*

Table 4.2 Joint modelling and frame global analysis

<table>
<thead>
<tr>
<th>MODELLING</th>
<th>TYPE OF FRAME ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous</td>
<td>Elastic analysis</td>
</tr>
<tr>
<td>Semi-continuous</td>
<td>Rigid-plastic analysis</td>
</tr>
<tr>
<td>Simple</td>
<td>Elastic-plastic analysis</td>
</tr>
</tbody>
</table>

- **Elastic analysis**
  - **Continuous**
  - **Semi-continuous**
  - **Simple**

- **Rigid-plastic analysis**
  - **Rigid**
  - **Semi-rigid**
  - **Pinned**

- **Elastic-plastic analysis**
  - **Full-strength**
  - **Partial-strength**
  - **Semi-rigid/full-strength**
  - **Semi-rigid/partial-strength**
  - **Pinned**
Figure 4.2 Modelling of joint by rotational springs

Table 4.3 Coefficients $\xi$ and $\zeta$ and lever arm $r$ for estimation of initial stiffness and bending moment resistance of beam to column joints and column bases, see Q&A 4.1

<table>
<thead>
<tr>
<th>Joint beam to column</th>
<th>Coefficient</th>
<th>Joint beam to column, base plate</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$13,0$</td>
<td>$\infty$</td>
<td>$7$</td>
</tr>
<tr>
<td>$r$</td>
<td>$7,5$</td>
<td>$6$</td>
<td>$7$</td>
</tr>
<tr>
<td>$r$</td>
<td>$8,5$</td>
<td>$7$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$3$</td>
<td>$10$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$3,5$</td>
<td>$35$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$11,5$</td>
<td>$15$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$11,5$</td>
<td>$14$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$6,0$</td>
<td>$40$</td>
<td>$-$</td>
</tr>
<tr>
<td>$r$</td>
<td>$5,5$</td>
<td>$20$</td>
<td>$5$</td>
</tr>
</tbody>
</table>
Q&A 4.1 Preliminary Design of Connections

prEN 1993-1-8 gives rules for determining the behaviour of major axis, beam-to-column steel moment connections. Is there any other method, which can be used for preliminary design?

A simple way of predicting connection behaviour was developed by Steenhuis for preliminary design [Steenhuis, 1999]. Estimation of stiffness and resistance of the joint is based on the weakest component. The stiffness may be estimated by

\[ S_{\text{j,ini.app}} = \frac{E}{\xi} r^2 t_{fc}, \]  

(4.1)

where \( t_{fc} \) is the thickness of the column flange or base plate. The lever arm \( r \) is estimated as the distance between the centres of its beam flanges, see Table 4.3.

The moment resistance of the joint may be based on the column flange thickness \( t_{fc} \) which is assumed to be the weakest element

\[ M_{\text{j,ini.app}} = \frac{\varsigma f_{y,fc} r^2 t_{fc}}{\gamma_{M0}}. \]  

(4.2)

The factor \( \varsigma \) can be found in Table 4.3. To ensure the column flange is the weakest component it is assumed the end plate thickness \( t_p \) is thicker than column flange \( t_{fc} \geq t_{fc} \). The thickness of the column web stiffener \( t_{wc} \) is \( t_{wc} \approx t_{fb} \) and the diameter of the bolts is larger than the thickness of the column flange \( d \geq t_{fc} \).

Q&A 4.2 Use of Elastic Theory for Global Analysis of Structures

Is it allowed to use elastic methods for analysing a structure with connections designed by means of plastic theory?

Elastic global analysis may be used with connection designed plastically provided that the appropriate connection stiffness is taken into account in the elastic global analysis.

For example if the moment capacity of a connection is based on \( M_{j,el} \) the tangent stiffness \( S_{j,el} \) should be used in the analyses. However, if the moment capacity is based on \( M_{j,uli} \), the secant stiffness \( S_{j,sec} \) should be used.

In practice, the resistance of the elements is often based on a plastic stress distribution performed with elastic global analysis. The rotation capacity of a plastic hinge cross sections is implemented by classification of the section using the slenderness of the web and flanges. Class 2 is required for elastic analysis and the resistance is based on a plastic stress distributions. This procedure is simple and practical. It is based on engineering experience and not on an exact procedure of analysis. It is expected that the Ultimate Limit State will be reached on limited occasions only. The same procedure can be applied to connections. The resistance of all structural elements, members and connections, must satisfy the design criteria.
The elastic behaviour of an element is expected at the Serviceability Limit State. The load ratio of loads at the Ultimate Limit State and Serviceability Limit State for steel structures can be estimated as \((1*1.35 + 3*1.50)/4)/1.00 = 1.46\) and the ratio of plastic and elastic resistance of an I cross-sections is about \(1.18/1.00 = 1.18\). Hence the check of elastic behaviour at the Serviceability Limit State is not necessary. This is not the case for composite members, where the check of elastic response at the Serviceability Limit State is part of the standard design procedure. The same principle is applied to the design of connections. It is reported [Zoetemeijer, 1983] that the beginning of non-linear behaviour of the end plates may be estimated at \(2/3\) of the plastic bending moment resistance of a beam of rectangular cross section. For other connection types this ratio was observed in tests results. This estimation is conservative and safe when the yield stress \(f_y\) is used in the prediction model, see Figure 4.4.

![Figure 4.4 Prediction of the joint behaviour](image)

A plastic force distribution is usually used in design, see Figure 4.5a. However, certain components can limit the rotational capacity of the connection. Methods to predict the rotational capacity of a connection from the deformation capacity of its component are currently under development. Therefore, simple deemed to satisfy criteria for determining a connections rotational capacity are given in prEN 1993-1-8: 2003.

The components in the connection can be divided into two categories: ductile components (plate in bending, column web in shear, and column web in tension) and brittle components (bolts in shear and tension, welds and reinforcing bars). It is good engineering practice to over design the brittle components to increase the deformation capacity and safety. Elastic distribution is applied in the case when the brittle component limits the resistance of the connection, see Figure 4.5c. If the brittle component (bolt in second bolt row, for example) is placed in the middle of the joint, then the lower components need to remain elastic. In this case, an elastic plastic force distribution is applied to the connection, see third bolt row on Figure 4.5b.

![Figure 4.5 The modelling of distribution of internal forces inside the bolted end plate joint, a) plastic force distribution, b) elastic plastic force distribution, c) elastic force distribution](image)
Q&A 4.3 Classification Criteria for Column Bases

Why are different limits used for the classification of beam-to-column connections and column bases in prEN 1993-1-8: 2003? Could you give the background to their classification system?

Connections may be classified according to their resistance, stiffness and rotation capacity, see Figure 4.6.

The boundaries for the stiffness between rigid and semi-rigid connections are based on the required accuracy of the design (e.g. of the global analysis) of the member forces and connection check. The minimum stiffness of joints in a structural frame, which causes a change of the internal forces within the required accuracy limits, may be evaluated. This stiffness is the limit for rigid connections and all joints with higher stiffness can be modelled as rigid joints. For practical reasons, the limits for beam to column joints given in standards are conservative. For simplicity these values are scaled to the bending stiffness of the connected beams, see Figure 4.7. Following the above procedure limits have been developed between rigid and semi-rigid connections. Connections are assumed to be rigid if the buckling load of the structure is not less than 97.5% the buckling load of the same structure with rigid connections. The need to check deflections at the Serviceability Limit State is the reason for the different limits for unbraced \( (\bar{S}_{j,ini,u} = 25) \) and braced frames \( (\bar{S}_{j,ini,u} = 8) \). The pinned connection cannot be designed by independent checks on stiffness, resistance and rotation capacity. It is characterized by low stiffness \( (\bar{S}_{j,ini,u} = 0.5) \), low bending moment resistance and high rotation capacity \( (\bar{\phi}_{cd} = 60 \text{ mrad}) \).

Similar calculation were performed on frames with semi-rigid column bases, see [Wald, Jaspart, 1999], to determine the classification limit for column bases. In this case the stiffness of the
The column base is defined in terms of the stiffness of the connected column. Furthermore, the resistance of the column is limited by its slenderness. The minimum stiffness of the column base depends on the relative slenderness $\lambda$ of the column and is expressed as:

\[
\text{for } \lambda \leq 0.5 \quad \text{is the limit } \quad S_{j,\text{ini}} > 0, \quad (4.3)
\]

\[
\text{for } 0.5 < \lambda < 3.93 \quad \text{is the limit } \quad S_{j,\text{ini}} \geq 7 (2\lambda - 1) E I_c / L_c, \quad (4.4)
\]

\[
\text{and for } 3.93 \leq \lambda \quad \text{is the limit } \quad S_{j,\text{ini}} \geq 48 I_c / L_c. \quad (4.5)
\]

The limit (4.5) is a conservative approximation and can be used for all columns. The limiting stiffness $12 E I_c / L_c$ may be used for unbraced frames compared to columns with a slenderness lower than $\lambda = 1.36$, see Figure 4.8.

![Figure 4.8 Prediction of column resistance based on the lower support bending stiffness](image)

The limits for unbraced frames are banned on a limiting horizontal displacement. Base plates are assumed to be rigid if the horizontal displacement of the structure is not less than 90% of the horizontal displacement of the same structure with rigid connections. A study was undertaken on a portal frame with a flexible rafter. The results from this study are given in Figure 4.9.

![Figure 4.9 Classification of column bases based on bending stiffness](image)
**Q&A 4.4 Design of Connections Loaded by Low Forces**

In the case of a connection subject to low forces, is it necessary to design it for a certain "reasonable" level of force?

Structural connections should be designed to transmit environmental, accidental and nominal forces. In the case of very low applied forces the integrity of the structure and load cases at erection should be taken into account. The resistance depends on the type of connection.

To prevent progressive collapse under accidental loading the model of the tie forces was introduced, see [BCSA 1996]. The example of tying the columns of the building is given in Fig. 4.10. The minimum tie force is taken as 75 kN.

![Figure. 4.10 Example of tying the columns of the building](image)

**Q&A 4.5 Modelling of Joint Eccentricity in Frame Design**

Frames are usually modelled with a system of lines, joining the centres of sections. In the figure below, if the connection is designed as a pin may the column be designed as an axially loaded column?

![Figure](image)

The eccentricity of the connection is taken into account in the global analysis. Only the eccentricity of a bolted beam to column connection (end plate, web cleats and so on) to web of an open column may be neglected, see Figure 4.11. The eccentricity of a connection to a column flange as shown in the picture above needs to be taken into account.
The error in neglecting the eccentricity about the column weak axis is relatively high. The error may be estimated from the interaction of the normal force and bending moment (buckling of the column is neglected)

\[
\frac{N_{s,d}}{N_{pl,Rd}} + \frac{N_{s,d} e}{M_{pl,Rd}} \leq 1, \quad (4.6)
\]

The column normal force resistance is

\[
N_{pl,Rd} = A f_y / \gamma_{M0} = 7808 \cdot 235 / 1.10 = 1668 \cdot 10^3 \, N, \quad (4.7)
\]

and column bending moment resistance is

\[
M_{pl,z,Rd} = W_{pl,z} f_y / \gamma_{M0} = 200.3 \cdot 10^3 \cdot 235 / 1.10 = 42.8 \cdot 10^6 \, Nmm. \quad (4.8)
\]

The normal force resistance drops to 1419 kN with eccentricity \( e = 4.5 \, mm \), and the error in design resistance is 14.9%.

The error in neglecting bending about column strong axis can be calculated using column bending moment resistance

\[
M_{pl,y,Rd} = W_{pl,y} f_y / \gamma_{M0} = 642.5 \cdot 10^3 \cdot 235 / 1.10 = 137.2 \cdot 10^6 \, Nmm. \quad (4.9)
\]

The normal force resistance drops to 752.8*10^3 N and the error in design resistance for eccentricity \( e = 100 \, mm \) is 54.9%.