6 MOMENT CONNECTIONS

6.1 Introduction

The moment connections are transferring, except of shear and normal forces, bending moment (full or partial compare to connected element) to supports. Stiffness of connection may be modelled in global analysis as rigid or semi-rigid depending on relative stiffness of the connection (compared to the structure) and required accuracy.

First joints were developed through experimental observations. The results of tests were summarised into design tables and lead to development of simple engineering prediction models for hand calculation. The new set of tests summarised in databanks and application of information technique and ultimate limit state concepts allow prediction of joint properties by analytical models. The most important properties are bending moment resistance $M_{j.Rd}$, deformation/rotational stiffness S_j and deformation/rotational capacity $\phi_{Cd.}$ These properties can be predicted with high accuracy, see Fig. 6.1. The complexity of the problem (due to imperfections, residual stresses, friction, etc.) requires in some cases simulating of the behaviour by discrete models (FE).

6.2 Component method

The application of the component method requires three basic steps: listing of the joint components, evaluation of force-deflection diagram of each individual component, in terms of initial stiffness, strength and deformation capacity, and assembly of the components in view to evaluate the whole joint behaviour. Principles of the component method were based on Zoetemeijer's work [Zoetemeijer, 1983]. The components are loaded by tension, compression and shear, see Fig. 6.1.

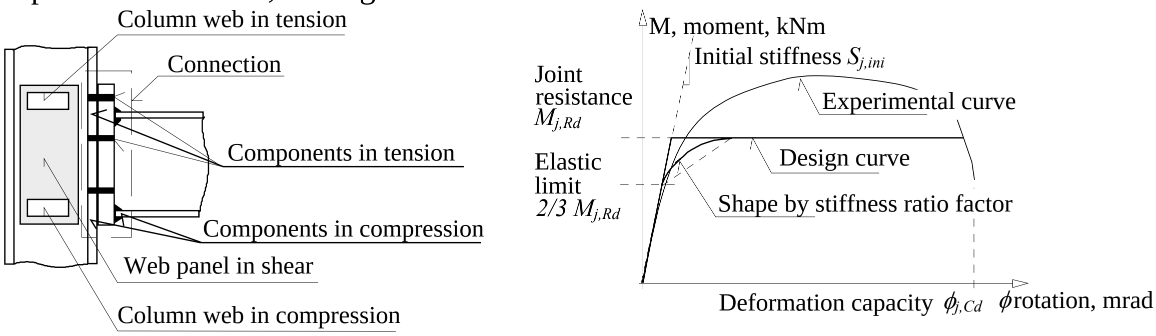


Fig. 6.1 Basic components of beam to column joint, moment – rotational curve

The accuracy of the method depends on accuracy of the description of the basic components by independent springs and on quality of assembling. The method enables very simple prediction for practical use as well as complex modelling for purposes of new developments. It is assumed, the component properties are independent and therefore they can be obtained easily. However, some components do not act independently but influence the others. This can be accounted for only in a simplified way, because general approach resulting in iterative calculation process would be too complicated for practical use.

Behaviour of the components can be described by non-linear force – deformation curve, see Fig. 6.2a). The behaviour is often non-linear and includes various effects (strain hardening, contact between elements, membrane effect, 3D tress distribution), but it can be simplified to bi-linear or tri-linear model. Parameters of the model can be derived from the component dimensions and material properties. The most important parameters include design resistance F_{Rd}, stiffness coefficient k and deformation capacity δ_{Cd}.

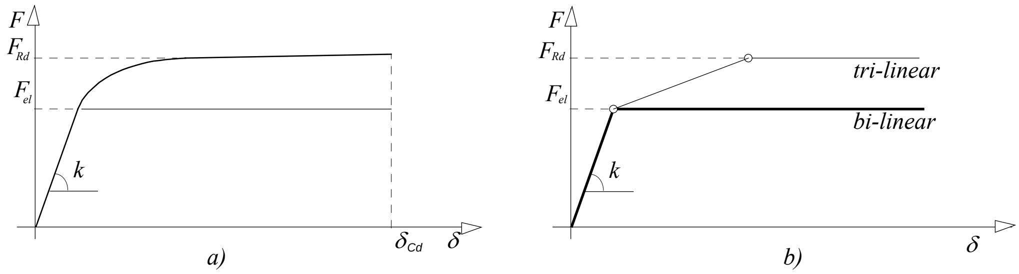


Fig. 6.2 Component behaviour, a) non-linear model, b) linear models

Properties of the joint are obtained by assembling properties of the individual components. When non-linear model of the component behaviour is used, incremental step by step assembling procedure can be used. This approach allows calculation of complete moment - rotation curve of the joint. In practical application, the designer usually needs only initial stiffness and bending moment resistance of the joint. These can be obtained easily by assembling the components, which are described by linear model.

Two general configurations of components can be found in structural joints: components connected in parallel and serial configuration, see Fig. 6.3.

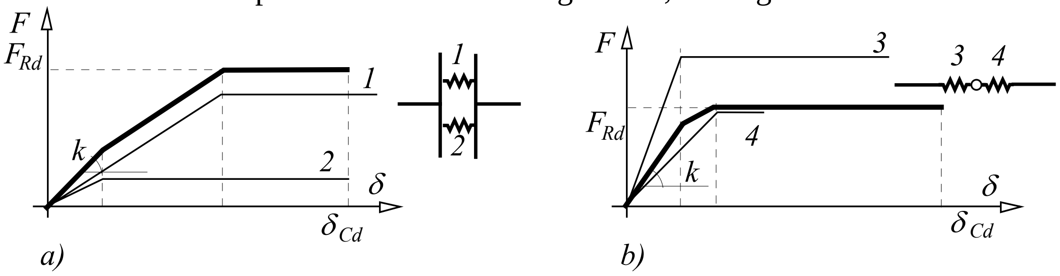


Fig. 6.3 Assembling of components,
a) parallel configuration, b) serial configuration

When the components are arranged in parallel configuration, the resulting properties of the assembly can be obtained from the following equations.

$$F_{Rd} = F_{1.Rd} + F_{2.Rd} \tag{6.1}$$

$$k = k_1 + k_2 \tag{6.2}$$

$$\delta_{Cd} = min\left(\delta_1 ; \delta_2\right) \tag{6.3}$$

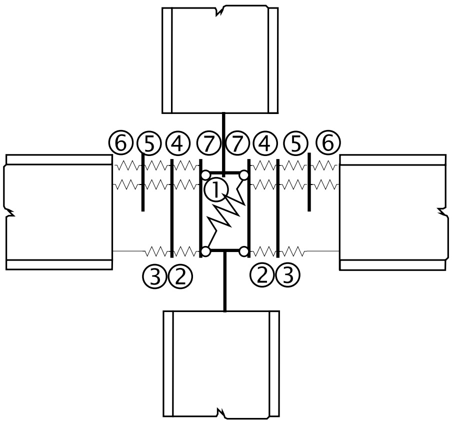
For components in serial configuration, the following formulas apply

$$F_{Rd} = min\left(F_{3.Rd} ; F_{4.Rd}\right) \tag{6.4}$$

$$k = \frac{k_3\ k_4}{k_3 + k_4} \tag{6.5}$$

$$\delta_{Cd} = \delta_3 + \delta_4 \tag{6.6}$$

Example of components for bolted beam to column connection is on Fig. 6.4.



① unstiffened column web in shear
② unstiffened column web in compression
③ beam flange in compression
④ column flange in bending
⑤ bolt row in tension
⑥ beam flange in bending
⑦ unstiffened column web in tension

Fig. 6.4 Components represented by springs on bolted beam to column joint

6.3 Resistance of joints

Bending moment resistance of joints is limited by the weakest component in tension, compression and shear.

Bending moment resistance of welded beam to column joint, see Fig. 6.5, can be obtained from

$$M_{j.Rd} = F_{t.Rd}\, z \,. \tag{6.7}$$

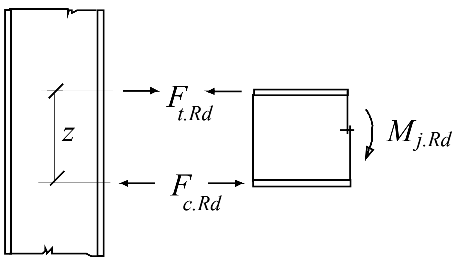


Fig. 6.5 Lever arm and resistance of components for welded joint

The resulting resistance of the tension zone $F_{t.Rd}$ is limited by column flange in bending $F_{t.fc.Rd}$, and by column web in tension $F_{t.wc.Rd}$. The resistance of the compression zone $F_{c.Rd}$ is obtained from: column web panel in shear $V_{wp.Rd} / \beta$, column web in compression $F_{c.wc.Rd}$, and beam flange in compression $F_{c.fb.Rd}$.

Bending moment resistance of bolted joint assuming plastic force distribution can be obtained from

$$M_{j.Rd} = \sum_i F_{ti.Rd}\, z_i \,, \tag{6.8}$$

where $F_{ti.Rd}$ is tension resistance of i-th bolt row and z_i is lever arm of the bolt row measured from centre of rotation. This is taken as centre of the compression flange of the beam for end plate joints, see Fig. 6.6.

Plastic force distribution in the bolted joints is assumed in most cases, however, elastic or elastic-plastic force distribution can also be used, see Fig. 6.6. For elastic force distribution, forces in the bolt rows are limited by tension resistance of the top bolt row. The bolt forces need to be checked to ensure the resistance is not exceeded.

When elastic distribution is assumed, shear force is transferred by all bolts and check for combined shear and tension is required. In this case, the tension bolt force should include the increase caused by prying of the bolts. For elastic-plastic and plastic force distributions, the shear force is usually assigned to bolt rows, which do not carry bending moment. As an alternative, combination of tension and shear can be considered.

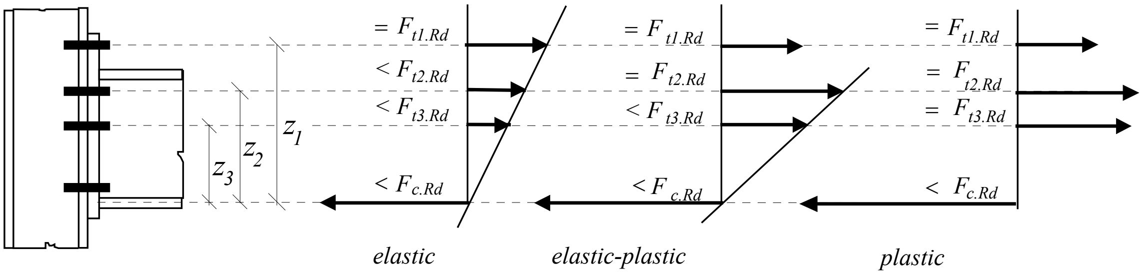


Fig. 6.6 Bending moment resistance of bolted beam to column joint ($F_{c.Rd}$ does not limit the resistance)

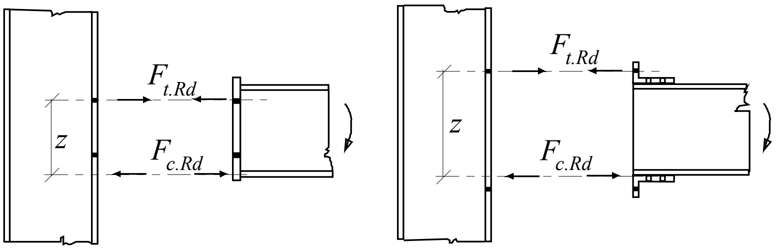


Fig. 6.7 Lever arm of bolted joints with one bolt row in tension

Resistance of single bolt row in tension is limited by column flange in bending $F_{t.fc.Rd}$, column web in tension $F_{t.wc.Rd}$, end plate in bending $F_{t.ep.Rd}$, and column web in tension $F_{t.wb.Rd}$. Total resistance of components in shear and compression is minimum of column web panel in shear V_{wpRd} / β, column web in compression $F_{c.wc.Rd}$, and beam flange in compression $F_{c.fb.Rd}$. The transformation parameter β is used to transfer the stiffness of the column web panel loaded by shear into connections on left and on right side of joint. For joints with two or more bolt rows, the resistance of bolt group in adjacent rows should be considered.

Influence of axial force in joints can be neglected, when the acting force is lower than 0,1 plastic design resistance of the connected beam ($0{,}1\ N_{pl.Rd}$).

When plastic force distribution on bolted joints is assumed, the resistance of single bolt row should be limited by $1{,}9\ B_{t.Rd}$ to ensure sufficient deformation capacity. When forming of plastic hinge in joint is required, strain hardening can cause overloading of the welds. In this case, the welds should be designed to carry bending moment increased to $1{,}4\ M_{j.Rd}$ for braced frames and $1{,}7\ M_{j.Rd}$ for unbraced frames.

6.3 Stiffness of joints

The component method can be used for calculation of initial stiffness of the joint $S_{j.ini}$. The stiffness is calculated by adding elastic deformations of single components. The deformation δ using stiffness coefficient k of the component is equal to

$$\delta_i = \frac{F_i}{k_i\ E}, \tag{6.9}$$

where F is force in the i-th component and E is modulus of elasticity.

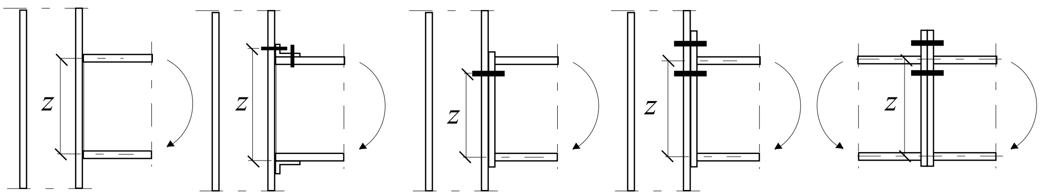


Fig. 6.8 Estimation of lever arm z for basic types of joints

Rotation of the joint can be calculated from deformation of the components taking into account lever arm between tension and compression zones, see Fig. 6.8.

$$\phi_j = \frac{\sum_i \delta_i}{z} . \tag{6.10}$$

Using the previous formula, the stiffness of the joint can be obtained

$$S_{j.ini} = \frac{M_j}{\phi_j} = \frac{F_i z}{\frac{\sum \delta_i}{z}} = \frac{F_i z^2}{\frac{F_i}{E} \sum \frac{1}{k_i}} = \frac{E\ z^2}{\sum \frac{1}{k_i}} . \tag{6.11}$$

Modifying the formula (6.11) to

$$S_j = \frac{E\ z^2}{\mu \sum \frac{1}{k_i}} , \tag{6.12}$$

description of complete moment - rotation curve can be obtained. For beam to column joints and beam splices, linear elastic behaviour is assumed up to $^2/_3$ of bending moment resistance of the joint. The coefficient μ indicates ratio of initial and secant stiffness, which is equal to one in elastic stage and larger than one for elastic - plastic part of the curve.

$$\mu = \frac{S_{j.ini}}{S_j} = \left(1{,}5\ \frac{M_{Sd}}{M_{j.Rd}} \right)^{\psi} \geq 1 , \tag{6.13}$$

The shape factor ψ is equal to *2,7* for welded joints and bolted end plate joints, and *3,1* for bolted flange cleats.

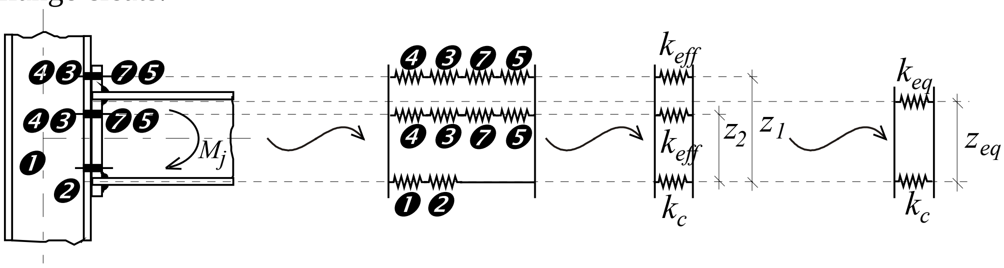


Fig. 6.9 Components of bolted beam to column joint, effective stiffness coefficient of bolt rows k_{eff} and equivalent stiffness coefficient of tension zone k_{eq}

The effective stiffness coefficient k_{eff} of every bolt row in tension should be calculated from

$$k_{eff} = \frac{1}{\sum_i \frac{1}{k_i}} , \tag{6.14}$$

where k_i are stiffness coefficient of basic components considered for the particular bolt row. Usually, these are unstiffened column web in tension, column flange in bending, bolts in tension and end plate in bending.

For end plate connections with two or more bolt rows in tension, the basic components of all bolt rows are represented by a single equivalent stiffness coefficient k_{eq} determined from

$$k_{eq} = \frac{\sum_j k_{eff.j}\ z_j}{z} . \tag{6.15}$$

The equivalent lever arm z_{eq} should be determined from

$$z_{eq} = \frac{\sum_j k_{eff.j} z_j^{\ 2}}{\sum_j k_{eff.j} z_j} . \tag{6.16}$$

6.4 Deformation capacity of joints

Required rotational capacity of joints depends on type of structure, but seldom exceeds *60 mrad*, see Fig. 6.10. Prediction of the available rotation capacity by component method, based on deformation capacity of each component, is under development. The end plate in bending and the column web panel in shear represent the ductile components. The bolt loaded in tension and shear and the reinforcement in tension are typical examples of brittle components. The prediction models allow separation of the ductile joints from brittle and danger solution not only on best engineering guess, but also on description of behaviour. The estimation of the upper limit of component resistance needs to be taken into account for the distribution of internal forces (for resistance calculation, the lower limit is sufficient). The basic rules for end plate joints and for welded joints assuring to reach the adequate rotational capacity are included in EN 1993-1-8 as deem to satisfy criteria. The rules limit brittle failure of bolts (mode 2 – bolt failure and plastic hinge in the plate, mode 3 – bolt failure) and weld failure only.

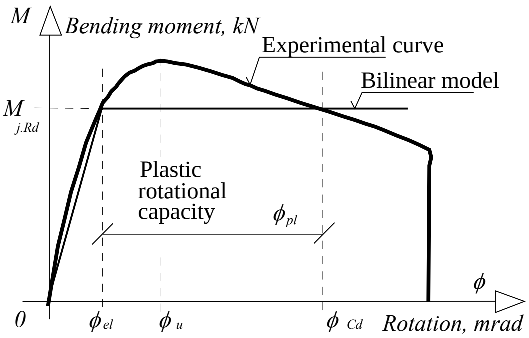


Fig. 6.10 Rotational capacity

6Q&A1 Stiffness Modification Coefficient η for End-Plated Connections
The values of stiffness modification coefficient η given in Table 5.2 do not cover wide range of different end-plate connections that can be used. For example do the values allow for connections into the web of a column/beam, thin end plates vs. thick end plates, extended vs. flush etc. Please, provide the background to this table.

To make an elastic global analysis according to clause 5.1.2 you may take into account a stiffness, which can be assumed as the initial stiffness divided by the stiffness modification coefficient η, see Fig. 6.12). The coefficient η is indicated in Table 5.2 [EN1993-1-8].

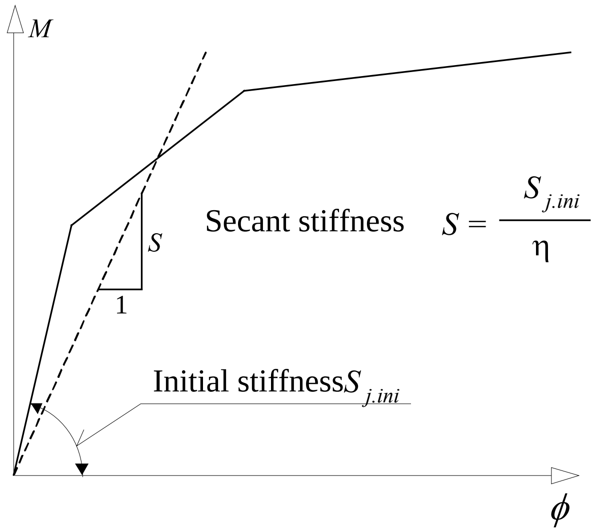


Fig. 6.11 Stiffness for global elastic analysis

The thickness of the end plates are much relevant for the initial stiffness but not for the stiffness modification coefficient η itself.

- For beams connected to the web of an unstiffened column or beam the stiffness modification coefficient η is not relevant because these joints have to be considered as hinges, see [Gomes, Jaspart, 1994] and [Gomes et al., 1994].
- For a beam connected to an unstiffened column web, the connection should be schematised as a hinge due to the limited rotational stiffness of the web.
- For a continuous beam connected on both sides of the column web, see Fig. 6.12a), the connection can be considered as a 'beam splice' with longer bolts.
- In case the beam is connected to a stiffened column web, see Fig. 6.12b), the stiffeners between the column flanges have a similar effect as a beam splice only in this case there is no web and the bending capacity is restricted to the bending capacity of the column.

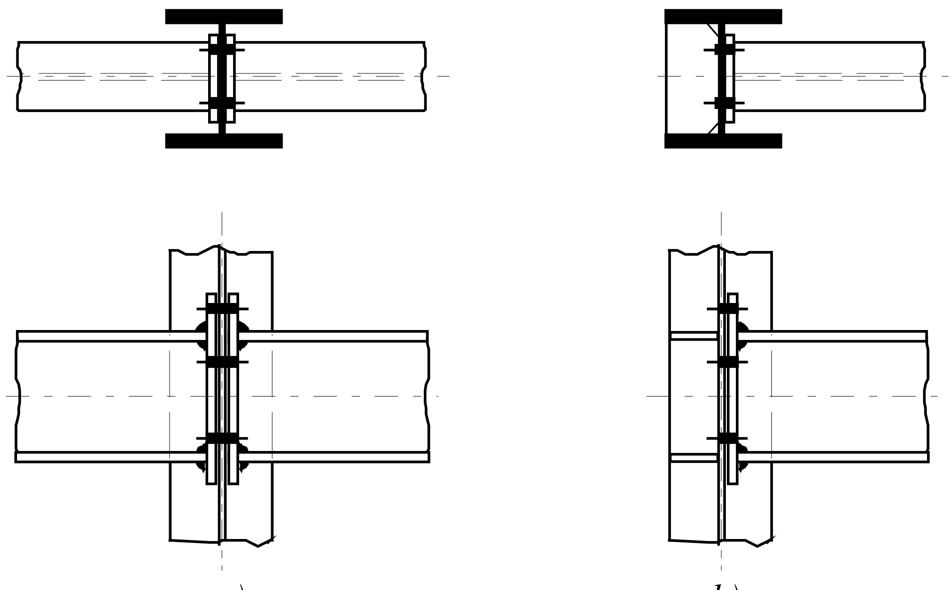


Fig. 6.12 Beam to column minor axis joints, a) beam splice, b) beam to stiffened column web

6Q&A2 Formula for Coefficient α of Effective Length of T-stub

Could you give a background information of curves for α used in calculation of effective length of a T-stub and equations for α depending on λ_1 and λ_2?

The background of these rules is model of plate solved by yield line theory. Details are given in the TU-Delft report written by Zoetemeijer [Zoetemeijer, 1990]. Note the α values in Figure 2.12 of the Zoetemeijer's publication need to be divided by 2 to compare with the Eurocode values.

The parallel parts of the curves in Fig. 6.14 of EN1993-1-8, correspond to the basic formulae in Tab. 6.6. With the values of m_1, m_2 and e, the values for λ_1 and λ_2 can be determined which gives the value of α. For the parallel parts of the curves $L_{eff} = \alpha\, m_1$ corresponds to the basic formula: $L_{eff} = 4\, m_1 + 1{,}25\, e$. In the above mentioned study of Zoetemeijer, the value of α did not exceeds 2 times π. The curves for α = 7 and 8 are added in the EN1993-1-8, see Fig. 6.14.

The curves for a constant value α, as illustrated in Fig. 6.12, follows from these equations:

$$\lambda_1 = \lambda_1^* + (1-\lambda_1^*)\left(\frac{\lambda_2^* - \lambda_2}{\lambda_2^*}\right)^{\frac{\alpha}{\sqrt{2}}} \qquad \text{in case } \lambda_2 < \lambda_2^* \tag{6.19}$$

$$\lambda_1 = \lambda_1^* \qquad \text{in case } \lambda_2 \geq \lambda_2^* \tag{6.20}$$

where

$$\lambda_1^* = \frac{1{,}25}{\alpha - 2{,}75}, \tag{6.21}$$

$$\lambda_2^* = \frac{\alpha\, \lambda_1^*}{2}. \tag{6.22}$$

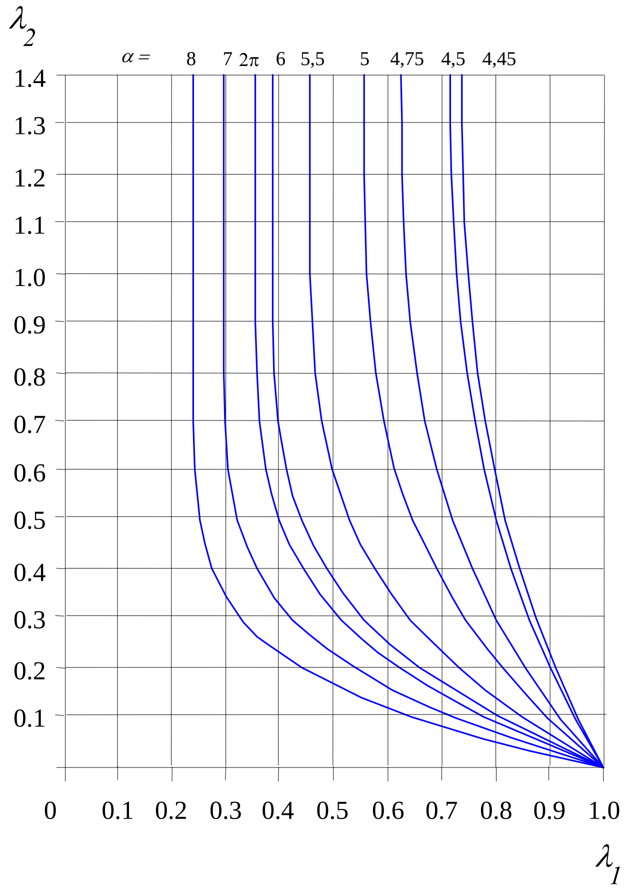


Fig. 6.14 Values of α for stiffened column flanges and end plates

Evaluation of coefficient α from the above equations requires iterative procedure. Simple method was developed to allow direct calculation [Sokol, 2000]. Curves on Fig. 6.14 were replaced by power functions

$$\lambda_1 = K + \frac{A\lambda_2}{\left(1+\left(B\lambda_2\right)^C\right)^{\frac{1}{C}}} . \tag{6.23}$$

Parameters K, A, B and C of these functions are summarised in Tab. 6.2. These functions represent contour lines of 3D surface with altitude equal to α.

Tab. 6.2 Parameters of power functions for coefficient α

α	K	A	B	C
8	0,9625419	-3,6454601	5,0308278	3,0586037
7	0,9636739	-2,6572564	3,9919931	2,9602062
2π	0,9726341	-1,9932994	3,2341635	2,7782066
6	0,9496498	-1,4803090	2,6267325	3,1078904
5,5	0,9785034	-1,1933579	2,3203117	2,9238812
5	0,9637229	-0,7743167	1,9115241	2,9607118
4,75	0,9946669	-0,7516043	1,9434759	1,9056765
4,5	1,2055126	-3,8888717	7,1149599	0,9088292
4,45	1,2265766	-3,2360189	6,2854489	1,0927973

As an example, coefficient α for $\lambda_1=0,60$ and $\lambda_2=0,35$ is shown on Fig. 6.15. Taking λ_2 as constant, section of the surface can be drawn using all functions from Tab. 6.2. The section is represented by multi-linear curve, see Fig. 6.15b). The value of α can be easily found by linear interpolation using the appropriate segment of the multi-linear curve.

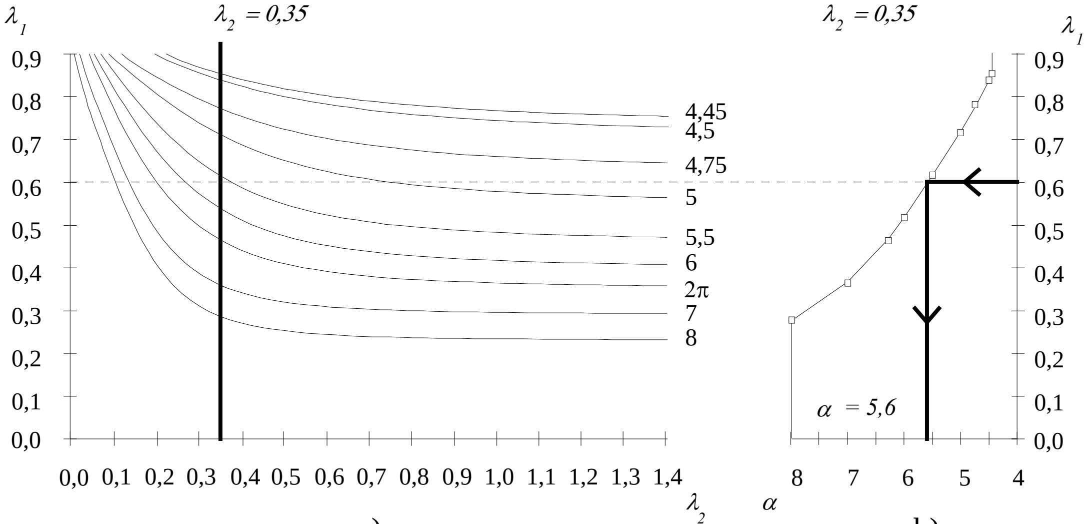


Fig. 6.15 Calculation of coefficient α, a) plan and section for λ_2=0,35, b) evaluation of α using the section

6Q&A3 Rules for Design of Haunched Connections

The current version of Annex J does not contain any rules of the design of portal frame haunched connections. Could you recommend simple and safe rules or reference where they could be found?

Two basic types can be distinguished, haunches designed to economise the rafter (in inclination of about 10%) and haunches used for increase of bending moment resistance in connection (in about 35%-40%). Similar questions raise in case of tapered built-up members. The current version of EN1993-1-8 contains component method, which can be used for all joints that can be schematised in the basic components. These are also the haunched connections. There are two questions related to component description and assembly of components in case of haunched connection: influence of inclination of the beam on the internal forces and resistance of the beam flange and column web in compression, see Fig. 6.16. The inclination needs to be taken into account for evaluation of component properties of the column web in compression and the end plate in bending (for welded connections, also of the column flange in bending and the column web in tension).

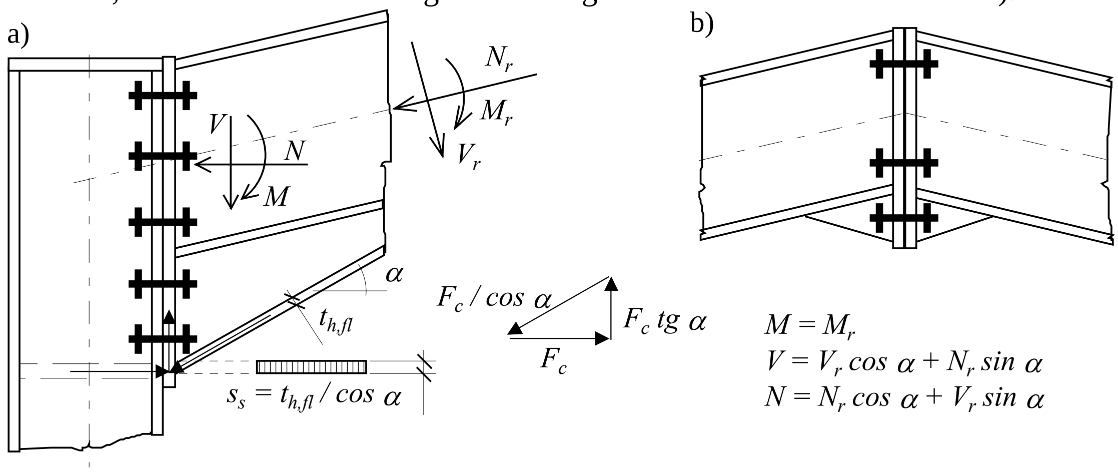


Fig. 6.16 Typical haunched joints in portal frame, a) stiffened haunch with flange, b) unstiffened haunch without flange

Precise details about haunches are given in Chapter 6.2.4.7 of prEN1993-1-8.
If the height of the beam including the haunch exceeds *600 mm* the contribution of the beam web to the compression resistance should be limited to *20%*.

The reinforcing haunches should be arranged with the following restrictions: the steel grade should match that of the member; the flange size and web thickness of the haunch should not be less than that of the member; the angle of the haunch flange to the flange of the member should not be greater than *45°*; and the length of stiff bearing s_s should be taken as equal to the thickness of the haunch flange parallel to the beam, see Fig. 6.16.

6Q&A4 Rules for Diagonal and K-stiffeners
Does it matter whether a (diagonal) stiffener of beam-to-column joint is loaded in tension or in compression?

There is difference in resistance calculation of the stiffener. For a stiffener loaded in tension, cross section resistance should be checked. For stiffener loaded in compression, plate buckling verification is required. As a simplifications, it is possible to use the following rules.

- The plate thickness of the stiffener is chosen the same size as the flange of the beam.
- The b/t ratio of the stiffener is chosen for the yield strength (at least a class 3 cross section is assumed).

K-stiffeners are loaded in tension and in compression. Both aspects must be checked as described above.

Fig. 6.17 Plate buckling design of a web stiffener

6Q&A5 Yield line Patterns for Row with 4 Bolts

How to extend EC3 rules to bolt-rows with 4 bolts instead of 2?

For tension, all bolts near the beam flange under tension can be taken for bending moment resistance calculation (a1-4 and b1-4), see Fig. 6.18. The bolts c2 and c3 could also be considered. However the bolts c1 and c4 can <u>not be considered for transfer of tension</u> due to the limited stiffness of the end plate. These bolts together with the bolt row d could be useful in shear.

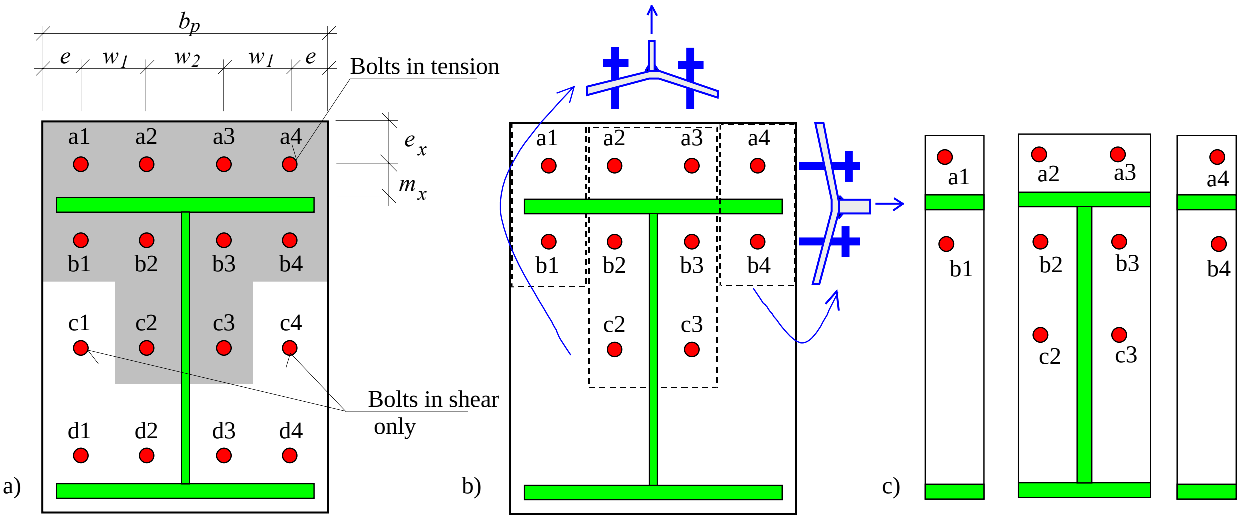
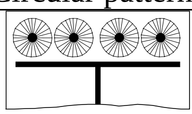
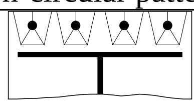
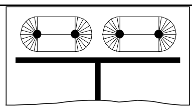
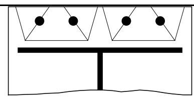
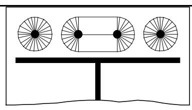
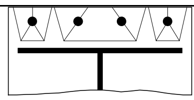


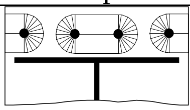
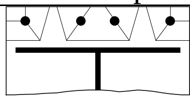
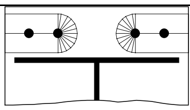
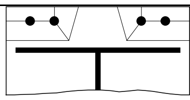
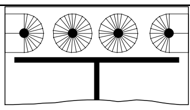
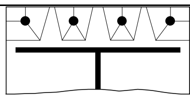
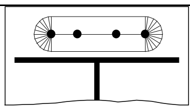
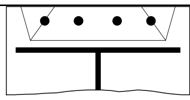
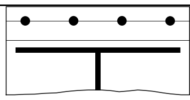
Fig. 6.18a) End-plate with 4 bolt is row, b) separation into T stubs in tension; b) division of top bolt rows into separate T stubs

Depending on size of the end plate and bolt spacing, there is several possibilities for yield line patterns for bolts in rows a and b.

The bolt row a can be considered as bolt row outside tension flange of the beam. The simplified approach of Eurocode 3 Annex J can be adopted with modifications of yield lines for bolt group, see tab. 6.3. The most simplified conservative solution is the fully independent bolt rows, see Fig. 6.18c), which avoid solving of the complex patterns of bolts rows found the top flange.

Tab. 6.3 Yield line patterns and effective length of T-stub for bolt rows with four bolts next to unstiffened beam flange

Circular patterns	Non-circular patterns
 $L_{eff.cp} = 4\,\pi\,m_x$	 $L_{eff.op} = 8\,m_x + 2{,}5\,e_x$
 $L_{eff.cp} = 2\,\pi\,m_x + 2\,w_1$	 $L_{eff.op} = 4\,m_x + 1{,}25\,e_x + w_1$
 $L_{eff.cp} = 3\,\pi\,m_x + w_2$	 $L_{eff.op} = 6\,m_x + 1{,}875\,e_x + 0{,}5\,w_2$

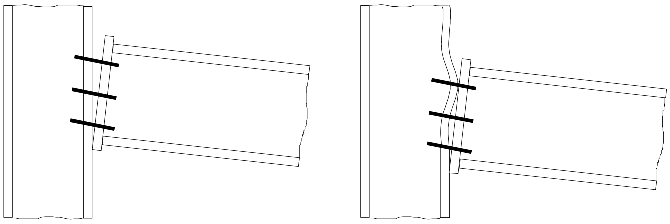
Circular patterns	Non-circular patterns
 $L_{eff.cp} = 2\pi\, m_x + w_2 + 2e$	 $L_{eff.op} = 4\, m_x + 1{,}25\, e_x + e + 0{,}5\, w_2$
 $L_{eff.cp} = \pi\, m_x + 2\, w_1 + 2e$	 $L_{eff.op} = 2\, m_x + 0{,}625\, e_x + e + w_1$
 $L_{eff.cp} = 3\pi\, m_x + 2\, e_1$	 $L_{eff.op} = 6\, m_x + 1{,}875\, e_x + e$
 $L_{eff.cp} = \pi\, m_x + 2\, w_1 + w_2$	 $L_{eff.op} = 2\, m_x + 0{,}625\, e_x + w_1 + 0{,}5\, w_2$
	 $L_{eff.op} = 0{,}5\, b_p$

6Q&A6 Plastic Distribution of Forces on End Plated Connection with Very Thick Plate

Is it permitted to use a plastic distribution of internal forces for a partial strength beam-to-column connection if very thick end plates are used? If not, are there any criteria for what thickness is required for the elastic design?

Bolt failure will be critical in cases when too thick end plates are used. As a result of that, brittle failure will occur, see Fig. 6.19a). Following correctly the procedure given in EN1993-1-8, the designer will be warned and forced to make changes in the design. This brittle mode of failure is not allowed because the rules for having sufficient rotational capacity are not fulfilled. In case the flange of the column is not very thick it could deform and deliver sufficient rotational capacity, see Fig. 6.19b).

For end plates and column flanges with $\rho > 2$ the bolts will cause brittle failure without sufficient rotational capacity. This is only allowed when the joint is designed to carry 1,2 times the beam strength. In that case rotational capacity will come from the beam.



a) *b)*

Fig. 6.19 Influence of column flange thickness on rotational capacity of joint,
a) thick end plate results in a brittle bolt failure,
b) rotational capacity is found in the column flange

6Q&A7 Distribution of Shear Forces on Bolted Connection

A moment and a shear force usually load an end-plated connection. How is the shear force distributed over the bolts?

In general, the force distribution over the bolts can be chosen in the most optimal way.

So, it is possible to distribute the shear force equally over all bolts, see Fig. 6.20. In that case, however, the bolts in tension and shear have to be checked with the rules for combined tension and shear, see EN 1993-1-8, as

$$\frac{F_{v.Sd}}{F_{v.Rd}} + \frac{F_{t.Rd}}{1{,}4F_{t.Rd}} \leq 1{,}0\ ,$$

where $F_{v.Sd}$ is shear force for one bolt; $F_{v.Rd}$ bolt shear resistance; $F_{t.Rd}$ tension force for one bolt (including prying action); $F_{v.Sd}$ bolt tensile resistance. For that reason, the shear force is most often distributed over the bolts in the compression part of the joint. If the capacity of these bolts is sufficient enough, the other bolts only need to be checked for tension. As simplification it may be verified that the search force transferred by the bolts does not exceed the (*0,4 / 1,4*) times the total shear resistance of those bolts that are also required to resist tension, on Fig. 6.19 is marked as “Remaining bolt shear resistance”.

Furthermore, to get sufficient deformation capacity, it is important that the capacity of the bolts in shear is higher than the bearing capacity of the bolts in the end plate or the column flange.

In case of pre-stressed bolts, the applied tensile force is counterbalanced by a contact force on the compression side of the connection, no reduction of the slip resistance is required.

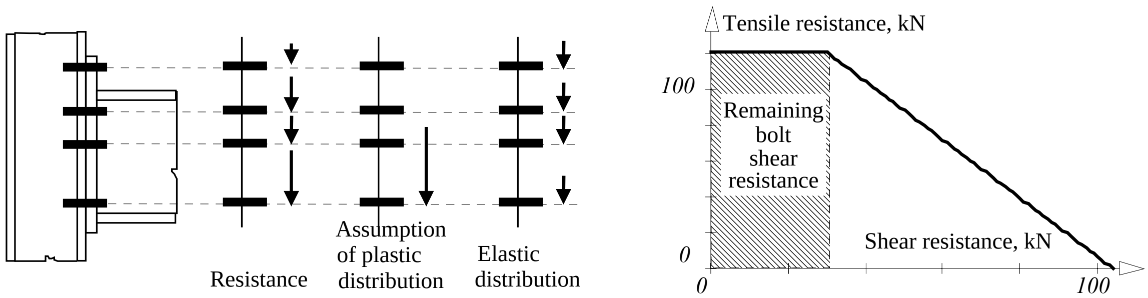


Fig. 6.20 Example of distribution of shear forces in connection, interaction of shear and tension bolt resistance

6Q&A8 Prying Force of T-stub in Fatigue Design

The effect of prying of bolts is included in formulas for resistance of bolt rows. However, in case of fatigue, the effect of prying forces in the bolts should be known in order to verify the bolts. How to cope with this?

In case of fatigue, bolts should always be pre-tensioned. Detailing is then of high importance. Transfer of the varying loads should pass directly through a stiff contact surface and not via the bolts. This is illustrated in Fig. 6.21. Influence of even small preloading on minimising the prying forces is shown on Fig. 6.21 and Fig. 6.22.

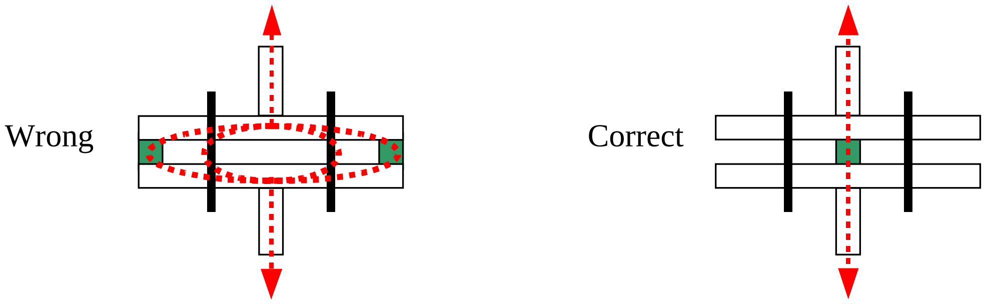


Fig. 6.21 Wrong and correct detailing of a pre-tensioned joint; the flow of the varying force through the joint is illustrated by dotted lines

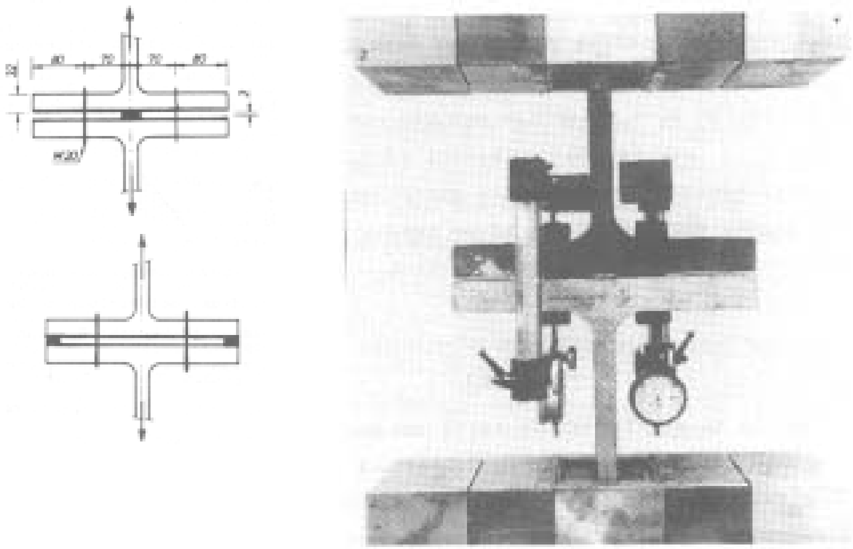


Fig. 6.22 Test specimens and assembling of a T-stub

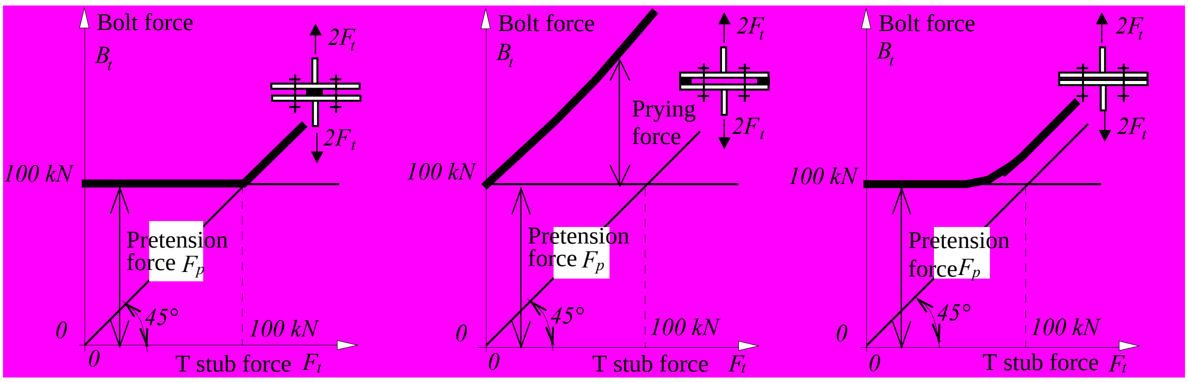


Fig. 6.23 Test results of T-stub; force in bolts F_b is indicated with bold line; F_t T-tub force

6Q&A9 Calculation of Joint Properties Loaded by Bending Moment and Axial Force
Clause J.3.1.3 limits the use of the calculation of bending moment resistance to connections with axial load less than 10% of axial load resistance. What approach should be applied to haunched connection used in a portal frame?

The questions include two aspects, which will be discussed separately: What is the influence of an angle on the design of the joint? What is the influence of an axial load on the moment resistance of a joint?

EC3 rules apply when the forces have been resolves in horizontal (parallel) and vertical (perpendicular) components. The slope of the beam causes change in geometry of the joint. Larger level arm should be taken into account.

The moment resistance of a joint loaded with an axial load can be determined from a linear interaction line between M_{Sd} and N_{Sd}, as shown in Fig. 6.24. The linear interaction line is found by determining the extreme values of the moment resistance (M_{Rd}) with no axial load and the axial load resistance (N_{Rd}) without bending moment. The design resistance of the joint should be checked with the following condition

$$\frac{N_{Sd}}{N_{Rd}} + \frac{M_{Sd}}{M_{Rd}} \leq 1\,. \tag{6.24}$$

This is a conservative approach especially for non-symmetrical joints, see Fig. 6.24. Point ① represents the maximal bending resistance; ② bending resistance in case of zero axial force; ③ maximal resistance in tension; ④ resistance in compression in case of zero bending moment; ⑤ negative bending in case of zero axial force; ⑥ maximal negative bending resistance; ⑦ point of activation of second bolt row; ⑧ axial compression; ⑨ point of activation of second bolt row.

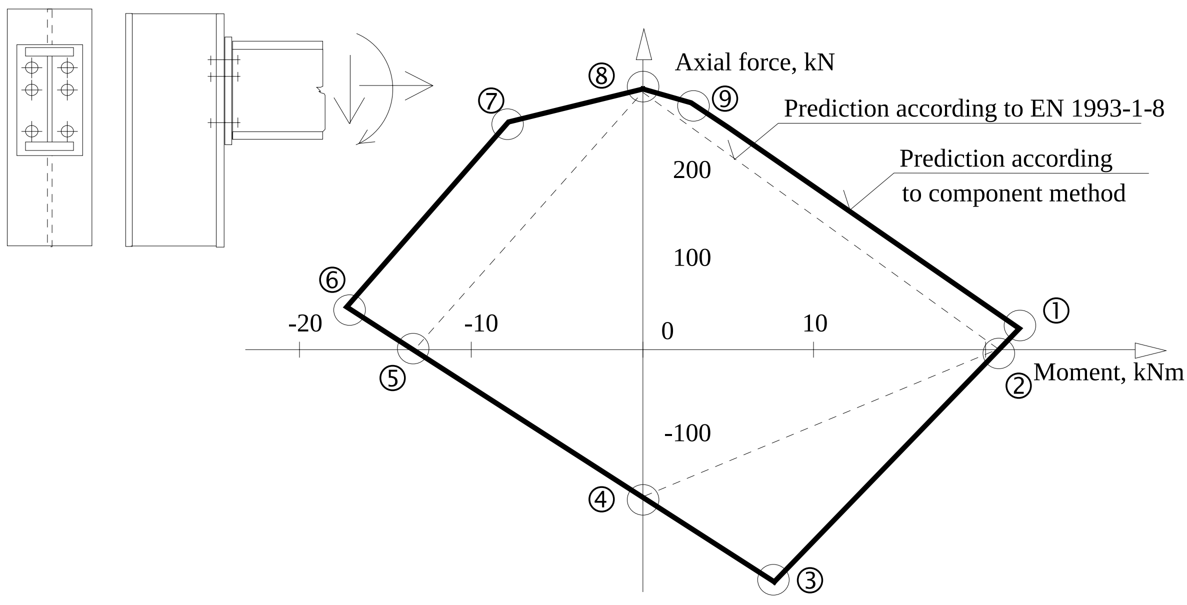


Fig. 6.24 Moment – axial load interaction curve, prediction according to EN 1993-1-8 and by component method

Another approach to combined compression and bending is based on extended component method for base plates used in EN 1993-1-8. Properties of the components should be

evaluated in the same way as for joints without axial force and are used in modified assembly procedure to calculate resistance and stiffness of the joint [Sokol et al., 2002].
Two typical loading paths may be distinguished, see Fig. 6.25. In case of non-proportional loading, the normal force is applied to the end plate connection in first step followed by application of the moment. In case of proportional loading, the normal force and the bending moment are applied simultaneously with constant ratio between the moment and normal force. In case of non-proportional loading, the initial stiffness of the joint is higher than for non-proportional loading. This effect is caused by presence of the normal force, which keeps the end plate in the contact at low bending moments, therefore only the components in compression contribute to deformation of the joint.

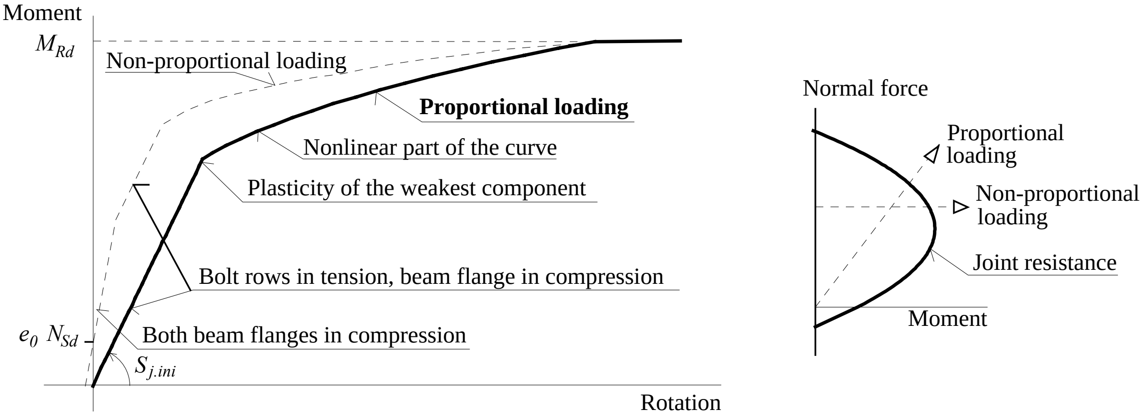


Fig. 6.25 Moment - rotation curve for proportional and non-proportional loading and loading paths on the moment - axial force interaction diagram

The size and shape of the contact area between the end plate and the column flange are based on effective rigid area [Wald, 1995]. Position of the neutral axis can be evaluated from equilibrium equations taking into account resistance of the tension and compression parts $F_{t.Rd}$ and $F_{c.Rd}$ respectively, and the applied normal force N_{Sd} and bending moment M_{Sd}. Plastic distribution of internal forces is assumed for the calculation, see Fig. 6.26.

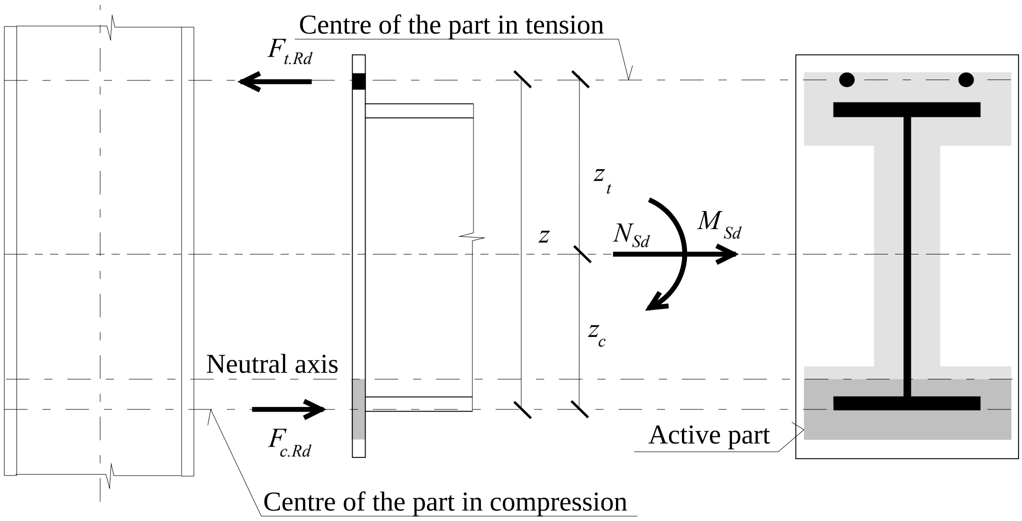


Fig. 6.26 The equilibrium in the joint

The simplified model takes into account only the effective area at beam flanges [Steenhuis, 1998] and the effective area at the beam web is neglected, as shown in Fig. 6.27. It is assumed the compression force acts at the centre of the flange in compression also in cases of limited size of outstand of the plate. The tension force is located in the bolt row in tension. In case of two or more bolt rows in tension part, the resistance of the part in tension is obtained as the resulting force of the active bolt rows.

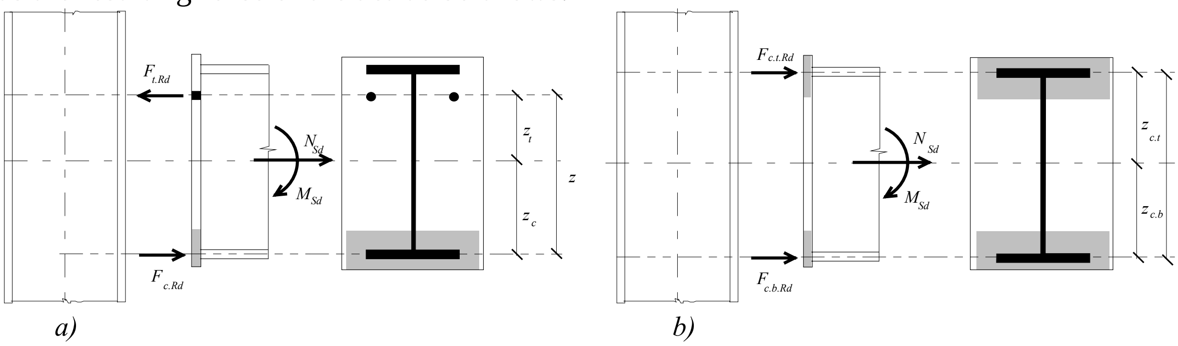


Fig. 6.27 The simplified model with the effective area at the flanges only; a) one bolt row in tension; b) no bolts in tension

The forces represent resistances of the components in tension $F_{t.Rd}$, and in compression $F_{c.t.Rd}$, $F_{c.b.Rd}$. For simplicity, the model will be derived for proportional loading only. Using equilibrium equations, the following formulas may be derived from Fig. 6.27a) assuming the eccentricity $e = \dfrac{M_{Sd}}{N_{Sd}} \leq -z_c$.

$$\frac{M_{Sd}}{z} + \frac{N_{Sd}\ z_c}{z} \leq F_t \tag{6.25}$$

and

$$\frac{M_{Sd}}{z} - \frac{N_{Sd}\ z_t}{z} \leq -F_c\ . \tag{6.26}$$

Since $e = \dfrac{M_{Sd}}{N_{Sd}} = \dfrac{M_{Rd}}{N_{Rd}} = const$ for proportional loading, equations (6.25) and (6.26) may be rearranged to

$$M_{j.Rd} = min\left\{\begin{array}{c} \dfrac{F_{t.Rd}\ z}{\dfrac{z_c}{e}+1} \\ \dfrac{F_{c.Rd}\ z}{1-\dfrac{z_{t.l}}{e}} \end{array}\right\}. \tag{6.27}$$

When the eccentricity $e = \dfrac{M_{Sd}}{N_{Sd}} \geq -z_c$, see Fig. 6.27b), there is no tension force in the bolt row, but both parts of the connection are loaded in compression. In this case, the equation (6.27) needs to be modified to

$$M_{j.Rd} = min\left\{\begin{array}{c} \dfrac{-F_{c.t.Rd}\ z}{\dfrac{z_{c.b}}{e}+1} \\ \dfrac{-F_{c.b.Rd}\ z}{\dfrac{z_{c.t}}{e}-1} \end{array}\right\}. \tag{6.28}$$

The rotational stiffness of the connection is based on the deformation of the components.

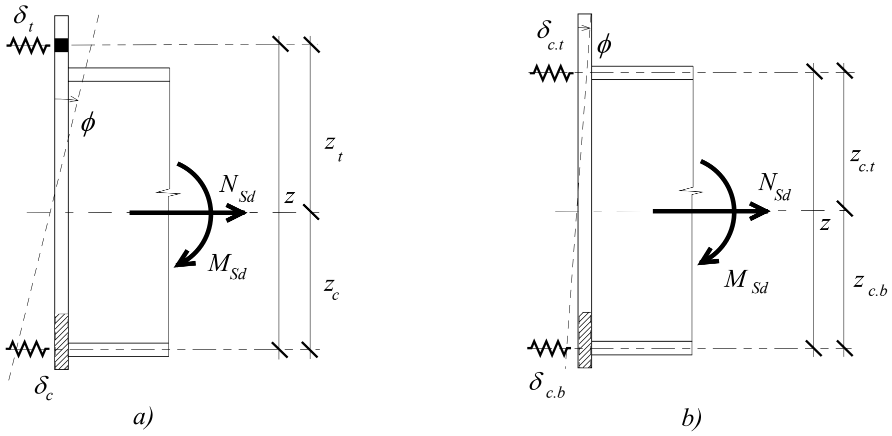


Fig. 6.28 The mechanical model of the end plate

The elastic deformation of the components in tension and compression parts, see Fig. 6.28a), may be expressed as

$$\delta_t = \frac{\dfrac{M_{Sd}}{z} + \dfrac{N_{Sd}\ z_c}{z}}{E\ k_t} = \frac{M_{Sd} + N_{Sd}\ z_c}{E\ z\ k_t}, \tag{6.29}$$

$$\delta_c = \frac{\dfrac{M_{Sd}}{z} - \dfrac{N_{Sd}\ z_t}{z}}{E\ k_{c.r}} = \frac{M_{Sd} - N_{Sd}\ z_t}{E\ z\ k_c}, \tag{6.30}$$

and the joint rotation is calculated using deformation of these components

$$\phi = \frac{\delta_t + \delta_c}{z} = \frac{1}{E\ z^2}\left(\frac{M_{Sd} + N_{Sd}\ z_c}{k_t} + \frac{M_{Sd} - N_{Sd}\ z_t}{k_c}\right). \tag{6.31}$$

The rotational stiffness of the joint depends on the bending moment, which is induced by the normal force applied with constant eccentricity e

$$S_{j.ini} = \frac{M_{Sd}}{\phi}. \tag{6.32}$$

The stiffness is derived by substitution of the rotation of the joint (6.31) into equation (6.32)

$$S_{j.ini} = \frac{M_{Sd}}{M_{Sd} + N_{Sd}\ e_0}\ \frac{E\ z^2}{\left(\dfrac{1}{k_c} + \dfrac{1}{k_t}\right)} = \frac{e}{e + e_0}\ \frac{E\ z^2}{\sum \dfrac{1}{k}}, \tag{6.33}$$

where the eccentricity e_0 is defined as follows

$$e_0 = \frac{z_c\ k_c - z_t\ k_t}{k_c + k_t}. \tag{6.34}$$

The non-linear part of the moment-rotation curve may be modelled by introducing the stiffness ratio μ, which depends on ratio γ of the acting forces and their capacities

$$\mu = (1{,}5\ \gamma)^{2{,}7} \geq 1. \tag{6.35}$$

Assuming the lever arms z_t and z_c of the components are approximately equal to $h/2$, i.e. one half oh the height of the connected beam, the factor γ can be defined as

$$\gamma = \frac{M_{Sd} + 0{,}5\ h\ N_{Sd}}{M_{Rd} + 0{,}5\ h\ N_{Sd}}, \tag{6.36}$$

and by substituting the eccentricity e, this can be simplified to

$$\gamma = \frac{e + \frac{h}{2}}{\left(\frac{M_{Rd}}{M_{Sd}} \right) e + \frac{h}{2}} \,. \tag{6.37}$$

Using the above described factor μ allows modelling of the moment-rotation curve of the joint, which is loaded by proportional loading, in the form

$$S_j = \frac{e}{e + e_0} \frac{E\, z^2}{\mu \sum \frac{1}{k}} \,. \tag{6.38}$$

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List of symbols

a	throat thickness of fillet weld
b_{eff}	effective width
b_b	width of beam flange
b_c	width of column flange
b_p	width of end plate
e	eccentricity
e_0	eccentricity of the joint
e	distance from bolt to edge of T-stub
e_x	distance from bolt to edge of end plate
f_y	yield stress
k	stiffness coefficient
k_c	total stiffness coefficient of the compression zone
k_{eff}	total stiffness coefficient of one bolt row in tension
k_{eq}	total stiffness coefficient the tension zone
m_x	distance from bolt to beam flange
m_1, m_2	distances from bolt to web of T-stub
t_p	thickness of end plate
w_1, w_2	distance between bolts
z	lever arm
z_{eq}	equivalent lever arm
z_c	lever arm of compression zone
$z_{c.t}$	lever arm of compression zone at top of the joint
$z_{c.b}$	lever arm of compression zone at bottom of the joint
z_t	lever arm of tension zone
$B_{t.Rd}$	design resistance of bolt in tension
E	modulus of elasticity
F	force
F_{el}	elastic limit
F_{Rd}	design resistance
$F_{c.Rd}$	design resistance in compression
$F_{c.b.Rd}$	design resistance in compression in bottom zone of the joint
$F_{c.t.Rd}$	design resistance in compression in top zone of the joint
$F_{t.Rd}$	design resistance in tension
$F_{t.ep.Rd}$	design resistance of end plate in bending
$F_{t.fc.Rd}$	design resistance of column flange in bending
$F_{t.wb.Rd}$	design resistance of beam web in tension
$F_{t.wc.Rd}$	design resistance of column web in tension
$F_{c.fb.Rd}$	design resistance of beam flange in compression
$F_{c.wc.Rd}$	design resistance of column web in compression
L_{eff}	effective length of a T-stub
M	bending moment
$M_{j.Rd}$	bending moment resistance of joint
M_{Sd}	applied bending moment
N_{Sd}	applied axial force
$N_{pl.Rd}$	plastic design resistance of the connected beam in tension
S_j	stiffness of joint
$S_{j.ini}$	initial stiffness of joint
$V_{wp.Rd}$	design resistance of column web panel in shear

α	coefficient of effective length of a T-stub
β	transformation parameter for shear loading
γ_{M0}	partial safety factor for material
δ	deformation
δ_c	deformation of components in compression zone
$\delta_{c.t}$	deformation of components in compression zone at top of the joint
$\delta_{c.b}$	deformation of components in compression zone at bottom of the joint
δ_t	deformation of components in tension zone
δ_{Cd}	deformation capacity
Φ_j	rotation of the joint
μ	stiffness ratio
ψ	shape factor
η	stiffness modification coefficient
λ_1, λ_2	dimensions of the T-stub