

Prestressed Concrete

Part 4

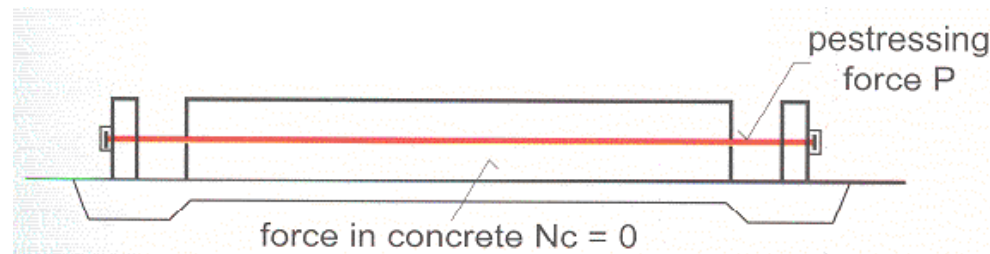
(Losses of prestress)

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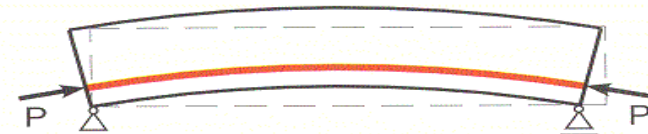
Department of Concrete and Masonry Structures

Prestressing force

- Prestressing force varies along the length of the tendon and also over the time
- Prestressing force significantly influences the behaviour of the structure
- Necessary to know the value of prestressing force in every point of the tendon
- We derive it from the stress of reinforcement before anchoring in the section at the stressed end

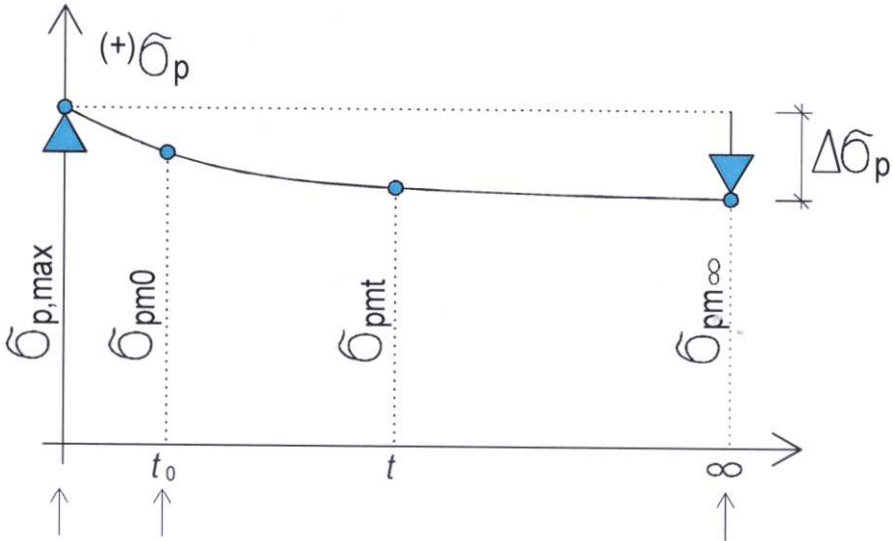
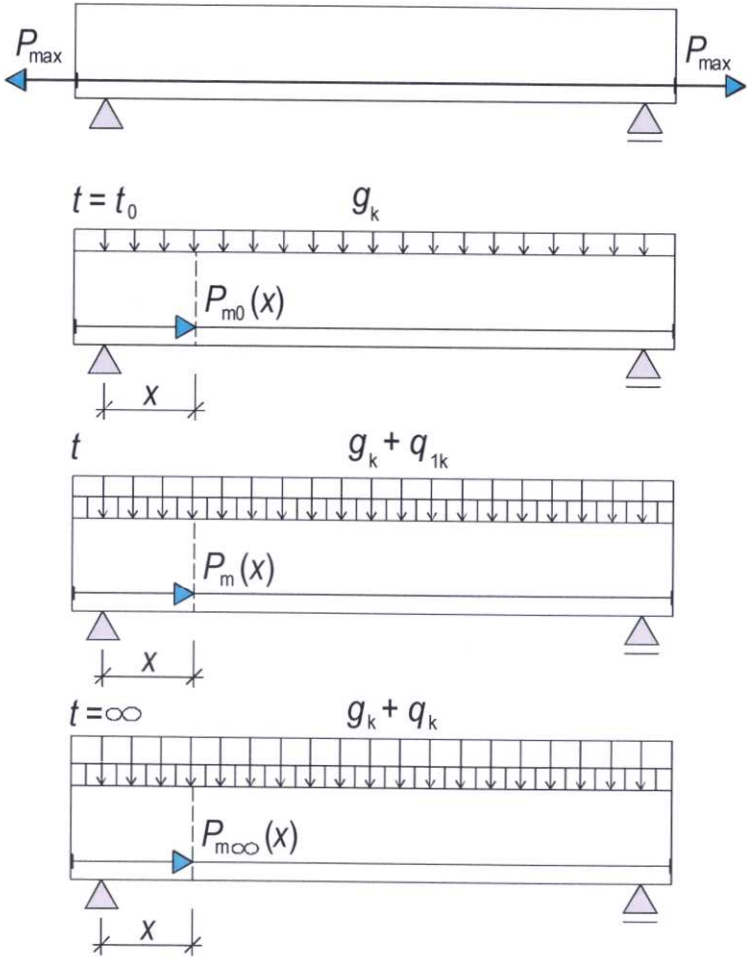


stress before anchoring of reinforcement




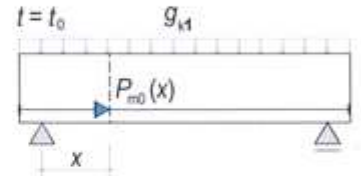
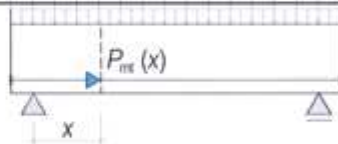
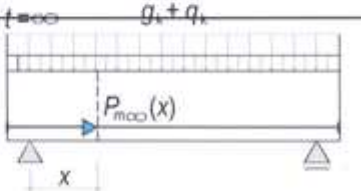
stress introduced by the jack

Mean value of prestressing force at the end of member

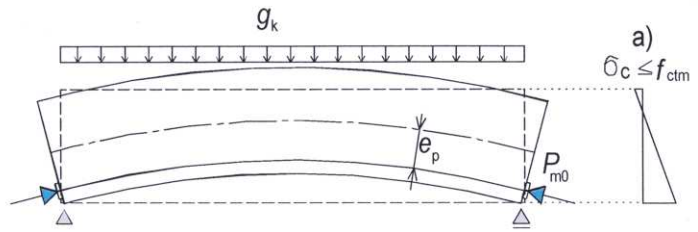
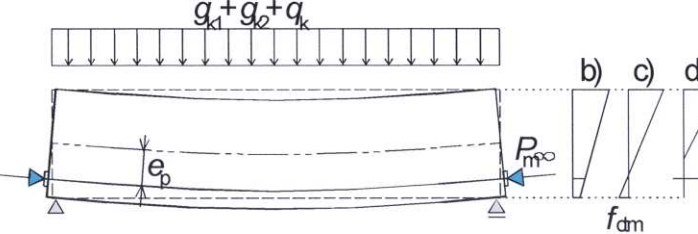
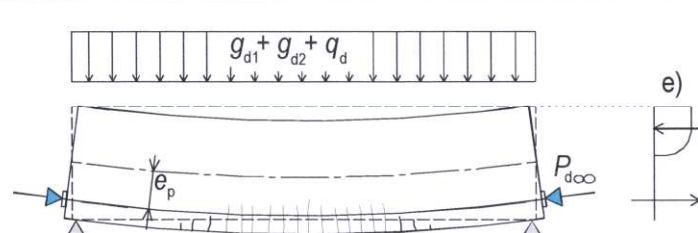


mean value of prestressing stress at the end of member

Mean values of prestressing force and stress during time

time	scheme	force *)	stress *)	limitation of stress
stretching $t = 0$		P_{max}	$\sigma_{p,max}$	$\sigma_{p,max} \leq \min(0,8 f_{pk}; 0,9 f_{p0,1k})$
after anchoring $t = t_0$		$P_{m0}(x)$	$\sigma_{pm0}(x)$	$\sigma_{pm0}(x) \leq \min(0,75 f_{pk}; 0,85 f_{p0,1k})$
during service life t		$P_{mt}(x)$	$\sigma_{pmt}(x)$	
after service life $t = \infty$		$P_{moo}(x)$	$\sigma_{pmt}(x)$	$\sigma_{pmoo}(x) \leq 0,75 f_{pk}$
*) mean values acting in tendons at time t and distance x				

Scheme of considering states of prestressed members

State	Scheme	Prastressing force	Criteria - check
<p style="text-align: center;">Stadium of prestressing</p> <p style="text-align: center;">SLS</p>		$P_{ki} = r_i P_{m0}$	<p style="text-align: center;">stress in concrete σ_c and in prestressing steel σ_p</p>
<p style="text-align: center;">Stadium of service</p> <p style="text-align: center;">SLS</p>		$P_{ki} = r_i P_{m0}$	<p style="text-align: center;">stress in concrete σ_c and in prestressing steel σ_p width of crack w_k deflection f_d</p>
<p style="text-align: center;">ULS</p>		$P_{d00} = \gamma_p P_{m0}$	<p style="text-align: center;">ultimite resistance</p> <p style="text-align: center;">M_{Rd} N_{Rd} V_{Rd}</p>
<p>a) to e) distribution of stresses in cross section ; r_i - allowance for possible variation in prestress</p>			

- **Maximum stress in tendon before anchoring** (at the active end during prestressing, acts for a short time period - EN 1992-1-1):

$$\sigma_{p,\max} = \min(0,8 f_{pk} ; 0,9 f_{p0,1k})$$

f_{pk} characteristic tensile strength of prestressing reinforcement

$f_{p0,1k}$ characteristic 0,1% proof-stress of prestressing reinforcement

- **Changes occur at the time of prestressing, during anchoring and after anchoring (short and long term losses) along the length of the tendon**

Short-term (immediate) losses due to:

- **friction between tendon and wall of duct,**
- **anchorage set loss (friction anchorage),**
- **immediate elastic strain in the concrete during stressing,**
- **sequential prestressing (phases of prestress.),**
- **relaxation of prestressing reinforcement (before transfer),**
- **deformation of end abutments of stressing bed,**
- **differences in temperature of prestressing reinforcement end stressing bed,**
- **bearing pressure on concrete – circumferential tendons with small radius of curvature**

Long-term (service life) losses caused by:

- **relaxation of prestressing reinforcement (after transfer),**
- **shrinkage of concrete,**
- **creep of concrete,**
- **creep of concrete due to many repeated cyclic load,**
- **immediate elastic strain in concrete due to variable load**

Simplifying assumptions for calculation of losses:

- Concrete and prestressing steel for short term effects of load – assume ideally elastic matters. Steel stress reaches the highest value at the instant of stressing, under low stresses behaves linearly. Instantaneous inelastic concrete strain is very small.
- Perfect bond between concrete and steel only for bonded prestressing steel (the strain of adjoining steel fibre and concrete fibre are the same).
- Prestressing force acts in the centre of gravity of the area of prestressing reinforcement.
- Short-term losses are calculated separately.

Transformed (idealized) cross section

Fictitious substitute cross section consisting of:

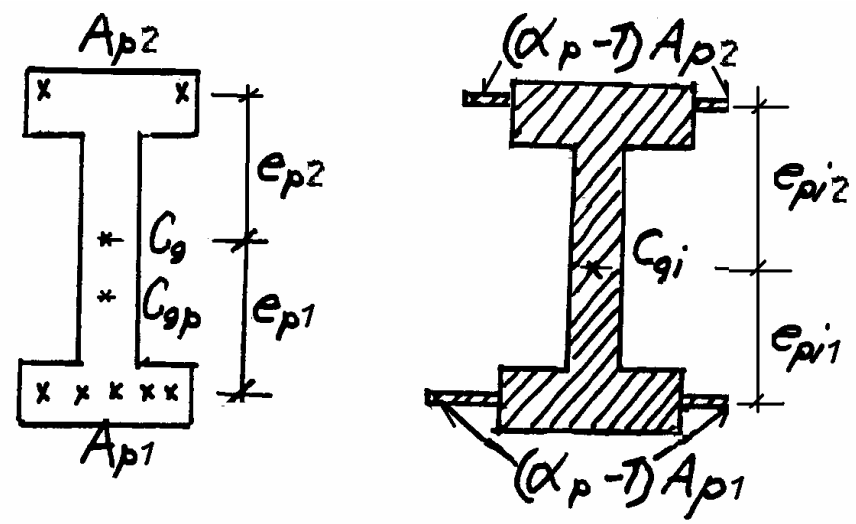
- a) concrete compressed part of the cross section,
- b)- α_p multiple of sectional area of prestressing reinforcement,
- c)- α_e multiple of sectional area of reinforcement,

where $\alpha_p = E_p/E_{cm}$; $\alpha_e = E_s/E_{cm}$

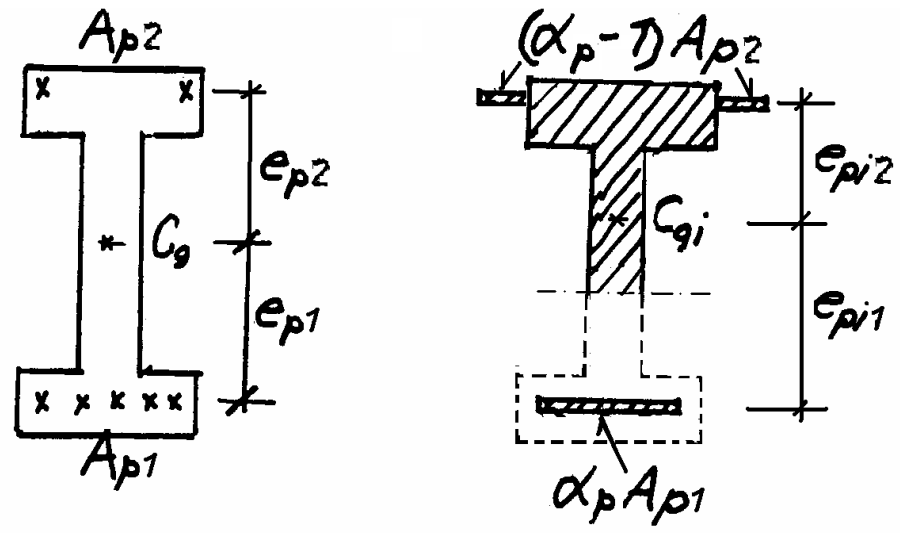
Calculate:

$$A_i = A_c + \alpha_p A_p + \alpha_e A_s \quad ; \quad I_i \quad ; \quad W_i \quad \text{etc.}$$

Idealized cross section – acts full concrete cross section

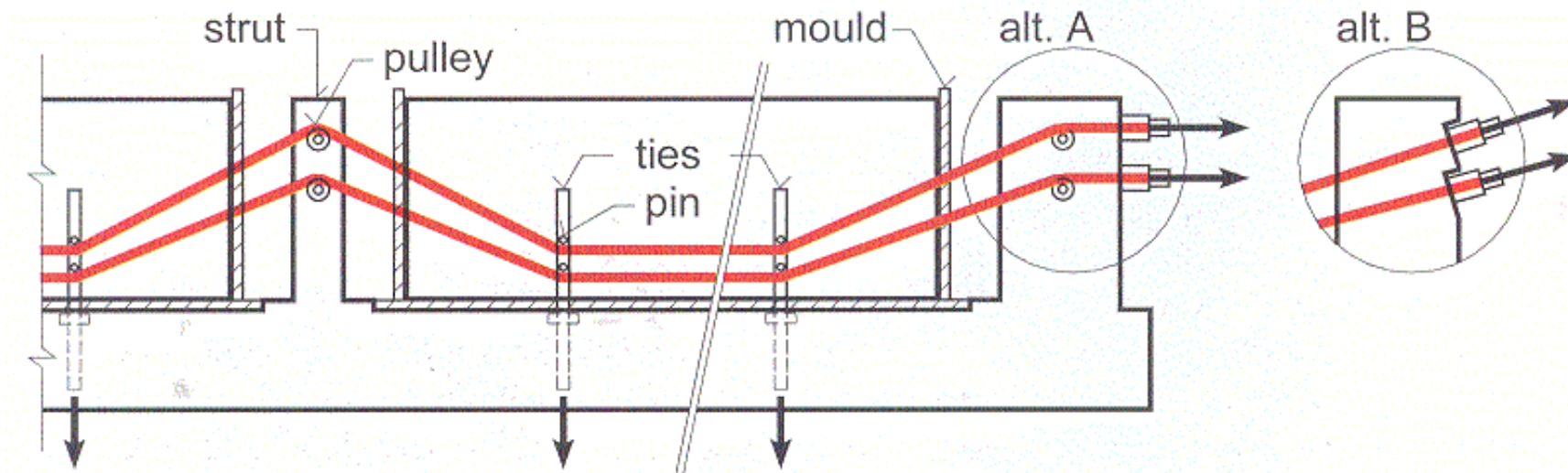


Idealized cross section – acts part of concrete cross section

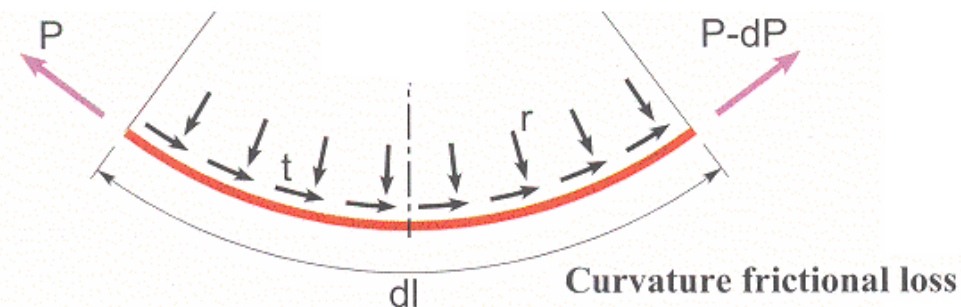


Loss of prestressing due to friction

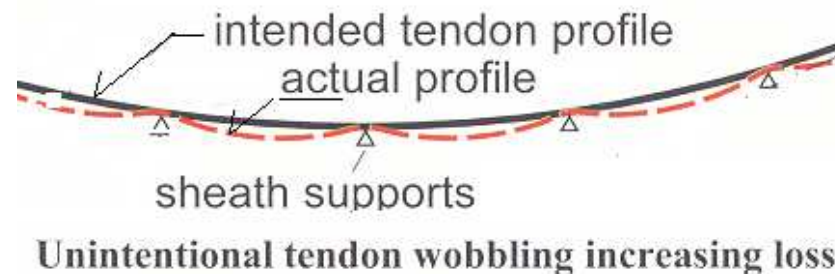
- Pre-tensioned prestressed concrete – rarely, (only at elevating prestressing steel)



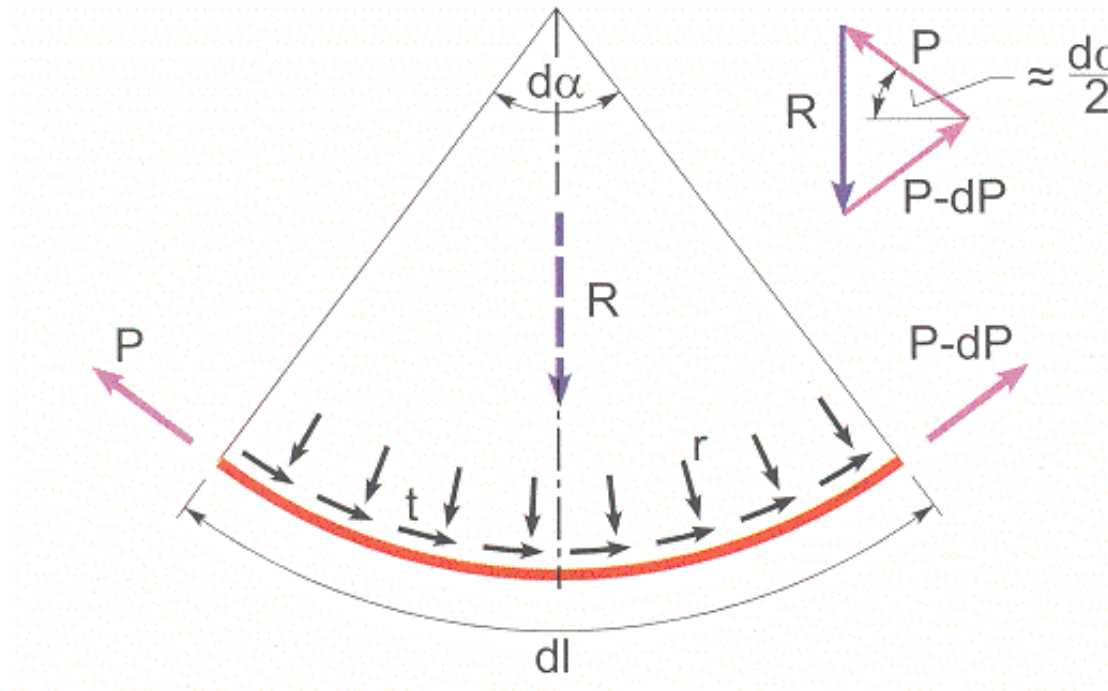
- Post-tensioned prestressed concrete –especially, during the stressing occurs friction between the tendon and wall of the tendon duct
- The loss consists of two components:
 - curvature frictional loss (curved part of tendon)



- wobble frictional loss



Intentional change of direction – curvature frictional loss



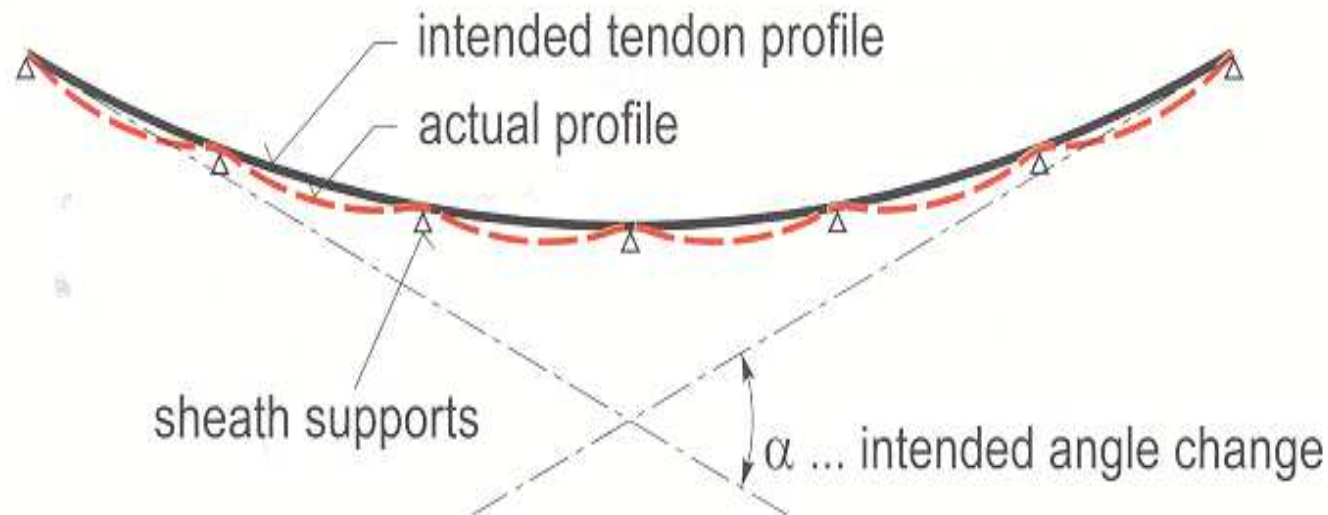
$t = \mu r$; μ is the friction coefficient;

r the force per unit length perpendicular to tendon

radial force R is in equilibrium with forces $P \Rightarrow R \approx P d\alpha$

$$dP = -\mu P d\alpha$$

Unintentional change of direction – wobble frictional loss



Tendon duct fixed by spacers – between them the duct deforms due to its self-weight and to occasional unevenness, thus forming an unintentional wobbles

**Unintentional angular change over the length d/l is expressed as $k d/l$;
 k – empirically determined unintended angular change per unit length of the tendon**

$$dP = -\mu P d\alpha - \mu P k dl :$$

The total change in prestressing force between the points A and B

$$\int_{P_A}^{P_B} \frac{dP}{P} = -\mu \int_0^{\alpha} d\alpha - \mu k \int_0^l dl = -\mu \left(\int_0^{\alpha} d\alpha - k \int_0^l dl \right)$$

P_A is the prestressing force in point A,

P_B is the prestressing force in point B,

α is the total intended angular change along the length A and B,

L is the total length of the tendon between the points A and B

Note: L can be substitute by perpendicular projection of the tendon into the horizontal axis l

From the solution we receive $P_B = P_A e^{-\mu(\alpha - kl)}$

From the difference of forces we obtain loss and then the loss of prestressing stress related to the stress at stressed end $\sigma_{p0,0}$

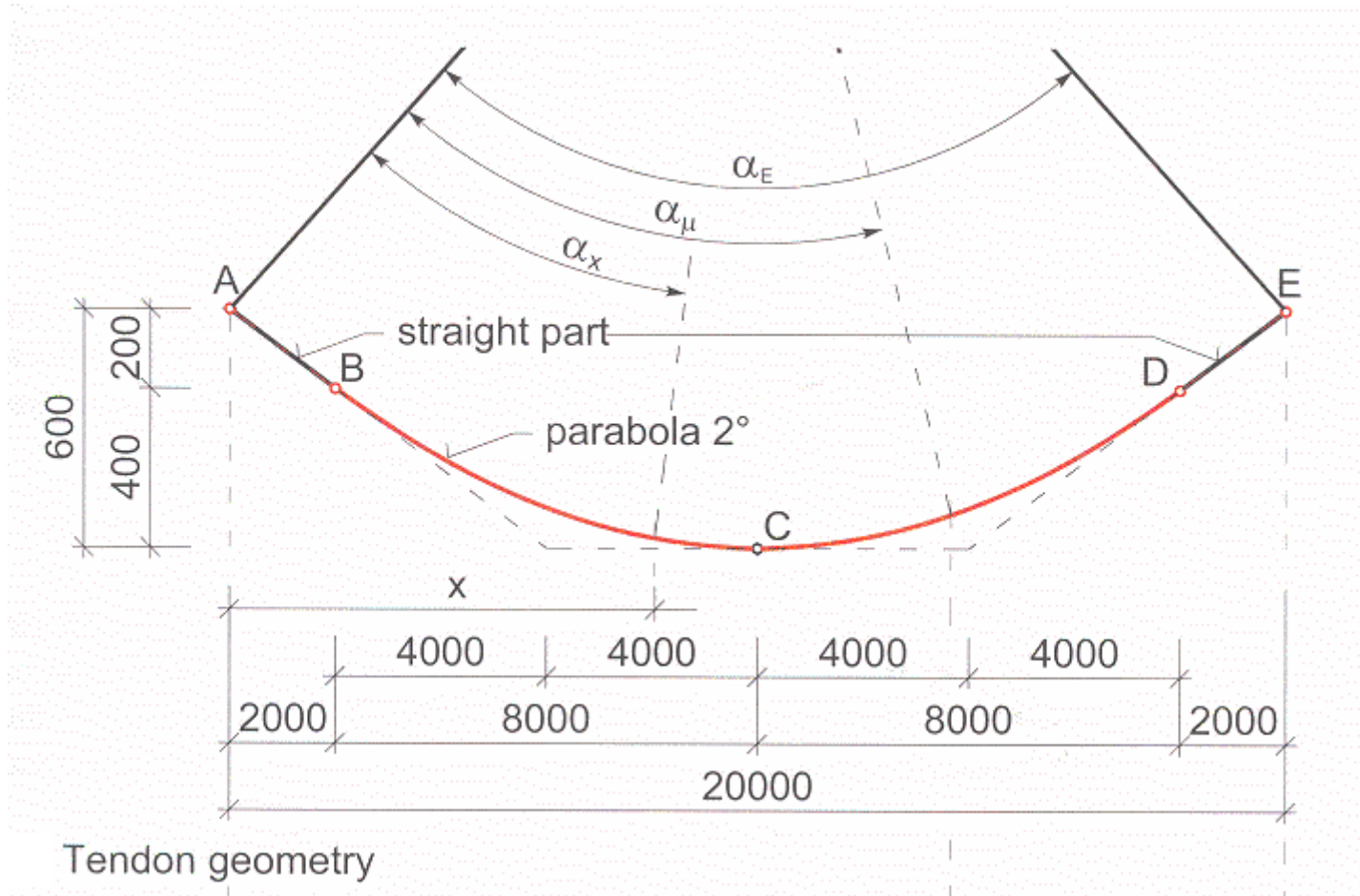
$$\Delta\sigma_{p\mu}(l) = -\sigma_{p0,0} \left(1 - e^{-\mu(\alpha + kl)} \right)$$

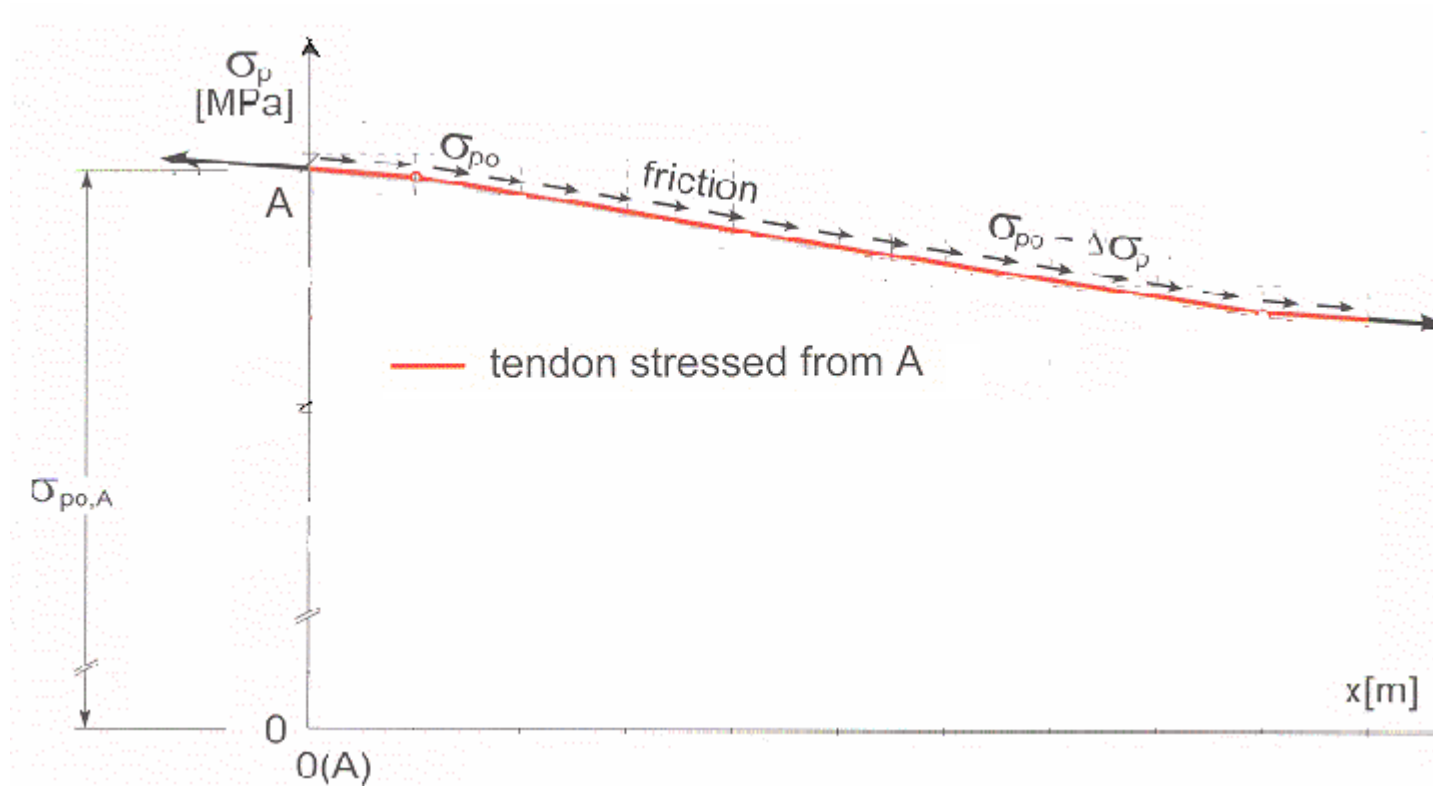
Coefficients of friction μ of post-tensioned internal tendons and external unbonded tendons

	Internal tendons 1)	External unbonded tendons			
		Steel duct/ non lubricated	HDPE duct/ non lubricated	Steel duct/ lubricated	HDPE duct/ lubricated
Cold drawn wire	0,17	0,25	0,14	0,18	0,12
Strand	0,19	0,24	0,12	0,16	0,10
Deformed bar	0,65	-	-	-	-
Smooth round bar	0,33	-	-	-	-
1) for tendons which fill about half of the duct					

Note: HPDE - High density polyethylene

Values k for unintended regular displacements for internal tendons will generally be in the range $0,005 < k < 0,01$ per meter

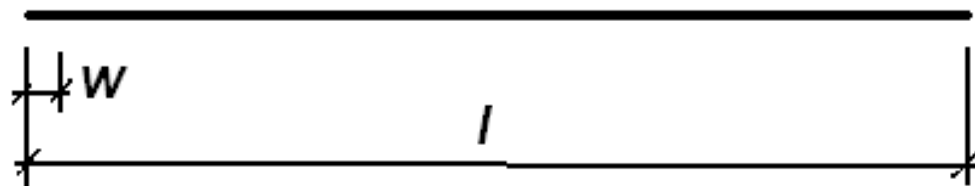




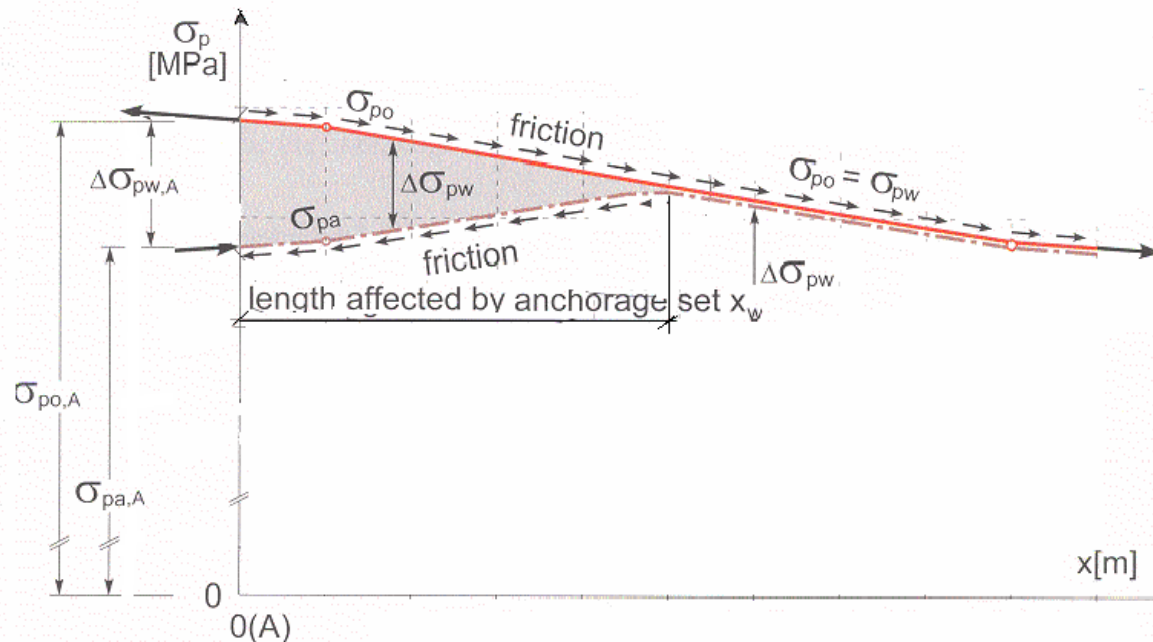
Anchorage set loss

- Drawn-in the wedge and the strand in the anchor head result in the reduction of stress in the prestressing reinforcement – anchorage set loss
- Anchorage set loss - friction not taken in account
pre-tensioned prestressed concrete
 - set w decreases elongation Δl of the stressed tendon with the length $l \Rightarrow$ anchorage set loss

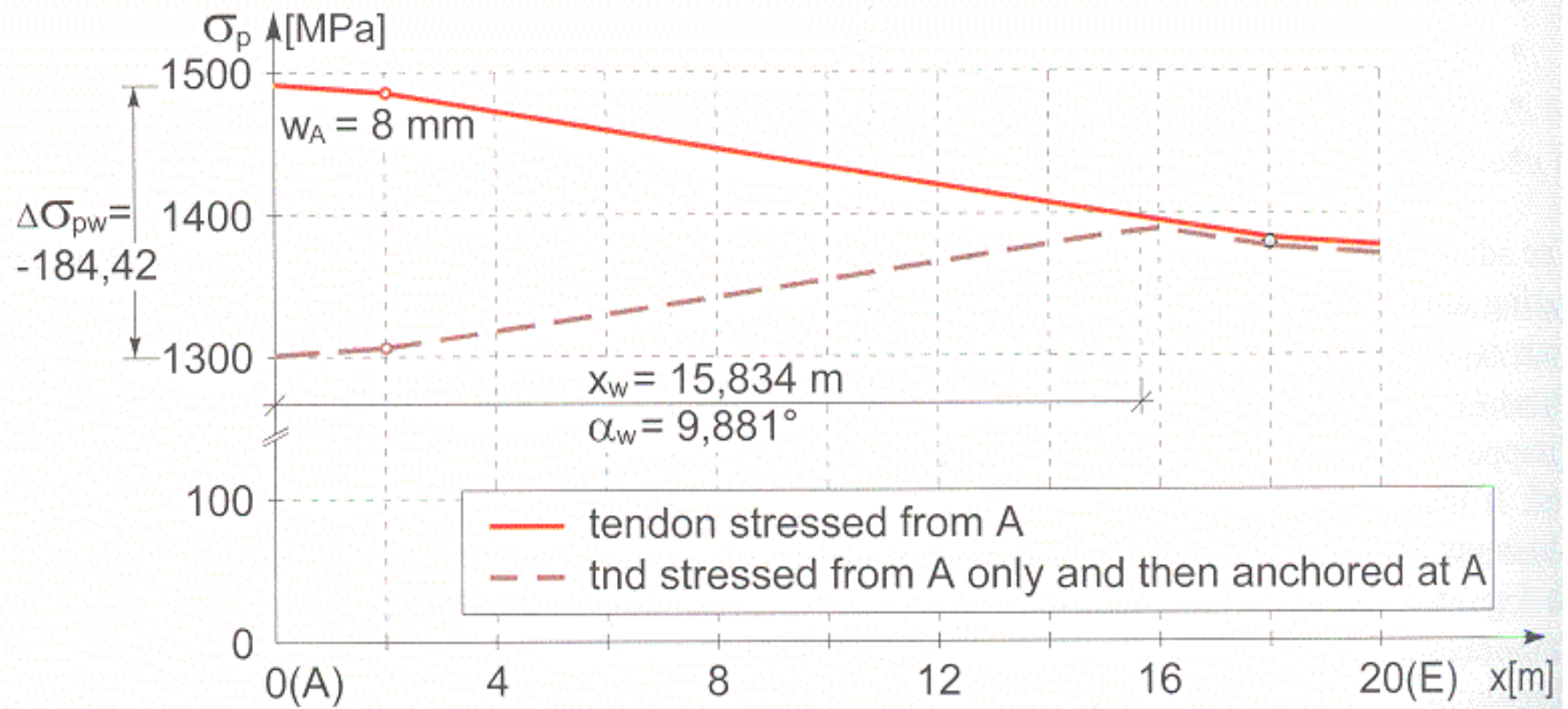
$$\Delta\sigma_{pw} = -\frac{wE_p}{l}$$



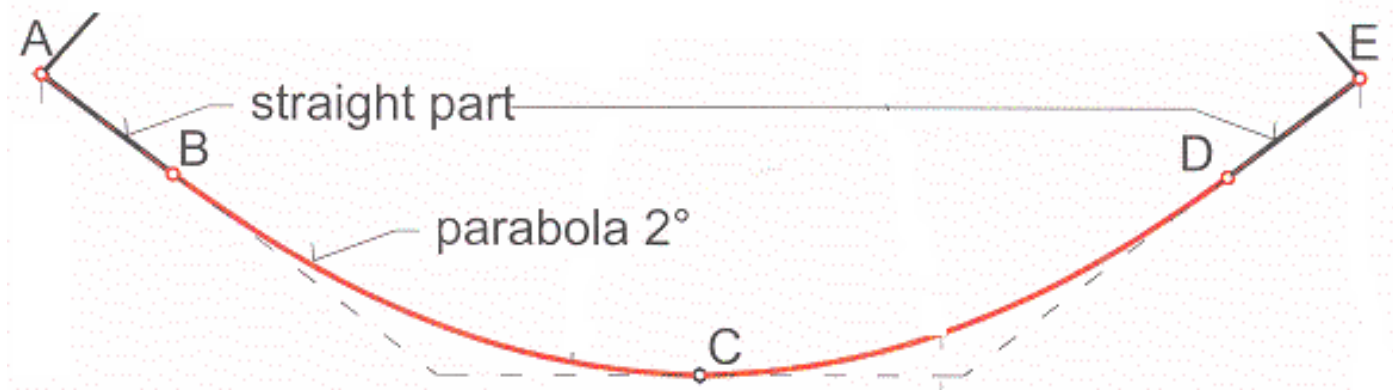
- **Anchorage set loss when tendon is stressed from one end only – friction take into account**
- Post-tensioned prestressed concrete
- When the tendon is anchored, stress $\sigma_{p0,A}$ on the stressed end decreases due to movement of the tendon to the non-stressed anchor \Rightarrow arises the friction in the opposite direction

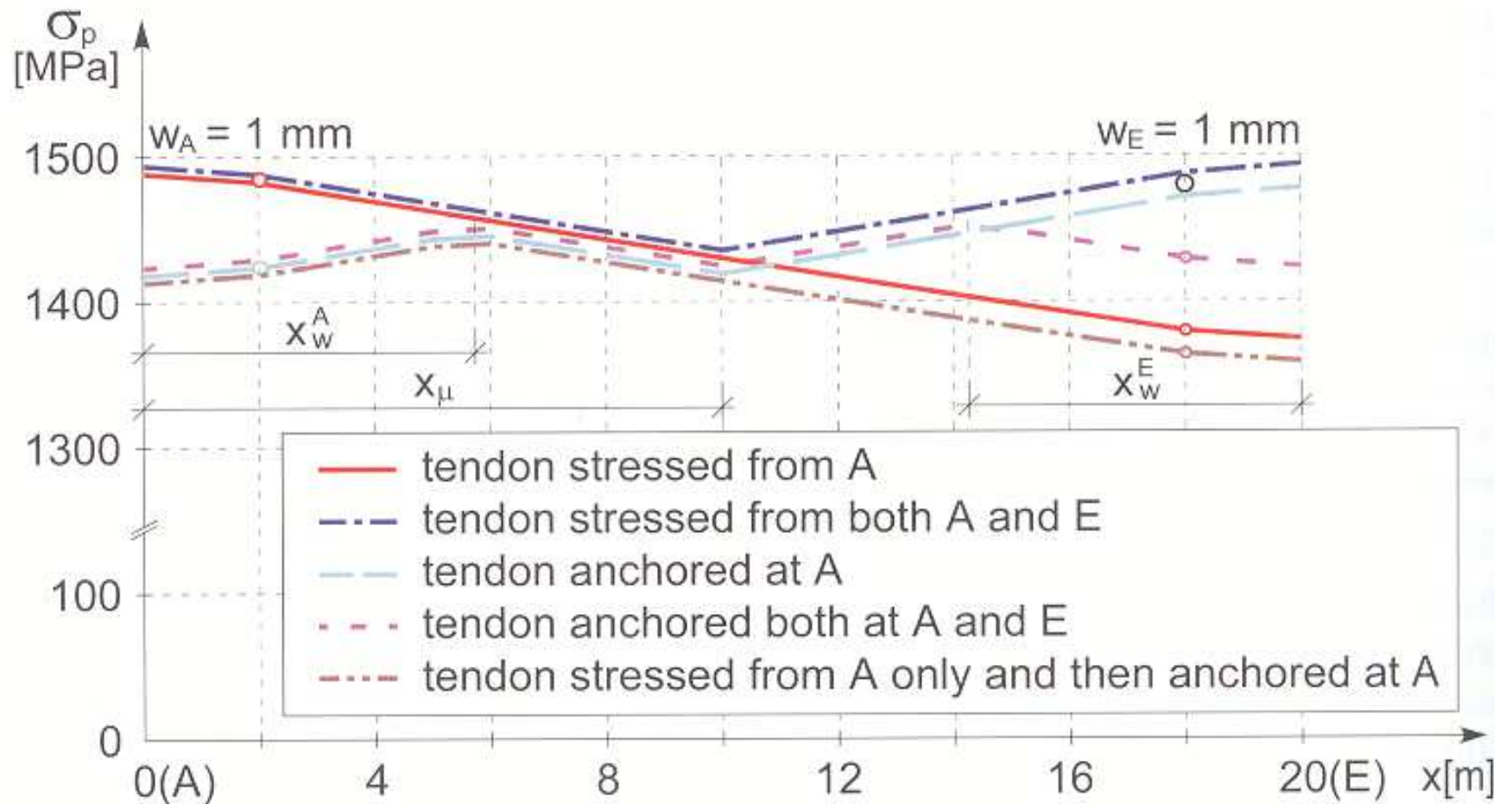


$$\Delta\sigma_{pw} = \sigma_{pa,A} e^{\mu(a_x + kx_w)} - \sigma_{p0,A} e^{-\mu(a_x + kx_w)}$$



- **Anchorage set loss when tendon is stressed from both ends**
- Post-tensioned prestressed concrete
- Let us suppose the following procedure of stressing:
 - stressing of the tendon from end A
 - stressing of the tendon from end E
 - anchoring the tendon at end A
 - anchoring the tendon at end E
- Decrease of the stress due to friction



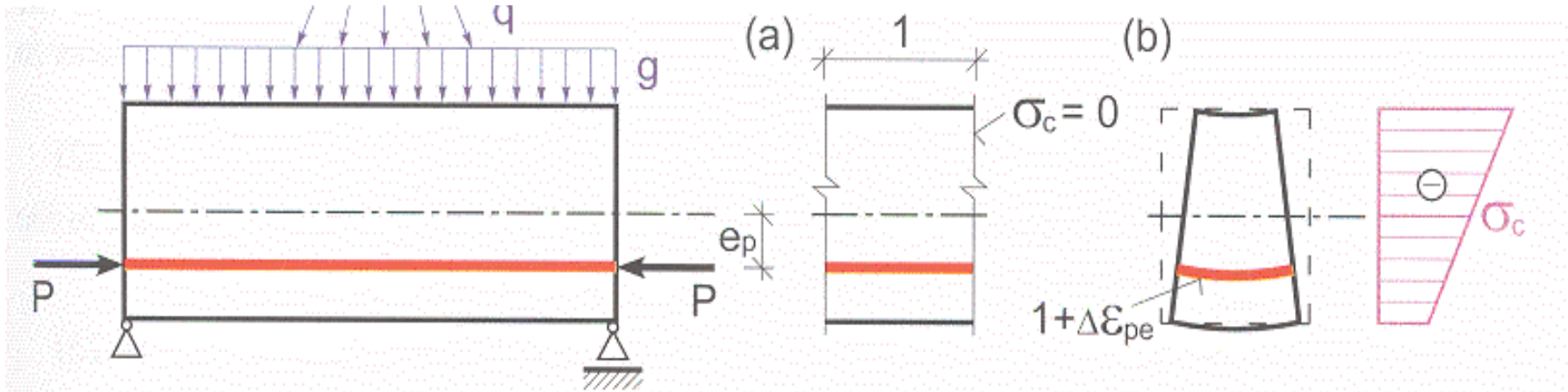


Friction and anchorage set loss when tendon stressed from both ends, anchorage set losses disappear along the length of the tendon

The advantage depends on the size of anchorage set during anchoring

Loss of prestressing due to immediate elastic strain in concrete

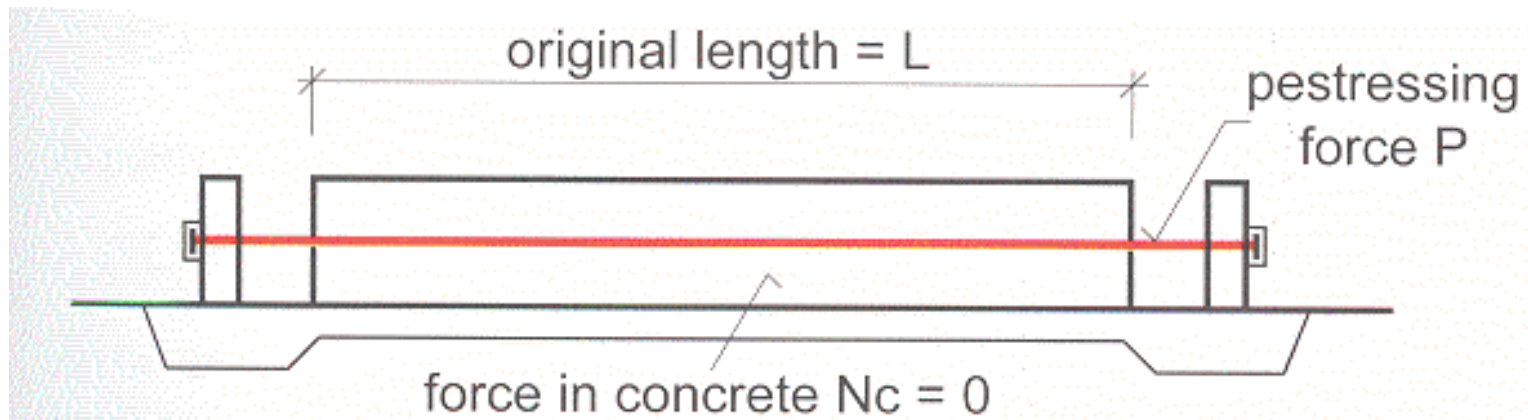
- **Load from prestressing** – one of many possible loads
- This load changes the stress state of the tendon and results in losses in prestressing
- **The pre-tensioned strand** is at the moment of the introduction of prestressing already the part of the prestressed element and resists the load introduced by itself \Rightarrow the loss of pressing due to immediate strain in concrete takes place
- **The pre-tensioned strand in duct** connects at the moment of the introduction of prestressing with the concrete only at the anchorages \Rightarrow the loss of pressing due to immediate strain in concrete at the moment of the introduction of prestressing = 0 (the loss is equilibrated by re-stretching of strand)



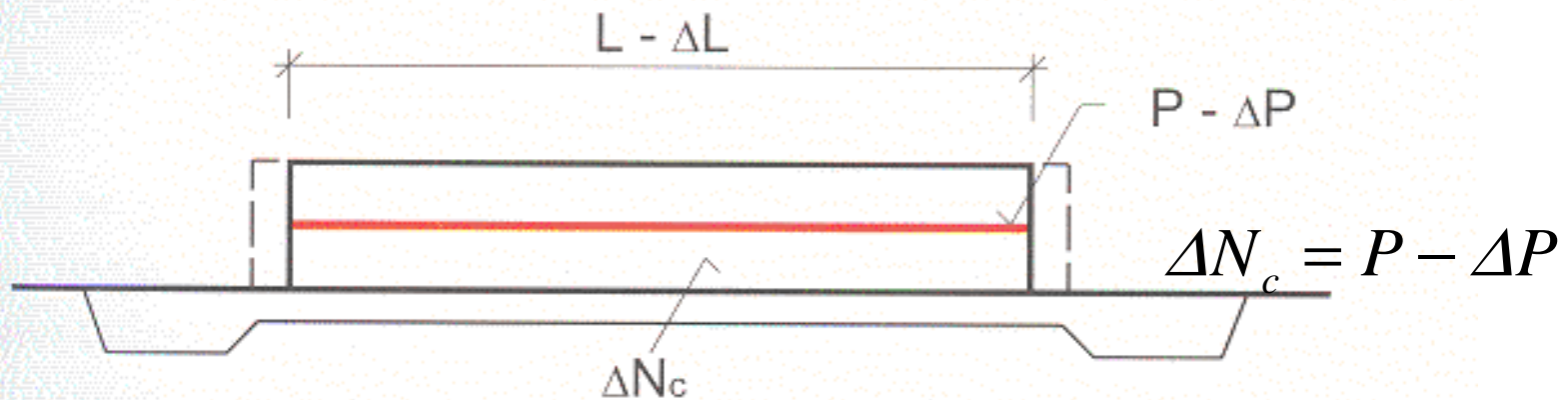
Loss (change) of prestressing due to elastic strain in concrete

Pre-tensioned prestressed concrete – loss due to immediate elastic strain in concrete

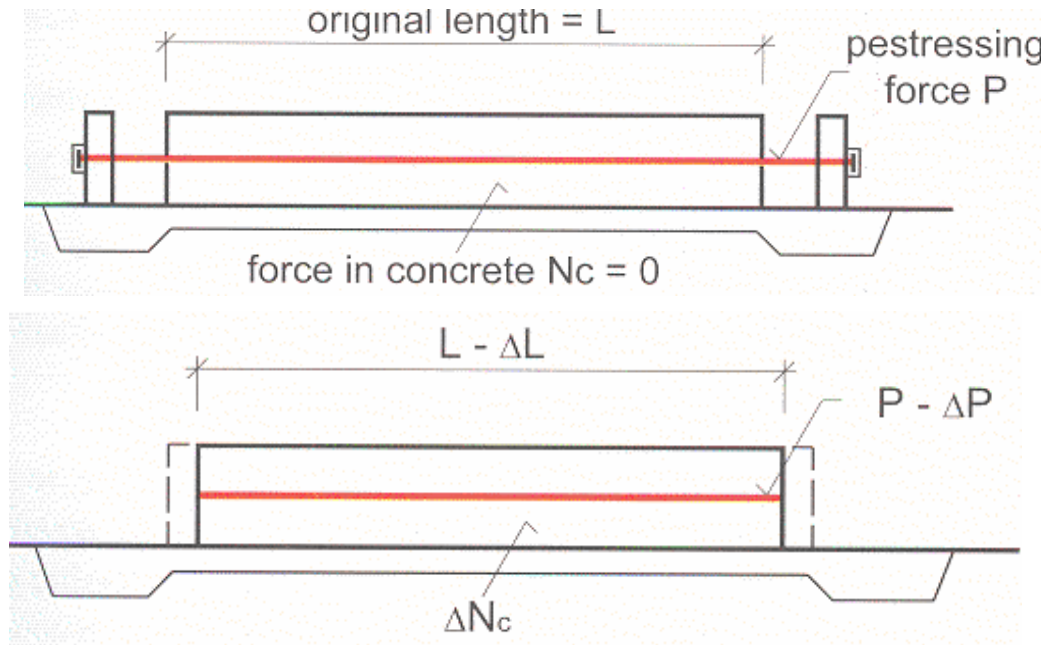
- Transfer of prestressing – by releasing the pre-tensioned prestressing reinforcement from anchorage abutments, the reinforcement acts with the concrete as a part of element \Rightarrow any load results in an immediate deformation of the element and thus deformation of the tendon



(a) Distribution of internal forces before release of strands by cutting



(b) Distribution of internal forces after transfer of prestressing

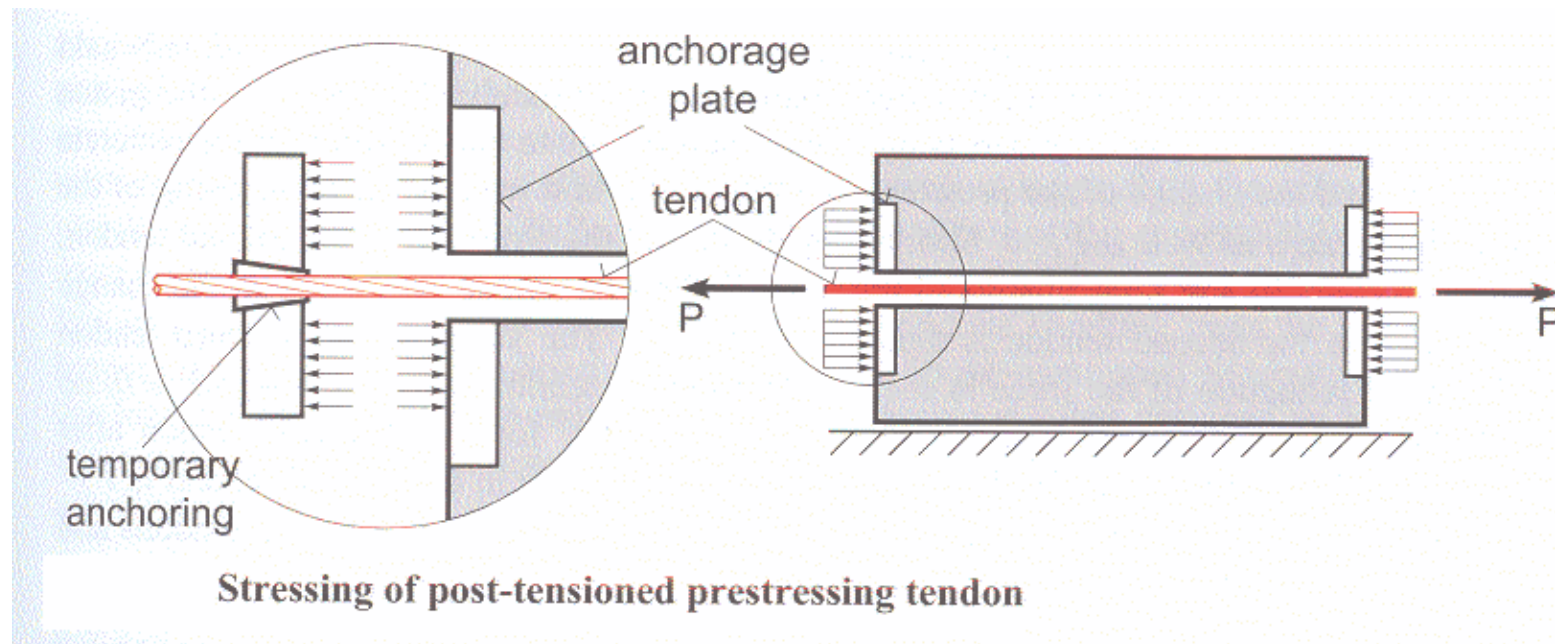


$$\Delta N_c = P - \Delta P \quad \Delta \epsilon_c = \frac{\Delta N_c}{A_c E_c}; \quad \Delta \epsilon_p = \frac{P - \Delta N_c}{A_p E_p}$$

$$\frac{\Delta N_c}{A_c E_c} = \frac{P - \Delta N_c}{A_p E_p} \Rightarrow \Delta N_c = \frac{P A_c E_c}{A_c E_c + A_p E_p} = \frac{P}{1 + \frac{A_p E_p}{A_c E_c}}$$

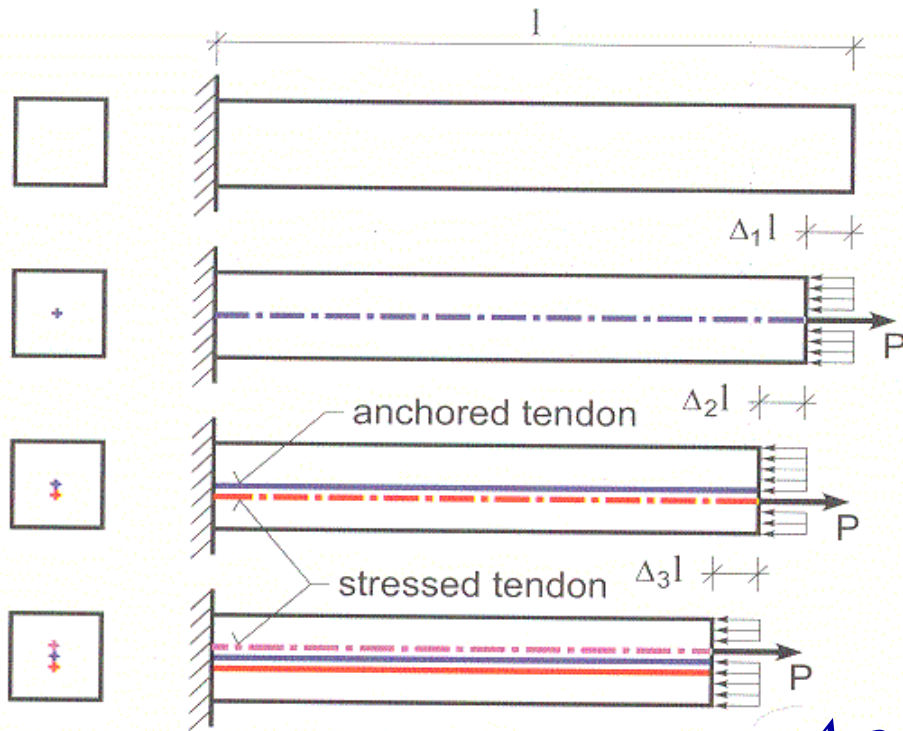
Post-tensioned prestressed concrete – loss due to immediate elastic strain in concrete

The tendon is not a part of prestressed element at the moment of stressing and does not resist the load introduced by itself. The tendon is re-stressed to the original value of the prestressing force \Rightarrow the loss of prestressing due to immediate elastic strain at the moment of the introduction of prestressing in the concrete is zero.



Loss of prestressing due to sequential stressing

Post-tensioned prestressed concrete – results of immediate elastic strain in the concrete and already anchored reinforcement during successive stressing of other tendons.
 Tendons stressed and anchored simultaneously \Rightarrow loss = 0



Loss due to sequential stressing, idealisation for centric prestressing

$A_c; A_p, ..$ stressing force P_0
1. tendon: $A_p/m..$
 $P=P_0/m..Δ_1/l$ deformation eliminated by the by piston stroke of jack $\Rightarrow Δ_1 P = 0$
2. tendon: $Δ_2 P = 0; Δ_1 P \neq 0$
 $P_1 = P - Δ_2 P -$
j. tendon: $P_1 = P - Δ_2 P - \dots - Δ_j P$

$$\Delta_j \epsilon_c = \frac{-\Delta_j P}{E_p \frac{A_p}{m}}; \Delta_j P = \frac{P A_p E_p}{m A_c E_c + (j-1) A_p E_p}$$

When the j -th tendon is being stressed, loss $\Delta_j P \neq 0$ takes place in all $(j-1)$ tendons and results in their shortening

$$\Delta_j \varepsilon_c = \frac{-\Delta_j P}{E_p \frac{A_p}{m}}$$

The total increment of the force acting in the concrete is $P - (j-1)\Delta_j P$ and leads to the shortening of the concrete

$$\Delta_j \varepsilon_c = \frac{-(P - (j-1)\Delta_j P)}{A_c E_c}$$

The loss of force in each of the $(j-1)$ tendons during stressing of the j -th tendon is

$$\Delta_j P = \frac{P A_p E_p}{m A_c E_c + (j-1) A_p E_p}$$

This loss due to sequential stressing may be assumed as a mean loss in each tendon as follows:

$$\Delta P_{el} = A_p \cdot E_p \cdot \sum \left[\frac{j \cdot \Delta \sigma_c(t)}{E_{cm}(t)} \right]$$

where

$\Delta \sigma_c(t)$ is the variation of stress at the centre of gravity of the tendons applied at time t

j is a coefficient equal to

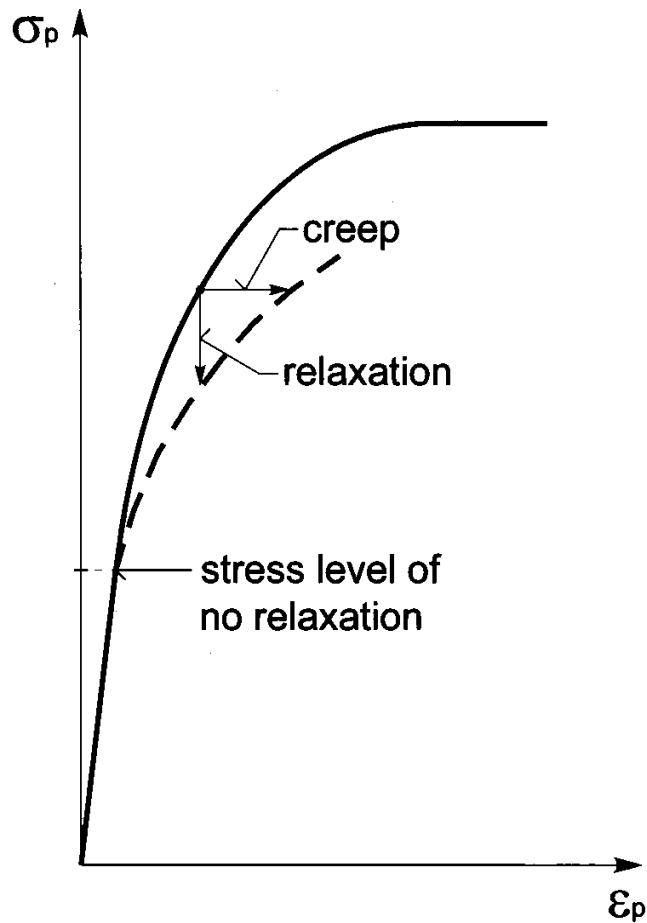
$j = (n - 1)/2n$ where n is the number of identical tendons successively prestressed; as an approximation j may be taken as $1/2$

$j = 1$ for the variations due to permanent actions applied after prestressing.

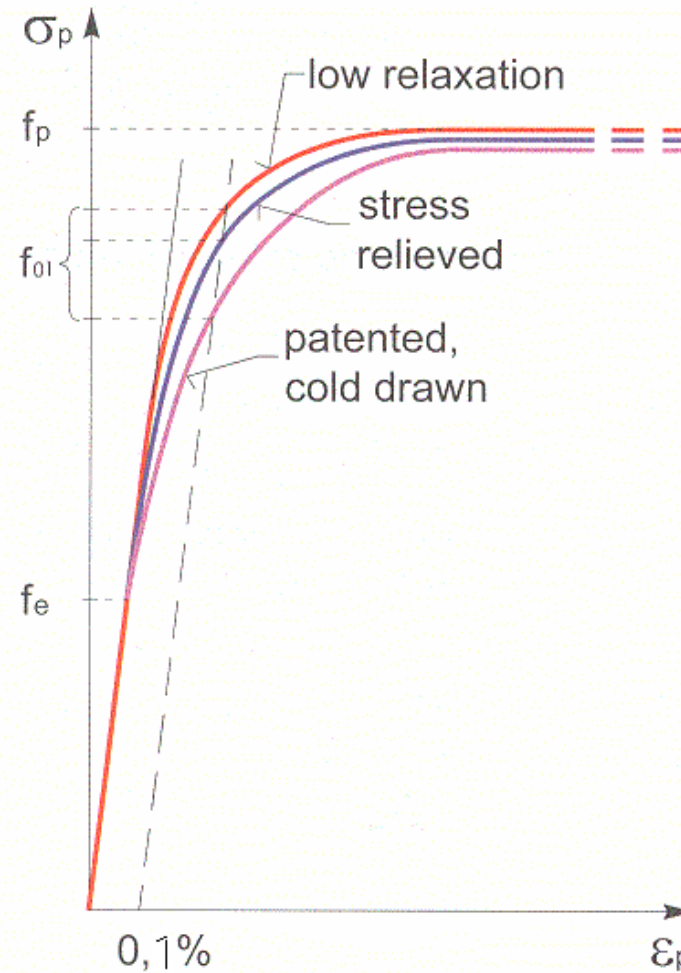
Loss of prestressing due to elastic strain in concrete resulting from external load

- **Losses in service life** – takes place after anchoring, or after transfer
- The **action of external load** (superimposed dead load, variable load, etc.) is **similar to the action of pre-tensioned prestressed strand during the introduction of prestressing** – the prestressing reinforcement is a part of the element \Rightarrow the loss of prestressing due to strain in concrete takes place

Loss of prestressing due to relaxation of prestressing reinforcement



Time dependent properties of prestressing steel



Real stress-strain diagram of prestressing wires/strands

In EN 1992-1-1, three classes of relaxation are defined:

Class 1: wire or strand - ordinary relaxation

Class 2: wire or strand - low relaxation

Class 3: hot rolled and processed bars

$$\text{Class 1} \quad \frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 5,39 \rho_{1000} e^{6,7 \mu} \left(\frac{t}{1000}\right)^{0,75 (1-\mu)} 10^{-5}$$

$$\text{Class 2} \quad \frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66 \rho_{1000} e^{9,1 \mu} \left(\frac{t}{1000}\right)^{0,75 (1-\mu)} 10^{-5}$$

$$\text{Class 3} \quad \frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 1,98 \rho_{1000} e^{8 \mu} \left(\frac{t}{1000}\right)^{0,75 (1-\mu)} 10^{-5}$$

$\Delta\sigma_{pr}$ is absolute value of the relaxation losses of the prestress

σ_{pi} for post-tensioning σ_{pi} is the absolute value of the initial prestress

$\sigma_{pi} = \sigma_{pm0}$; for pre-tensioning σ_{pi} is the maximum tensile stress applied to the tendon minus the immediate losses occurred during the stressing process

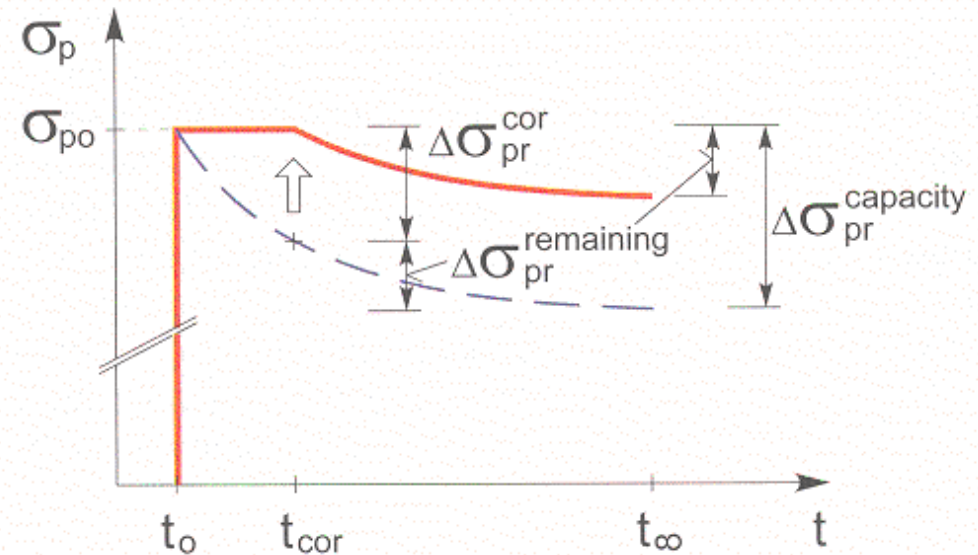
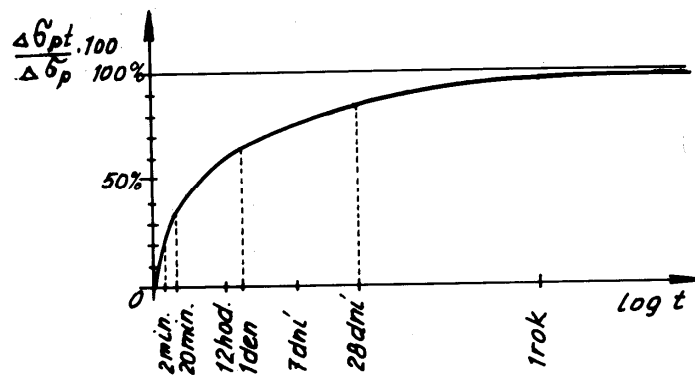
t the time after tensioning (in hours)

$\mu = \sigma_{pi} / f_{pk}$, where f_{pk} is the characteristic value of the tensile strength of the prestressing steel

ρ_{1000} the value of relaxation loss (in %), at 1000 hours after tensioning and at a mean temperature of 20°C.

Loss of prestressing due to relaxation

- Applies to both production and service stages
- The size of relaxation depends on the
 - class, indicating the relaxation behaviour
 - level of introduced prestressing
 - time



Correction of relaxation through keeping constant stress during stressing

Loss of prestressing due to deformation of end abutments of stressing bed

- Pre-tensioned prestressed concrete – abutments deform due to sequential stressing
- Assume m identical and identically stressed strands results in total force P . If P is introduced as a stroke, the distance l_p between anchorage abutments would shorten by value Δl_p . The strain in tendons is $\epsilon_p = \Delta l_p / l_p$; in one tendon $\epsilon_{p1} = \epsilon_p / m$, the loss in in all $j-1$ anchored strands

$$\Delta\sigma_{pA1} = -E_p \epsilon_{p1} = -E_p \frac{\Delta l_p}{l_p m}$$

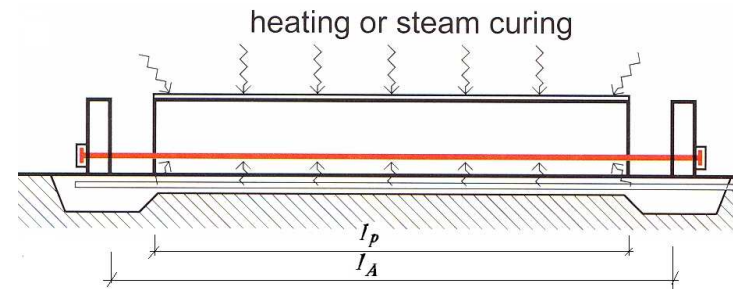
- Average loss in each strand after sequential stressing

$$\Delta\sigma_{pA} = \frac{1}{m} \sum_{i=1}^m \left(-E_p \left(\frac{m-1}{m} \right) \frac{\Delta l_p}{l_p} \right) = -E_p \frac{m-1}{2m} \frac{\Delta l_p}{l_p}$$

Loss due to differences in the temperature of prestressing reinforcement and stressing bed

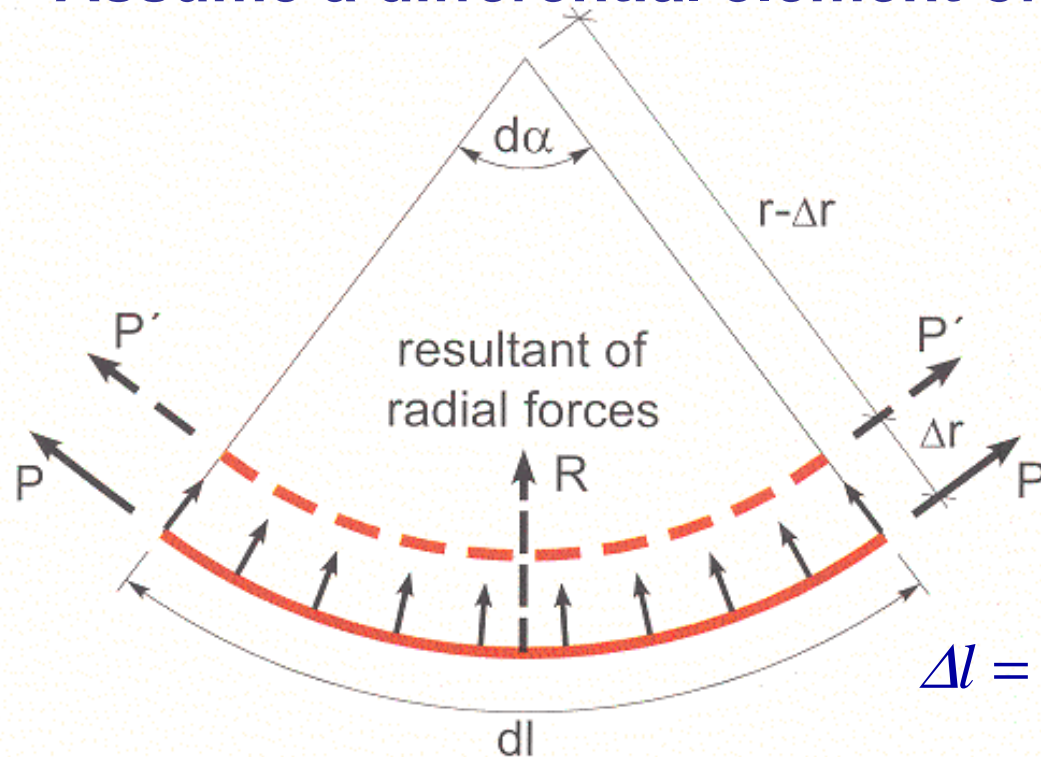
- Pre-tensioned prestressed concrete – eg. by steam curing of concrete
- Assume the distance between abutments l_A , the length of the prestressing reinforcement l_p with the difference of temperature ΔT_p . Due to change in temperature the distance between anchorage abutments will be $\Delta l_p = \alpha_p \Delta T_p l_p$, where α_p is the coefficient of thermal expansion of the prestressing reinforcement
- The difference strain in tendons $\Delta \varepsilon_{pT} = \Delta l_p / l_A$; the loss will be

$$\Delta \sigma_{pT} = -E_p \varepsilon_p = -E_p \frac{\alpha_p \Delta T_p l_p}{l_A}$$



Draw-in loss of prestressing

- Wire wound structure – with small radius of curvature
- Assume a differential element of the wound tendon



Draw-in loss of prestressing

$$\Delta l = (r - \Delta r)d\alpha - rd\alpha = -\Delta r d\alpha$$

$$\Delta\sigma_{po} = E_p \frac{-\Delta r d\alpha}{rd\alpha} = -E_p \frac{\Delta r}{r}$$

- **Maximum stress in tendon after anchoring**
(stage of transfer the prestressing force to concrete):

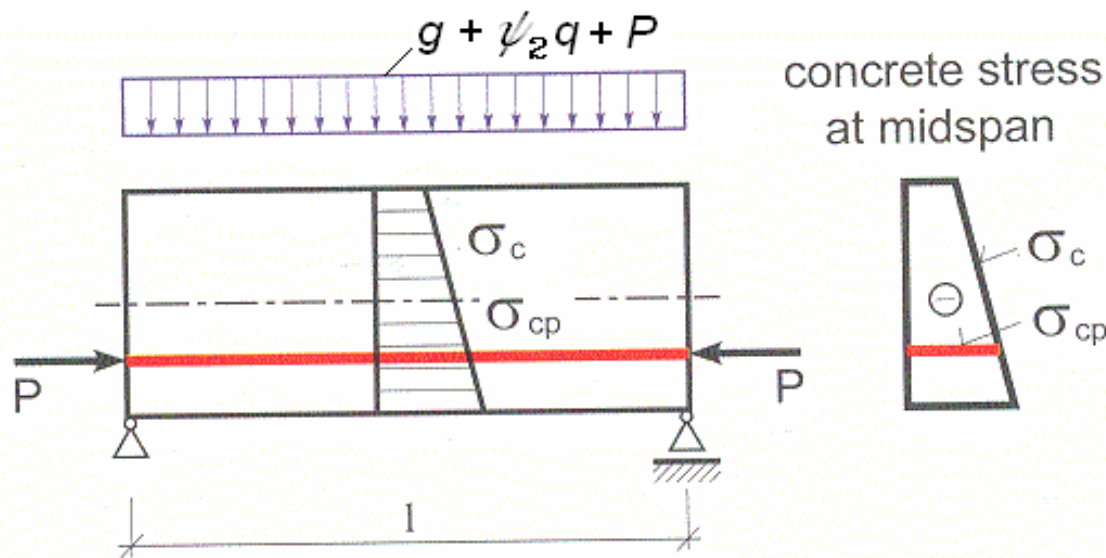
$$\sigma_{p,\max} = \min(0,75 f_{pk} ; 0,85 f_{p0,1k})$$

f_{pk} characteristic tensile strength of prestressing reinforcement

$f_{p0,1k}$ characteristic 0,1% proof-stress of prestressing reinforcement

Long term main losses of prestressing due to shrinkage and creep of concrete and relaxation of prestressing steel

- Calculation is generally complex
- Creep – long term loads

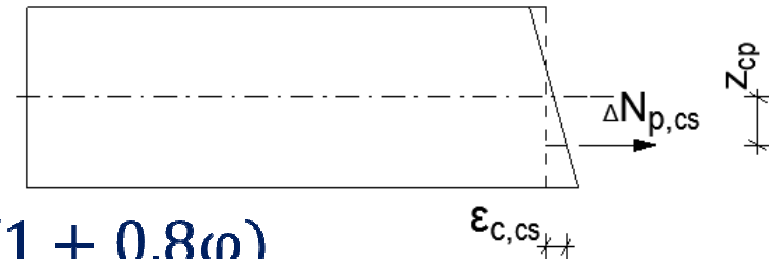
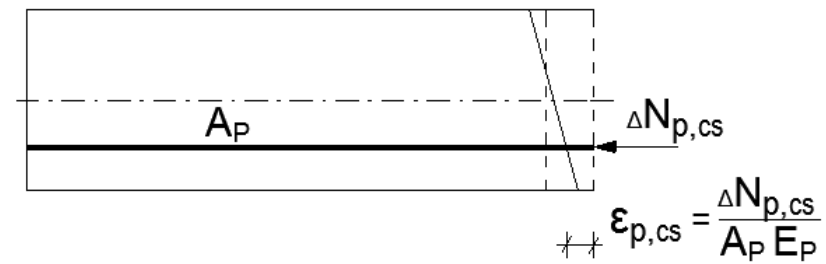
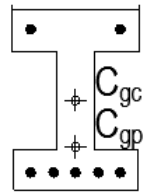
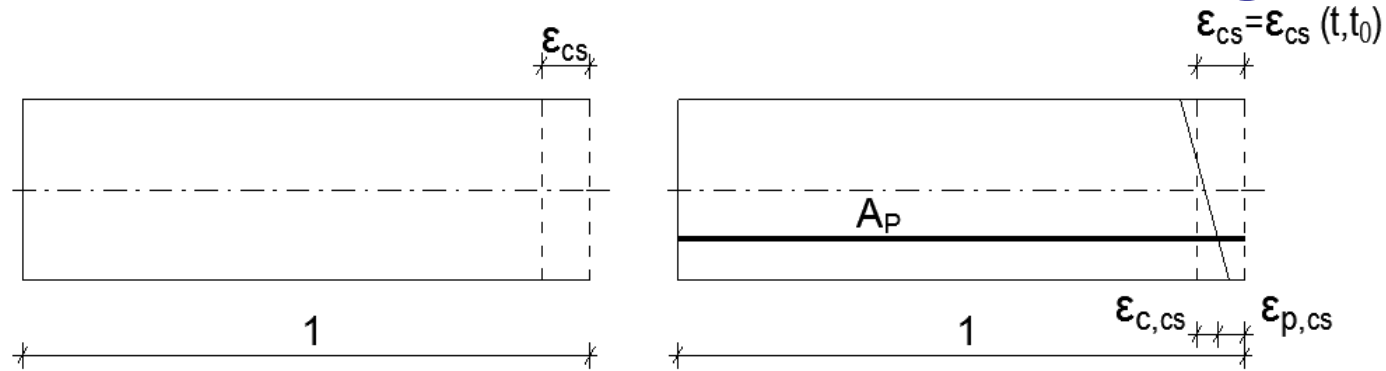


Stress state due to long-term loads ($g + \psi_2 q + P$) affecting the creep of concrete

in time interval (t_0, t)
constant stress σ_{cp}

$$\epsilon_{cc} = \frac{\sigma_{cp}}{E_c(t)} \varphi(t_0, t)$$

Derivation of basic formula for shrinkage



$$\epsilon_{c,cs} = \left(\frac{\Delta N_{p,cs}}{A_c} + \frac{\Delta N_{p,cs} z_{cp}^2}{I_c} \right) \frac{(1 + 0,8\varphi)}{E_{cm}}$$

Derivation of basic formula for shrinkage

$$\varepsilon_{cs} = \frac{\Delta N_{P.cs}}{A_P E_P} + \frac{\Delta N_{P.cs}}{A_C E_{cm}} \left(1 + \frac{A_C}{I_C} z_{pc}^2 \right) (1 + 0,8\varphi)$$

$$\Delta N_{P.cs} = \frac{\varepsilon_{cs} A_P E_P}{1 + \frac{A_P E_P}{A_C E_{cm}} \left[\left(1 + \frac{A_C}{I_C} z_{pc}^2 \right) (1 + 0,8\varphi) \right]}$$

$$\Delta \sigma_{p.cs} = \frac{\varepsilon_{cs} A_P E_P}{1 + \alpha_e \frac{A_P}{A_C} \left(1 + \frac{A_C}{I_C} z_{pc}^2 \right) (1 + 0,8\varphi)}$$

$$\Delta \sigma_{p.cs} = \frac{\varepsilon_{cs} A_P E_P}{A}$$

$$\Delta P_{c+s+r} = A_p \Delta \sigma_{p,c+s+r} = A_p \frac{\varepsilon_{cs} E_p + 0,8 \Delta \sigma_{pr} + \frac{E_p}{E_{cm}} \varphi(t, t_0) \cdot \sigma_{c, QP}}{1 + \frac{E_p}{E_{cm}} \frac{A_p}{A_c} \left(1 + \frac{A_c}{I_c} z_{cp}^2\right) [1 + 0,8 \varphi(t, t_0)]}$$

$\Delta \sigma_{p,c+s+r}$ is the absolute value of the variation of stress in the tendons due to creep, shrinkage and relaxation at location x , at time t

ε_{cs} is the estimated shrinkage strain in absolute value

E_p is the modulus of elasticity for the prestressing steel

E_{cm} is the modulus of elasticity for the concrete

$\Delta \sigma_{pr}$ is the absolute value of the variation of stress in the tendons at location x , at time t , due to the relaxation of the prestressing steel; it is determined for a stress of $\sigma_p = \sigma_p(G + P_{m0} + \psi_2 Q)$; where $\sigma_p = \sigma_p(G + P_{m0} + \psi_2 Q)$ is the initial stress in the tendons due to initial prestress and quasi-permanent actions

$\phi(t, t_0)$ is the creep coefficient at a time t and load application at time t_0

$\sigma_{c,QP}$ is the stress in the concrete adjacent to the tendons, due to self-weight and initial prestress and other quasi-permanent actions where relevant; the value of $\sigma_{c,QP}$ may be the effect of part of self-weight and initial prestress or the effect of a full quasi-permanent combination of action ($\sigma_c(G+P_{m0}+ \psi_2 Q)$), depending on the stage of construction considered

A_p is the area of all the prestressing tendons at the location x

A_c is the area of the concrete section

I_c is the second moment of area of concrete section

z_{cp} is the distance between centre of gravity of the concrete section and tendons

Prestressing force

The initial prestress force $P_{m0}(x)$ at time $t = t_0$ applied to the concrete immediately after tensioning and anchoring (post-tensioning) or after transfer of prestressing (pre-tensioning)

$$P_{m0}(x) = P_{\max} - \sum \Delta P_i(x)$$

where P_{\max} is the force at tensioning

$$(\sigma_{pm,\max} = \min(0,80 f_{pk}; 0,90 f_{p0,1k}))$$

$\Delta P_i(x)$ are the immediate losses

Limitation:

$P_{m0}(x)$ should not exceed the following value

$$P_{m0}(x) = A_p \cdot \sigma_{pm0}(x) \leq A_p \cdot \sigma_{pm0,\max}$$

where

$$\sigma_{pm0,\max} = \min(0,75 f_{pk}; 0,85 f_{p0,1k})$$

Immediate main losses of prestress for

a) pre-tensioning

During the stressing process:

- loss due to friction - in case of curved tendons $\Delta P_{\mu}(x)$
- losses due to wedge draw-in of the anchorage devices ΔP_w

Before the transfer of prestress to concrete:

- loss due to relaxation of the pre-tensioning tendons (period between tensioning of tendons and transfer) ΔP_r
- in case of heat curing, losses due to shrinkage and relaxation are modified; direct thermal effect should also be considered ΔP_T

At the transfer of prestress to concrete:

- loss due to elastic deformation of concrete as the result of the action of pre-tensioned tendons when they are released from the anchorages ΔP_{el}

a) post-tensioning

Short time losses:

- losses due to friction $\Delta P_{\mu}(x)$
- losses due to wedge draw-in of the anchorage devices ΔP_w
- loss due to elastic deformation of concrete in case of sequential prestressing ΔP_{el}
- loss due to bearing pressure on concrete – only by circumferential tendons with small radius of curvature

Long term losses

The mean value of the prestress force $P_{m,t}(x)$ at the time $t > t_0$ should be determined with respect to the prestressing method

In addition to the immediate losses the time-dependent losses of prestress $\Delta P_{c+s+r}(x)$ as a result of creep and shrinkage of the concrete and the long term relaxation of the prestressing steel should be considered

$$P_{m,t}(x) = P_{m0}(x) - \Delta P_{c+s+r}(x)$$