Thermomechanics
Tasks
(in progress)

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Chapter 1

Heat transport

1.1 Conduction

1. A wall consists of two layers:

   (a) bricks on inner side with thermal conductivity $\lambda_1 = 0{,}8\,\mathrm{W\,K^{-1}\,m^{-1}}$ and thickness d_1 = 0,45 m

   (b) styropore insulation on outer side with thermal conductivity $\lambda_2 = 0{,}04\,\mathrm{W\,K^{-1}\,m^{-1}}$ and thickness d_2 = 0,1 m

   Area of the wall is S = 80 m^2.
   Calculate thermal resistances of each layer (i.e. R_1 and R_2) and total thermal resistance of the wall R_T (excluding surface thermal resistances).

2. In the task 1 calculate total thermal resistance of the wall R_T *including* surface thermal resistances. Surface thermal resistances are as follows: internal $R_{si} = 0{,}13$, external $R_{se} = 0{,}04$. Calculate also amount of heat which penetrates during one hour through the wall. Temperature of external air is θ_{ex}= -15 °C, temperature of internal air is $\theta_{ia} = 20\,°\mathrm{C}$.

3. In the task 2 calculate temperature θ_1 on the interface of the bricks and the insulation.

4. Ceramic furnace is insulated by rock mineral wool hight temperature board (Knauf Insulation power-tek bd 700).
   Inner surface temperature of the board $\theta_{\mathrm{si}} = 700\,°\mathrm{C}$.
   External surface temperature of the board $\theta_{\mathrm{se}} = 50\,°\mathrm{C}$.
   Thickness of the board $d = 5\,\mathrm{cm}$, area of the board $A = 2\,\mathrm{m}^2$.
   Thermal conductivity of the wool in relation to temperature $\lambda\left(\theta\right)$ can be found in the table:

$\theta/°\mathrm{C}$	50	100	200	300	400	500	600	700
$\lambda/\left(\mathrm{W\,K^{-1}\,m^{-1}}\right)$	0,041	0,045	0,059	0,075	0,095	0,119	0,147	0,178

   Calculate heat losses of the furnace as a heat flow passing through the board Φ in watts.
   Omit corner effects.

   Hints:

   (a) express $\lambda\left(\theta\right)$ in a form of second order polynomial i.e. $\lambda\left(\theta\right) = \lambda_0 + \lambda_1\theta + \lambda_2\theta^2$ as a *best fit* of the tabular data given in the data sheet. The best fit can be made by the least square method using a calculator or a spreadsheet (Excel).

   (b) using *separation of variables* solve differential equation (Fourier's law) $q = \lambda\left(\theta\right)\frac{\mathrm{d}\theta}{\mathrm{d}x}$ at steady state conditions ($q = \mathrm{const.}$).

5. Calculate and (using a spreadsheet) draw a graph $\theta\left(x\right)$, i.e. temperature versus thickness of the wall for:

   (a) ordinary planar wall

   (b) pipe (cylindrical) wall (inner diameter $r_{\mathrm{i}} = 0.05\,\mathrm{m}$, outer diameter $r_{\mathrm{e}} = r_{\mathrm{i}} + d$)

   (c) spherical wall (wall of spherical vessel; inner diameter $r_{\mathrm{i}} = 0.05\,\mathrm{m}$, outer diameter $r_{\mathrm{e}} = r_{\mathrm{i}} + d$)

   In all three cases: $\lambda = 0.04\,\mathrm{W\,m^{-1}K^{-1}}$; surface temperatures: $\theta_{\mathrm{si}} = 25\,°\mathrm{C}$, $\theta_{\mathrm{se}} = 0\,°\mathrm{C}$; steady state; thickness of the wall $d = 0.1\,\mathrm{m}$.

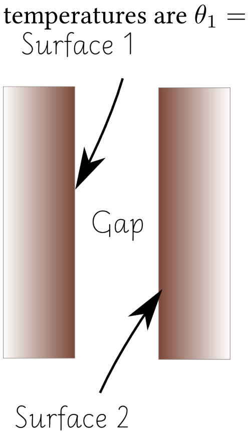
6. Find the thickness d of an insulation of a steel pipe at which total thermal resistance $R_{\mathrm{l,T}}$ is *minimal*:
   Inner radius of a steel pipe: $r_1 = 1$ mm.
   Outer radius (without insulation): $r_2 = 2$ mm.
   Thermal conductivity of steel $\lambda_{\mathrm{st}} = 50 \ \mathrm{W\,m^{-1}K^{-1}}$.
   Thermal conductivity of insulation $\lambda_{\mathrm{iz}} = 0,050 \ \mathrm{W\,m^{-1}K^{-1}}$.
   Heat transfer coefficient at the outer surface $h_{\mathrm{se}} = 8\,\mathrm{W\,m^{-2}K^{-1}}$.
   The result is required with an accuracy of ±0,1 mm!
   Hint: $R_{\mathrm{l,T}} = R_{\mathrm{l,tr}} + R_{\mathrm{l,iz}} + R_{\mathrm{l,se}} = \frac{\ln\frac{r_2}{r_1}}{2\pi\lambda_{\mathrm{st}}} + \frac{\ln\frac{r_2+d}{r_2}}{2\pi\lambda_{\mathrm{iz}}} + \frac{1}{2\pi h_{\mathrm{se}}(r_2+d)}$
   Draw $R_{\mathrm{l,T}}$ as a function of d in a spreadsheet!

7. Find the thickness d of a wall of a steel pipe of a radiator at which the thermal power of the radiator is maximal:
   inner radius: $r_1 = 10$ mm
   thermal conductivity of steel $\lambda = 50 \ \mathrm{W\,m^{-1}K^{-1}}$
   suppose, that surface heat transfer coefficient is constant: $h_{\mathrm{se}} = 8\,\mathrm{W\,m^{-2}K^{-1}}$

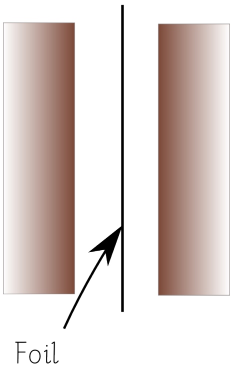
Chapter 2

Radiation

1. Calculate radiative heat flux between two gray planar surfaces. Use Kirchhoff's Law and Eq. 19.3 in [2] . Surface temperatures are $\theta_1 = 20\,°\mathrm{C}$, $\theta_2 = 10\,°\mathrm{C}$, emissivities are $\varepsilon_1 = 0,8$, $\varepsilon_2 = 0,9$.



2. Assume, that a low emissivity foil (with emissivity $\varepsilon_f = 0,05$) is inserted into the gap between the surfaces in Example 1and calculate radiative heat flux in this case. What is the equilibrium temperature θ_f of the foil?



3. Calculate equilibrium temperature of the Earth, assuming that there is no atmosphere. Albedo of the Earth surface is 0,3, solar constant is 1.361 kilowatts per square meter (kW/m^2) and emissivity of the Earth surface is ε_E =0,95.

Bibliography

[1] http://www.ewp.rpi.edu/hartford/∼ernesto/S2006/CHT/Notes/ch03.pdf

[2] Radiation Heat Transfer (Heat transfer by thermal radiation)