Prestressed Concrete

Part 7

(ULS Ultimate limit states)

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Ultimate Limit State (ULS)

associated with collapse, or with forms of structural failure which may pose a danger to people, properties, etc. ULS which may require consideration include:

EQU: loss of equilibrium of the entire structure or any part, which is considered as a rigid body,

STR: failure by rupture, excessive deformation, or loss of stability of the entire structure or any part, including supports and foundations,

GEO: failure of excessive deformation of the ground where the strength of the soil or rock are significant in providing resistance.

In ultimate limit states STR:

it shall be verified that

$$E_d \leq R_d$$

where

E_d is the design value of an internal force or moment,

R_d the corresponding design resistance, associating all structural properties with the respective design values

Design situations

are classified as:

1. **persistent situations which refer to the condition of normal use,**

2. **transient situations which refer to the temporary conditions of the structure, such as execution or repair,**

3. **accidental situations which refer to exceptional conditions of the structure or to its exposure, e.g. to fire, explosion, or an impact,**

4. **seismic situations which refer to exceptional conditions of the structure when subjected to seismic events.**

Effects of loads

Persistent and transient situations - may be expressed as

$$E_d = E\{\gamma_{G,j} G_{k,j} ; \gamma_p P ; \ \gamma_{Q,1} Q_{k,1} ; \gamma_{Q,i} \psi_{0,i} Q_{k,i}\} \quad j \geq 1; i > 1$$

The combination of **effects of actions** to be considered should be based on

- the design value of the **leading variable action** and

- the design combination values of **accompanying variable actions**.

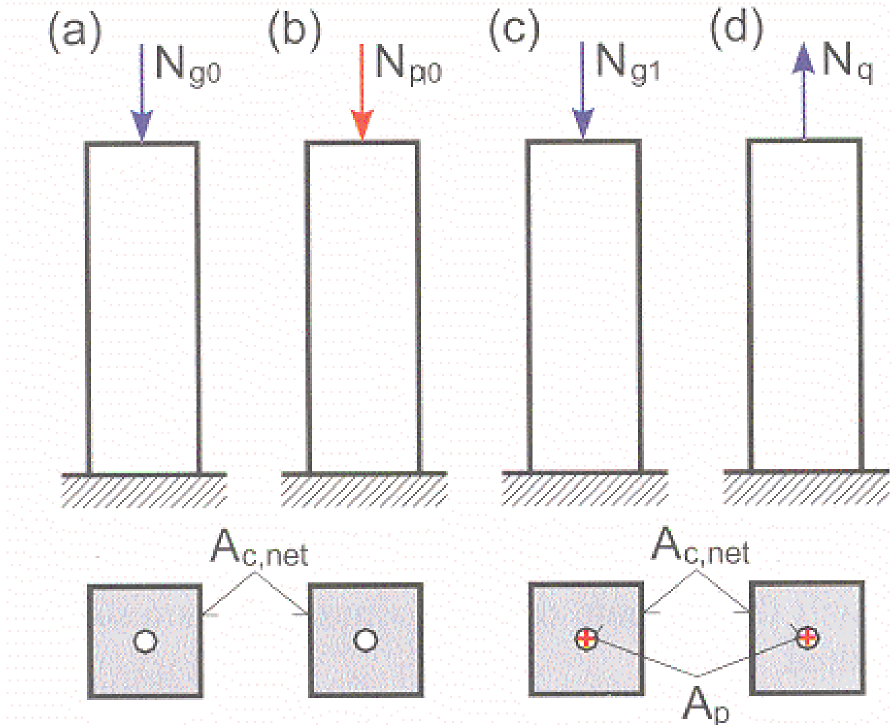
The combination of actions in the brackets "{}" in equation may be expressed either as

$$\sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_p P + \gamma_{Q,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \qquad (1)$$

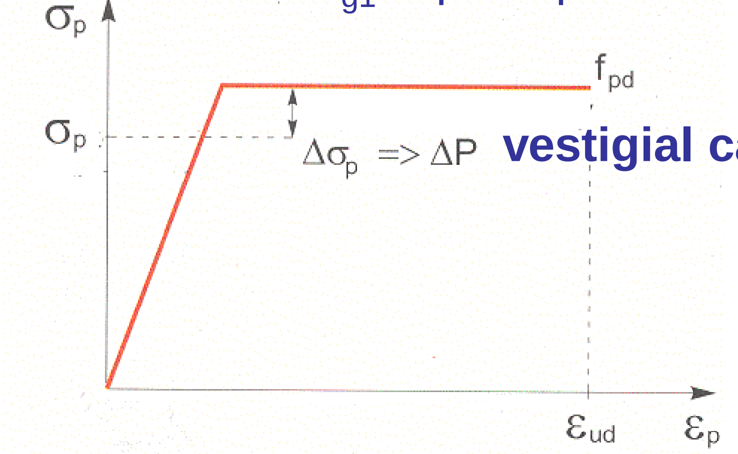
or, **alternatively for (STR) limit states**, the less favourable of the two following expressions (1a)(1b) where; the latter procedure gives more concise values and therefore is recommended for use.

$$\begin{cases} \sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_p P + \gamma_{Q,1} \psi_{0,i} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} & (1a) \\ \sum_{j \geq 1} \xi_j \gamma_{G,j} G_{k,j} + \gamma_p P + \gamma_{Q,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} & (1b) \end{cases}$$

Prestressed element subjected to tensile axial load



N$_{g1}$ superimposed load



vestigial capacity

Concrete:

$$\sigma_{cg0} = \frac{N_{g0}}{A_{c,net}}; \varepsilon_{cg0} = \frac{\sigma_{cg0}}{E_{cm}}; copr.$$

$$\sigma_{cp0} = \frac{N_{p0}}{A_{c,net}}; \varepsilon_{cp0} = \frac{\sigma_{cp0}}{E_{cm}}; copr.$$

$$\sigma_{cg1} = \frac{N_{g1}}{A_i}; \varepsilon_{cg1} = \frac{\sigma_{cg1}}{E_{cm}}; copr.$$

$$\sigma_c = \sum \sigma_{ci}; \varepsilon_c = \sum \varepsilon_{ci};$$

Steel:

$$\sigma_{pg0} = 0$$

$$\sigma_{p0} = \frac{N_{p0}}{A_p}; \varepsilon_{p0} = \frac{\sigma_{p0}}{E_p}; tens.$$

$$\sigma_{pg1} = \frac{N_{g1} E_p}{A_i E_{cm}}; \varepsilon_{pg1} = \frac{\sigma_{pg1}}{E_p}; copr.$$

$$\sigma_p = \sum \sigma_{pi}; \varepsilon_p = \sum \varepsilon_{pi};$$

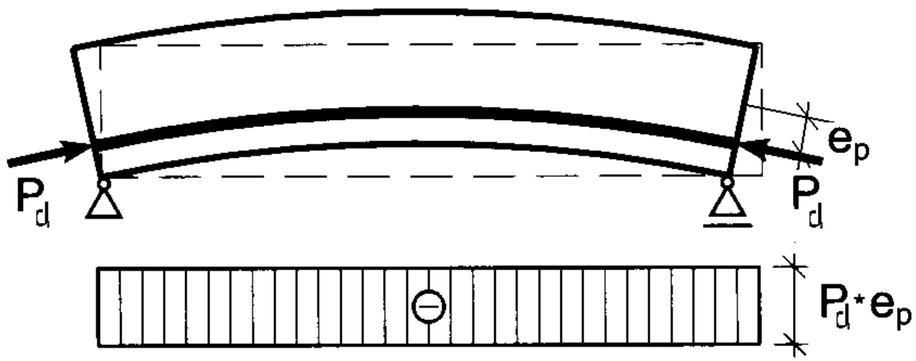
$\sigma_c; \sigma_p$ – **initial stress**

Capacity for tension variable load:

$$N_q = A_{c,net}\sigma_c + A_p(f_{pd} - \sigma_p)$$

- **Effect of prestressing at ULS**

Prestressing force acts as the external force.



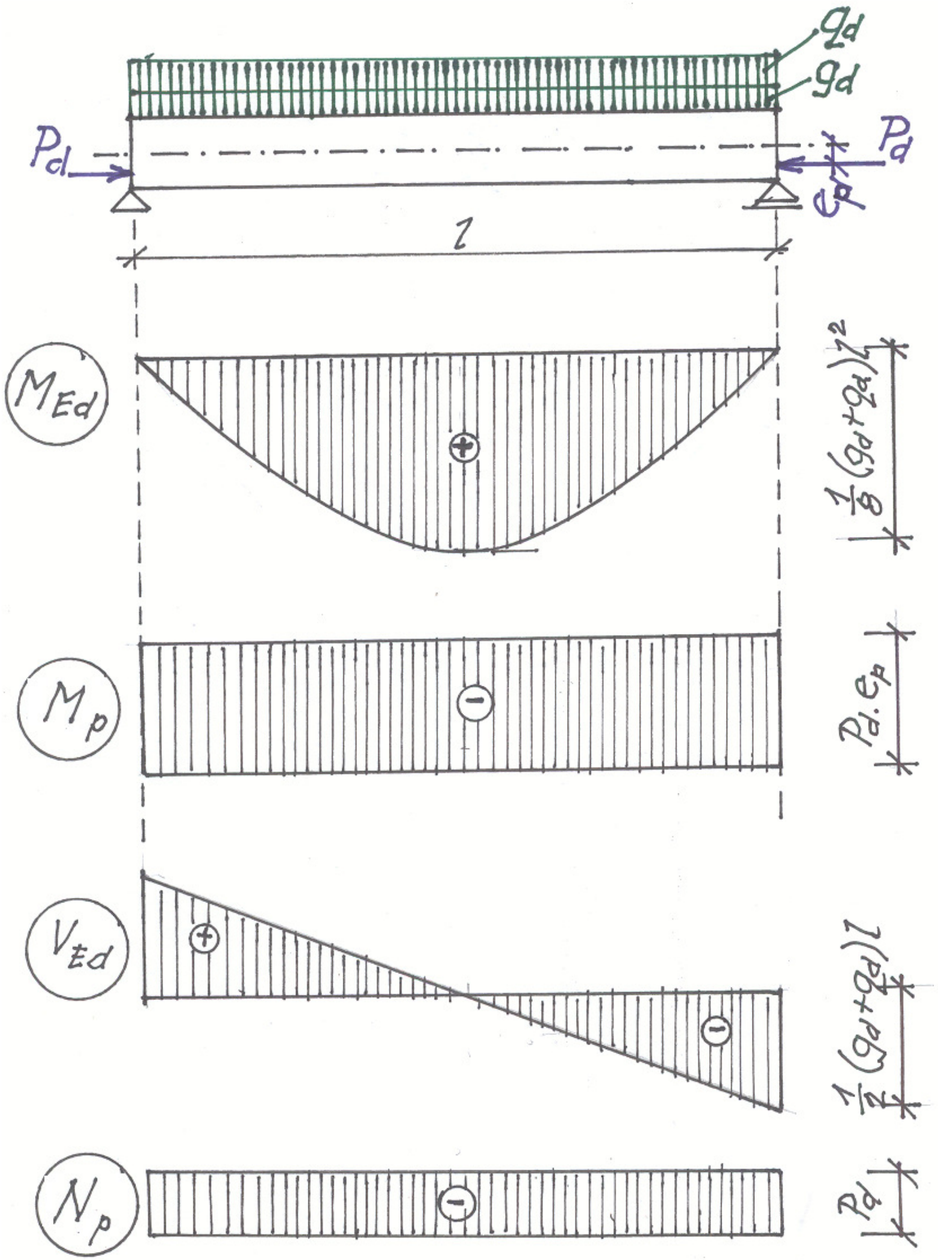
Design value of prestressing force:

$$P_d = \gamma_P P_{m,t}(x)$$

Prestress in most situations is intended to be favourable. The design value of prestress may be based on the mean value of the prestressing force and then $\gamma_P = 1,0$.

- **In the verification of local effects $\gamma_{P,unfav}$ should be used. The recommended value is $\gamma_{P,unfav} = 1,2$**

Effects of loads – simply supported beam



In the middle of the beam:

$M_{Ed} – M_p = (g_d + q_d)\, l^2 / 8 – P_d\, e_p$

$N_p = P_d$

In the support of the beam:

$V_{Ed} = (g_d + q_d)\, l / 2$

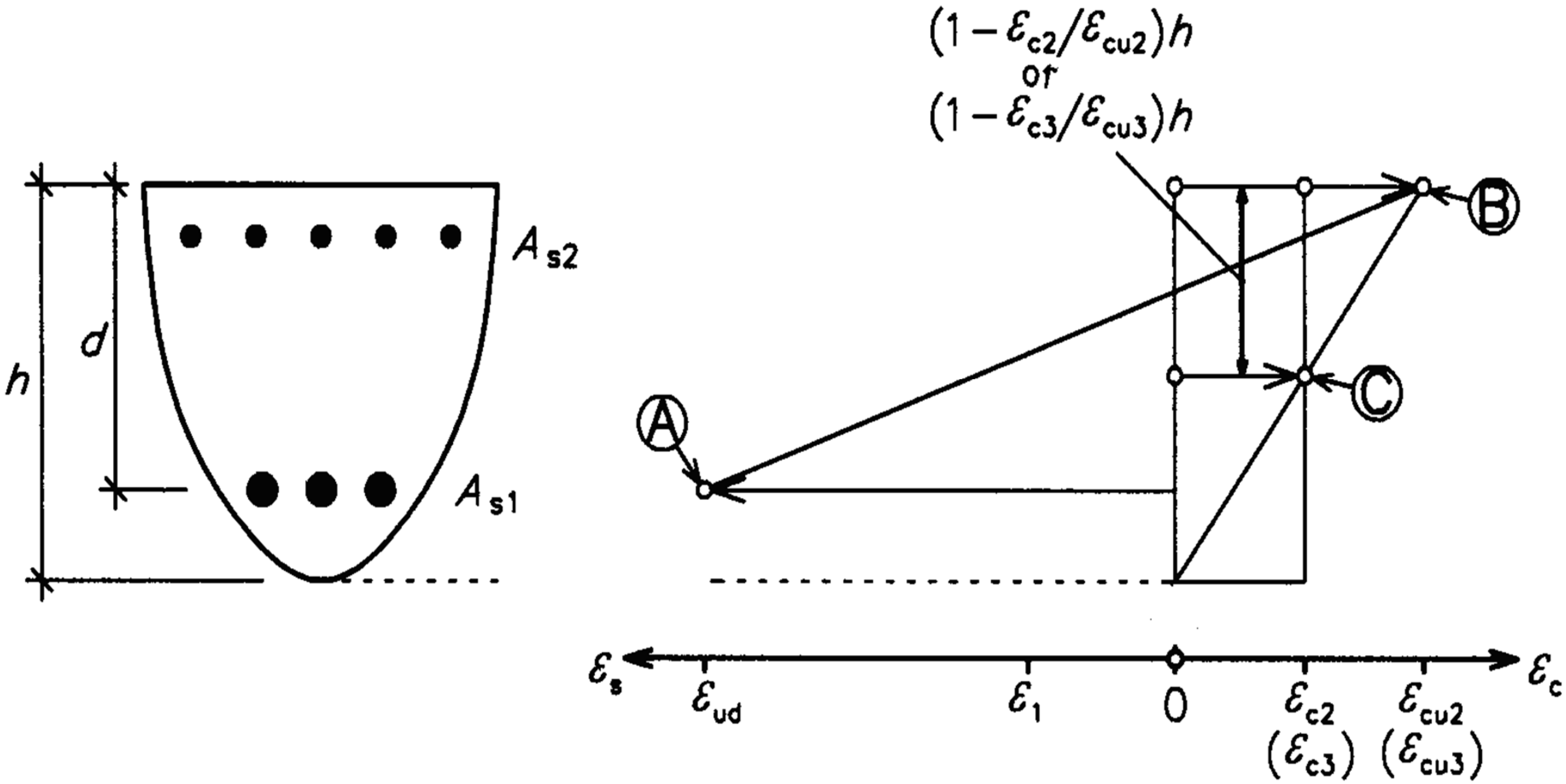
$N_p = P_d$

ULS – Normal force and bending moment

The resisting forces of the sections should be calculated by applying the following **assumptions**:

1. The **section** perpendicular to the axis of bending which are **plane** before bending **remain plane** after bending.
2. The **strain in reinforcement is equal to the strain in the concrete** at the same level.
3. The **tensile strength** of concrete is **ignored.**
4. The **stresses** in the concrete and reinforcement can be computed from the strains **using stress-strain diagrams** for concrete and steel,
5. The **ultimate strain is reached** at the extreme compressed concrete fibres, and/or in extreme tensioned steel fibres

Possible strain distributions in the ultimate limit state



A - reinforcing steel tension strain limit

B - concrete compression strain limit

C - concrete pure compression strain limit

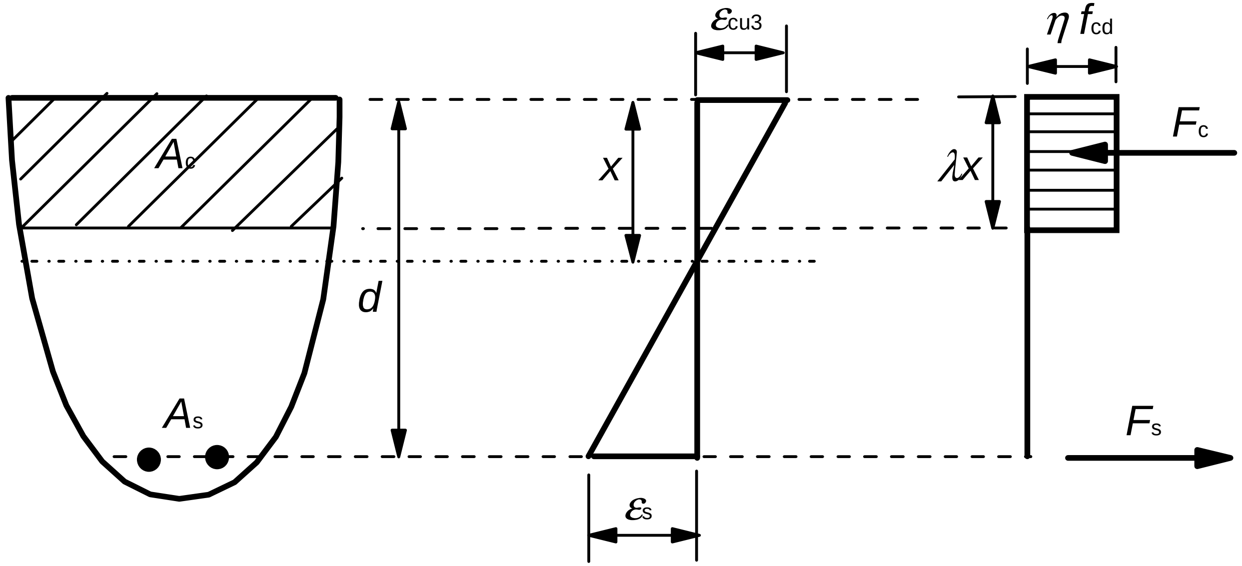
Design strength for concrete

The value of the design compressive strength

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C,$$

γ_C is the partial safety factor for concrete; the recommended values for the ultimate limit state is 1.5 in persistent and transient

α_{cc} the coefficient taking account of long-term effects on the compressive strength and of unfavourable effects resulting from the way the load; the recommended value is 1.0.



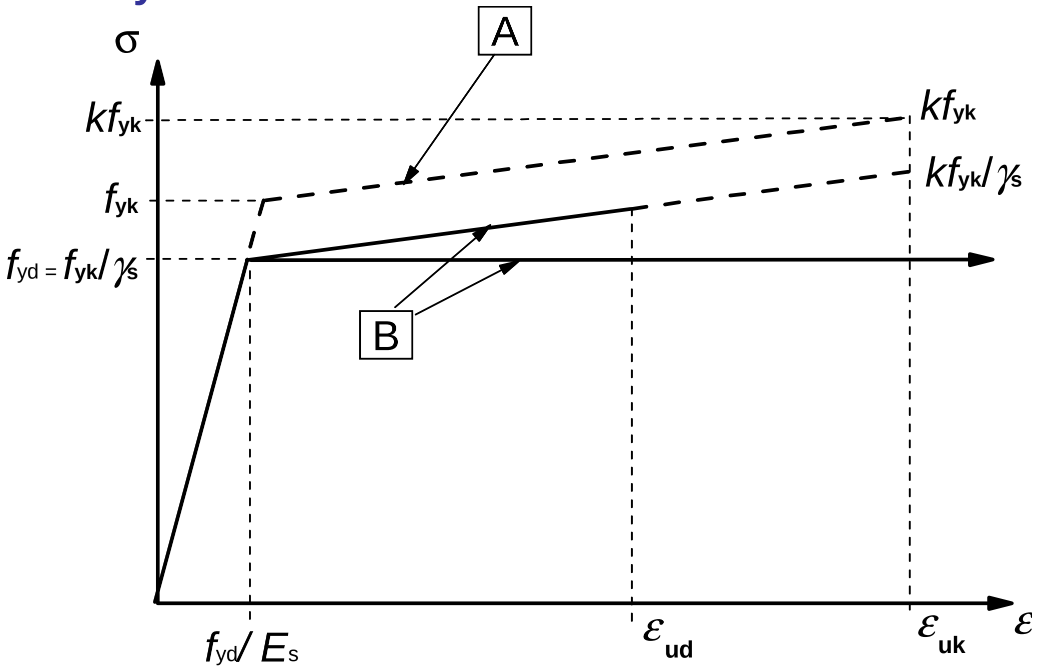
Design strength for reinforcing steel

The value of the design compressive and tensile yield strength of reinforcing steel

$$f_{yd} = f_{yk} / \gamma_S,$$

γ_S is the partial safety factor for steel; the recommended value for ultimate limit state is 1.15,

f_{yk} the characteristic yield strength, either taken from f_y or $f_{0.2}$.



$k = (f_t/f_y)_k$

A Idealised

B Design

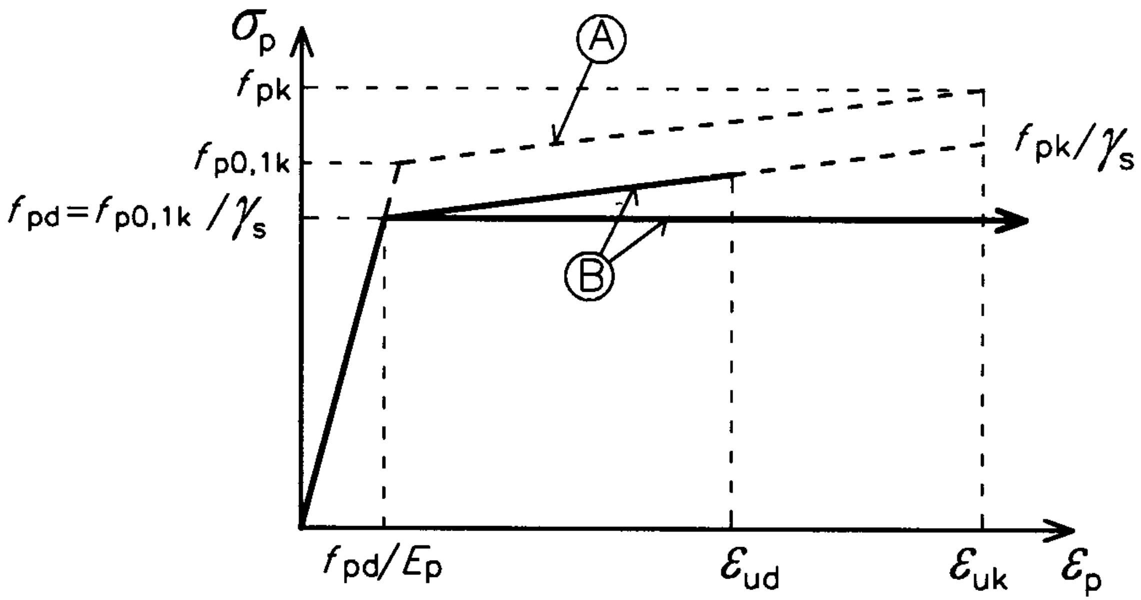
Design strength for prestressing steel

The value of the design compressive and tensile strength of prestressing steel

$$f_{pd} = f_{p0,1k} / \gamma_S,$$

γ_S **is** **the partial safety factor for steel; the recommended value for ultimate limit state is 1.15,**

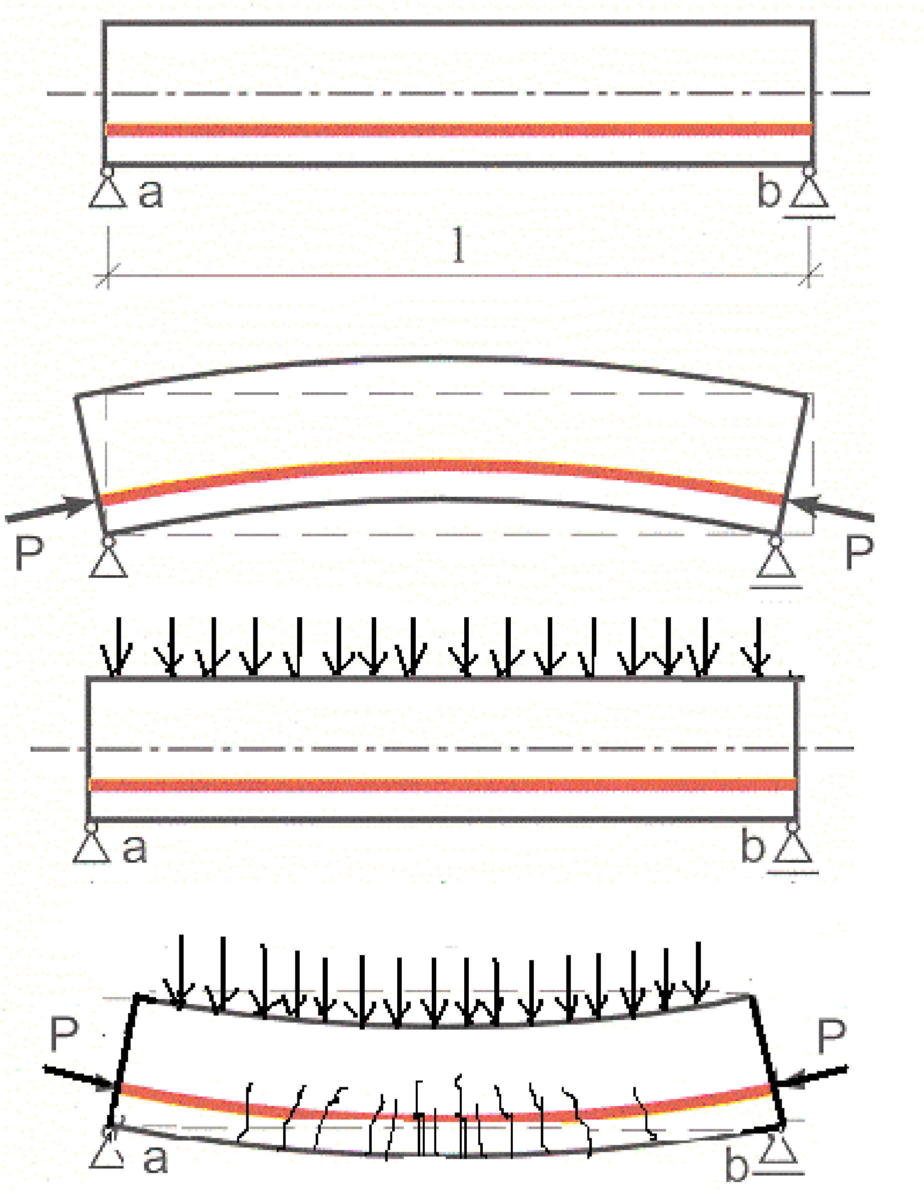
$f_{p0,1k}$ **the characteristic value of proof stress**



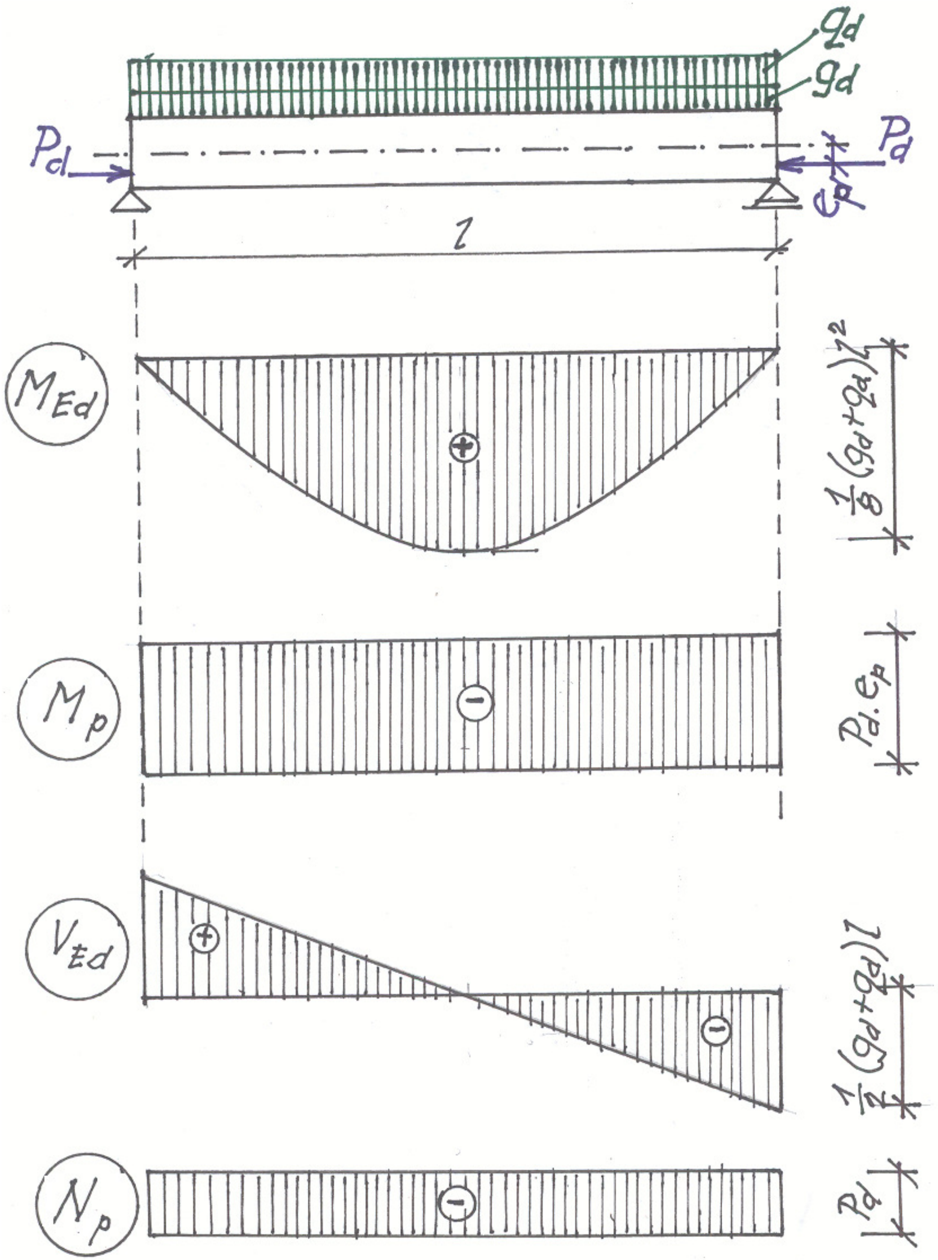
A Idealised
B Design

(absolute values are shown for tensile stress and strain)

Prestressed element subjected to bending



Effects of loads – simply supported beam



In the middle of the beam:

$M_{Ed} - M_p = (g_d + q_d)\, l^2 / 8 - P_d\, e_p$

$N_p = P_d$

In the support of the beam:

$V_{Ed} = (g_d + q_d)\, l / 2$

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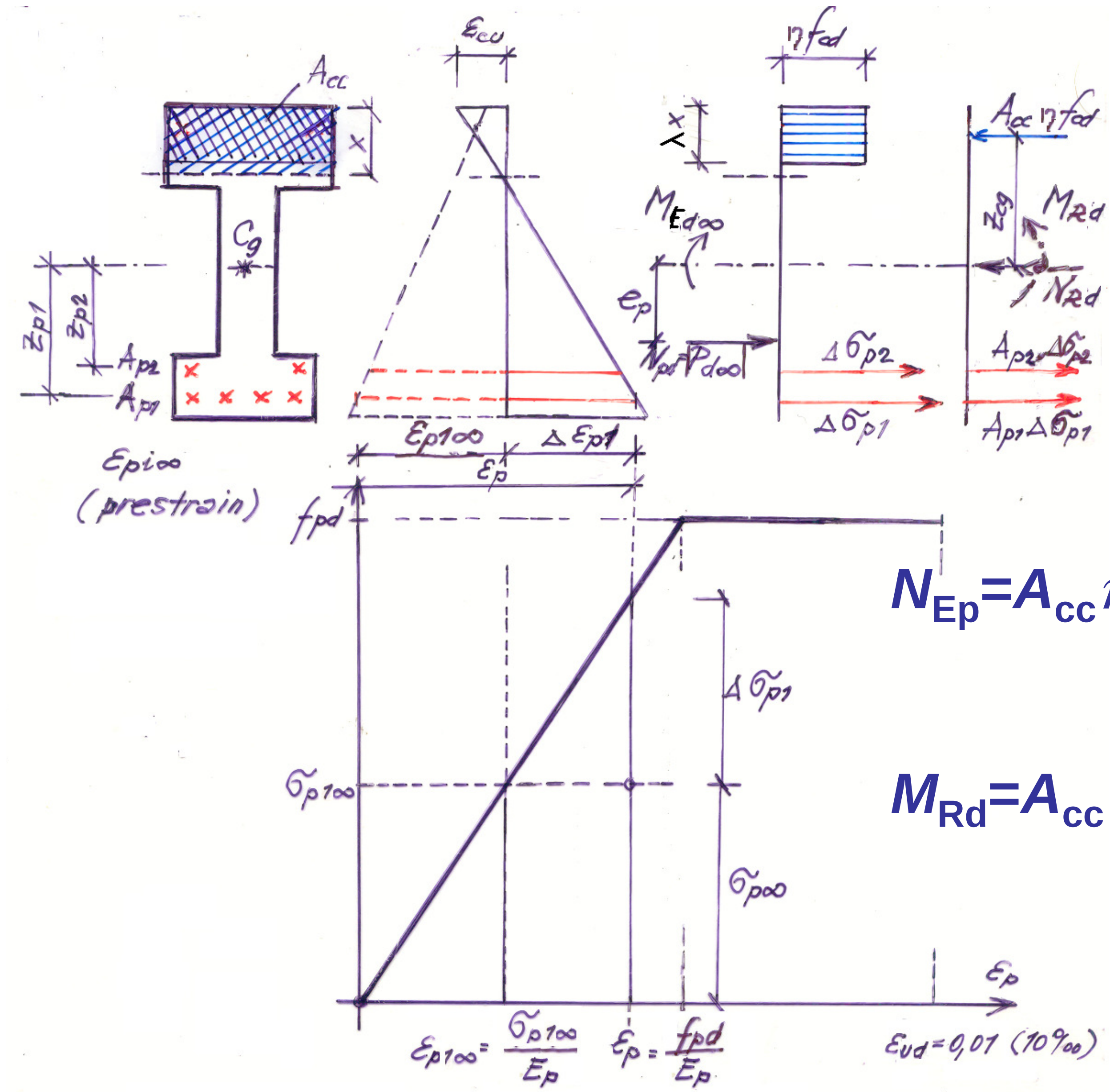


$M_{Ed} - M_{Ep} =$
$(g_d + q_d).l^2/8 - P_d\, e_p$
$N_{Ep} = P_d$

Assuming $N_{Rd} = N_{Ep}$:

$$N_{Ep} = A_{cc}\eta f_{cd} + \sum A_{pi}\Delta\sigma_{pi} \qquad (1)$$

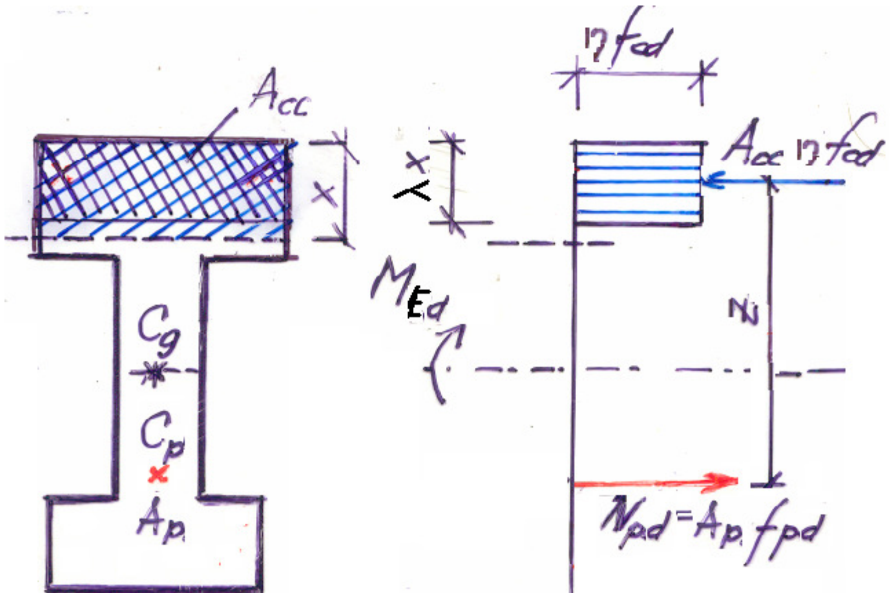
$$\Downarrow$$

$$x$$

$$M_{Rd} = A_{cc}\eta f_{cd} z_{gc} + \sum A_{pi}\Delta\sigma_{pi} z_{pi} \qquad (2)$$

Condition of safety:

$M_{Ed} - M_{Ep} \leq M_{Rd}$



Sometimes in practice it is assumed that the prestressing force acts as <u>internal force</u>, then

$$A_{cc}\eta f_{cd} + A_p f_{pd} = 0 \qquad (1)$$

$$\Downarrow$$

$$x$$

$$M_{Rd} = A_{cc}\eta f_{cd} z \qquad (2)$$

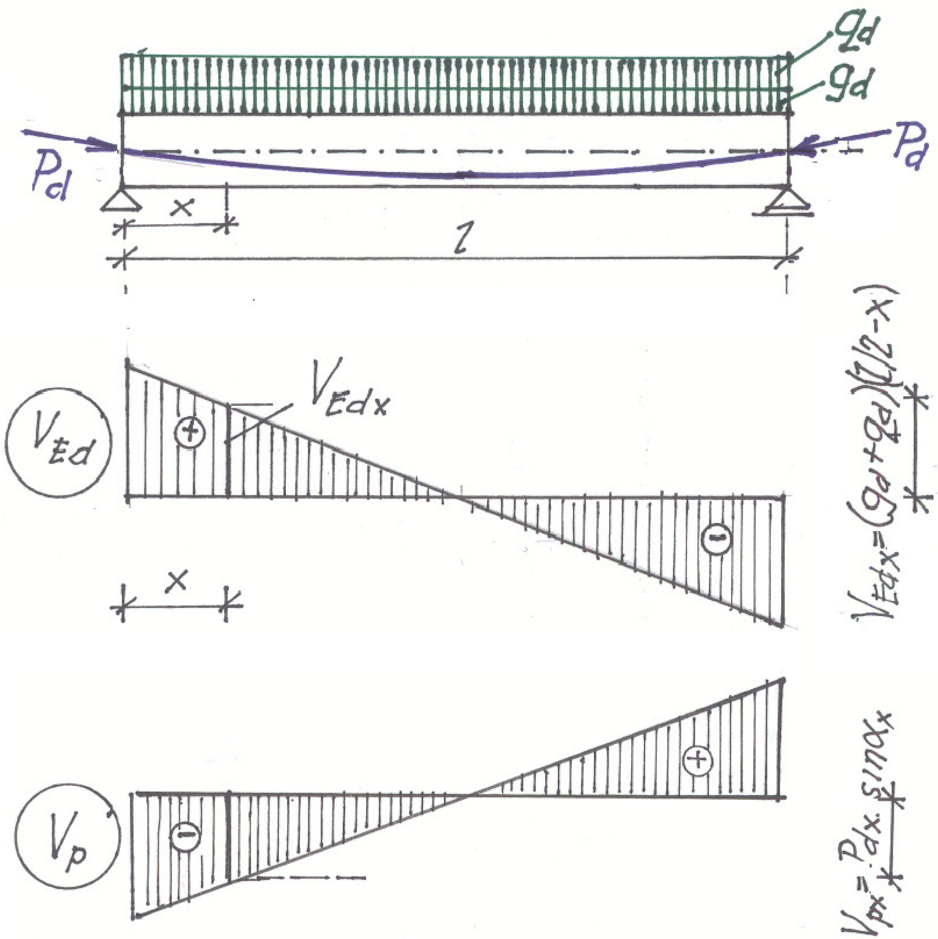
Condition of safety:

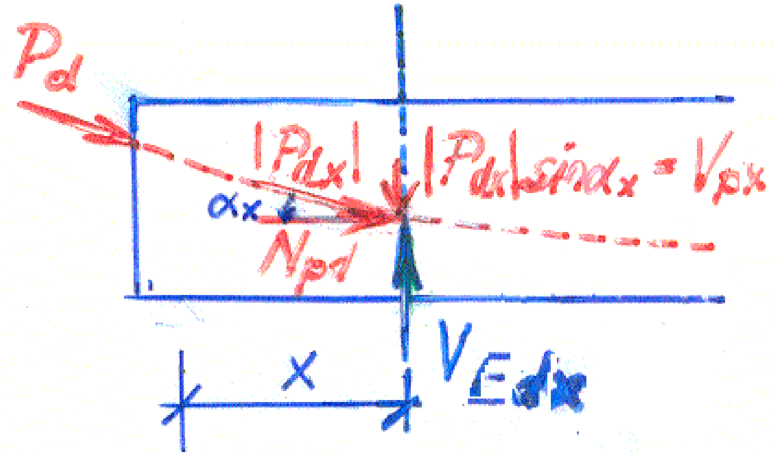
$$M_{Ed} \leq M_{Rd}$$

It is assumed: $P_d = A_{cc}\eta f_{cd} - A_p(f_{pd} - \sigma_{p\infty})$

ULS – Shear force

Shear forces

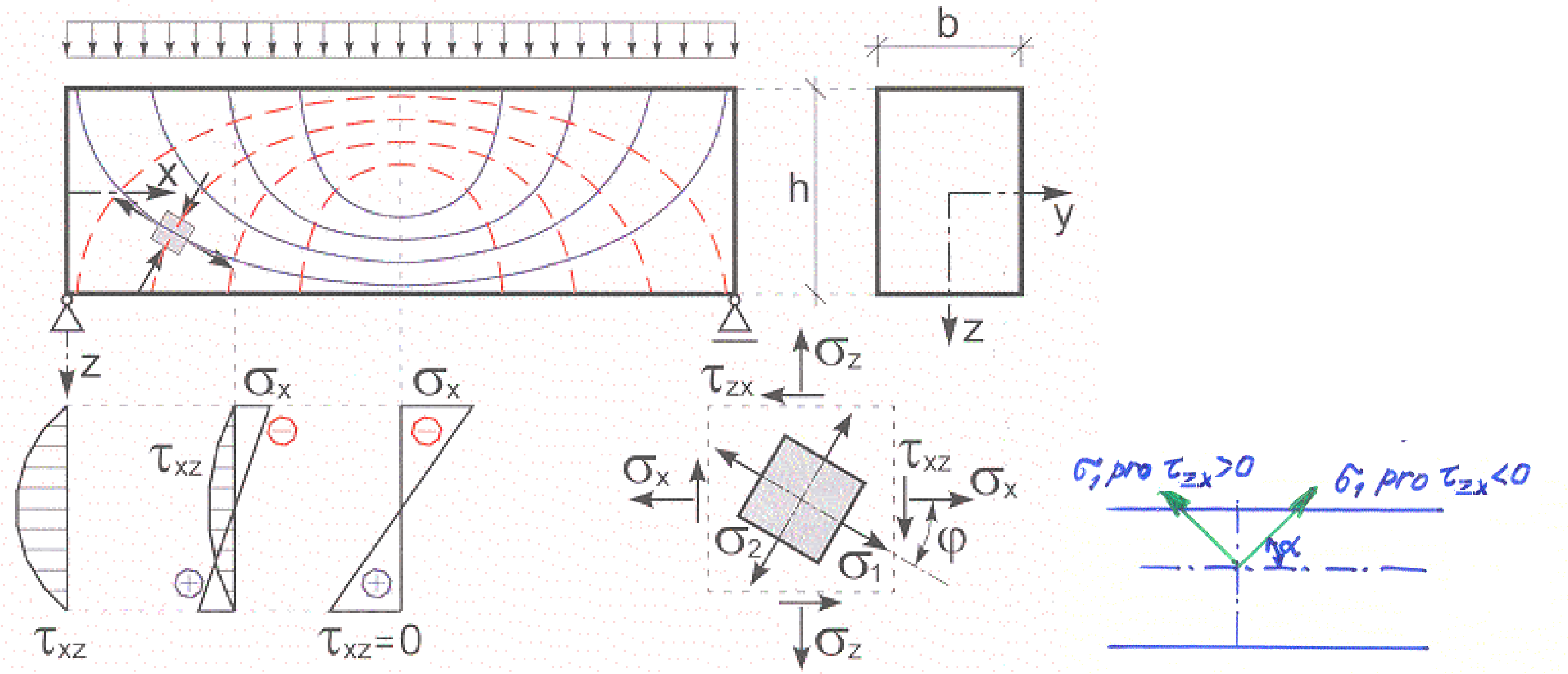




Lift-up tendon reduces the shear force V_{Ed}:

$$V_{Edx} - P_{dx} \sin\alpha_x$$

Trajectories of principal stress

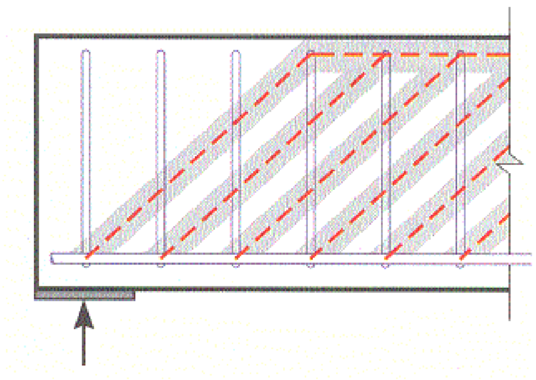


$$\sigma_{1.2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau^2_{xz}}\,; tg\,2\alpha = \frac{-2\tau_{xz}}{\sigma_x - \sigma_z}$$

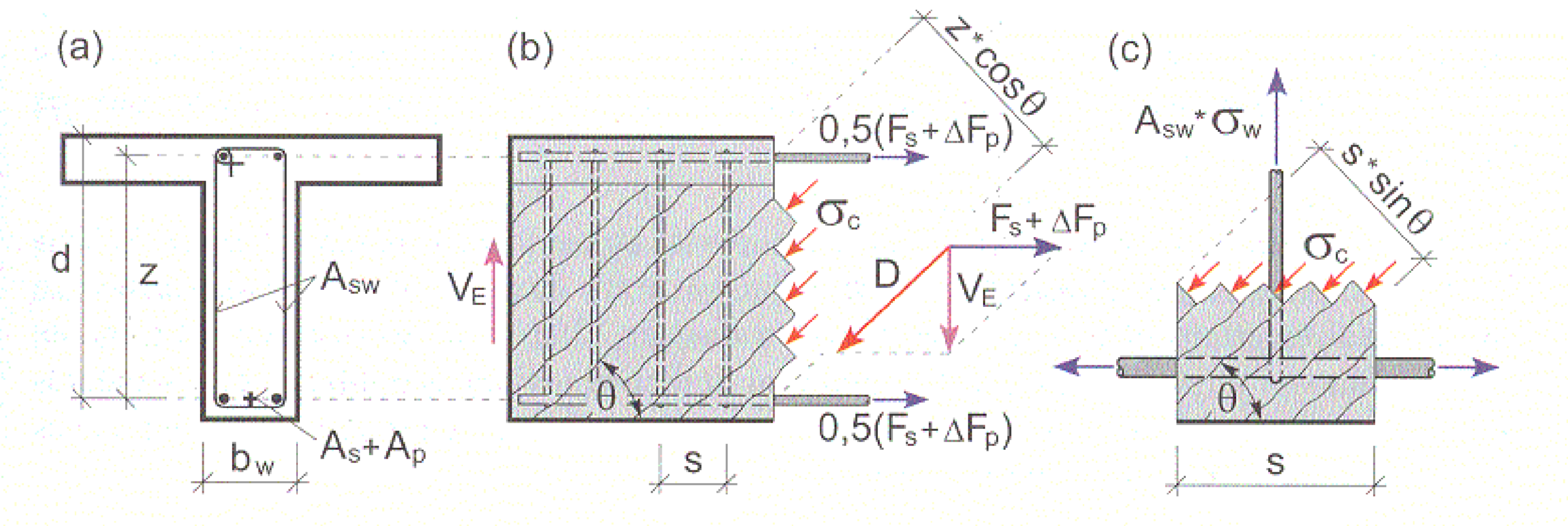
If σ_x is compression stress $\Rightarrow$ decreases the principal tension stress

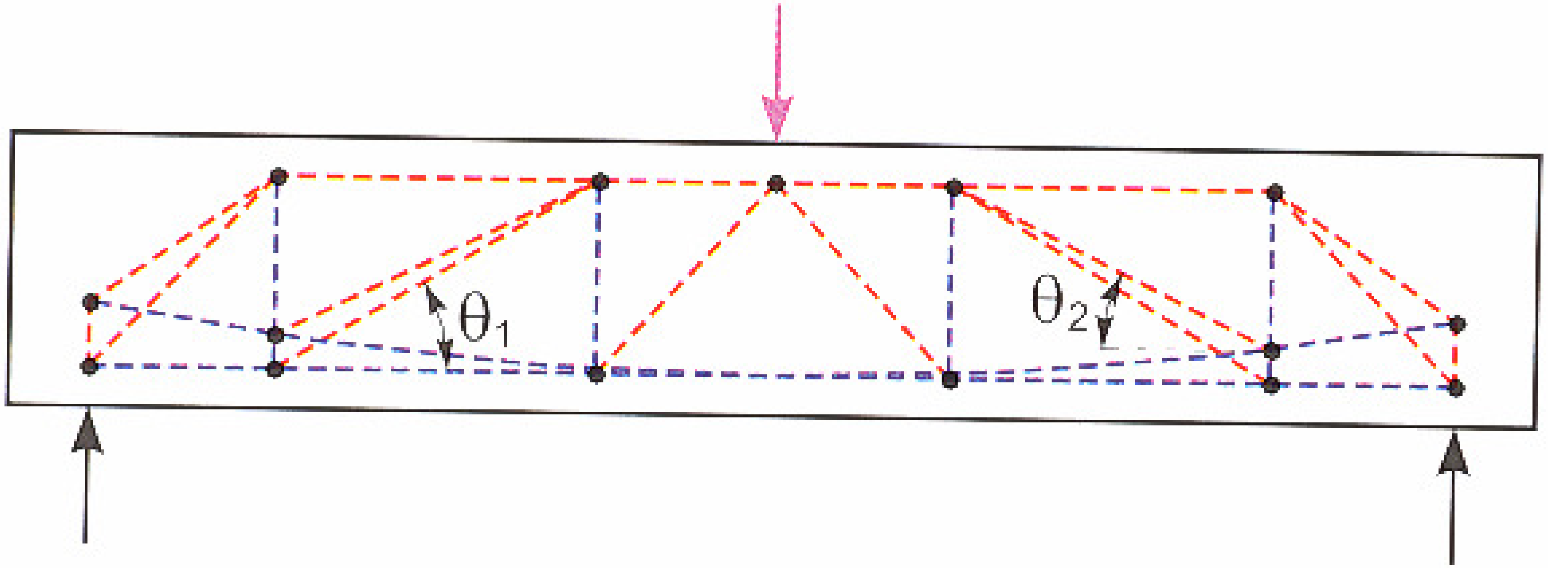
ULS – Shear force

Truss model

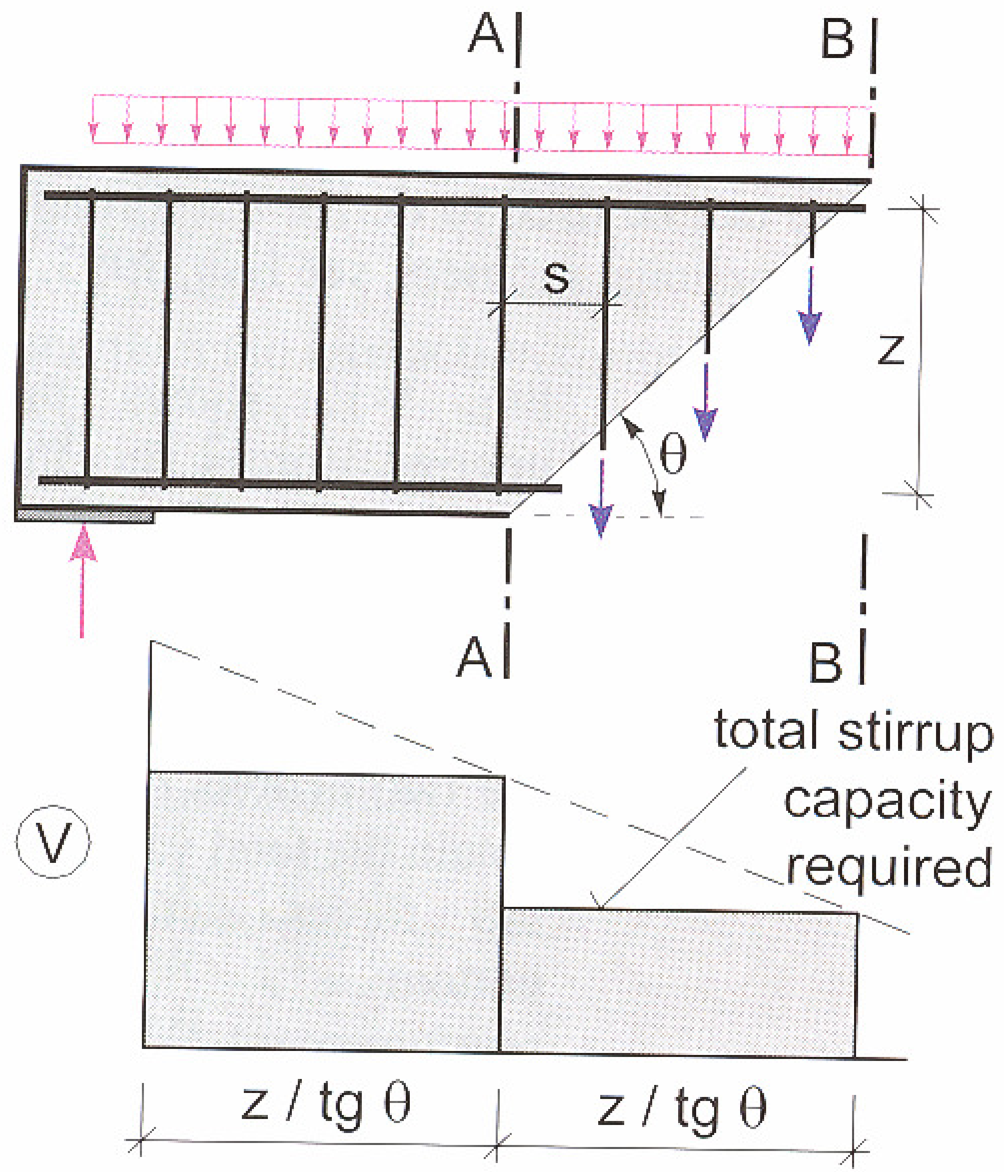


Equilibrium conditions of the truss model





Truss model of prestressed beam



The staggering concept for shear design

For the verification of the shear resistance the following symbols are defined:

$V_{Rd,c}$ is the design shear resistance of the member without shear reinforcement.

$V_{Rd,s}$ the design value of the shear force which can be sustained by the yielding shear reinforcement.

$V_{Rd,max}$ the design value of the maximum shear force which can be sustained by the member, limited by crushing of the compression struts.

The shear resistance of a member with shear reinforcement

$$V_{Rd} = V_{Rd,s}$$

Members not requiring design shear reinforcement

$$V_{Ed} \leq V_{Rd,c}$$

The design value for the shear resistance $V_{Rd,c}$ is given by:

$$V_{Rd,c} = [C_{Rd,c} k(100\, \rho_l\, f_{ck})1/3 + k_1\, \sigma_{cp}]^{1/3}\, b_w d$$

with a minimum of

$$V_{Rd,c} = (v_{min} + k_1 \sigma_{cp})\, b_w d$$

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$$ with d in mm ; $$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

A_{sl} the area of the tensile reinforcement, which extends $\geq (l_{bd} + d)$ beyond the section considered

b_w the smallest width of the cross-section in the tensile area [mm]

σ_{cp} $= N_{Ed}/A_c < 0{,}2\, f_{cd}$ [MPa]

N_{Ed} the axial force in the cross-section due to loading or prestressing [in N] (N_{Ed}>0 for compression).

A_C the area of concrete cross section [mm^2]

$V_{Rd,c}$ is [N]

The recommended values for:

$C_{Rd,c}$ is $0{,}18/\gamma_c$, $v_{min} = 0{,}035\, k^{3/2} \cdot f_{ck}^{1/2}$; $k_1 = 0{,}15$

In regions uncracked in bending (where the flexural tensile stress is smaller than $f_{ctk,0,05}/\gamma_c$) the shear resistance should be limited by the tensile strength of the concrete. In these regions the shear resistance is given by:

$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \sigma_{cp} f_{ctd}} \qquad (6.4)$$

I is the second moment of area

B_w the width of the cross-section at the centroidal axis, allowing for the presence of ducts

S the first moment of area above and about the centroidal axis

α_l $= l_x/l_{pt2} \le 1{,}0$ for pretensioned tendons
$= 1{,}0$ for other types of prestressing

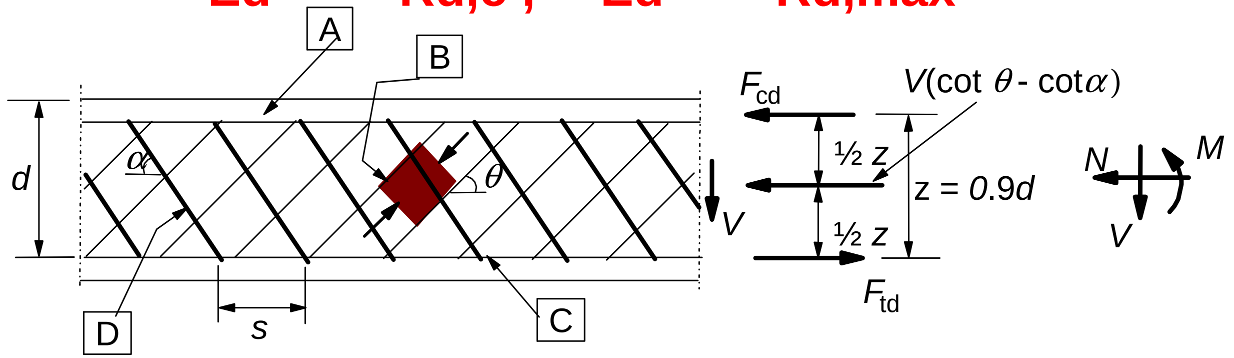
l_x the distance of section considered from the starting point of the transmission length

l_{pt2} the upper value of the transmission length of the prestressing element

σ_{cp} the concrete compressive stress at the centroidal axis due to axial loading and/or prestressing ($\sigma_{cp} = N_{Ed} / A_c$ in MPa, $N_{Ed} > 0$ in compression)

Members requiring design shear reinforcement

$$V_{Ed} > V_{Rd,c}\,;\; V_{Ed} \leq V_{Rd,max}$$



A - compression chord, B - struts, C - tensile chord, D - shear reinforcement

α is the angle between shear reinforcement and the beam axis perpendicular to the shear force (measured positive- in Fig.),

θ the angle between the concrete compression strut and the beam axis perpendicular to the shear force,

F_{td} the design value of the tensile force in the longit. reinforcement

F_{cd} the design value of the concrete compression force in the direction of the longitudinal member axis,

B_w the minimum width between tension and compression chords

z the inner lever arm, for a member with constant depth, corresponding to the bending moment in the element under consideration. In the shear analysis approximate value $z = 0{,}9d$ may be used.

The angle θ should be limited. The recommended limits are:

$$1 \leq \cot\theta \leq 2{,}5$$

<u>For members with vertical shear reinforcement</u>, the shear resistance, V_{Rd} is the smaller value of:

$$V_{Rd,s} = \frac{A_{sw}}{s} z \, f_{ywd} \cot\theta$$

and

$$V_{Rd,max} = \alpha_{cw} \, b_w \, z \, \nu_1 \, f_{cd} / (\cot\theta + \tan\theta)$$

A_{sw} is the cross-sectional area of the shear reinforcement

s the spacing of the stirrups

f_{ywd} the design yield strength of the shear reinforcement

ν_1 a strength reduction factor for concrete cracked in shear

α_{cw} a coefficient taking account of the state of the stress in the compression chord

The recommended value of ν_1 is $\nu = 0{,}6(1- f_{ck} / 250)$, f_{ck} in MPa.