Thermomechanics

Lectures

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Available online:

http://people.fsv.cvut.cz/~vydra/fyzb.html#literatura

Schedule

Basic Concepts of Thermodynamics

- System, Process, State, Equations of State
- 1^{th} Law, Energy Balance Equation, specific heat capacity

Heat Transfer

- heat conduction, radiation, convection
- Newton's law of cooling

Mass Transfer

water vapour difussion, condensation

Bonus – Basics of Technical Typography

Fonts, units, variables

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The System

- Any volume separated by a boundary from the surroundings
  - air in a room
  - a concrete wall
  - a building
  - steam or exhaust gases in an engine
  - a liquid in a pipe
- Types of the system
  - open – mass crosses the boundary (windows open)
  - closed – a fixed quantity of mass (windows closed)
  - insulated (adiabatic) – heat does not cross the boundary
    - quality insulation, symmetry planes
    - it is a relative term - the point is that the exchange of heat with the environment is insignificant compared to the processes inside

State of the system

State variables

θ – temperature (K, °C)	p – pressure (Pa)	V – volume (m^3)
ρ – density ($kg\,m^{-3}$)	σ – stress (Pa)	l – length (m)

Equations of state – describe relationship between state variables

$pV = nRT$	– ideal gas law
$\sigma = E\varepsilon\|_{\theta=\text{konst.}}$	– Hook's law
$l = l_0\left(1+\alpha\theta\right)\|_{\sigma=\text{konst.}}$	– linear thermal expansion

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Ideal gas law

$pV = nRT$

,

- p – pressure of the gas (Pa)
- V – volume of the system
- $n = \frac{m}{\mu}$ – the amount of substance (mol)
  - where μ is the molar mass of the gas
- $R = 8.31\,J\,K^{-1}\,mol^{-1}$ – universal gas constant

Ideal gas law

Example: Chimney draft

Determine pressure at the bottom of a chimney

- height of the chimney $h = 50\,\mathrm{m}$,
- temperature inside of the chimney $\theta_\mathrm{i} = 60\,^\circ\mathrm{C}$,
- ambient (external) temperature $\theta_\mathrm{e} = -10\,^\circ\mathrm{C}$

Solution

Let's consider two systems – outside (e) and inside (i)

Both systems are in contact on top of the chimney – there is the same pressure (p_0)

The difference in pressures outside and inside at the chimney base is therefore only due to the difference in hydrostatic pressures outside and inside

Ideal gas law

Chimney draft – continuation

- hydrostatic pressure at the chimney base inside:
  $p_\mathrm{i} = p_0 + \rho_\mathrm{i} g h,$
- hydrostatic pressure at the chimney base outside:
  $p_\mathrm{e} = p_0 + \rho_\mathrm{e} g h,$
- the difference: $\Delta p = p_\mathrm{e} - p_\mathrm{i} = (\rho_\mathrm{e} - \rho_\mathrm{i})\, gh$
- density of air (or flue gases)
  $pV = \frac{m}{\mu}\mathrm{R}T \Rightarrow p = \frac{\rho}{\mu}\mathrm{R}T \Longrightarrow \rho = \frac{p\mu}{\mathrm{R}T}$
  because $\Delta p \ll p_0$, so $\rho_\mathrm{i} = \frac{p_0 \mu_\mathrm{i}}{\mathrm{R}T_\mathrm{i}}$ and $\rho_\mathrm{e} = \frac{p_0 \mu_\mathrm{e}}{\mathrm{R}T_\mathrm{e}}$

Ideal gas law

Chimney draft – cont.

- so $\rho_\mathrm{i} = \frac{p_0\mu_\mathrm{i}}{\mathrm{R}T_\mathrm{i}}$ and also $\rho_\mathrm{e} = \frac{p_0\mu_\mathrm{e}}{\mathrm{R}T_\mathrm{e}}$
- we know the temperature and pressure, the molar mass remains to be determined
  - google for the air
  - molar mass of flue gases should be calculated
    - depend on the fuel and excess air
    - approximately we can assume $\mu_\mathrm{i} \doteq 28\,\mathrm{g\,mol^{-1}}$
- $\Delta p = (\rho_\mathrm{e} - \rho_\mathrm{i})\,gh = \frac{p_0 gh}{\mathrm{R}}\left(\frac{\mu_\mathrm{e}}{T_\mathrm{e}} - \frac{\mu_\mathrm{i}}{T_\mathrm{i}}\right) = \frac{1{,}013\cdot 10^5 9{,}81\cdot 50}{8{,}31}\left(\frac{28{,}97}{263{,}15} - \frac{28}{333{,}15}\right)\cdot 10^{-3} = 156\,\mathrm{Pa}$

Thermodynamic Process

Process = system change.

The process is described by changing the status parameters!

Types of processes

- isochoric (izovolumic) ($V = \text{konst.}$)
- isothermic ($\theta = \text{konst.}$)
- isobaric ($p = \text{konst.}$)
- adiabatic (well insulated) ($\mathrm{d}Q = 0$)
- relaxation - the system is moving to thermodynamic equilibrium

Relaxation Process

Thermodynamic equilibrium

- The condition of the system surroundings does not change
- The system is moving into a state of equilibrium
- The process is called a relaxation process
- Time how long it takes - relaxing time

Relaxation time - how quickly a warm body cools down

- depending on the size, capacity and thermal conductivity
  - cathedral x pin
- thermometer – must be in thermodynamic equilibrium with the surroundings!

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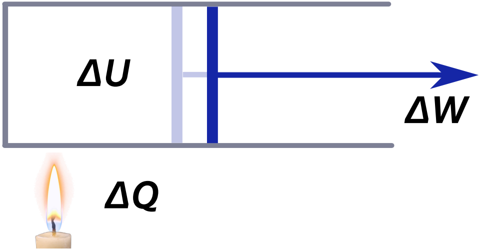
Energy Balance Equation

Example: gas confined by a piston in a cylinder (like an engine)



1st Law of Thermodynamic

$\Delta Q = \Delta U + \Delta W$

ΔQ – heat added to the system

ΔU – internal energy change (stored energy)

ΔW – the work done by expanding gas on the piston

Energy Balance Equation

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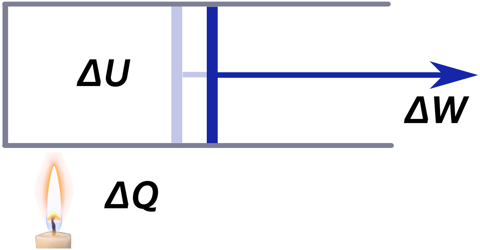

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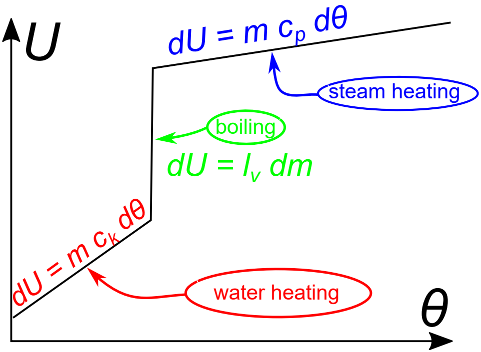
Internal Energy U

It is a measure of the total energy of particles (atoms, molecules)



- depends on
  - phase of matter
    $dU = l \cdot dm$
    (l is latent heat)
  - temperature
    $dU = m \cdot c \cdot d\theta$
    (c is specific heat)
  - amount of mass
    (open systems)

Specific Heat Capacity

Definice

- *The amount of heat required to heat 1 kg of mass up 1 °C*

- $c = \frac{1}{m}\frac{\mathrm{d}Q}{\mathrm{d}\theta}$
- we know that $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}W$, so
- $c = \frac{1}{m}\frac{\mathrm{d}Q}{\mathrm{d}\theta} = \frac{1}{m}\left(\frac{\mathrm{d}U}{\mathrm{d}\theta} + \frac{\mathrm{d}W}{\mathrm{d}\theta}\right)$

Specific Heat Capacity Depends on Type of the Process

c_v – specific heat capacity at constant volume

- $c = \frac{1}{m}\frac{dQ}{d\theta} = \frac{1}{m}\left(\frac{dU}{d\theta} + \frac{dW}{d\theta}\right)$
- $dW = 0$ (izochoric process!) so
- $c_v = \frac{1}{m}\frac{dU}{d\theta} + 0$

c_p – specific heat capacity at constant pressure

- during heating, the body usually expands while doing work $dW > 0$
- so $c_p > c_v$

Specific Heat Capacity

material	$\frac{c_p}{\mathrm{kJ\,kg^{-1}K^{-1}}}$	$\frac{c_v}{\mathrm{kJ\,kg^{-1}K^{-1}}}$	$\frac{c_p}{\mathrm{kJ\,m^{-3}K^{-1}}}$
air	1,005	0,718	1,285
argon	0,52	0,312	0,924
flue gases	1	?	?
water steam	1,97	1,5	?
concrete	0,8	0,8	~2300
bricks	0,8	0,8	by type
water	4,18	4,18	4200
steal	0,45	0,45	3530
ice	2,11	2,11	1940

The heat capacity depends on the temperature

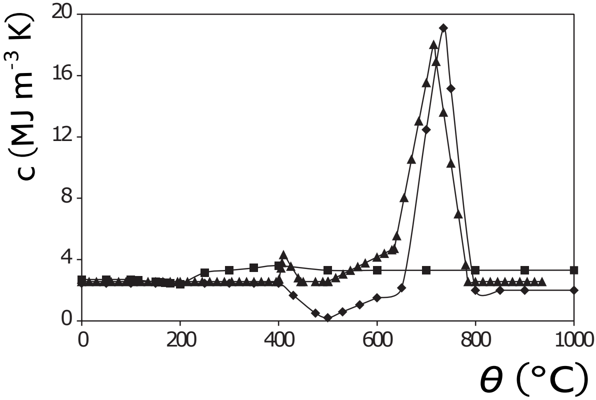
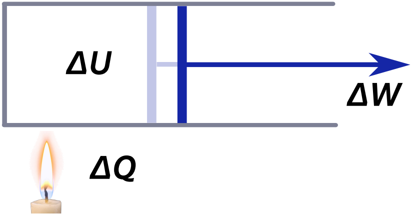


Figure: Concrete capacity on temperature. The capacity increase above 600 °C corresponds to the endothermic decomposition of the limestone aggregate.

Energy Balance in a System (like Engine)



Energy balance equation (1^{th} law)

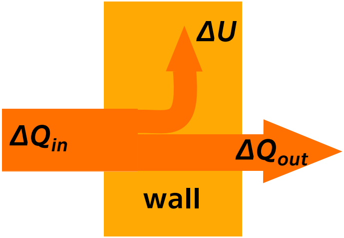
$\Delta Q = \Delta U + \Delta W$

Energy can't disappear (conservation of energy law!):
energy transferred to the system in the form of heat (ΔQ)
is either

1. stored in a form of internal energy (ΔU) increasing the temperature of the system etc.
2. or used to do a work (ΔW) on the piston

Energy Balance in a Control Volume (Concrete Wall)

Instead of the engine we take another system - like a concrete wall



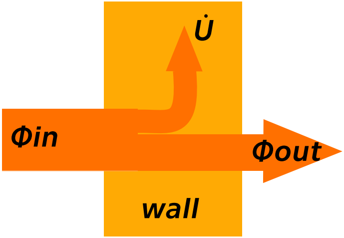
Energy balance equation

$\Delta Q = \Delta Q_{in} - \Delta Q_{out} = \Delta U + 0(!!)$

- ΔQ_{in} – heat “flowing” into the wall
- ΔQ_{out} – heat “leaking” from the wall on the other side
- ΔU – the surplus of Q is stored in the wall
  in a form of internal energy (temperature increases!)
- $\Delta W = 0$ (wall can't work, there isn't any piston!)

Heat Transfer – Energy Balance in a Wall (in Watts)

Expressing energy balance per unit of time - we obtain heat flows (in watts)



Energy balance eq.

$$\frac{d}{d\tau}\left(\Delta Q_{in} - \Delta Q_{out}\right) = \frac{d}{d\tau}\left(\Delta U + \Delta W\right)$$

$$\Phi_{in} - \Phi_{out} = \dot{U} + 0$$

- ΔQ is the total amount of heat delivered (J)
- Φ is the heat flow per unit of time (heat flow rate) $\Phi = \frac{dQ}{d\tau}(\mathrm{W})$
- $\dot{U}$ is the energy storage rate $\dot{U} = \frac{dU}{d\tau}\ (\mathrm{W})$

Heat Transfer – Heat Flow

- Φ – heat flow rate (aka heat rate, thermal flow) (W)
- q – heat flux ($\mathrm{W\,m^{-2}}$)

Energy balance in a wall

$\Phi_{in} - \Phi_{out} = \dot{U}$ (heat surplus is stored in the wall)

Energy balance in a wall in steady state

$\dot{U} = 0$ (steady state, nothing changes with time) $\Rightarrow$

$\Phi_{in} - \Phi_{out} = 0$

$\Phi_{in} = \Phi_{out}$ (flow out is equal to the flow in)

The same balance applies to surfaces (surface cannot store heat!)

Mechanisms of Heat transfer

Conduction

- 1D: the wall (homogeneous + multi-layer)
- 2D: the tube (cylindrical symmetry)
- 3D: the sphere (spherical symmetry)

Radiation

- heat transfer from a surface to another surface

Convection

- heat transfer from a surface to a room
- heat transfer from a surface to another surface

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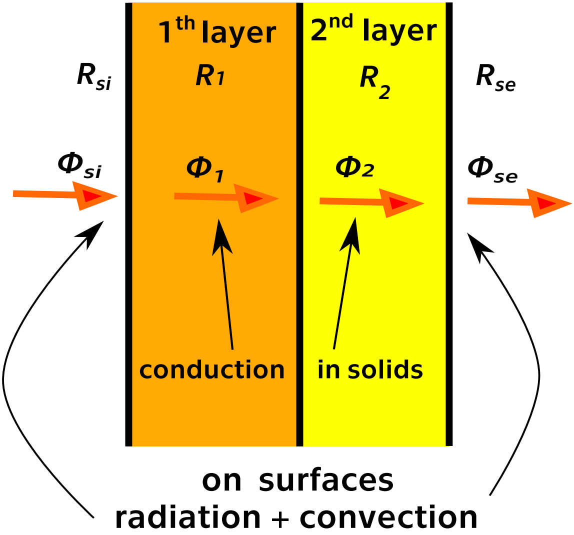
A Wall - Mechanisms of Heat transfer



Conduction

- Always from a warm to a cold body
  - by the way which body is cold/warm 🤔?
- Bodies must be in direct contact

Principle: atoms share kinetic energy:

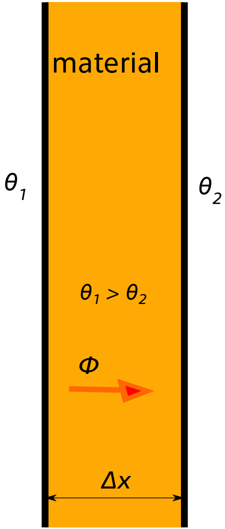
- by means of collisions (in gases and liquids)
- by means of diffusion of electrons (in metals)
- by means of vibrations (in solids)

1D Conduction (a Wall), Steady State

Fourier's law

$$\Phi = \frac{\Delta Q}{\Delta t} = -S\lambda\frac{\theta_2-\theta_1}{\Delta x} = -S\lambda\frac{\Delta\theta}{\Delta x}\ (\mathrm{W})$$

- Φ – heat flow rate (W)
- θ_1, θ_2 – temperature of surfaces
- λ – thermal conductivity (material property)
- S – area of the wall (perpendicular to the flow)

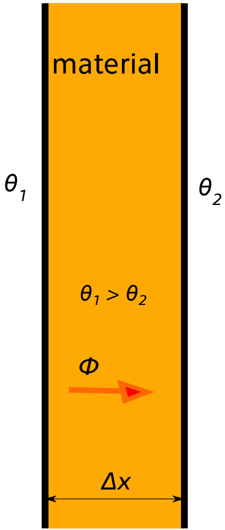


1D Conduction (Wall), Steady State

Heat flow expressed in civil engineering annotation

$$\Phi = -SU\Delta\theta = -S\frac{\Delta\theta}{R} \qquad (1)$$

- $U = \frac{\lambda}{\Delta x}$ – overall heat transfer coefficient
  aka thermal transmittance
  (in $\mathrm{W\,m^{-2}\,K^{-1}}$)
- $R = \frac{\Delta x}{\lambda}$ – thermal resistance
  aka R-value
  (in $\mathrm{K\,m^2\,W^{-1}}$)



Differential Form of Fourier's Law

In the limit of very thin wall ($\Delta x \to 0$):

Heat flow rate Φ

$$\Phi = -\lim_{\Delta x \to 0} S\lambda \frac{\Delta\theta}{\Delta x} = -S\lambda \frac{\mathrm{d}\theta}{\mathrm{d}x} \; (\mathrm{W})$$

In the limit of very small area ($S \to 0$):

Heat flux q

$$q = \lim_{S \to 0} \frac{\Phi}{S} = -\lambda \frac{\mathrm{d}\theta}{\mathrm{d}x} \; (\mathrm{W\,m^{-2}}) \qquad (2)$$

- Heat flux can be defined locally (point-wise)
  - it may vary from place to place ⇒ 3D conduction

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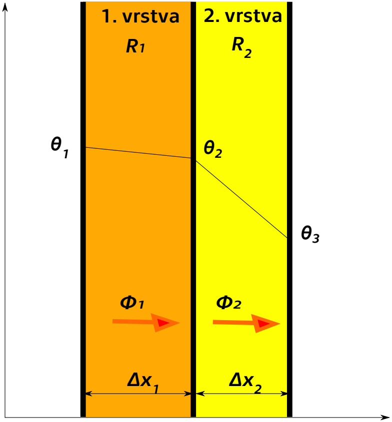
- Heat flux can be defined locally (point-wise)
  - it may vary from place to place $\Rightarrow$ 3D conduction

Multi-layer Wall, Steady State

Let suppose 1D conduction (no corners, no thermal bridges)



Fourier's law

- 1^{th} layer: $\Phi_1 = S\frac{\theta_1 - \theta_2}{R_1}$
- 2^{nd} layer: $\Phi_2 = S\frac{\theta_2 - \theta_3}{R_2}$

plus continuity equation

- $\Phi_1 = \Phi_2 = \Phi$

Multi-layer Wall

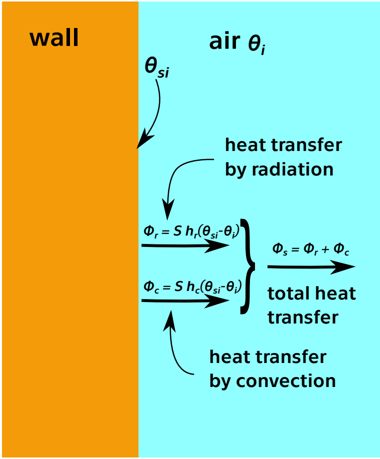
Solving above equations one gets:

$$\Phi = S\frac{\theta_1 - \theta_3}{R_1 + R_2} \tag{3}$$

and comparing with equation (1) one gets total thermal resistance of the wall:

$$R_\text{T} = R_1 + R_2$$

Heat Transfer from Surface to a Room



Heat is transfer from the surface to a room by means of

- radiation (Φ_r)
- convection (Φ_c)

Summing up both flows one gets total heat flow Φ_s at the surface

Surface heat transfer

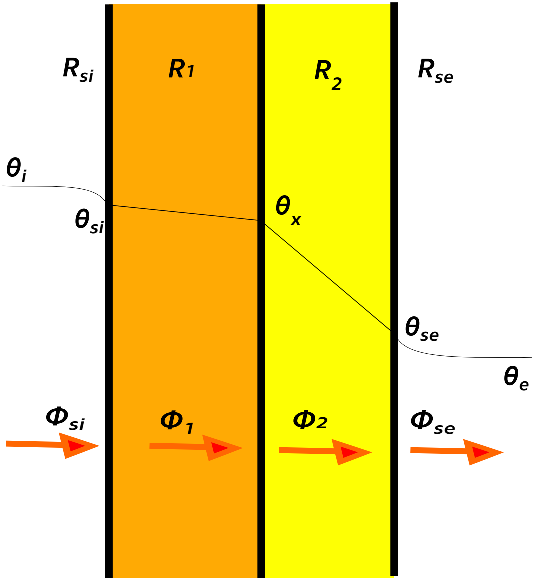
$$\Phi_s = \Phi_r + \Phi_c = S\,(h_r + h_c)\,(\theta_{si} - \theta_i)$$

$$\Phi_s = S\,h_s\,(\theta_{si} - \theta_i) = S\frac{(\theta_{si} - \theta_i)}{R_s}$$

h_s – heat transfer coefficient

R_s – surface thermal resistance

Total Thermal Resistance R_T



Steady-state heat flow rate

$$\Phi_{si} = \Phi_1 = \Phi_2 = \Phi_{se} = \Phi$$

$$\Phi = S \frac{\theta_i - \theta_e}{R_{si} + R_1 + R_2 + R_{se}}$$

Total R_T of the wall

$$R_T = R_{si} + R_1 + R_2 + R_{se}$$

Heat Conductivity λ vs. Temperature

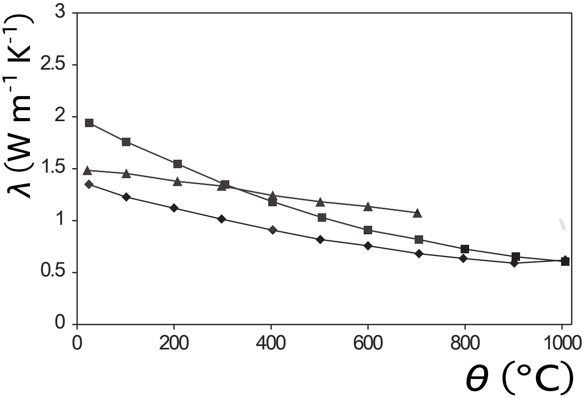


Figure: Heat conductivity of concrete vs. temperature

Question: is it important to take it into account 🤔? When?

Heat Conductivity λ vs. Temperature

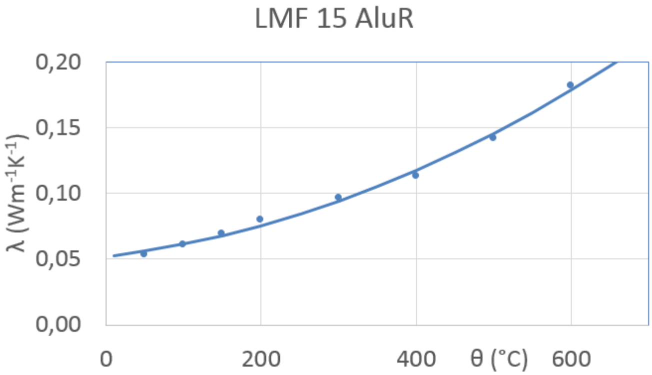


Figure: Heat conductivity of rock mineral wool vs. temperature

Determining Thermal Resistance of a Wall

Fourier's law: $q = -\lambda(\theta)\frac{d\theta}{dx}$ and $q = \text{const.}$ (steady state), the eq. can be integrated easily:

$\int_0^d q\,dx = -\int_{\theta_1}^{\theta_2} \lambda(\theta)\,d\theta$

Example for $\lambda(\theta) = \lambda_0 + a\cdot\theta + b\cdot\theta^2$, where λ_0, a, b are material parameters

$q\,[x]_0^d = -\left[\lambda_0\theta + a\frac{\theta^2}{2} + b\frac{\theta^3}{3}\right]_{\theta_1}^{\theta_2}$

$q\cdot d = -\left(\lambda_0(\theta_2-\theta_1) + a\frac{\theta_2^2-\theta_1^2}{2} + b\frac{\theta_2^3-\theta_1^3}{3}\right)$

$q = -\frac{\lambda_0(\theta_2-\theta_1)+a\frac{\theta_2^2-\theta_1^2}{2}+b\frac{\theta_2^3-\theta_1^3}{3}}{d}$,

and so: $R = \frac{(\theta_1-\theta_2)\cdot d}{\lambda_0(\theta_2-\theta_1)+a\frac{\theta_2^2-\theta_1^2}{2}+b\frac{\theta_2^3-\theta_1^3}{3}}$

Thermal Resistance of a Wall

Example - thermal insulation of a boiler

material: Knauf Insulation HTB 700,

$\lambda_0 = 0,514,\ a = 7,7 \cdot 10^{-5},\ b = 2,21 \cdot 10^{-7},$

$d = 5\,\mathrm{cm}$

1. hot boiler

   $\theta_1 = 700\,°\mathrm{C}, \theta_2 = 100\,°\mathrm{C}$

   after substitution: $R = 0.4\,\mathrm{K\,m^2\,W^{-1}}$

2. cold boiler

   $\theta_1 = 30\,°\mathrm{C}, \theta_2 = 20\,°\mathrm{C}$

   after substitution: $R = 0.935\,\mathrm{K\,m^2\,W^{-1}}$

Fourier's Law in More Dimensions

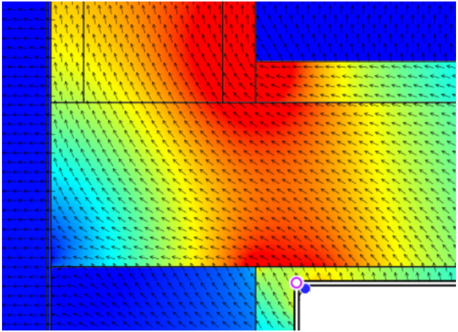
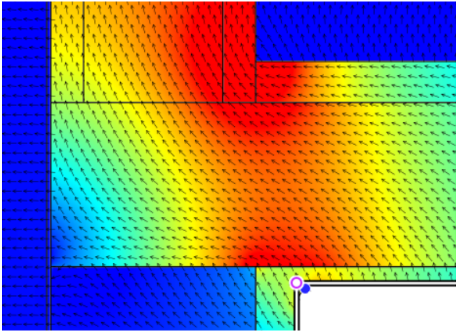


Figure: Heat flows in direction of temperature gradient – in corners, near thermal bridges, etc.

Question: Is direction of the heat flow correct?

Answer: No. Heat flow should be perpendicular to isotherms!!

Fourier's Law in More Dimensions



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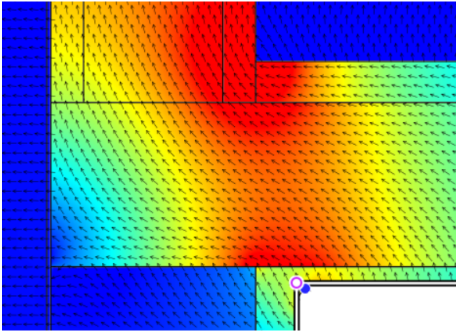

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Fourier's Law in More Dimensions

- 3D Fourier's law in Cartesian coordinates

$$
\begin{aligned}
q_x &= -\lambda \frac{\partial \theta}{\partial x} \\
q_y &= -\lambda \frac{\partial \theta}{\partial y} \\
q_z &= -\lambda \frac{\partial \theta}{\partial z}
\end{aligned}
$$

- or symbolically written

$$
\vec{q} = -\lambda \mathrm{grad}\theta
$$

- Heat flows in opposite direction of temperature gradient!

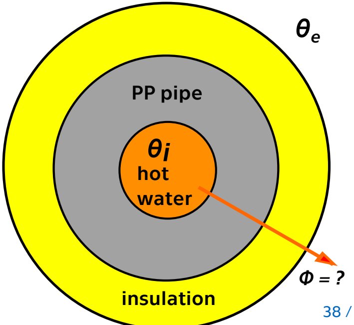
Problem: Heat Loss of a Hot Water Piping

Calculate heat loss of an insulated 20 m long pipe!

- pipe: PP PN20 Ø 20 mm, $\lambda_{tr} = 0.22\,W\,K^{-1}\,m^{-1}$
- hot water temperature $\theta_i = 80\,°C$
- insulation: URSA RS 1/Alu 20 mm, $\lambda_{iz} = 0.0359\,W\,K^{-1}\,m^{-1}$
- ambient temperature $\theta_e = 10\,°C$

Calculation procedure:

1. heat resistance of PP pipe
2. + heat resistance of insulation
3. + heat resistance of surface layer

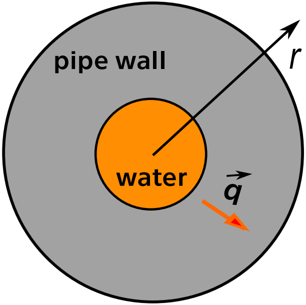


Conductive Heat Loss through Cylinder or Pipe Wall

1^{th} step: a pipe without insulation, surface temperatures are known

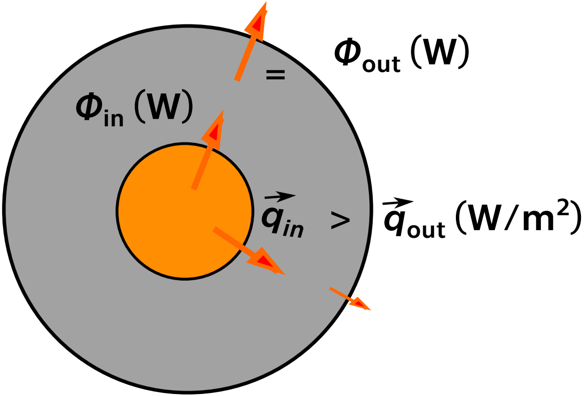
- Suppose cylindrical symmetry - the temperature gradient is only in the radial direction (and the same in all radial directions)
- Therefore heat conduction is a one-dimensional problem:

$$q = q_r = -\lambda \frac{d\theta}{dr}$$



Pipe Wall – Energy Balance in Steady State

- Heat flow rate (in watts!) through the inner surface of the wall must be the same as heat flow through the outer surface
- generally: $\Phi = S\,q = -2\pi r\,l\,\lambda \frac{\mathrm{d}\theta}{\mathrm{d}r} = \text{konst.}$

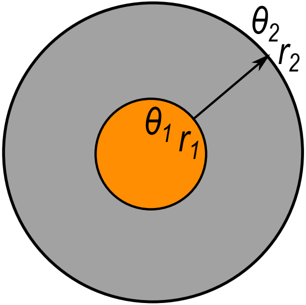


Pipe Wall – Steady State

Let's solve the eq. $\Phi = -2\pi r\, l\, \lambda \frac{d\theta}{dr} = \text{konst.}$

1. Separating variables we get: $\Phi \frac{dr}{r} = -2\pi l \lambda d\theta$
2. Integration using boundary conditions on inner and outer surfaces i.e. $\theta(r_1) = \theta_1, \theta(r_2) = \theta_2$:

$$\int_{r_1}^{r_2} \Phi \frac{dr}{r} = -2\pi l \lambda \int_{\theta_1}^{\theta_2} d\theta$$



$$\Phi \left(\ln r_2 - \ln r_1\right) = -2\pi l \lambda \left(\theta_2 - \theta_1\right)$$

$$\boxed{\Phi = -l \cdot \frac{2\pi\lambda(\theta_2 - \theta_1)}{\ln \frac{r_2}{r_1}}}$$

Pipe Wall – Steady State

Let's quantify the first step of the problem defined on slide 38

Example: PP pipe, 20 meters long, data as follows:

- $\theta_1 = 80\,^{\circ}\mathrm{C}$ (hot water)
- Inner diam. $d_1 = 13.2\,\mathrm{mm}$, outer diam. $d_2 = 20.0\,\mathrm{mm}$
- Thermal conductivity $\lambda = 0.22\,\mathrm{W\,K^{-1}\,m^{-1}}$
- Temp. of outer surface $\theta_2 = 76.5\,^{\circ}\mathrm{C}$ (to be calculated later!)

Solution

$$\Phi = -l \cdot \frac{2\pi\lambda(\theta_2 - \theta_1)}{\ln\frac{r_2}{r_1}}$$

$$\Phi = -20 \cdot \frac{2 \cdot \pi \cdot 0{,}22 \cdot (76{,}5 - 80)}{\ln\frac{20}{13{,}2}}$$

$$\Phi \doteq 230\,\mathrm{W}$$

Pipe Wall – Heat Resistance

- Compare heat flow formulas for "wall" and the pipe wall.
- Generally: *Heat Flow* = *transverse dimensions* x *some function of longitudinal dimensions and conductivity* (U, R^{-1}) x *temperature difference*

Wall

$$\Phi = -S \cdot \frac{\lambda}{d} \cdot (\theta_2 - \theta_1)$$

$$\Phi = -S \cdot \frac{1}{R} \cdot \Delta\theta$$

Pipe wall

$$\Phi = -l \cdot \frac{2\pi\lambda}{\ln \frac{r_2}{r_1}} \cdot (\theta_2 - \theta_1)$$

$$\Phi = -l \cdot \frac{1}{R_l} \cdot \Delta\theta$$

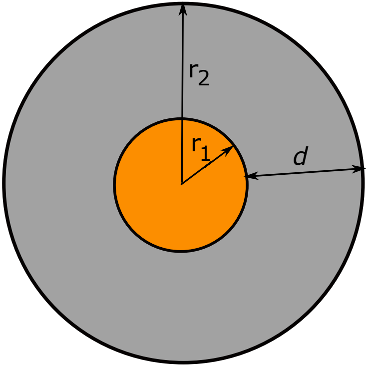
- As a transverse dimension, there is length l of the pipe!
- The area S perpendicular to heat flux is not constant in the case of pipelines!

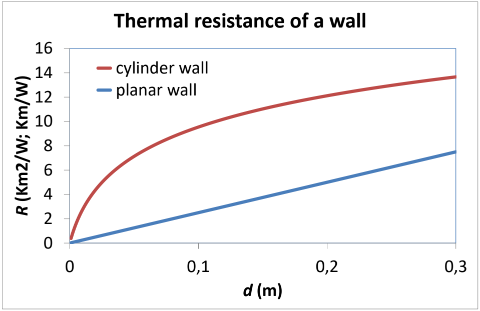
Pipe Wall – Heat Resistance

Thermal resistance per length

$$R_\text{l} = \frac{\ln\frac{r_2}{r_1}}{2\pi\lambda} = \frac{\ln\frac{d+r_1}{r_1}}{2\pi\lambda}(\text{K}\,\text{m}^{1}\text{W}^{-1}) \qquad (4)$$

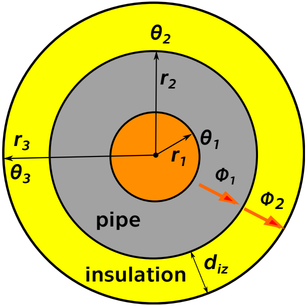
Thermal resistance grows logarithmically with wall thickness!





Insulated Pipe Wall

To calculate the thermal resistance of a two-layer pipe wall, we use a procedure similar to that used in calculating multi-layer wall resistance.



- The heat flow rate can be calculated for each layer separately:

$$\Phi_1 = -\frac{2\pi l \lambda_{tr}}{\ln \frac{r_2}{r_1}} (\theta_2 - \theta_1)$$

$$\Phi_2 = -\frac{2\pi l \lambda_{iz}}{\ln \frac{r_3}{r_2}} (\theta_3 - \theta_2)$$

- temperature θ_2 and heat flow rates Φ_1 and Φ_2 are unknown.

Insulated Pipe Wall

- Supposing steady-state one gets:
  $\Phi_1 = \Phi_2$ (continuity equation)
- and excluding the unknown θ_2:
  $\Phi = l\,\frac{\theta_3-\theta_1}{R_{l,tr}+R_{l,iz}}$
  where
  $R_{l,tr} = \frac{\ln\frac{r_2}{r_1}}{2\pi\lambda_{tr}}, \quad R_{l,iz} = \frac{\ln\frac{r_3}{r_2}}{2\pi\lambda_{iz}},$ l is the length of the pipe

Finally:

Thermal resistance of the pipe's wall per length

$R_{l,c} = R_{l,tr} + R_{l,iz}$

Insulated Pipe Wall

Let's quantify the second step of the problem defined on slide 38

1. PP pipe, inner temper. $\theta_1 = 80\,°\mathrm{C}$:
   - Inner diam. $d_1 = 13.2\,\mathrm{mm}$, outer diam. $d_2 = 20.0\,\mathrm{mm}$
   - Thermal conductivity $\lambda_{tr} = 0.22\,\mathrm{W\,K^{-1}\,m^{-1}}$
2. Insulation, outer temper. $\theta_3 = 21.2\,°\mathrm{C}$ (to be calculated later!):
   - Inner diam. $d_2 = 20.0\,\mathrm{mm}$, outer diam. $d_3 = 60.0\,\mathrm{mm}$
   - Thermal conductivity $\lambda_{iz} = 0.0359\,\mathrm{W\,K^{-1}\,m^{-1}}$

$$R_{l,tr} = \frac{\ln \frac{r_2}{r_1}}{2\pi\lambda_{tr}} = \frac{\ln \frac{20}{13{,}2}}{2\cdot\pi\cdot 0{,}22\cdot} = 0.30\,\mathrm{m\,K\,W^{-1}}$$

$$R_{l,iz} = \frac{\ln \frac{r_3}{r_2}}{2\pi\lambda_{iz}} = \frac{\ln \frac{60}{20}}{2\cdot\pi\cdot 0{,}0359\cdot} = 4.87\,\mathrm{m\,K\,W^{-1}}$$

$$R_{\mathrm{l,c}} = R_{\mathrm{l,tr}} + R_{\mathrm{l,iz}} = 5.17\,\mathrm{m\,K\,W^{-1}}$$

$$\Phi = -l\frac{1}{R_{l,c}}\Delta\theta = -\frac{1}{5{,}17}(21{,}2 - 80) \doteq 230\,\mathrm{W}$$

Surface Thermal Resistance

Heat flow rate Φ from the surface of the pipe to the air, can be express by means of thermal resistance R_{se} or by the heat transfer coefficient h_{se} as we did in the case of heat flow through the wall:
$\Phi = l\frac{\Delta\theta}{R_{l,\mathrm{se}}} = h_{\mathrm{se}} S \Delta\theta$, where

- $S = 2\pi r_3 l$ is area of the surface,
- $\Delta\theta$ is temperature difference between surface and ambient air,
- r_3 is external radius of the insulated pipe.

By comparing one gets:

Surface thermal resistance of the pipe's wall per length

$$R_{\mathrm{l,se}} = R_{\mathrm{se}} \frac{1}{2\pi r_3} = \frac{1}{2\pi h_{\mathrm{se}} r_3}$$

Total Thermal Resistance of the Pipe's Wall

Let's quantify the third step of the problem defined on slide 38

Surface thermal resistance

- $h_{se} = 5.42\,\mathrm{W\,m^{-2}\,K^{-1}}$(to be calculated later!)
- $R_{l,se} = R_{se}\frac{1}{2\pi r_3} = \frac{1}{2\pi h_{se} r_3} = \frac{1}{2\pi \cdot 5{,}42 \cdot 0{,}030} = 0.98\,\mathrm{K\,m\,W}$

Total thermal resistance

- $R_{l,T} = R_{l,tr} + R_{l,iz} + R_{l,se} = 0,3 + 4,87 + 0,98 = 6.15\,\mathrm{K\,m\,W}$

Heat flow

$$\Phi = -l\frac{1}{R_{l,T}}\Delta\theta = -\frac{1}{6{,}15}(10-80) \doteq 230\,\mathrm{W}$$

Heat Loss of the Pipe – Summary

The problem is defined on slide 38

Now we can evaluate temperatures at the interfaces used in the 1th and the 2nd step of solution:

1. Using the formula $\Phi = -l \cdot \frac{2\pi\lambda_{tr}(\theta_2 - \theta_1)}{\ln\frac{r_2}{r_1}}$ we can express θ_2

   $$\theta_2 = \theta_1 - \Phi \ln\frac{r_2}{r_1} \cdot \frac{1}{2\pi\lambda_{tr} l}$$

   $$\theta_2 = 80 - 230 \cdot \ln\frac{20}{13{,}2} \cdot \frac{1}{2\pi \cdot 0{,}22 \cdot 20} = 76.5\,^\circ\text{C}$$

2. Using the formula $\Phi = -l \frac{1}{R_{l,c}} (\theta_3 - \theta_1)$ we can express θ_3

   $$\theta_3 = \theta_1 - \Phi \frac{R_{l,c}}{l}$$

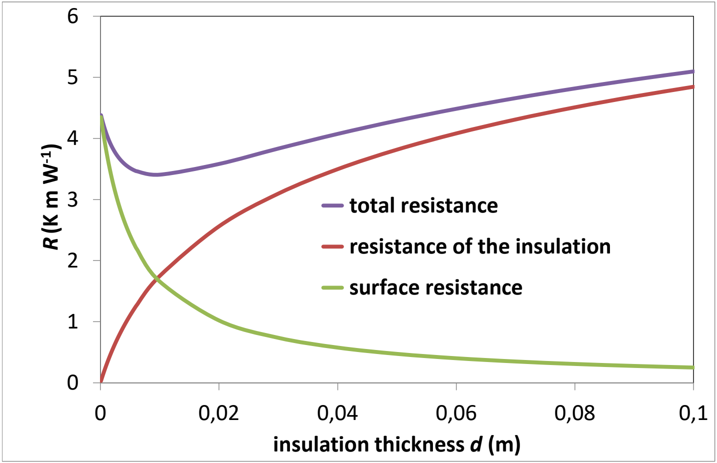
   $$\theta_3 = 80 - 230 \cdot \frac{5{,}17}{20} = 20.5\,^\circ\text{C}$$

   This seems to be a bit inaccurate...

   (hopefully rounding errors 🤔?)

Total Thermal Resistance of the Pipe's Wall

$$R_{\mathrm{l,T}} = R_{\mathrm{l,tr}} + R_{\mathrm{l,iz}} + R_{\mathrm{l,se}} = \frac{\ln \frac{r_x}{r_1}}{2\pi\lambda_{\mathrm{tr}}} + \frac{\ln \frac{r_2}{r_x}}{2\pi\lambda_{\mathrm{iz}}} + \frac{1}{2\pi h_{\mathrm{se}} r_2}$$



Optimal Thickness of Insulation of the Pipe

Home work

Find the thickness of thermal insulation d, at which the overall thermal resistance is minimal:

- inner radius: $r_1 = 1\,\mathrm{mm}$
- external radius (without insulation): $r_x = 2\,\mathrm{mm}$
- thermal conductivity of insulation $\lambda_{\mathrm{iz}} = 0{,}050\,\mathrm{W\,m^{-1}K^{-1}}$
- heat transfer coefficient $h_{\mathrm{se}} = 8\,\mathrm{W\,m^{-2}K^{-1}}$
- calculate with accuracy better than $\pm 0{,}1\,\mathrm{mm}$!

Optimal Thickness of a Steel Pipe of a Radiator

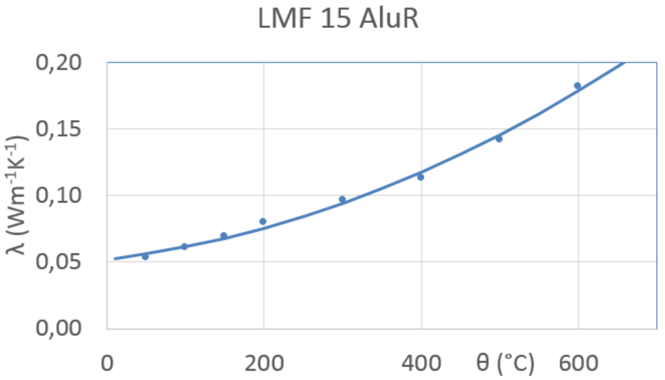
Home work

Find the thickness d of a wall of a steel pipe of a radiator at which the thermal power of the radiator is maximal:

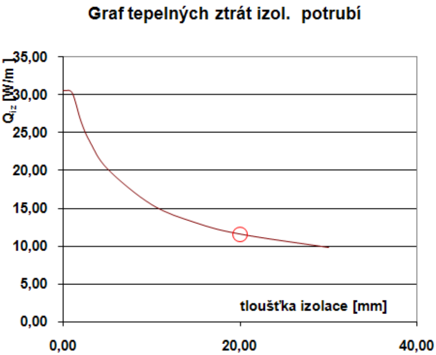
- inner radius: $r_1 = 10\,\mathrm{mm}$
- thermal conductivity of steel $\lambda = 50\ \mathrm{W\,m^{-1}K^{-1}}$
- suppose, that surface heat transfer coefficient is constant: $h_{se} = 8\,\mathrm{W\,m^{-2}K^{-1}}$

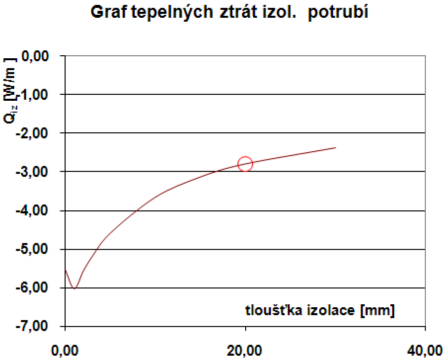
Thermal Conductivity is a Function of Temperature

1. heat transfer coefficient h_{se} depends on temperature!!! (more later)
2. thermal conductivity λ of thermal insulation depends on temperature too (example below is for rock-wool):



Cu 12 mm pipe





hot pipe: 90/15 °C

cold pipe: 5/25 °C

software: URSABIL 2.2 [3]

The Temperature Drop Through a Hot Water Piping

Problem

The hydraulic line is sitting in $\theta_e = 10\,^{\circ}\mathrm{C}$ ambient air. The fluid is flowing through the line defined in problem on slide 38 at fluid flow $\dot{m} = 0.5\,\mathrm{kg\,s^{-1}}$ and the inlet temperature is known to be $\theta_{\mathrm{in}} = 80\,^{\circ}\mathrm{C}$. Find the outlet temperature θ_{out}, length of the line is $l = 20\,\mathrm{m}$.

The Temperature Drop Through a Hot Water Piping

- $\mathrm{d}\theta$ denotes temperature drop of the fluid in length $\mathrm{d}l$
- The amount of heat lost by the cooling fluid per unit of time can be determined using the calorimetric equation $\mathrm{d}\dot{Q} = c\dot{m}\mathrm{d}\theta$
- The same amount of heat passes through the pipe wall in the form of heat loss: $\mathrm{d}\dot{Q} = -\mathrm{d}l\,\frac{\theta-\theta_e}{R_{\mathrm{l,T}}}$, therefore

Energy balance equation

$$c\,\dot{m}\,\mathrm{d}\theta = -\mathrm{d}l\,\frac{\theta-\theta_e}{R_{\mathrm{l,T}}}$$

The Temperature Drop Through a Hot Water Piping

Let's solve the equation $c\,\dot{m}\,\mathrm{d}\theta = -\mathrm{d}l\,\frac{\theta-\theta_\mathrm{e}}{R_{\mathrm{l,T}}}$

Separating variables: $\frac{c\,\dot{m}\,\mathrm{d}\theta}{(\theta-\theta_\mathrm{e})} = -\mathrm{d}l\,\frac{1}{R_{\mathrm{l,T}}}$

Integrating through the piping length: $\int_{\theta_\mathrm{i}}^{\theta} \frac{c\,\dot{m}\,\mathrm{d}\theta}{(\theta-\theta_\mathrm{e})} = -\frac{1}{R_{\mathrm{l,T}}} \int_0^l \mathrm{d}l$

$[\ln(\theta-\theta_\mathrm{e})]_{\theta_\mathrm{i}}^{\theta} = -\frac{1}{c\,\dot{m}\,R_{\mathrm{l,T}}}\,[l]_0^l$

$\ln\frac{(\theta-\theta_\mathrm{e})}{(\theta_\mathrm{i}-\theta_\mathrm{e})} = -\frac{l}{c\,\dot{m}\,R_{\mathrm{l,T}}}$

Temperature drop through the piping

$\theta(l) = \theta_\mathrm{e} + (\theta_\mathrm{i}-\theta_\mathrm{e})\exp\left(-\frac{l}{c\,\dot{m}\,R_{\mathrm{l,T}}}\right)$

at $l = 20\,\mathrm{m}$: $\theta(20) = 79.89\,^\circ\mathrm{C}$

Heat flow in Spherical Symmetry

In case of steady-state:

$\Phi(r) = \text{konst.}$ heat flow through the spherical surface is independent of its radius!

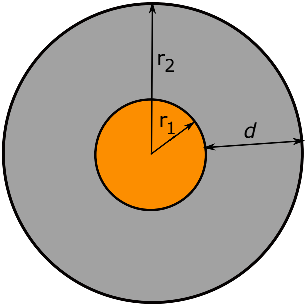
$\Phi(r) = S(r)q(r) = \text{konst.}, S(r) = 4\pi r^2$

$q(r) = -\lambda \frac{\partial \theta}{\partial r}$ (Fourier's law)

so: $\Phi(r) = -4\pi r^2 \lambda \frac{\partial \theta}{\partial r} = \text{konst.}$

Separating the variables we get:

$$-4\pi\lambda \mathbf{d}\theta = \Phi \frac{\mathbf{d}r}{r^2} \tag{5}$$



Heat flow in Spherical Symmetry

integrating the equation 5 we get:

$-4\pi\lambda \int_{\theta_1}^{\theta_2} \mathbf{d}\theta = \Phi \int_{r_1}^{r_2} \frac{\mathbf{d}r}{r^2}$

$-4\pi\lambda(\theta_2 - \theta_1) = -\Phi(\frac{1}{r_2} - \frac{1}{r_1})$

so the thermal flow: $\Phi = 4\pi\lambda \frac{(\theta_2 - \theta_1)}{(\frac{1}{r_2} - \frac{1}{r_1})}$

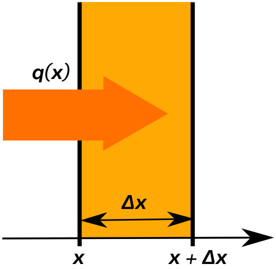
and thermal resistance: $R = \frac{(\frac{1}{r_2} - \frac{1}{r_1})}{4\pi\lambda} \left(\mathrm{K\,W^{-1}}\right)$

Note the thermal resistance units – it is related neither to unit area nor length.

Do you understand why 🤔?

1D Energy Balance

- Heat flows in direction x only
- q is a function of x: $q = q(x)$
  - $q(x)$ is the flux in point x
  - $q(x + \Delta x)$ is the flux in point $x + \Delta x$



Balance of heat in a layer

- Into the layer flows: $\Delta Q = q(x) \cdot \Delta\tau \cdot S$
- Within the same time flows out: $q(x + \Delta x) \cdot \Delta\tau \cdot S$
- The difference remains in the layer:

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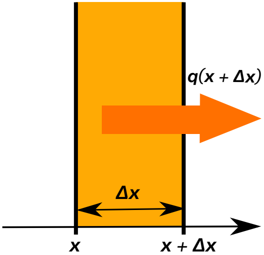

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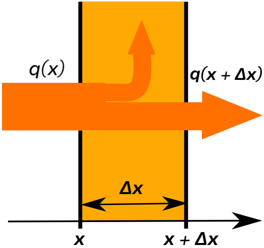

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- Within the same time flows out: $q(x + \Delta x) \cdot \Delta\tau \cdot S$
- The difference remains in the layer:

$$-\Delta q \cdot \Delta\tau \cdot S = -\left(q\left(x + \Delta x\right) - q\left(x\right)\right) \cdot \Delta\tau \cdot S = \Delta Q$$

Balance of Heat in Material Layer

- Amount of heat remaining in the layer per unit of time:

$$\frac{\Delta Q}{\Delta \tau} = -S \cdot \Delta q$$

- 1^{th} law of thermodynamic: $\Delta Q = \Delta U$, therefore

$$\frac{\Delta U}{\Delta \tau} = -S \cdot \Delta q$$

- Internal energy U depends on temperature $\Delta U = m\,c\,\Delta\theta$, therefore

$$\frac{m\,c\,\Delta\theta}{\Delta \tau} = -S \cdot \Delta q$$

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Balance of Heat in Material Layer

- Because $m = \rho \cdot V$ and the volume of the layer can be expressed $V = S \cdot \Delta x$ we get

$$\frac{\rho\, c\, \Delta\theta \cdot S \cdot \Delta x}{\Delta\tau} = -S \cdot \Delta q$$

- and finally

$$\frac{\rho\, c\, \Delta\theta}{\Delta\tau} = -\frac{\Delta q}{\Delta x}$$

- Energy balance in this form can be used directly to solve the transient conduction problem by means of numerical methods such as Finite Difference Method or Finite Volume Method

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$$\frac{\rho\, c\, \Delta\theta}{\Delta\tau} = -\frac{\Delta q}{\Delta x}$$

- Energy balance in this form can be used directly to solve the transient conduction problem by means of numerical methods such as Finite Difference Method or Finite Volume Method

In the limit $\Delta x \to 0$ and $\Delta\tau \to 0$

Heat Equation

$$\lim_{\Delta\tau \to 0} \frac{\rho\, c\, \Delta\theta}{\Delta\tau} = -\lim_{\Delta x \to 0} \frac{\Delta q}{\Delta x}$$
$$\rho c \frac{\partial\theta}{\partial\tau} = -\frac{\partial}{\partial x} q$$
$$\rho c \frac{\partial\theta}{\partial\tau} = -\frac{\partial}{\partial x}\left(-\lambda \frac{\partial\theta}{\partial x}\right) \quad (6)$$
$$\frac{\partial\theta}{\partial\tau} = \frac{\lambda}{\rho c} \frac{\partial^2\theta}{\partial x^2}$$
$$\frac{\partial\theta}{\partial\tau} = a \frac{\partial^2\theta}{\partial x^2}$$

using Fourier's law: $q = -\lambda \frac{\partial\theta}{\partial x}$

Trivial Example

Water in pot on stove

- Pour water of $m = 0{,}7\,\mathrm{kg}$ into the pot,
- temperature of water $\theta_1 = 20\,^\circ\mathrm{C}$.
- Power of heating $P = 3\,\mathrm{kW}$ for $\tau = 5$ minutes
- Heat losses to the surroundings are $Q_z = -600\,\mathrm{kJ}$
- What is the final temperature of the water θ_x?

Water in pot on stove

Energy balance equation

- $\Delta Q = P \cdot \tau + Q_z$
- $\Delta U = (\theta_x - \theta_1)\, mc$
- because $\Delta U = \Delta Q$ so:

$$\theta_x = \frac{P \cdot \tau + Q_z + \theta_1 mc}{mc} = 122\,°\mathrm{C}$$

- Isn't that too much?

Water in pot on stove

Solution

- The final temperature can't be higher then boiling point θ_v, so $\theta_x = \theta_\text{v}$
- Recalculate the internal energy change for $\theta_x = \theta_\text{v}$ and add the change in inner energy caused by the change of state
  $\Delta U = (\theta_\text{v} - \theta_1)\, mc + m_\text{v} l_\text{v}$
- Using equality $\Delta U = \Delta Q$ we can determine the amount of evaporated water m_v:
  $m_\text{v} = \frac{P \cdot \tau + Q_z - (\theta_\text{v} - \theta_1) mc}{l_\text{v}}$
  $m_v = 0{,}029\,\text{kg}$

Solving the Heat Equation $\frac{\partial \theta}{\partial \tau} = \frac{\lambda}{\rho c} \frac{\partial^2 \theta}{\partial x^2}$

It is a partial differential equation, can be solved:

- Analytically – possible only in few simple cases
- Numerically
  - by FEM (Finite Element Method) aka MKP
  - by FDM (Finite Difference Method) aka MKD

Analytical solution in a trivial case

Let's solve the heat equation in a trivial case of heat conduction in a homogeneous wall of thickness d in a steady state, with boundary conditions $\theta(0) = \theta_{si}$, $\theta(d) = \theta_{se}$.

- It is steady state, so $\frac{\partial\theta}{\partial\tau} = 0$ and the equation:

$$
\begin{aligned}
0 &= \frac{\lambda}{\rho c}\frac{\partial^2\theta}{\partial x^2} \\
0 &= \frac{\partial^2\theta}{\partial x^2} \\
c_1 &= \int \frac{\partial^2\theta}{\partial x^2}\mathrm{d}x \\
c_1 x + c_2 &= \int \frac{\partial\theta}{\partial x}\mathrm{d}x
\end{aligned}
$$

$$
\boxed{c_1 x + c_2 = \theta(x)}
$$

Analytical solution in a trivial case

Thus, the temperature in the wall is linear

$$\theta(x) = c_1 x + c_2$$

The integration constants remain to be determined c_1 a c_2 .

We determine them by substituting boundary conditions:

- $x = 0$ so $\theta(0) = c_1 \cdot 0 + c_2 = c_2 = \theta_{si}$
- $x = d$ so $\theta(d) = c_1 \cdot d + c_2 = c_1 d + \theta_{si} = \theta_{se}$
  - so $c_1 = \frac{\theta_{se} - \theta_{si}}{d}$ and finally

$$\theta(x) = \frac{\theta_{se} - \theta_{si}}{d} x + \theta_{si}$$

Boundary Conditions

- Boundary conditions can be
  - constant
  - time dependent (for example periodically)

Dirichlet boundary condition

$\theta(0, \tau) = f_1(\tau)$

$\theta(d, \tau) = f_2(\tau)$

where f is a known function defined on the boundary

Contact of two solid bodies

At the interface:

flow from left = flow to right

$-\lambda_1 \frac{d\theta}{dx}\Big|_{\text{from left}} = -\lambda_2 \frac{d\theta}{dx}\Big|_{\text{from right}}$

Boundary Conditions

- Boundary conditions can be
  - constant
  - time dependent (for example periodically)

Dirichlet boundary condition

$\theta(0, \tau) = f_1(\tau)$

$\theta(d, \tau) = f_2(\tau)$

where f is a known function defined on the boundary

Contact of two solid bodies

At the interface:

flow from left = flow to right

$-\lambda_1 \frac{d\theta}{dx}\Big|_{\text{from left}} = -\lambda_2 \frac{d\theta}{dx}\Big|_{\text{from right}}$

Boundary Conditions

Newton boundary condition

- At the boundary the heat transfer coefficient h_{se} is known and also ambient temperature θ_e
- Flow from left = flow to right:

$$-\lambda \frac{d\theta}{dx}\Big|_{\text{at boundary}} = h_{se}\left(\theta_e - \theta\right)$$

Neumann boundary condition

- At the boundary heat flux is known

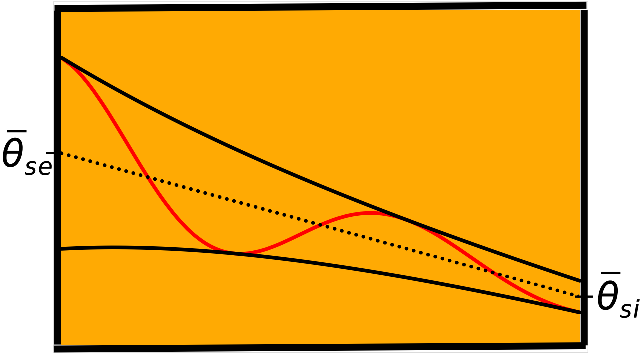
  $q\left(0,\tau\right) = f_1\left(\tau\right)$

  $q\left(d,\tau\right) = f_2\left(\tau\right)$
- Special cases: well insulated body or plane of symmetry:

  $q\left(0,\tau\right) = q\left(d,\tau\right) = 0$ so $\lambda \frac{d\theta}{dx}\Big|_{\text{at boundary}} = 0$

Periodic (sinusoidal) boundary conditions

- To determine the dynamic behavior of the structure in summer
- The amplitude of temperature variation on the external surface is attenuated in the structure
- There is also delay (phase shift) of maxima and minima



Thermal performance of building components according to EN ISO 13786

Periodic boundary conditions are defined as follows

$$\theta_e(\tau) = \bar{\theta}_e + \theta_{e0} \cos\left(\frac{2\pi}{T}\tau + \varphi\right)$$

$$\theta_i(\tau) = \bar{\theta}_i + \theta_{i0} \cos\left(\frac{2\pi}{T}\tau + \varphi + \psi\right)$$

θ_{e0} – **amplitude of external temperature**

$\bar{\theta}_e$ – **average external temperature**

ψ – **phase shift (lag)** of the inner temperature versus external temperature

T – **period** (24 hours; one year...)

Dynamic behavior according to EN ISO 13786

- Any complex number $\hat{a}$ can be written in a trigonometric form as follows:

$$\hat{a} = |\hat{a}| \cdot (\cos\alpha + i\sin\alpha) = |\hat{a}| \cdot e^{i\alpha}$$

- Then the periodic part of the external temperature $\theta_e(\tau) = \bar{\theta}_e + \left|\hat{\theta}_e\right| \cos\left(\frac{2\pi}{T}\tau + \varphi\right)$
- can be written as a real part of a complex number $\hat{\theta}_e = \left|\hat{\theta}_e\right| \cdot e^{\frac{2\pi}{T}\tau} \cdot e^{i\varphi}$
- so $\theta_e(\tau) = \bar{\theta}_e + \mathrm{Re}\left(\left|\hat{\theta}_e\right| \cdot e^{\frac{2\pi}{T}\tau} \cdot e^{i\varphi}\right)$

The periodic parts of the temperatures and fluxes are treated as complex numbers $\hat{\theta}_e$, $\hat{\theta}_i$, $\hat{q}_{si}$, $\hat{q}_{se}$...

Dynamic behavior according to EN ISO 13786

The standard EN ISO 13786 defines various quantities, eg:

Periodic thermal admittance on inner and outer surfaces

Periodic thermal transmittance

Periodic capacity on inner and outer surfaces

Periodic heat flux through the structure in a given direction (usually from the outside to the inside)

Decrement factor ratio of the modulus of the periodic thermal transmittance to the steady-state thermal transmittance U

Periodic thermal admittance

Periodic thermal admittance on inner surface

- Complex quantity defined as the complex amplitude of the heat flux through the inner surface, divided by the complex amplitude of the inner temperature, when the temperature on the other side is held constant
- $\hat{\theta}_\mathrm{i}$ is the complex amplitude of the inner temperature
- $\hat{q}_\mathrm{si}$ the complex amplitude of the heat flux through the inner surface

$$\hat{Y}_\mathrm{ii} = \left.\frac{\hat{q}_\mathrm{si}}{\hat{\theta}_\mathrm{i}}\right|_{\theta_e=\mathrm{konst.}} \left(\mathrm{W\,m^{-2}K^{-1}}\right)$$

Periodic thermal admittance – capacity

$\hat{\kappa}_{\mathrm{si}}$ **heat capacity** amount of heat acumulated into inner surface during one period per square metre and per 1 K temperature diffrence.

$$\hat{\kappa}_{\mathrm{si}} = \frac{\hat{Y}_{\mathrm{ii}} - \hat{Y}_{\mathrm{ie}}}{\omega}$$

Periodic thermal admittance

Periodic thermal admittance on inner surface

structure	modulus ($W\,m^{-2}K^{-1}$)	lag (h)	periodic capacity ($kJ\,m^{-2}K^{-1}$)
bricks 45 cm	4,7	+1,3	66
ditto + ETICS	4,7	+1,3	66
Porotherm 44 cm	3,3	+2,6	46
light structure OSB	1,6	+4,6	19

Heavy structure on the inner side of the perimeter wall provides greater resistance of the interior against overheating caused for example by solar power through windows

Periodic thermal transmittance

Periodic thermal transmittance

- Complex quantity defined as the complex amplitude
  of the heat flux through the inner surface,
  divided by the complex amplitude of the external
  temperature,
  when the temperature on the other side is held constant
- $\hat{\theta}_\mathrm{e}$ is the complex amplitude of the external temperature
- $\hat{q}_\mathrm{si}$ is the complex amplitude of the heat flux through the inner surface

$$\hat{Y}_\mathrm{ie} = \left.\frac{\hat{q}_\mathrm{si}}{\hat{\theta}_\mathrm{e}}\right|_{\theta_i=\mathrm{konst.}}$$

Periodic thermal transmittance

f – decrement factor – ratio of the modulus of the periodic thermal transmittance $\hat{Y}_{ie}$ to the steady-state thermal transmittance U

$$f = \frac{\hat{Y}_{ie}}{U} = \left|\frac{\hat{q}_{si}}{\hat{\theta}_e}\right| \cdot \left|\frac{\overline{\theta}_i - \overline{\theta}_e}{\overline{q}_{si}}\right|$$

δ – periodic penetration depth depth at which the amplitude of the temperature variations are reduced by the factor "e" in a homogeneous material of infinite thickness subjected to sinusoidal temperature variations on its surface

$$\delta = \sqrt{\frac{\lambda T}{\pi \rho c}}$$

Thermal transmittance

Thermal transmittance

structure	static	periodic $\hat{Y}_{ie}$		decrem.	attenuation
	U	modulus	lag	factor f	$\left\lvert\frac{\hat{\theta}_{si}}{\hat{\theta}_{se}}\right\rvert$
	$W\,m^{-2}K^{-1}$	$W\,m^{-2}K^{-1}$	h	-	-
bricks 45 cm	1,34	0,10	-16	0,08	75
ditto + ETICS	0,17	0,003	-18	0,02	2190
Porot. 44 cm	0,32	0,008	-23	0,03	980
light	0,13	0,12	-3	0,92	63

Radiation of solids and liquids

Surface of each body emits energy

- in the form of elektromagnetic radiation
- in a wide range of wavelength λ

Radiation can be

• infrared (IR) • visible • ultraviolet (UV)

Basic physical quantities

- **Radiant power** Φ (W) – radiant energy emitted per unit time.
- **Radiant flux** $H = \frac{d\Phi}{dS}$ (W/m^2) – radiant power per unit area.
- **Spectral Radiant flux** $H_\lambda = \frac{dH}{d\lambda}$ – Radiant flux per unit wavelength.

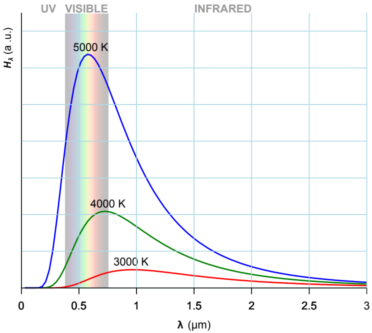
Planck's law for ideal surfaces

Radiation of ideal (aka absolutely black) body is governed by Planck's law

Planck's law

$$H_{\lambda\check{c}} = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)} \quad (\mathrm{W\,m^{-2}\,m^{-1}})$$

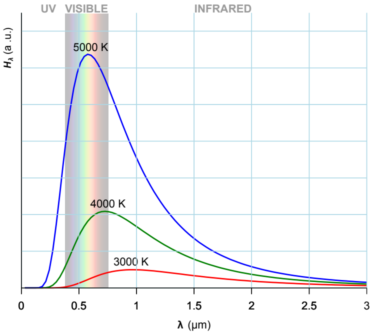
- T– temperature of the body (in Kelvins!!)
- λ – wavelength
- the rest are physical constants



Planck's law for ideal surfaces

Radiation of ideal (aka absolutely black) body is governed by Planck's law

- The warmer the body is, the more it emits at shorter wavelengths - the bluer it is
- Cooler bodies emit more at longer wavelengths - so they are red
- or even radiate only in the invisible infrared region.



Planck's law for real surfaces

- Real surfaces emit radiation worse than ideal surfaces
- The ability to emit is determined by a property called emissivity e
- Spectral Radiant flux of a real surface can be expressed as

$$H_\lambda = e(\lambda) H_{\lambda\check{c}}$$

- $H_{\lambda\check{c}}$ is spectral Radiant flux of a black body

Emissivity $e\,(\lambda)$

- emissivity is lower then one and greater then zero: $0 \leq e\,(\lambda) \leq 1$
- it depends on the wavelength of the emitted radiation
- the surface at some wavelengths may radiate better than at others

Emissivity – examples

The ideal emitter (absolutely black body)

- the ideal emitter has an emissivity equal to one for all wavelengths
- it is called an "**absolutely black body**"
- Why?
  - the ideal emitter is also the ideal absorber
  - it is **absolutely black** in the incident light!

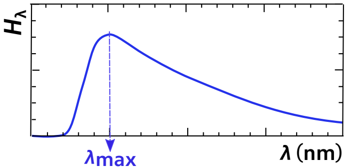
Examples of real surfaces

- the emissivity of the polished metals in the infrared range is very low
  - this can be used to reduce heat loss by radiation
  - aluminum under-roof foils, etc.

Wien's displacement law

- The spectral radiant flux of black-body radiation peaks at the wavelength $\lambda_{\max}$:

$$\lambda_{\max} = \frac{b}{T} \quad (\mathrm{m})$$



- where b = 2,8978 10^{-3}(m K) is Wien's constant

Surface of	The Sun	A Human Body
Temperature	5780 K	310 K
$\lambda_{\max}$/nm	500 (blue-green light)	9300 (IR)

Radiant flux of a Surface over Full Wavelength Range

- Integrating Planck's law over the full wavelength range (from zero to infinity)
- The result is the Stefan-Boltzmann law.

Stefan-Boltzmann law

$$H = \int_0^\infty H_\lambda \mathrm{d}\lambda = \int_0^\infty e(\lambda) \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1 \right)} \mathrm{d}\lambda = e\left(T\right) \sigma T^4 \quad \left(\mathrm{W\,m^{-2}}\right)$$

Radiant flux of a Surface

Stefan-Boltzmann law

$$H = e(T) \cdot \sigma T^4 \quad (\mathrm{W\,m^{-2}})$$

- $\sigma = 5,67\,10^{-8}\,(\mathrm{W\,m^{-2}\,K^{-4}})$ is Boltzmann constant.

Emissivity $e(T)$

- This time as a function of surface temperature, not a function of wavelength!
- Examples:
  - absolutely black body:
    $e(\lambda) = 1$ for all wavelengths, therefore $e(T) = 1$
  - „gray“ body:
    $e(\lambda) = \text{const.} < 1$ for all wavelengths, therefore $e(T) = \text{const.}$

Radiant flux of a Surface and IR cameras

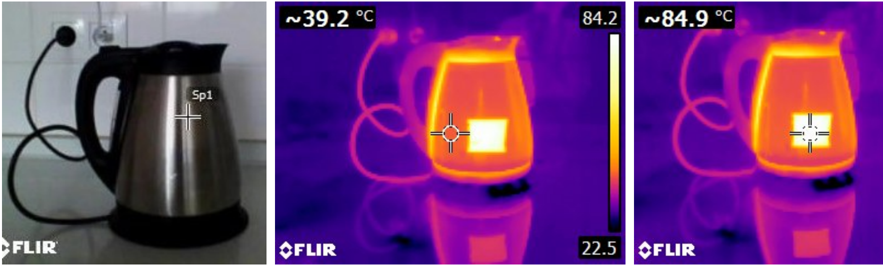
- Thermal cameras determine surface temperature by measuring radiant flux H of the surface as

$$T = \sqrt[4]{\frac{H}{e \cdot \sigma}}$$

- The determination of the temperature is therefore strongly influenced by emissivity of the surface
- We have to know the emissivity of the surface and set the camera correctly!

Surface temperature determined by IR camera

- Example - a kettle with hot water
  - a piece of adhesive tape is stuck on the kettle



  - Low emissivity steel radiates much less compared to the tape
  - The thermal camera shows much lower apparent temperature!
  - Despite the fact, that the temperature of metal is higher (it has lower heat loss to the surroundings)!
  - Q: Why is the cable red?

Radiation absorbed by the body surface

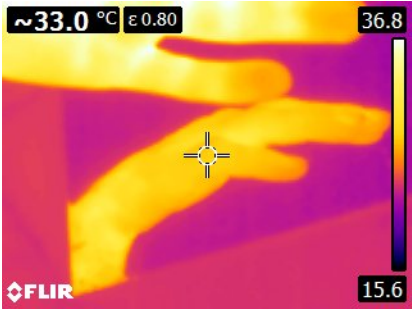
- We can already calculate radiant power
  of a surface at a certain temperature.
- But what is going on with the emitted radiation?
  - irradiates other surfaces
  - where it is either reflected or absorbed

- Incident radiation (irradiation) is marked as: $E \ \left(\mathrm{W\,m^{-2}}\right)$.

- The ability of a surface to absorb incident radiation is called absorptivity (absorption factor) a.
  - its value lies between zero and one $a \in \langle 0, 1 \rangle$
  - for most materials $a\left(\lambda\right) = e\left(\lambda\right)$
  - absorptivity of an "absolutely black body" $a = 1$.

Reflected radiation

- Incident radiation E is partially absorbed ($E_p = a \cdot E$)
- the rest $E_r = E - E_p$, is reflected
- Compare the hand reflection by means of thermal camera
  1. in a mirror (i.e. on a glass with high absorption)
  2. on aluminum foil with low absorption



- aluminum foil reflects in the IR region markedly better!

Kirchhoff's law

Because $a(\lambda) = e(\lambda)$ also $a(T_1) = e(T_2)$ for $|T_1 - T_2| < 1000\,\mathrm{K}$ (so called Kirchhoff's law)

Table: Emisivitty and absorptivity at different temperatures

Temperature	$\approx$ **300 K**	$\approx$ **6000 K**
radiation	IR	UV + visible + IR
material	$a(T) = e(T)$	
white paint	$\approx 0{,}8$	$< 0{,}1$
clean metal	$< 0{,}1$	$\approx 0{,}1$
glass	0,837	transparent
black paint	$\approx 0{,}8$	$> 0{,}9$
selektive absorber	$\approx 0{,}05$	$\approx 0{,}95$

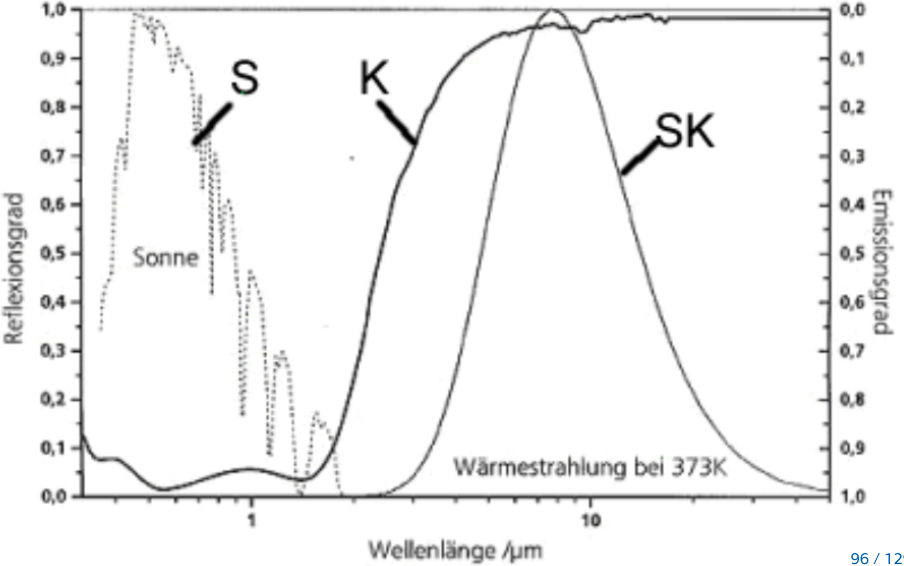
Selective absorbers for solar collectors

- Materials that have high absorptivity for solar iradiation
  - high use of sunlight
- low absorptivity (and emissivity!!!) for infrared radiation
  - low heat loss

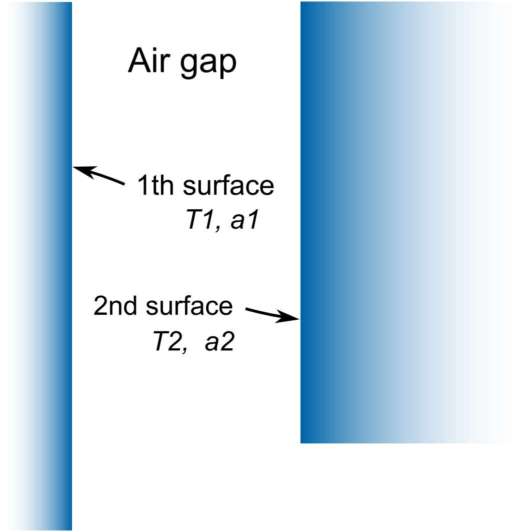
Table: Emisivity and absorptivity of selective absorbers

Temperature	$\approx$ **350 K**	$\approx$ **6000 K**
radiation	IR	UV + visible + IR
material	$a\,(T) = e\,(T)$	
$Ni_xAl_yO_z$	$\approx 0{,}1$	0,92 - 0,97
Crystal Clear™	0,04 - 0,09	0,94 – 0,96
TiNOX	$\approx 0{,}05$	$\approx 0{,}91$

TiNOX – Selective absorber for solar collectors



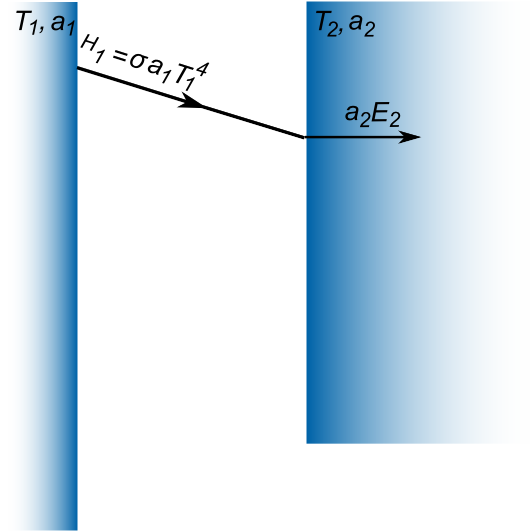
Two opposing planar surfaces



geometrically simplest case:

- Infinitely large planar parallel surfaces
- The first surface with temperature T_1 , emissivity and absorptivity a_1
- The second surface with temperature T_2 , emissivity and absorptivity a_2
- According to Kirchhoff's law: $a_1 = \epsilon_1$, $a_2 = \epsilon_2$

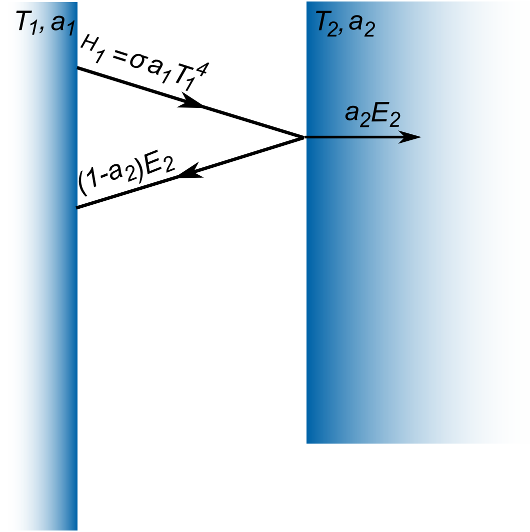
Two opposing planar surfaces



All radiation emitted by one surface falls on the opposite surface, so

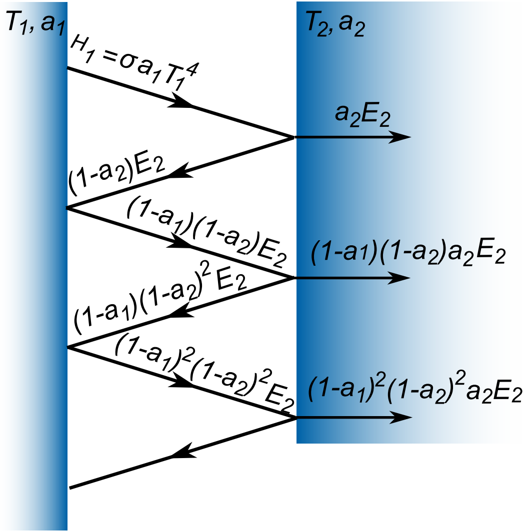
$H_1 = E_2$

Two opposing planar surfaces



Only part of the incident radiation E_2 is absorbed ($E_{p2} = a_2 E_2$), the rest is reflected

Two opposing planar surfaces

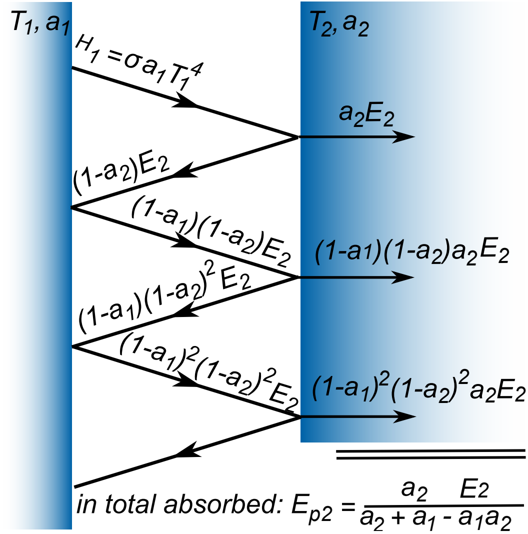


The radiation reflected from the right surface is partially reflected from the left and returned...

And again...

And again...

Two opposing planar surfaces



Total radiation absorbed by the right surface (sum of infinite geometric series):

$$E_{p2} = a_2 E_2 \left(1 + q + q^2 + \ldots\right)$$

$$q = (1 - a_1)(1 - a_2)$$

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

$$E_{p2} = \frac{a_2}{a_1 + a_2 - a_1 a_2} E_2$$

Two opposing planar surfaces

- we have finally radiation absorbed by the right surface:

$$E_{\mathrm{p}2} = \frac{a_2}{a_1 + a_2 - a_1 a_2} E_2 = \frac{a_2}{a_1 + a_2 - a_1 a_2} H_1 = \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} \sigma T_1^4$$

- analogously the left surface absorbs radiation emitted by the right surface:

$$E_{p1} = \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} \sigma T_2^4$$

- total radiation heat flow (from left to right):

$$\boxed{q_r = E_{p2} - E_{p1} = -\frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} \sigma \left(T_2^4 - T_1^4\right)}$$

Linearization

- The difference of the fourth powers of temperatures is not practical, so we express it linearly by means of Taylor's series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots$$

- In our case $f(x) = T^4$
- Using only the first linear term, we have

$$T_2^4 - T_1^4 \doteq 4T_1^3\left(T_2 - T_1\right) \doteq 4\overline{T}^3\left(T_2 - T_1\right) \doteq 4\overline{T}^3\left(\theta_2 - \theta_1\right)$$

$$\text{where } \overline{T} = \frac{T_2 + T_1}{2}$$

- The radiation heat flux can finally be approximately expressed as

$$q_r = -\frac{a_1 a_2}{a_1 + a_2 - a_1 a_2}\sigma 4\overline{T}^3\left(\theta_2 - \theta_1\right)$$

Linearization - heat transfer coefficient

- Heat flux transmitted by radiation q_r we expressed as

$$q_r = -\frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} \sigma 4 \overline{T}^3 \left(\theta_2 - \theta_1\right)$$

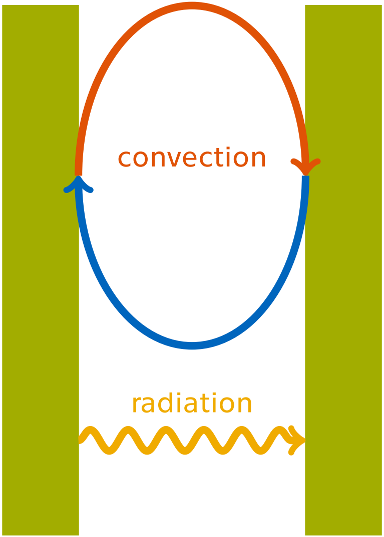
- It can also be written:

$$q_r = -h_\mathrm{r} \left(\theta_2 - \theta_1\right)$$

- Comparing both expressions we get h_r (radiation part of the heat transfer coefficient):

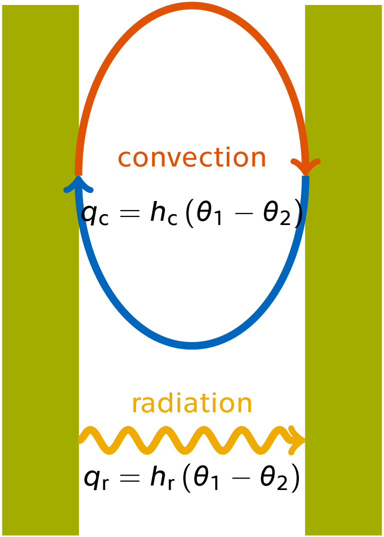
$$\boxed{h_\mathrm{r} = 4 \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} \sigma \overline{T}^3}$$

Heat transfer in the air gap



- The heat is transferred in the gap by
  - radiation
    and at the same time
  - convection (of air)
- q_c is the convective part of the flux
- q_r is the radiant part of the flux
- Total heat flux:
  $q_{\mathrm{T}} = q_{\mathrm{r}} + q_{\mathrm{c}} = (h_{\mathrm{r}} + h_{\mathrm{c}})(\theta_1 - \theta_2)$

Heat transfer in the air gap



- The heat is transferred in the gap by
  - radiation

    and at the same time
  - convection (of air)
- q_c is the convective part of the flux
- q_r is the radiant part of the flux
- Total heat flux:

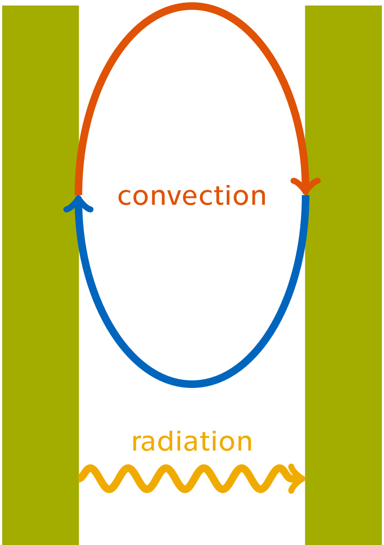
  $q_T = q_r + q_c = (h_r + h_c)(\theta_1 - \theta_2)$

Thermal Resistance R_g to the Heat Transfer in the Air Gap

- Total heat flux in the air gap
  is the sum of radiation and convection
- $q_T = q_r + q_c = (h_r + h_c)(\theta_1 - \theta_2) = h_T(\theta_1 - \theta_2) = \frac{(\theta_1 - \theta_2)}{R_m}$
- R_g is the thermal resistance of the air gap, obviously:

$$\boxed{R_g = \frac{1}{h_r + h_c}}$$

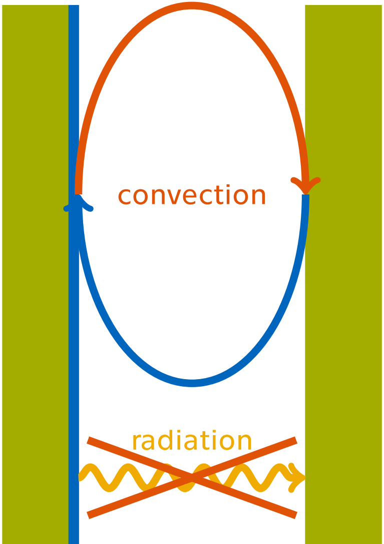
Reduction of the heat transfer in the gap



- Heat transfer occurs by
  - radiation
  - convection (including conduction)

- Radiation can be reduced
  but
- the convection remains unchanged...

Reduction of the heat transfer in the gap



- Heat transfer occurs by
  - radiation
  - convection (including conduction)

- Radiation can be reduced but
- the convection remains unchanged...

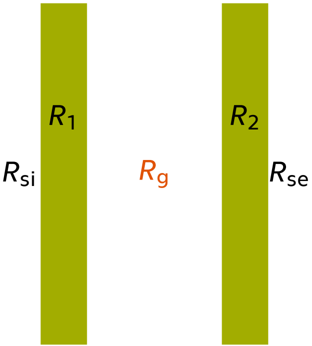
Reduction of the radiant heat transfer

How can we limit the heat transfer by radiation?

1. By reducing the emissivity of surfaces in the IR region
   - clean metal surfaces
   - metal foils (under-roof foils)
   - metallized surfaces (double glazing)

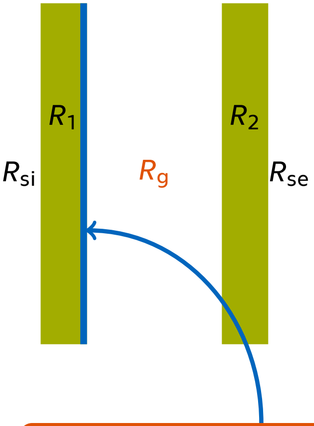
2. Inserting a screen into the gap

Double glazing



- Total thermal resistance of the double glazing is the sum of all resistances in series
- $R_T = R_{si} + R_1 + R_g + R_2 + R_{si}$
- emissivity of glass $e = 0,837$
- heat resistance of the gap R_g is enlarged by metal plating!
- emissivity of plating $e \simeq 0,05$

Double glazing



- Total thermal resistance of the double glazing is the sum of all resistances in series
- $R_T = R_{si} + R_1 + R_g + R_2 + R_{si}$
- emissivity of glass $e = 0,837$
- heat resistance of the gap R_g is enlarged by metal plating!
- emissivity of plating $e \simeq 0,05$

The inner surface of the glass is metallized on the warm side.

Double glazing - plating effect

- Resistances that do not change by plating:
  - R_{se}, R_{si}, $2\times$ glass resistance: $R_1 + R_2 = \frac{0{,}004}{1} + \frac{0{,}004}{1}$
  - also convective part of the heat transfer coeff. h_c remains nearly constant
  - there is no flow at the gap width $d = 12\,\text{mm}$, so $h_c = \frac{\lambda}{d} = \frac{0{,}025}{0{,}012} = 2.1\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$

- the heat transfer coefficient is strongly influenced by the surface plating

$$h_r = 4\frac{a_1 a_2}{a_1 + a_2 - a_1 a_2}\sigma \overline{T}^3$$

Double glazing - plating effect

No plating

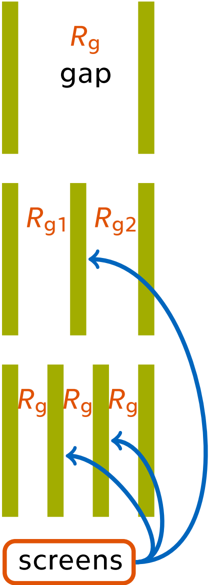
$$h_r = 4\frac{0{,}837 \cdot 0{,}837}{0{,}837+0{,}837-0{,}837 \cdot 0{,}837} \cdot 5,67 \cdot 10^{-8} \cdot 283^3$$
$$h_r = 3.7\,\mathrm{W\,K^{-1}\,m^{-2}}$$
$$R_g = \frac{1}{2{,}1+3{,}7} = 0.17\,\mathrm{K\,m^2\,W^{-1}}$$
$$R_{gT} = 0,13 + 0,004 + 0,17 + 0,004 + 0,04 = 0.35\,\mathrm{K\,m^2\,W^{-1}}$$

One glass plated

$$h_r = 4\frac{0{,}837 \cdot 0{,}05}{0{,}837+0{,}05-0{,}837 \cdot 0{,}05} \cdot 5,67 \cdot 10^{-8} \cdot 283^3$$
$$h_r = 0.25\,\mathrm{W\,K^{-1}\,m^{-2}}$$
$$R_g = \frac{1}{2{,}1+0{,}25} = 0.42\,\mathrm{K\,m^2\,W^{-1}}$$
$$R_{gT} = 0,13 + 0,004 + 0,42 + 0,004 + 0,04 = 0.60\,\mathrm{K\,m^2\,W^{-1}}$$

$$h_r = 4\frac{a_1 a_2}{a_1+a_2-a_1 a_2}\sigma \overline{T}^3, \quad R_g = \frac{1}{h_c+h_r}, \quad R_{gT} = R_{si} + R_1 + R_g + R_1 + R_{se}$$

Shielding radiation in the gap by screens



- Consider vacuum ⇒ heat transfer by radiation only!
- The same emissivity of all surfaces.
- Then the thermal resistance of the gap: $R_g = \frac{1}{h_r} = \left(4\frac{e}{2-e}\sigma\bar{T}^3\right)^{-1}$
- Inserting screens, the thermal resistance of each gap is approximately also R_g.
- The total resistance is therefore approx. $R_{gT} \doteq (n+1)\cdot R_g$, n is number of screens.
- In a vacuum, radiation shielding is effective!

Shielding in a vacuum gap (between two aluminum foils)

Numerically for $\theta_1 = 2.5\,°\mathrm{C}$, $\theta_2 = 17.5\,°\mathrm{C}$, $e = 0,05$

No screen: $\overline{\theta} = 10\,°\mathrm{C}$

$$R_g = \left(4\frac{0,05}{2-0,05}\cdot 5,67\cdot 10^{-8}\cdot 283^3\right)^{-1} = \frac{1}{0,131} = 7.6\,\mathrm{K\,m^2\,W^{-1}}$$

One screen: $\overline{\theta}_1 = 6.25\,°\mathrm{C}$, $\overline{\theta}_2 = 13.75\,°\mathrm{C}$

$$R_{g1} = \left(4\tfrac{0,05}{2-0,05}\cdot 5,67\cdot 10^{-8}\cdot 279^3\right)^{-1} = 7.92\,\mathrm{K\,m^2\,W^{-1}}$$

$$R_{g2} = \left(4\tfrac{0,05}{2-0,05}\cdot 5,67\cdot 10^{-8}\cdot 287^3\right)^{-1} = 7.27\,\mathrm{K\,m^2\,W^{-1}}$$

$$R_{gT} = R_{g1} + R_{g2} = 7,92 + 7,27 = 15.19\,\mathrm{K\,m^2\,W^{-1}} = 2R_g$$

Shielding in the air gap (between two aluminium foils)

- In the air gap, the convection or conduction is also involved
- Suppose a narrow gap – there is no convection, just conduction
- Then the thermal resistance of the gap $R_{\mathrm{g}} = \frac{1}{h_r + h_c} = \left(4\frac{e}{2-e}\sigma T^3 + \frac{\lambda}{d}\right)^{-1}$
- After inserting the screen, the thermal resistance of each gap is $R'_{\mathrm{g}} = \left(4\frac{e}{2-e}\sigma \overline{T}^3 + \frac{\lambda}{d/2}\right)^{-1}$ and $R_{\mathrm{gT}} = R'_{\mathrm{g1}} + R'_{\mathrm{g2}}$
- For temperatures from the previous example and $\lambda = 0.025\,\mathrm{W\,m^{-1}\,K^{-1}}$, $d = 12\,\mathrm{mm}$

Shielding in the air gap (between two aluminium foils)

No screen

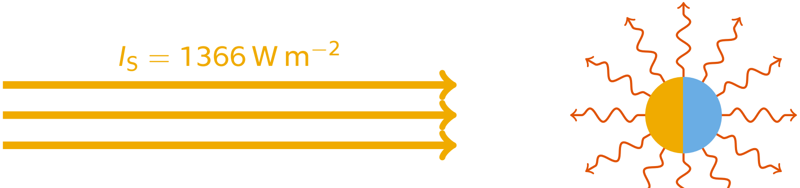
- $R_g = \left(4\frac{0{,}05}{2-0{,}05} \cdot 5,67 \cdot 10^{-8} \cdot 283^3 + \frac{0{,}025}{0{,}012}\right)^{-1} = 0.451\,\mathrm{K\,m^2\,W^{-1}}$

One screen (another aluminium foil)

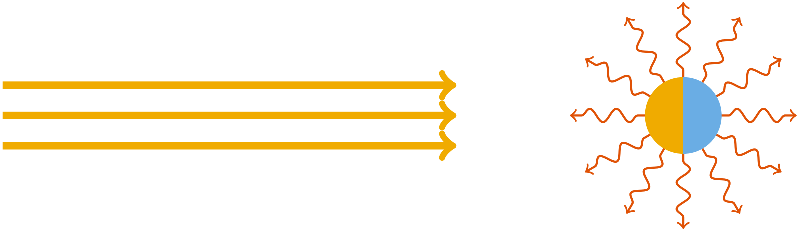
- $R'_{g1} = \left(4\frac{0{,}05}{2-0{,}05} \cdot 5,67 \cdot 10^{-8} \cdot 279^3 + \frac{0{,}025}{0{,}006}\right)^{-1} = 0.233\,\mathrm{K\,m^2\,W^{-1}}$
- $R'_{g2} = \left(4\frac{0{,}05}{2-0{,}05} \cdot 5,67 \cdot 10^{-8} \cdot 287^3 + \frac{0{,}025}{0{,}006}\right)^{-1} = 0.232\,\mathrm{K\,m^2\,W^{-1}}$
- $R_{gT} = 0,233 + 0,232 = 0.465\,\mathrm{K\,m^2\,W^{-1}} = 1,03R_g$

Earth's equilibrium temperature (without atmosphere)

- Calculate the Earth's equilibrium temperature T_Z under these assumptions
  - it has no atmosphere
  - it has no internal heat sources
  - the intensity of the Sun's radiation is $I_S = 1366\,\mathrm{W\,m^{-2}}$
  - the absorptivity of the earth's surface for solar radiation is $a = 0.7$
  - the surface emissivity in the IR region is $e = 0.97$



Earth's equilibrium temperature (without atmosphere)



- Solution
- In equilibrium heat input = heat output
  - $P_{\text{in}} = P_{\text{out}}$
  - the heat input of the Earth is $P_{\text{in}} = I_{\text{S}} \cdot a \cdot \pi R_Z^2$
  - the heat output of the Earth's surface is $P_{\text{out}} = 4\pi R_Z^2 \cdot e \cdot \sigma T_Z^4$
  - which means
    $T_Z = \sqrt[4]{\frac{I_S a}{4e\sigma}} = \sqrt[4]{\frac{1366 \cdot 0{,}7}{4 \cdot 0{,}97 \cdot 5{,}67 \cdot 10^{-8}}} = 256{,}8\,\text{K} = -16{,}3\,°\text{C}$

Heat transfer by convection

- Moving mass is used to transport heat

- Conductive heat transport is also an integral part
  - from surface to liquid
  - from one layer of fluid to another

- The convection is
  - forced (fan, wind)
  - natural, gravity (caused by temperature difference)

Forced Convection

EN ISO 6946

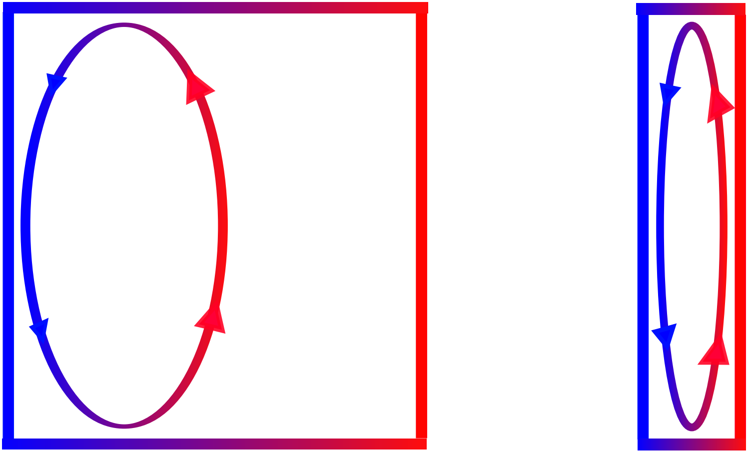
$h_{ce} = 4 + 4v$,

where v is wind velocity

Not subject of this lectures.

Heat transfer coefficient on the external side of the structure.

Natural gravity convection



Vertical wall-to-interior
(internal surface of a building envelope)

wall-to-wall transition
(air gap)

Natural gravity convection

Heat transfer by natural convection is a complex process whose modeling is based on several approaches.

Numerical solution

Numerical solution of differential equations describing convection (using specialized software).

Similarity theory

Heat transfer is determined experimentally only in certain cases and converted into other geometrically and physically similar cases using similarity theory. Whether situations are similar can be determined using similarity numbers (criteria).

Natural convection – numerical solution

- Balance equations need to be constructed to solve the problem
  - energy balance
  - mass balance
  - momentum balance
  - angular momentum balance
- You need to know and write
  - equation of state of flowing medium
  - dependence of material properties of media on state parameters
- So we have a set of many equations that need to be solved simultaneously
  this is not a simple task, but software that can do it exists ...

Nusselt Number

The situations can be considered geometrically and physically similar if their Nusselt numbers are equal

Nusselt Number

$$Nu = \frac{h_c l}{\lambda}$$

- λ – thermal conductivity of the liquid
- l – „characteristic size" of the body

If we know the Nusselt number for a given situation, then the heat transfer coefficient by convection (h_c) is determined using:

Determination of h_c from a known Nusselt number

$$h_c = \frac{Nu\,\lambda}{l} \quad (\mathrm{W\,m^{-2}\,K^{-1}}) \tag{7}$$

Rayleigh number

Empirical relations for Nusselt number (8, 9) contain the number Ra

$$Ra = \frac{g\Delta T l^3}{\overline{T} v a},$$

where g is the gravitational acceleration, ΔT is the temperature difference between the surface and the fluid, $\overline{T}$ s the mean temperature determined from surface and fluid temperature, l is the wall height , v the kinematic viscosity a is the coefficient of thermal conductivity of the flowing fluid, ie air.

The number expresses the proportion of heat transfer by convection and conduction.

- If small - the liquid does not flow.
- Larger - fluid flows laminarly.
- Even bigger - flows turbulently.

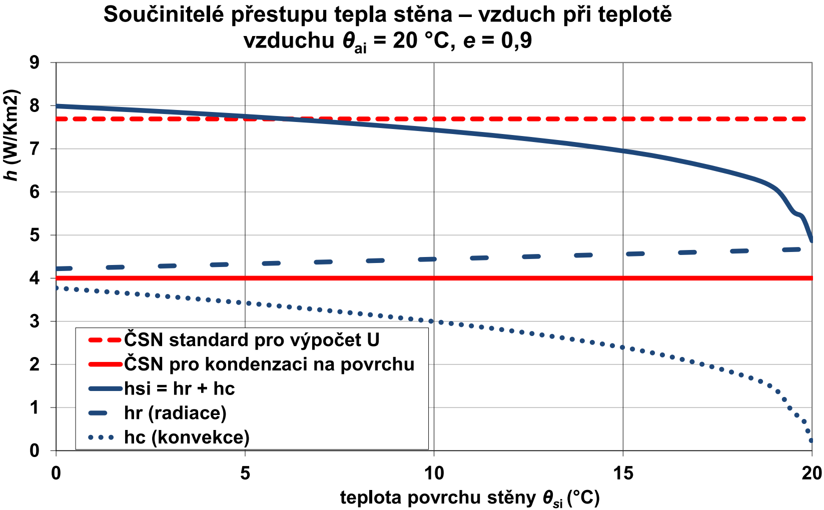
Heat transfer vertical wall – air

The Nusselt number was determined experimentally by many authors. Experimental values can be interleaved (fit) by some suitable function. Various forms of these functions can be found in literature, here is one of the simplest shapes for heat transfer from vertical wall to fluid:

Empirical relations for Nusselt number

$$\begin{aligned} Nu &= 1,18 \cdot Ra^{\frac{1}{8}} \qquad \left(\text{pro}\, 10^{-3} \leq Ra < 500\right) \\ Nu &= 0,54 \cdot Ra^{\frac{1}{4}} \qquad \left(\text{pro}\, 500 \leq Ra < 2 \cdot 10^{7}\right) \\ Nu &= 0,135 \cdot Ra^{\frac{1}{3}} \qquad \left(\text{pro}\, 2 \cdot 10^{7} \leq Ra < 10^{13}\right) \end{aligned} \tag{8}$$

Heat transfer vertical wall – air (h_{si})



software: koeficientPrestupuNaSteneaVeVzduchoveMezere.xlsx [3]

Wall to wall heat transfer (air gap)

Other empirical relationships apply to the heat transfer between two vertical walls separated by an air gap, e.g.

Empirical relations for Nusselt number

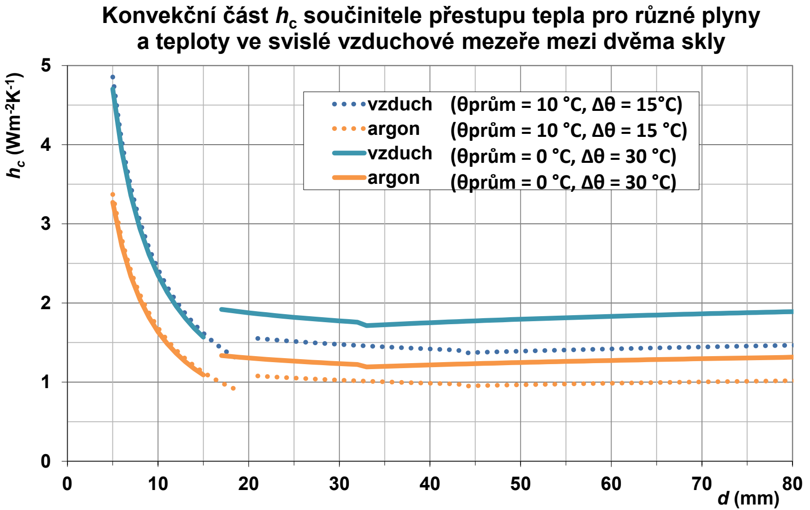
$$Nu = 1 \qquad \left(\text{for } Ra < 124\frac{a}{v}\left(0,952 + \frac{a}{v}\right)\frac{h}{l}\right)$$

$$Nu = 0,19 \cdot Ra^{\frac{1}{4}} \cdot \left(\frac{l}{h}\right)^{\frac{1}{9}} \qquad (\text{pro } 15000 \leq Ra < 150000) \qquad (9)$$

$$Nu = 0,071 \cdot Ra^{\frac{1}{3}} \cdot \left(\frac{l}{h}\right)^{\frac{1}{9}} \qquad (\text{pro } 150000 \leq Ra < 7200000)$$

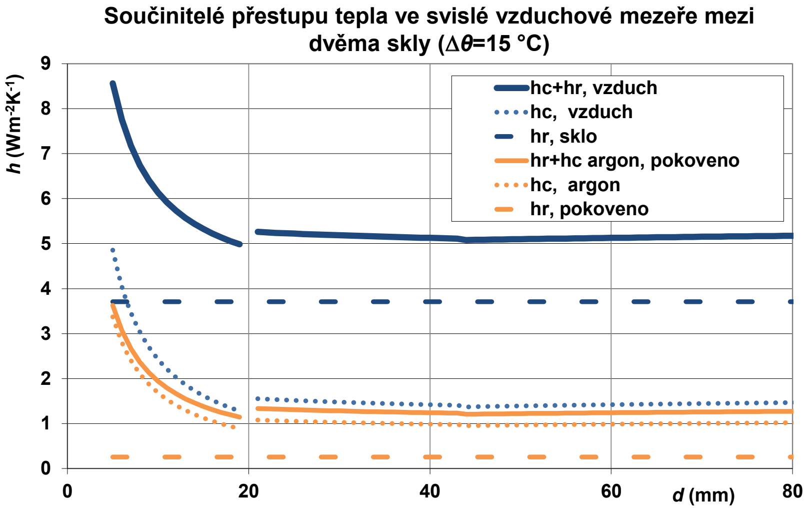
In equations, l is the width, h the height of the air gap.

Examples of calculated coefficients $h_c = Nu \cdot \frac{\lambda}{l}$



software: `koeficientPrestupuNaSteneaVeVzduchoveMezere.xlsx` [3]

Heat transfer in the gap between two glasses



Abbreviations I

Abbreviation	Quantity (czech/english)	Definition	Units
Q	teplo (heat)		J
Φ	celkový tok teplpřes plochu S (heat flow rate)	$\Phi = \frac{dQ}{d\tau}$	W
$\vec{q}$	hustota tepelného toku (heat flux)	$q = \frac{d\Phi}{dS}$	$\frac{W}{m^2}$
θ	teplota (temperature)		°C
T	absolutní teplota (absolute temperature)	$T = \theta + 273,15$	K
τ	čas (time)		s

Abbreviations II

φ	množství materiálu v objemu material in volume		$\frac{\text{kg}}{\text{m}^3}, \frac{\text{mol}}{\text{m}^3}$
m_s	hmotnost suchého materiál mass of dry material		kg
m_w	hmotnost kapalné vody mass of liquid water		kg
M_c	rychlost kondenzace condensation rate		$\frac{\text{kg}}{\text{m}^2\,\text{s}}$
μ	faktor difúzního odporu diffusion resistance factor		–

Abbreviations III

μ_{w}	molární hmotnost vody molar mass of water	$0{,}018\ \frac{\mathrm{kg}}{\mathrm{mol}}$	$\frac{\mathrm{kg}}{\mathrm{mol}}$
φ	relativní vlhkost vzduchu relative air humidity		– nebo %
u	vlhkost materiálu material moisture	$u = \frac{m_w}{m_s}$	– nebo %
v (index)	vodní pára (water vapour)		–
w (index)	kapalná voda (liquid water)		–

References I

[1] Prof. J.Biddle's lecture series on CPP (youtube)

[2] Prof. Sukhatme: Lecture Series on Heat and Mass Transfer

[3] Demonstrační software pro výuku

[4] John H. Lienhard: A Heat Transfer Textbook