Prestressed Concrete

Part 6

(SLS - Serviceability control)

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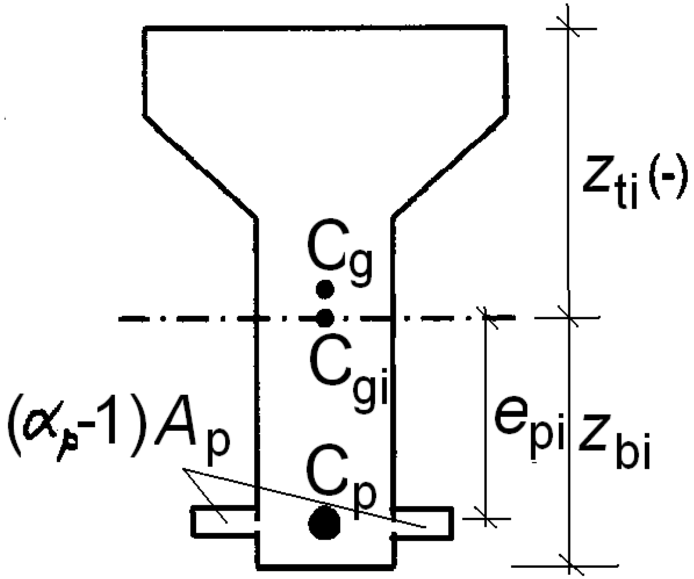
Check of the serviceability limit states:

a) Stress limitation

- Calculate for the transformed (idealised) cross section with full concrete section:

z_{cti} ; z_{cbi} ; e_{pi} ; A_i ; I_i ; σ_{cti} ; σ_{cbi} ;



the stresses:

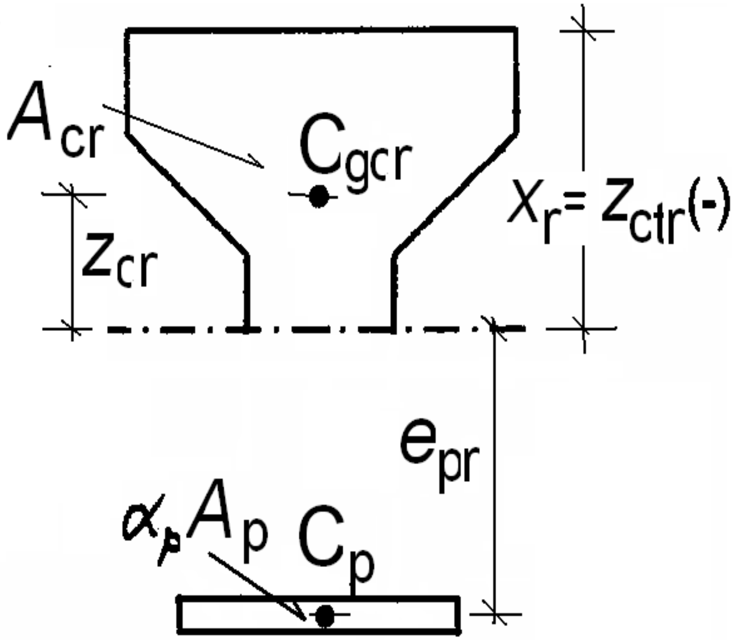
$$\Rightarrow \quad \sigma_{ci} = \frac{-P}{A} + \frac{-Pe_{pi} + M}{I_i} z_i$$

$$\sigma_{pi} = \alpha_p \left(\frac{-P}{A_i} + \frac{-Pe_{pi} + M}{I_i} e_{pi} \right)$$

and check with the stress limitation

If the tension concrete stress $\sigma_{cbi} > f_{ctm}$, it is necessary to assume idealised cross section, where does not act the concrete in tension

- **If the tension concrete stress $\sigma_{cbi} > f_{ctm}$, it is necessary to assume idealised cross section, where does not act the concrete in tension**
  - **determine the location of neutral axis x_r from the equation: $A_{cr} z_{cr} = \alpha_p A_p e_{pr}$**
  - **calculate: $z_{ctr} = x_r$; e_{pr} ; A_r ; I_r ; σ_{ctr} ; $\sigma_{pr} \Rightarrow$**
  - **check the stresses with stress limitations**



$$\sigma_{cr} = \frac{-P}{A_r} + \frac{-Pe_{pr} + M}{I_r} z_r \ldots\ldots P_{m0,sub} ; M_{g0}$$

$$\sigma_{pr} = \alpha_p \left(\frac{-P}{A_r} + \frac{-Pe_{pr} + M}{I_r} e_{pr} \right) \ldots\ldots P_{m\infty,\inf} ; M_k$$

$$\sigma_{ctr} \leq -0,6 f_{ck} \qquad \sigma_{pi} \leq f_{yk}$$

If the decompression is required for the quasi-permanent load combination, it is necessary check width of crack for frequent load combination

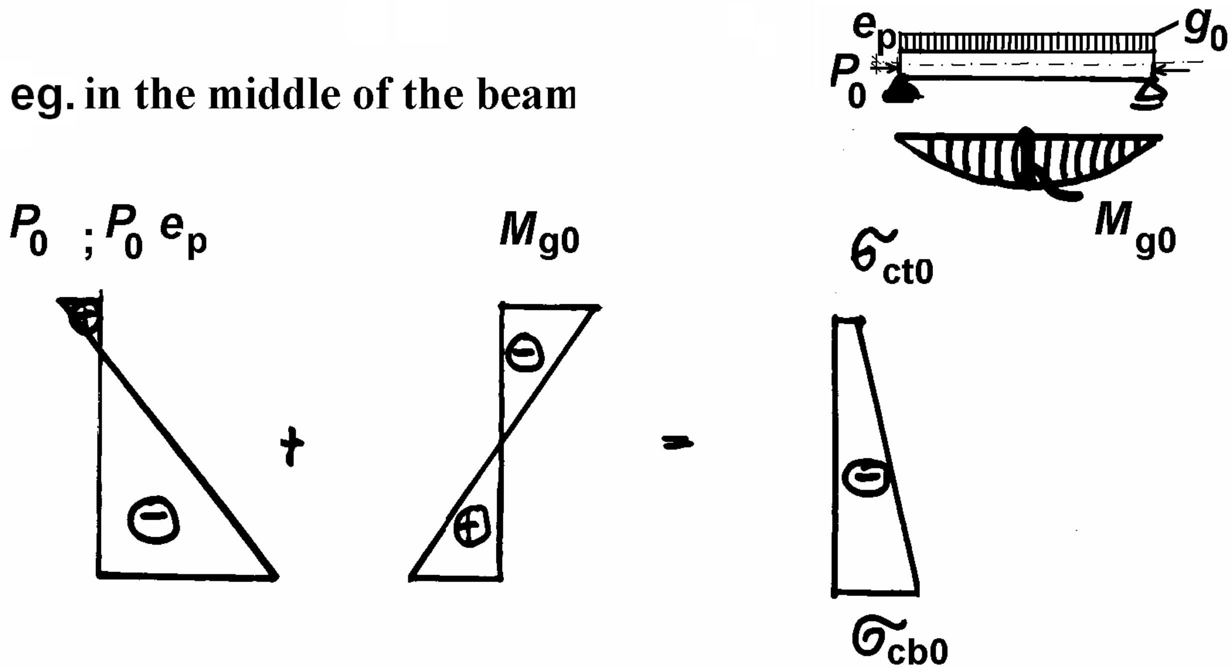
Stage of prestressing

after transfer of prestressing – introduction the prestressing force into concrete – act:

a) the initial prestress force P_0 at time t_0 with eccentricity e_p

b) b) usually only self-weight g_0 - Mg_0

eg. in the middle of the beam



σ_{ct0}

by prestressing:

full $|\sigma_{ct0}| \leq 0$

limited $\sigma_{ct0} \leq fctm$

partial $\sigma_{ct0} > fctm$

σ_{cb0}

$|\sigma_{cb0}| \leq 0{,}6\ f_{ck}$

by pre-tensioning

$|\sigma_{cb0}| \leq 0{,}7\ f_{ck}$

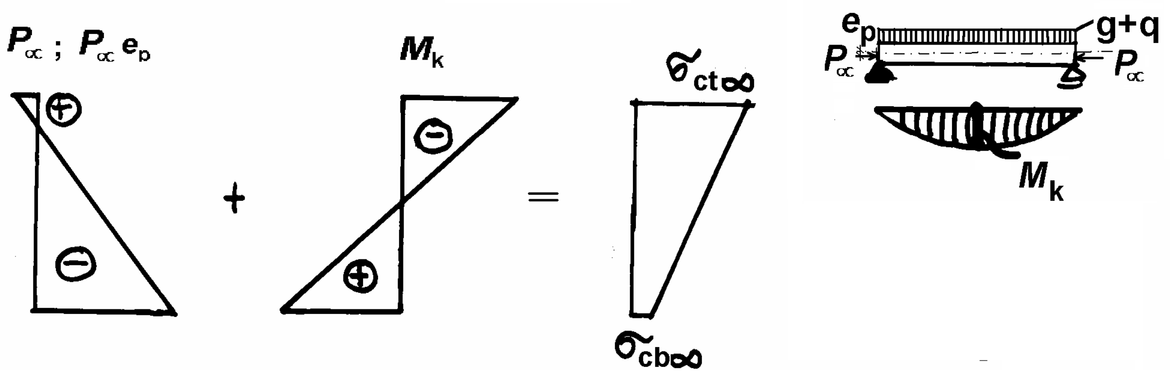
- **Stage of service**

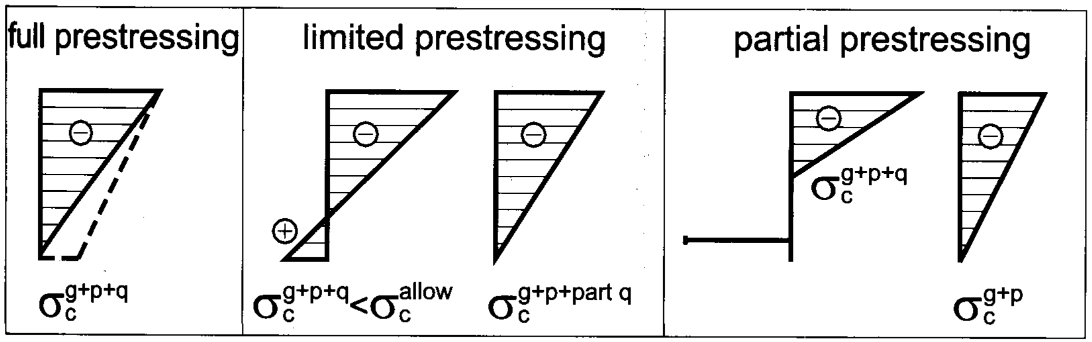
after all losses of prestressing force – act:

a) the prestress force P_{∞} at time $t = \infty$ with the eccentricity e_p; $t = \infty \cong$ 500 000 hours $\cong$ 57 years

b) service load in prescribed combination $g + q$ - M_k

eg. in the midle of the beam





$\sigma_{ct\infty}$

$|\sigma_{ct\infty}| \leq 0{,}6\, f_{ck}$

$\sigma_{cb\infty}$

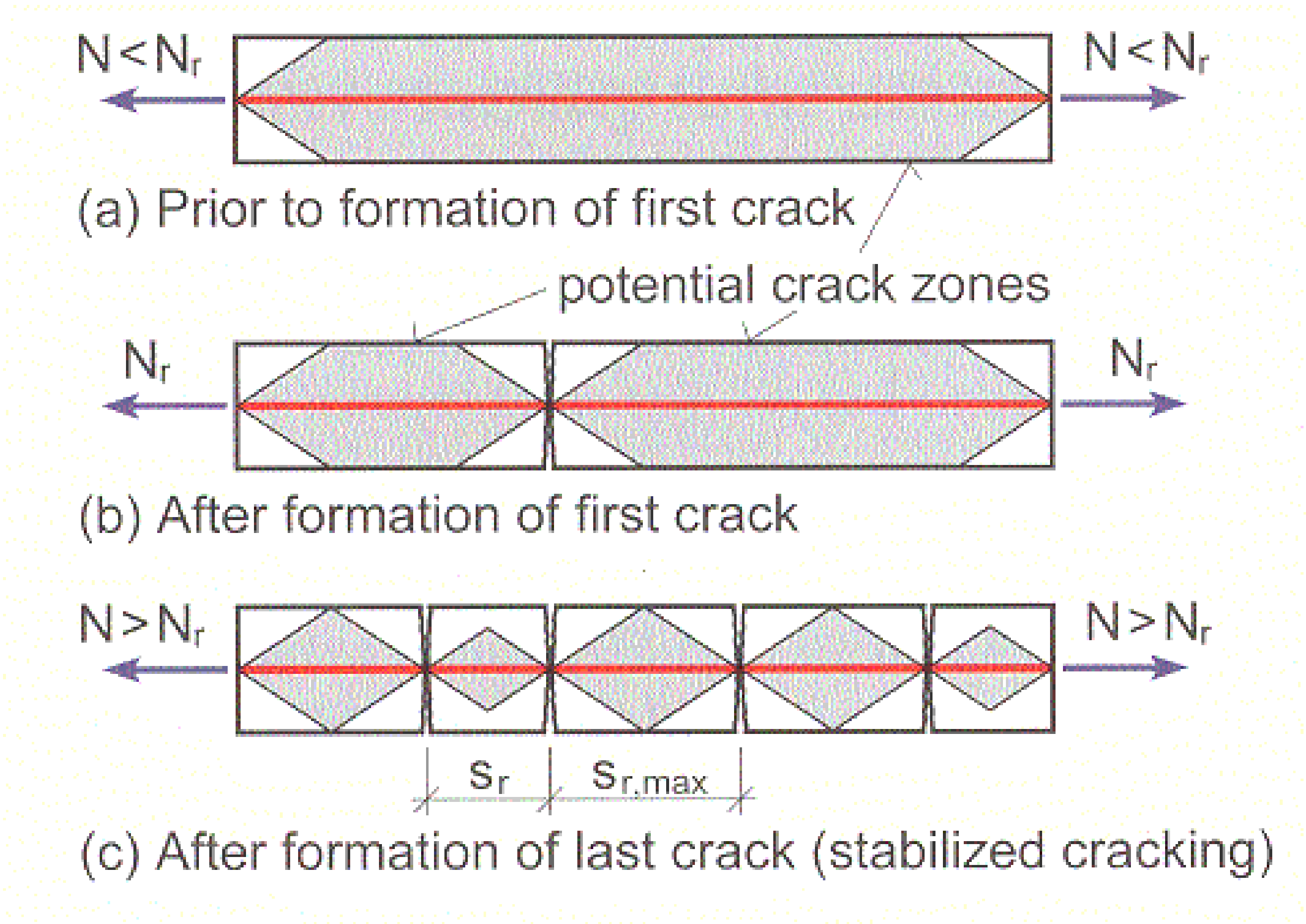
by pre-tensioning:

full $|\sigma_{cb\infty}| \leq 0$

limited $\sigma_{cb\infty} \leq f_{ctm}$

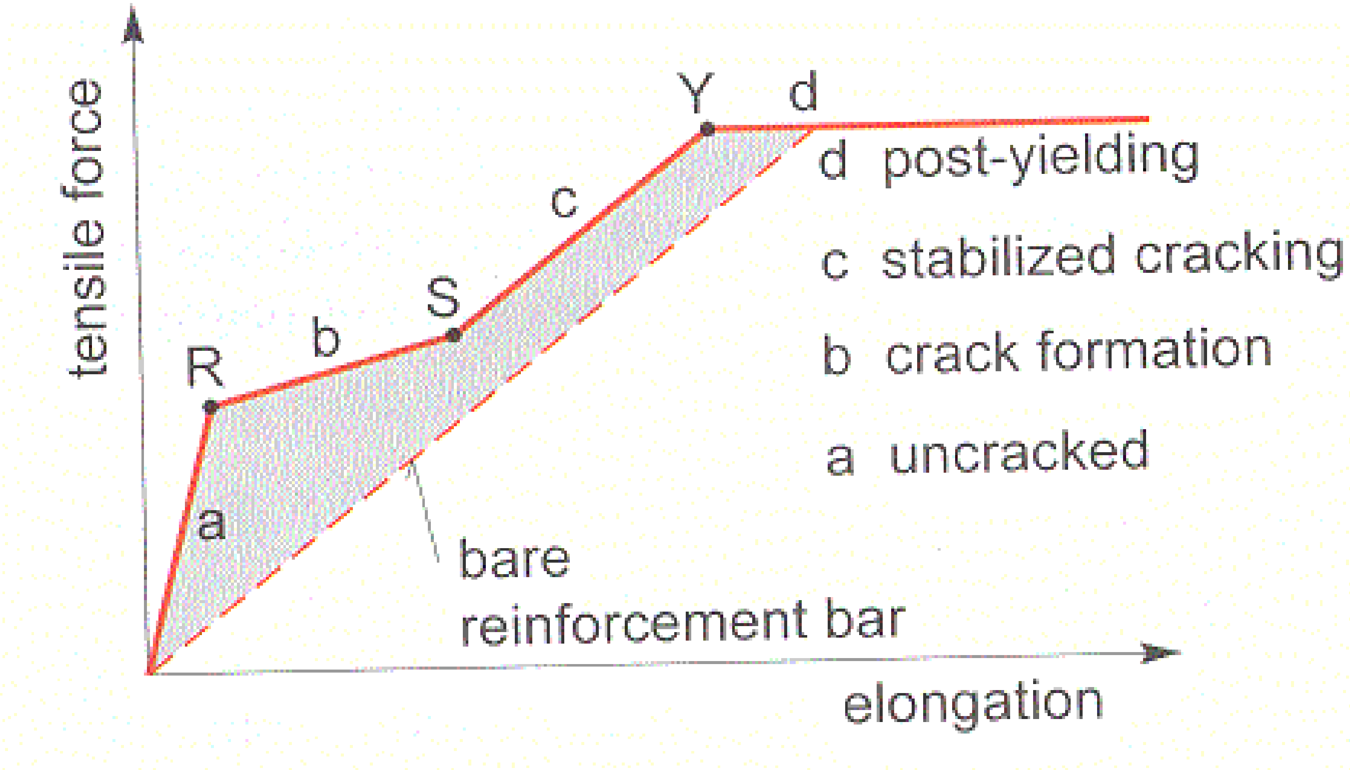
partial $\sigma_{cb\infty} > f_{ctm}$

b) Crack control

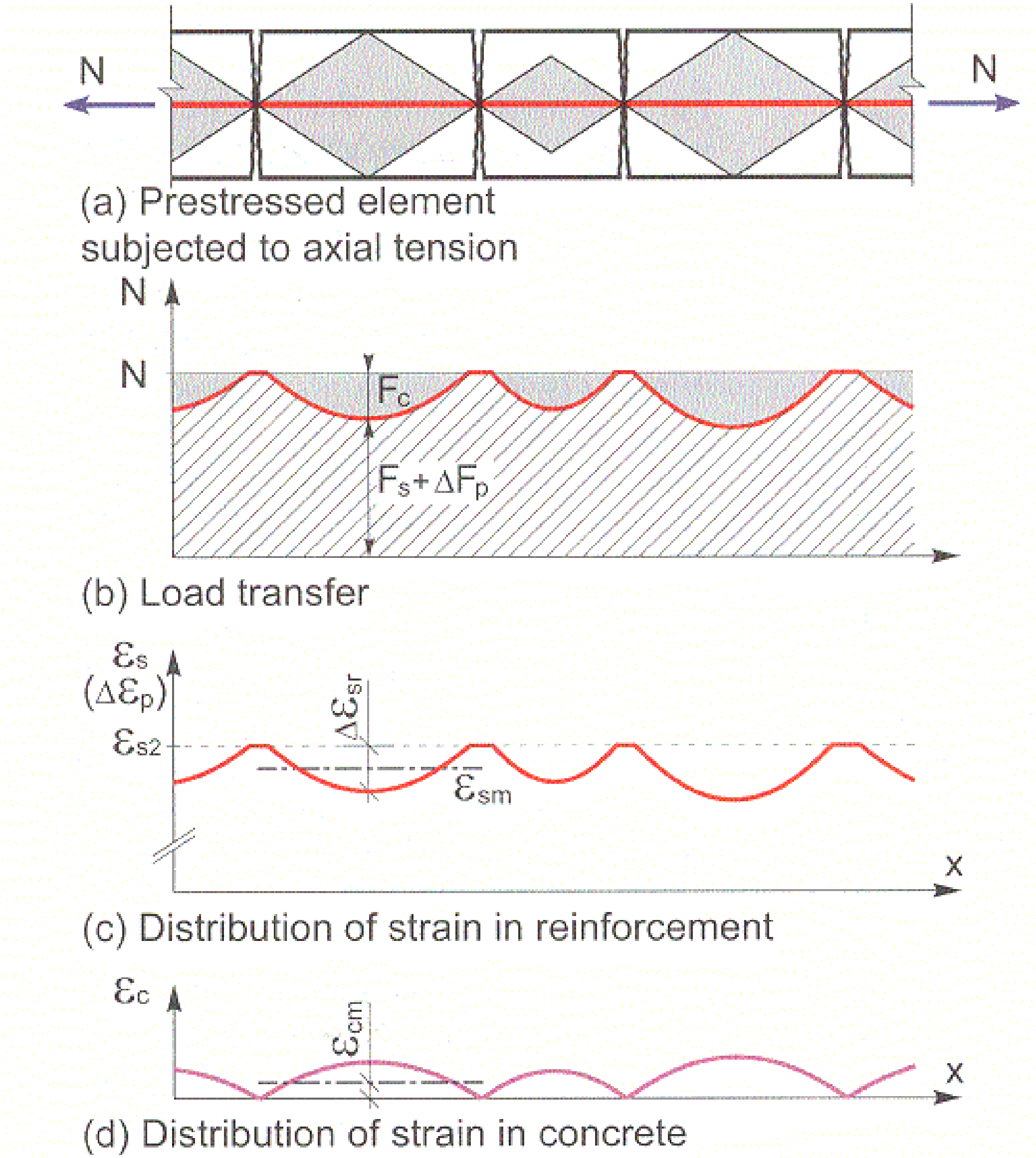


Crack formation

Formation of cracks



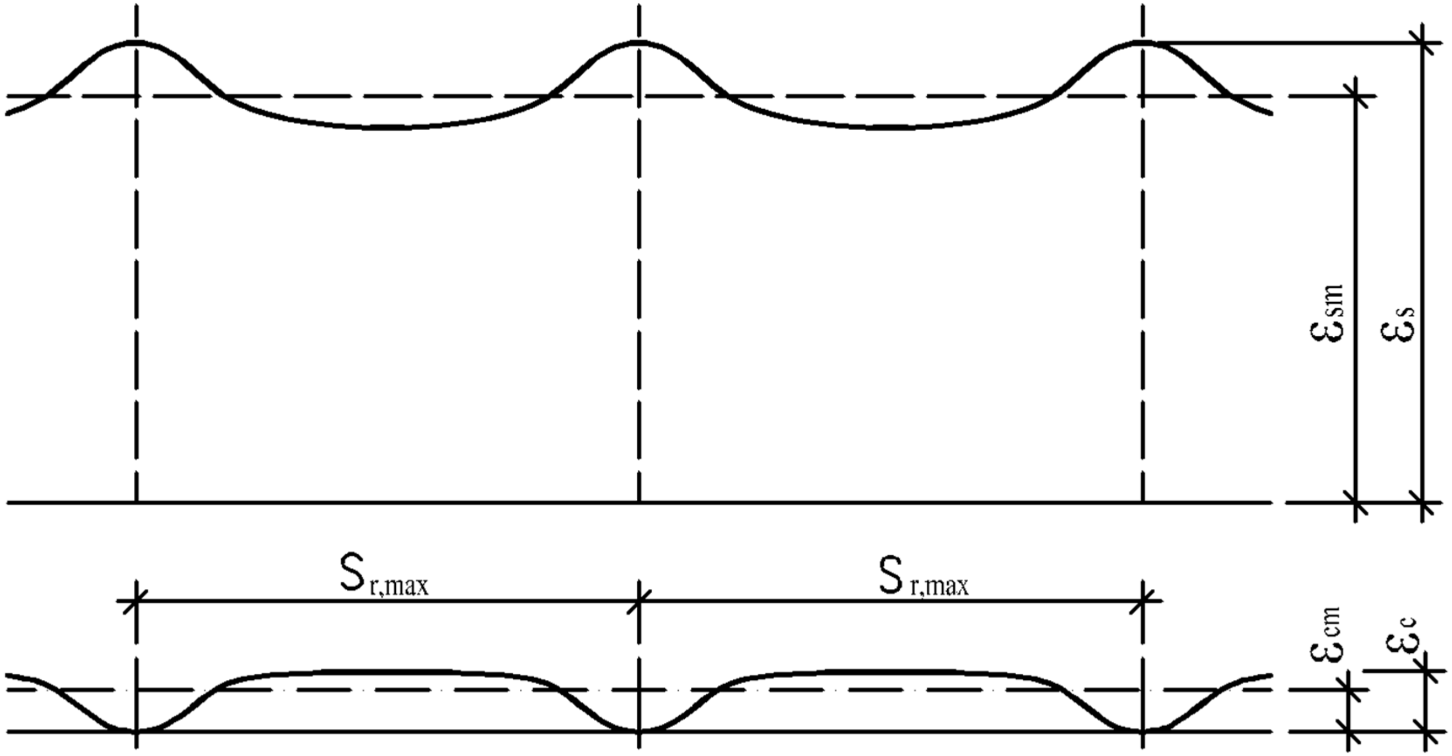
Idealized response of symmetrically reinforced element subjected to axial tension



$$w = \int_{s_s} \left(\varepsilon_{sx} - \varepsilon_{cx} \right) dx$$

Force and strain distribution in the phase of stabilized cracking

EN 1992-1-1



$$w_k = s_{r,\max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

$$w_k = s_{r,max}\,(\varepsilon_{sm} - \varepsilon_{cm})$$

$S_{r,max}$ maximum crack spacing,

ε_{sm} mean strain in the reinforcement under relevant combination of loads

ε_{cm} the mean strain in the concrete between cracks

$$s_{r,max} = k_3\, c + k_1 k_2\, k_4\, \phi / \rho_{p,eff}$$

c the cover to the longitudinal reinforcement

ϕ the bar diameter

$\rho_{p,eff} = (A_s + \xi_1^2 A_p')/A_{c,eff}$

$\xi_1 = \sqrt{(\xi\, \phi_s / \phi_p)}$; prestressing steel only $\xi_1 = \sqrt{\xi}$

ξ ratio bond reinforcement – see next

ϕ_s largest bar diameter of reinforcing steel

ϕ_p equivalent diameter of tendon,

$\phi_p = 1{,}6\sqrt{A_p}$ for bundles, A_p area of tendon

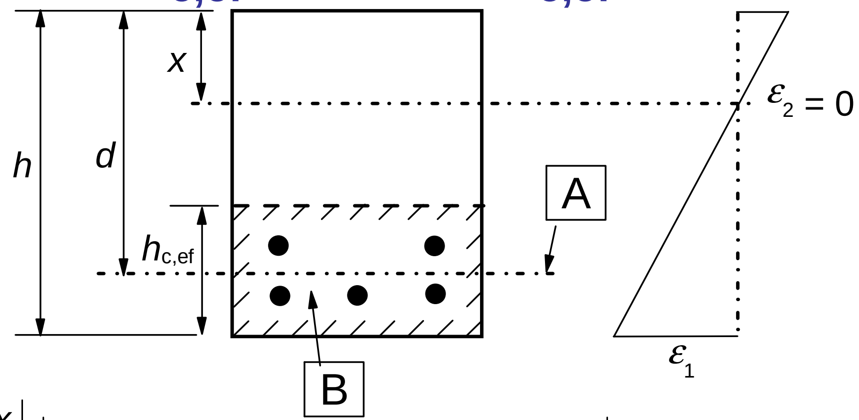
$\phi_p = 1{,}75\ \phi_{wire}$ 7 wire strand, $\varnothing_{wire}$ wire ϕ

Ratio of bond strength, ξ, between tendons and reinforcing steel

	ξ		
prestressing steel	pre-tensioned	bonded, post-tensioned	
		$\leq$ C50/60	$\geq$ C70/85
smooth bars and wires	Not applicable	0,3	0,15
strands	0,6	0,5	0,25
indented wires	0,7	0,6	0,3
ribbed bars	0,8	0,7	0,35
Note: For intermediate values between C50/60 and C70/85 interpolation may be used.			

A_p‘ is the area of pre or post-tensioned tendons within $A_{c,eff}$

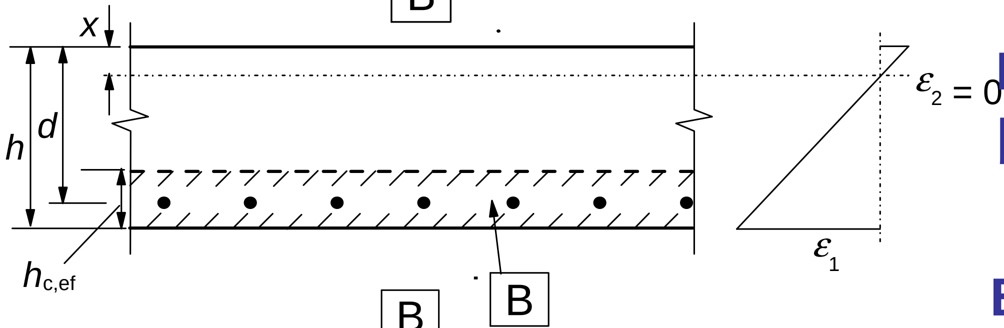
$A_{c,eff}$ the effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth, $h_{c,ef}$, where $h_{c,ef}$ is the lesser of 2,5(h-d), (h-x)/3 or h/2



A - level of steel centre of gravity

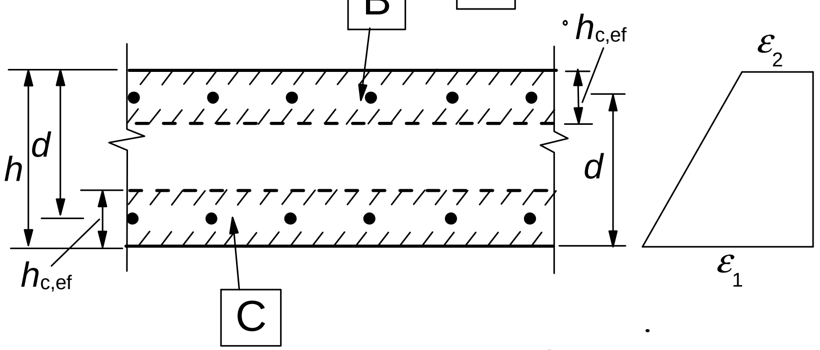
B - effective tension area, $A_{c,eff}$

a) Beam



B - effective tension area, $A_{c,eff}$

b) Slab



B - effective tension area for upper surface, $A_{ct,eff}$

C - effective tension area for lower surface, $A_{cb,eff}$

c) Member in tension

k_1 a coefficient which takes account of the bond properties of the bonded reinforcement:

= 0,8 for high bond bars

= 1,6 for bars with an effectively plain surface (e.g. prestressing tendons)

k_2 a coefficient which takes account of the distribution of strain:

= 0,5 for bending

= 1,0 for pure tension

For cases of eccentric tension or for local areas, intermediate values of k_2 should be used which may be calculated from the relation:

$$k_2 = (\varepsilon_1 + \varepsilon_2)/(2\varepsilon_1)$$

where ε_1 is the greater and ε_2 is the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section

$$\varepsilon_{sm} - \varepsilon_{cm} = \left(\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 + \alpha_e \rho_{p,eff}\right) \right) \frac{1}{E_s} \geq 0,6 \frac{\sigma_s}{E_s}$$

σ_s **the stress in tension reinforcement assuming a crack section; for pretensioned members, σs may be replaced by $\Delta\sigma_p$ the stress variation in prestressing tendons from the state of zero strain of the concrete at the same level**

$\rho_{p,eff}$ **the effective reinforcement ratio $\rho_{p,eff} = A_p{}' / A_{c,eff}$**

$A_{p'}$ **the area of the tension reinforcement in $A_{c,eff}$**

$A_{c,eff}$ **the effective tension area**

k_t **a factor dependent on duration of the load**

$k_t = 0,6$ for short term loading

$k_t = 0,4$ for long term loading

$$\boldsymbol{w_{\mathrm{k}} = s_{\mathrm{r,max}} \left(\varepsilon_{\mathrm{sm}} - \varepsilon_{\mathrm{cm}}\right)}$$

$$s_{\mathrm{r,max}} = k_3\, c + k_1 k_2\, k_4\, \phi / \rho_{p,eff}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \left(\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 + \alpha_e \rho_{p,eff}\right) \right) \frac{1}{E_s} \geq 0{,}6 \frac{\sigma_s}{E_s}$$

$$w_{\mathrm{k}} = \left(k_3\, c + k_1 k_2\, k_4\, \phi / \rho_{p,eff}\right)\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

$$w_{\mathrm{k}} = f\left(c;\ \phi;\ \rho_{p,eff};\ f_{ct,eff};\ \sigma_s; \rho_{p,eff}\right)$$

Calculation of the bar diameter ϕ

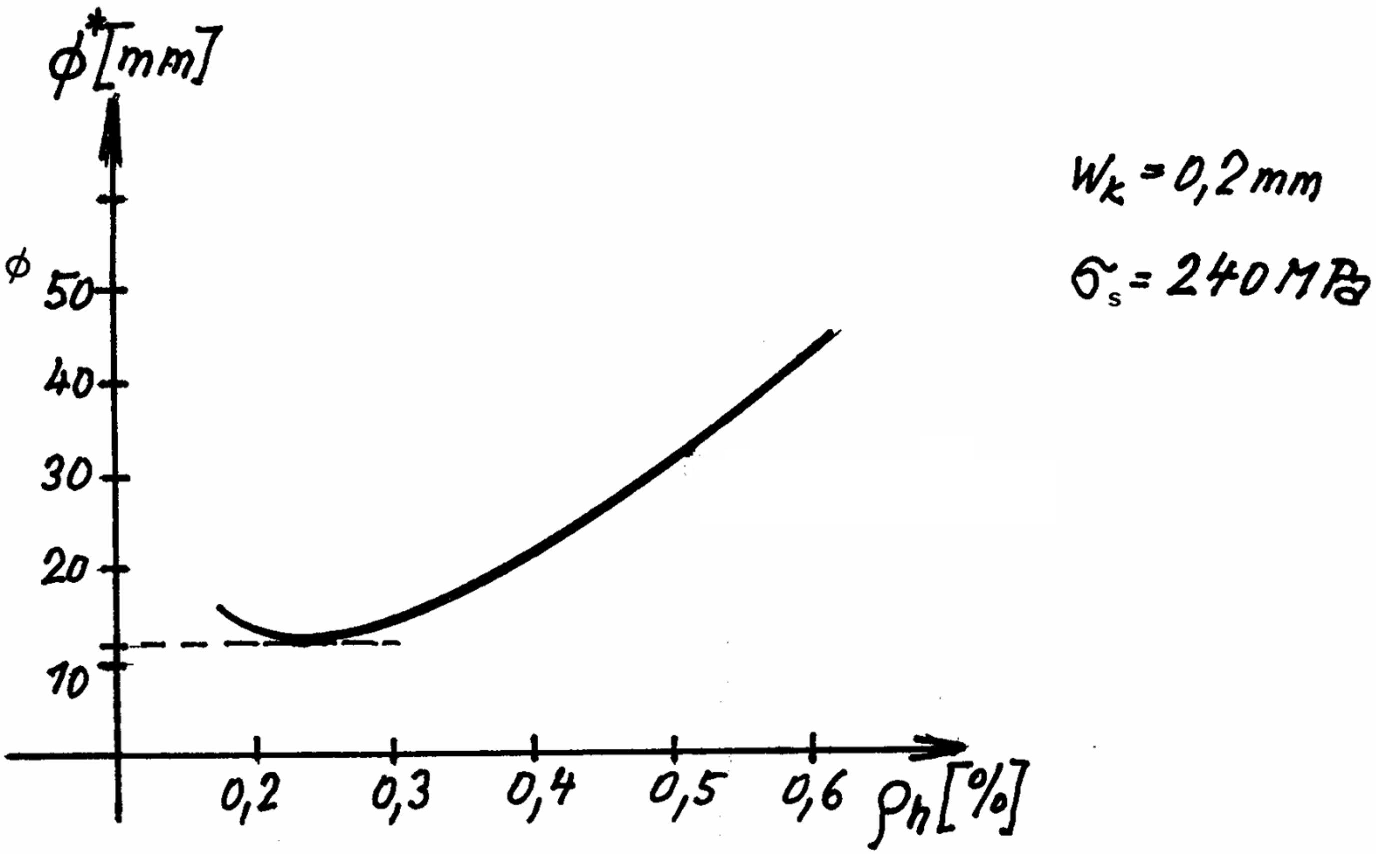
$$\rho_{p,eff} = A_s / \left(b\, h_{c,eff} \right) \qquad \rho_h = A_s / \left(b\, h \right)$$

$$\rho_{\mathrm{p,eff}} = \rho_{\mathrm{h}}\, h / h_{\mathrm{c,eff}}$$

$$\phi = \left\{ \frac{E_{\mathrm{s}}\ w_{\mathrm{k}}}{\sigma_s - k_{\mathrm{t}} \dfrac{f_{\mathrm{ct,eff}}}{\rho_{\mathrm{h}} \dfrac{h}{h_{\mathrm{c,eff}}}} \left(1 + \alpha_{\mathrm{e}}\ \rho_{\mathrm{h}} \dfrac{h}{h_{\mathrm{c,eff}}} \right)} - 3{,}4\, c \right\} \rho_{\mathrm{h}} \frac{h}{h_{\mathrm{c,eff}}} \frac{1}{0{,}425\, k_1\ k_2}$$

$$\phi = f\left(w_k ;\, f_{ct,eff} ; \sigma_s ; \rho_h \right)$$

c = 25 mm; $f_{ct,eff}$ = 2,9 MPa; $h_{c,eff}$ = 2,5 (h - d); h – d = 0,1 h;
k_1 = 0,8; k_2 = 0,5; k_t = 0,4



Maximum bar diameters ϕ_s^* for crack control[1]

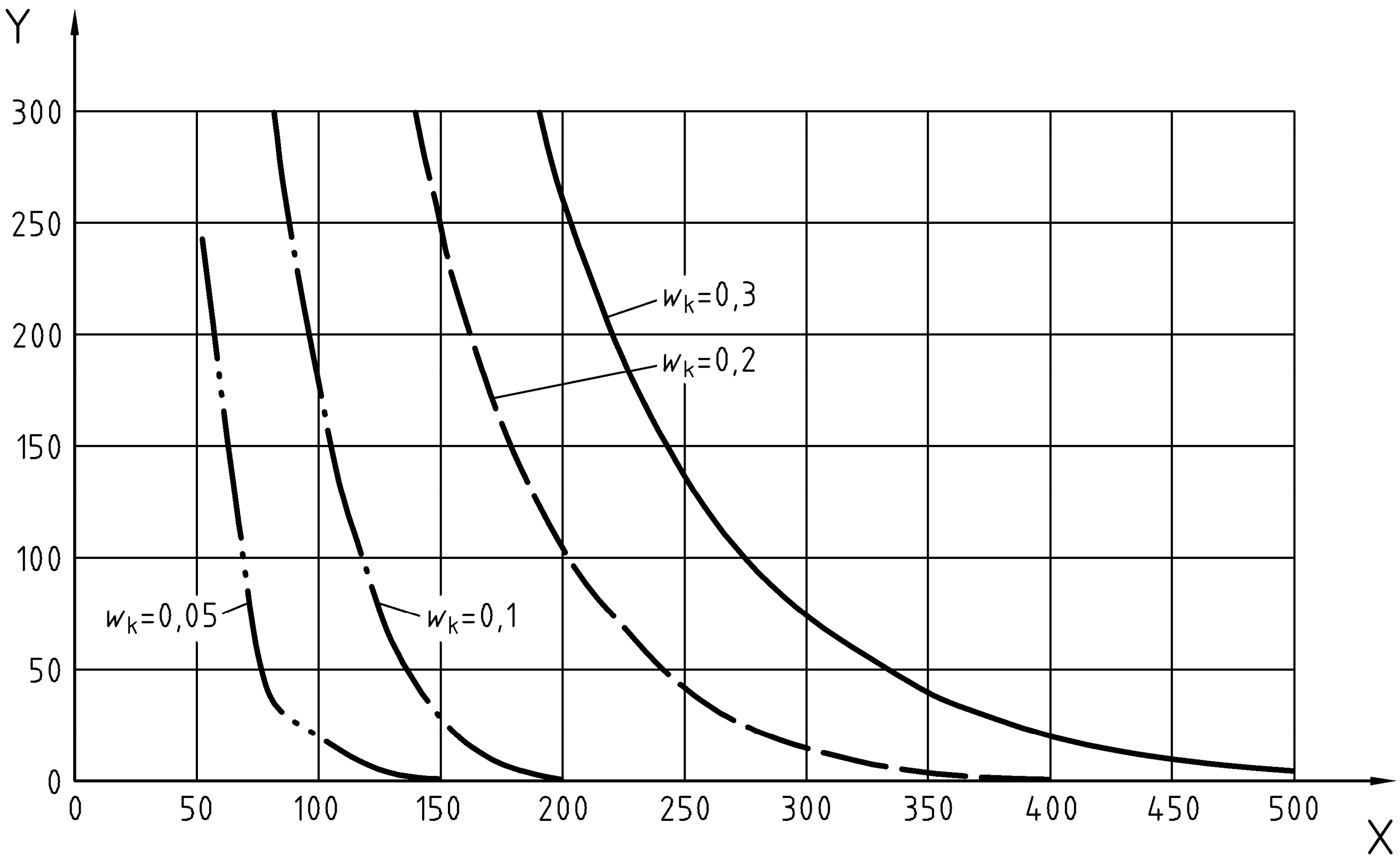
Steel stress[2] [MPa]	Maximum bar size [mm] w_k= 0,4 mm	w_k= 0,3 mm	w_k= 0,2 mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Notes: 1. The values in the table are based on the following assumptions:
c = 25mm; $f_{ct,eff}$ = 2,9MPa; h_{cr} = 0,5; $(h-d)$ = 0,1h; k_1 = 0,8; k_2 = 0,5; k_t = 0,4; k = 1,0

2. Under the relevant combinations of actions

Corrections: $\phi_s = \frac{f_{ct,eff}}{2,9}\frac{h_{cr}}{8(h-d)}\phi_s^*$ tension; $\phi_s = \frac{f_{ct,eff}}{2,9}\frac{k_c\, h_{cr}}{2(h-d)}\phi_s^*$ bending

EN 1992-3: Maximum bar diameters for crack control in members subjected to axial tension



Key X reinforcement stress, *s* (N/mm²)

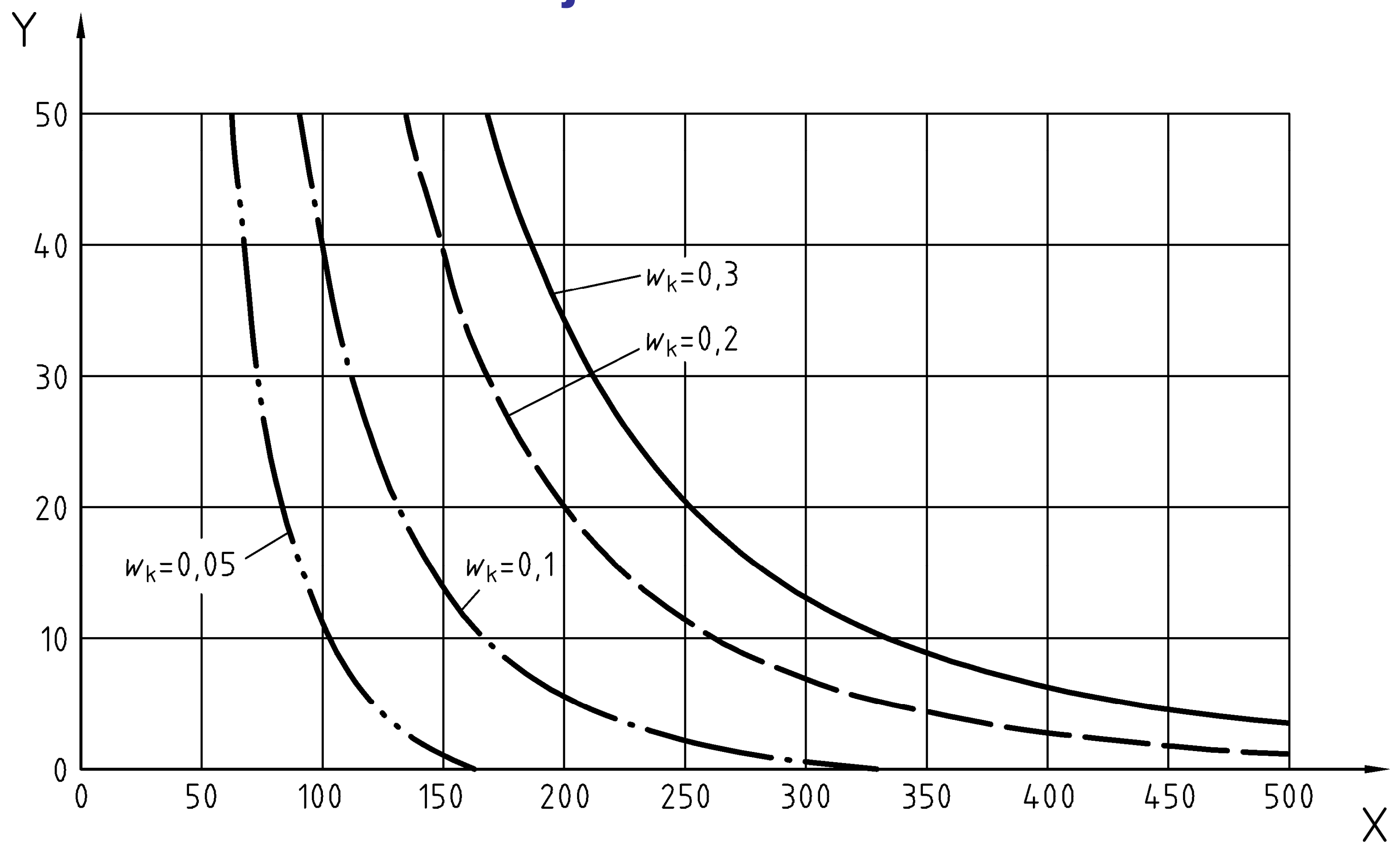
Y maximum bar diameter (mm)

$$\phi_s = \phi_s^* \frac{f_{ct,eff}}{2,9} \frac{h}{10(h-d)}$$

$$\rho_h = \pi\, \phi^2 / \left(4\, s\, h\right) \qquad s = \pi\, \phi^2 / \left(4\, \rho_\mathrm{h}\, h\right)$$

Steel stress σ_s [MPa]	**Maximum bar spacing *s* [mm]**		
	w_k = 0,4 mm	**w_k = 0,3 mm**	**w_k = 0,2 mm**
160	**300**	**300**	**200**
200	**300**	**250**	**150**
240	**250**	**200**	**100**
280	**200**	**150**	**50**
320	**150**	**100**	**-**
360	**100**	**50**	**-**

Maximum bar spacings for crack control in members subjected to axial tension



Key

X **reinforcement stress, *s* s (N/mm²)**

Y **maximum bar spacing (mm)**

- **The area of reinforcement by imposed deformation $(1+ \alpha_e \rho_{p,eff}) = 1$**

$$A_s = \frac{-K_b + \sqrt{K_b^2 - 4.K_a.K_c}}{2K_a}$$

kde

$$K_a = E_s w_k \qquad K_b = -3,4c\left(F_s - 0,4F_{cr}\right)$$

$$K_c = -0,425.k_1.k_2.\phi.A_{c,eff}\left(F_s - 0,4F_{cr}\right)$$

F_{cr} $= A_{c,eff}.f_{ct,eff}$ the force in effective area
F_s the tension force transmitted by reinforcement,
$f_{ct,eff}$ the effective tensile strength of concrete at the time when we can expect the rise in concrete; by the escape of hydratation heat we can assume
$f_{ct,eff} = 0,5\ f_{ctm}$,

- **E_s = 200 000 Mpa modulus of elasticity of steel,**
- **w_k characteristic width of crack.**

$$K_a = E_s w_k - 3,4k_t c f_{ct,eff}\alpha_e \quad K_b = -3,4c\left(F_s - 0,4F_{cr}\right) - 0,425k_1k_2k_t\phi F_{cr}\alpha_e$$

$\rho_{p,eff} = (A_s + \xi_1^2 A_p')/A_{c,eff}$

$\xi_1 = \sqrt{(\xi \varnothing_s / \varnothing_p)}$; **prestressing steel only** $\xi_1 = \sqrt{\xi}$

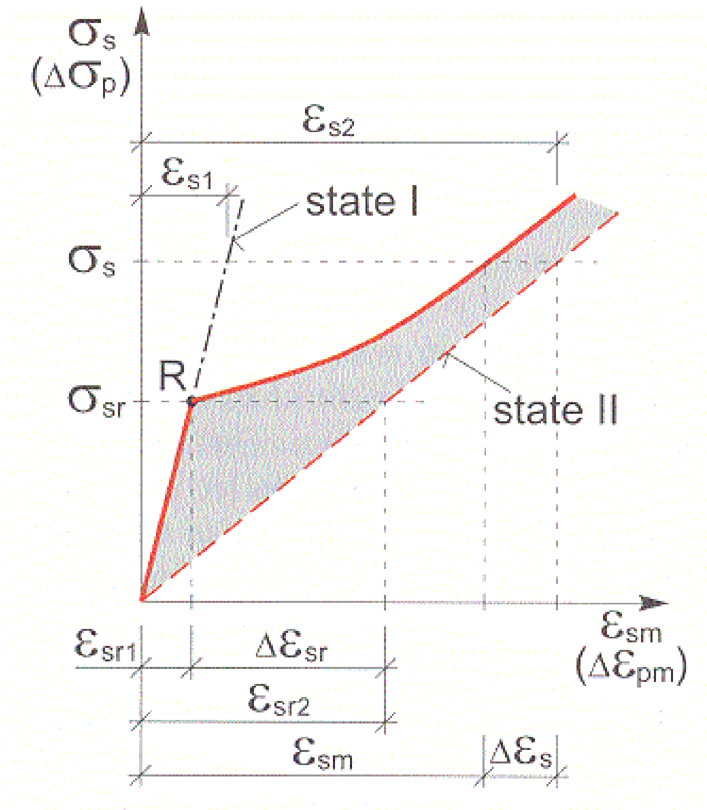
$\varnothing_s$ **largest bar diameter of reinforcing steel**

$\varnothing_p$ **equivalent diameter of tendon,**

$\varnothing_p = 1{,}6 \sqrt{A_p}$ **for bundles,** A_p **area of tendon**

$\varnothing_p = 1{,}75\ \varnothing_{wire}$ **7 wire strand,** $\varnothing_{wire}$ **wire** $\varnothing$

c) Deflection control



Simplified stress-strain relationship of embedded reinforcement

Members which are expected to crack a <u>deformation parameter may be calculated</u>

$$\alpha = \zeta\alpha_{II} + (1 - \zeta)\alpha_{I}$$

α **is the deformation parameter (strain, a curvature, a rotation, or may be a deflection)**

α_{I}, α_{II} **are the values of the parameter calculated for the uncracked and fully cracked conditions respectively**

ζ **a distribution coefficient (allowing for tensioning stiffening at a section)**

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_{s}}\right)^{2}$$

ζ **= 0 for uncracked section**

β is a coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain

= 1,0 for a single short-term loading

= 0,5 for sustained loads or many cycles of repeated loading

σ_s the stress in the tension reinforcement calculated on the basis of a cracked section

σ_{sr} the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking

Note: σ_s/σ_{sr} may be replaced by M_{cr}/M for flexure or N_{cr}/N for pure tension, where M_{cr} is the cracking moment and N_{cr} is the cracking force.

For loads with a duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete

$$E_{c,eff} = \frac{E_{cm}}{1+\varphi\left(\infty,t_0\right)}$$

$\varphi(\infty,t_0)$ **is the creep coefficient relevant for the load and time interval**

Shrinkage curvatures **may be assessed using expression**

$$\frac{1}{r_{cs}} = \varepsilon_{cs}\alpha_e \frac{S}{I}$$

$1/r_{cs}$ **is the curvature due to shrinkage**

ε_{cs} **the free shrinkage strain**

S **the first moment of area of the reinforcement about the centroid of the section**

I **the second moment of area of the section**

α_e **the effective modular ratio**