Prestressed Concrete

Part 3

(Design of prestressing)

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Design of prestressed members

- **Limit states:** **ULS – ultimate - associated with collapse, or with forms of structural failure**
  SLS – serviceability- beyond which specified service requirements are not met
- **Reinforced concrete structure:**
  ULS – design of size and reinforcement
  SLS – check of serviceability
  fulfilment of detailing and particular rules
- **Prestressed concrete structure:**
  SLS – design of size and prestressing force
  ULS – check of carrying capacity
  fulfilment of detailing and particular rules

Serviceability limit states (SLS)

Three combinations designated by the representative value of the dominant action are:

- **Characteristic (Rare) combination**

$$\sum_{j\geq 1} G_{k,j} \text{"+"} P \text{"+"} Q_{k,1} \text{"+"} \sum_{i>1} \psi_{0,i} Q_{k,i}$$

- **Frequent combination**

$$\sum_{j\geq 1} G_{k,j} \text{"+"} P \text{"+"} \psi_{1,1} Q_{k,1} \text{"+"} \sum_{i>1} \psi_{2,i} Q_{k,i}$$

- **Quasi-permanent combination**

$$\sum_{j\geq 1} G_{k,j} \text{"+"} P \text{"+"} \sum_{i\geq 1} \psi_{2,i} Q_{k,i}$$

G_k, Q_k **- characteristic value of permanent, variable load**;
Q_{k1} **- dominant value of variable load**;
P **- mean value of prestressing force**

SLS – (EN 1992-1-1)

a) Limitation of stresses

- **Concrete:**
  - compressive stress under characteristic combination of loads (longitudinal cracks) $\sigma_c \leq 0{,}6\ f_{ck}$

    if $0{,}45 f_{ck} \leq \sigma_c \leq 0{,}6 f_{ck}$ – non-linear creep
  - pretensioned members by transfer $\sigma_c \leq 0{,}7\ f_{ck}$
- **Reinforcement:**
  - tensile stresses under characteristic combination of loads $\sigma_s \leq 0{,}8\ f_{yk}$

    including imposed deformation $\sigma_s \leq 1{,}0\ f_{yk}$
- **Prestressed tendons:**
  - tensile stresses under characteristic combination of loads $\sigma_p \leq 1{,}0\ f_{pk}$

b) Crack control – limited and partial prestress

- Without cracks $\sigma_c \leq f_{ctm}$
- Crack width $w \leq w_{max}$

Recommended value w_{max} [mm] for prestressed members with bonded tendons

Exposure class	Frequent load combination
X0, XC1	0,2
XC2, XC3, XC4	0,2[1)]
XD1, XD2, XS1 XS2, XS3, XS4	Decompression [2)]

[1)] In addition, decompression should be check under quasi-permanent load combination

[2)] Prestressed tendon should be in compressed area

b) Deflection control

- **Deflection under quasi-permanent load combination** $y_{max} \leq l\ /\ 250$

- **Deflection that could demage adjacent parts of the structure (brittle partition walls etc.)**

  deflection after construction $y_{max} \leq l\ /\ 500$

where *l* is the span

Investigation stages of prestress element

- **Stage of prestress**
- **after transfer of prestressing – act: prestressing force P_{m0} and usually only self-weight – class of concrete by prestressing**
- **Servis stage**

after all losses of prestressing force – act: prestressing force $P_{m\infty}$ and service load in prescribed combination – class of concrete in service

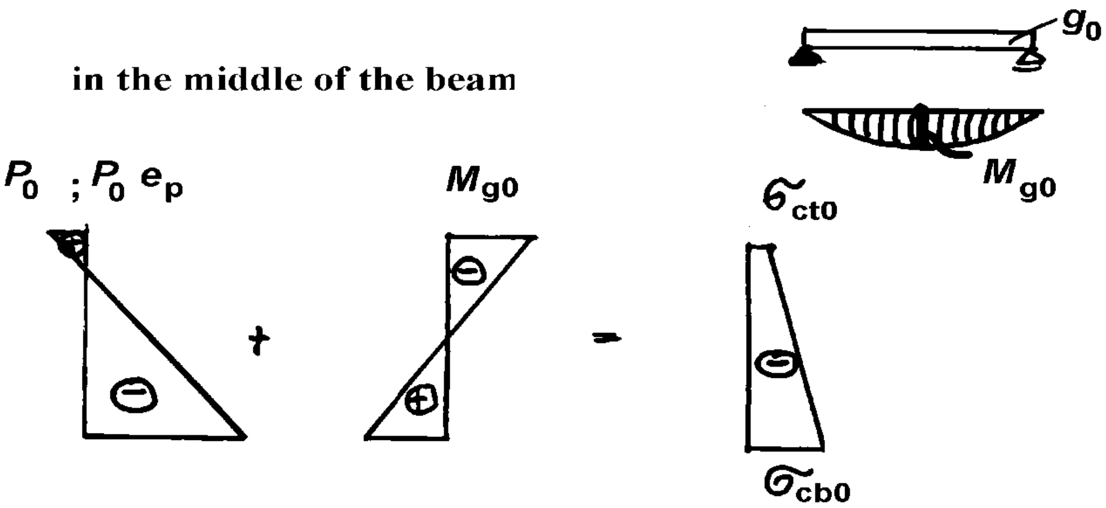
Investigation stages of prestress element

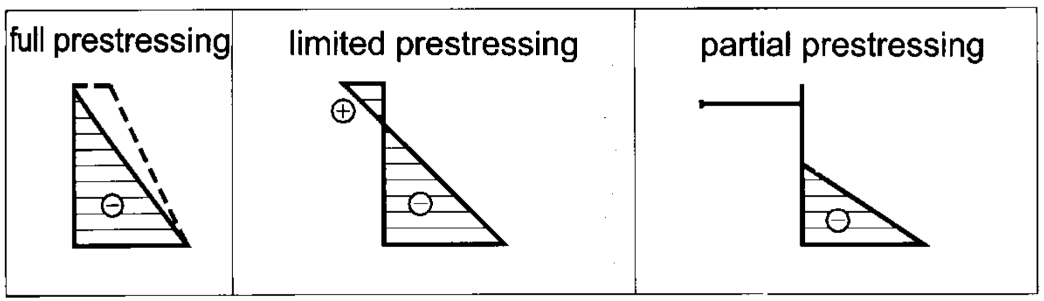
- **Stage of prestressing**

after transfer of prestressing – introduction the prestressing force into concrete – act:

a) the prestressing force P_{m0} at time t_0 with eccentricity e_p

b) usually only self-weight g_0 - M_{g0}





σ_{ct0} – by prestressing:

full $|\sigma_{ct0}| \leq 0$

limited $\sigma_{ct0} \leq f_{ctm}(t_0)$

partial $\sigma_{ct0} > f_{ctm}(t_0)$

σ_{cb0}

$|\sigma_{cb0}| \leq 0{,}6\ f_{ck}(t_0)$

by pre-tensioning

$|\sigma_{cb0}| \leq 0{,}7\ f_{ck}(t_0)$

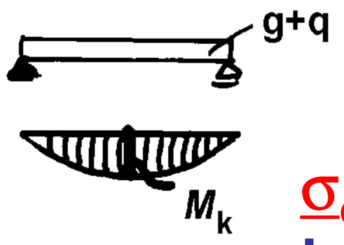
- **Stage of service**

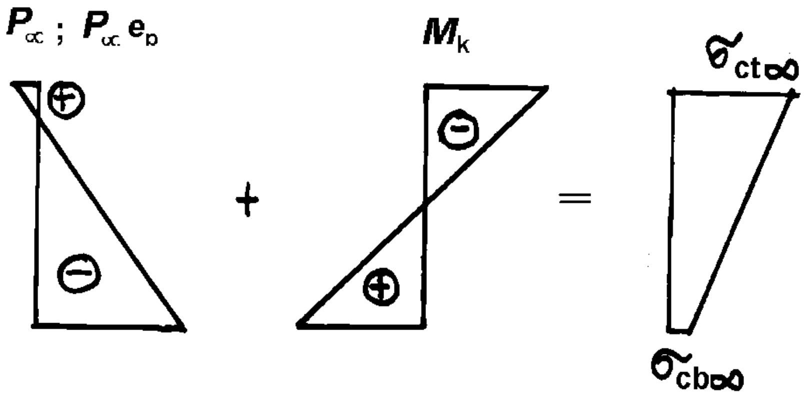
  after all losses of prestressing force – act:

  a) the prestressing force $P_{m\propto}$ at time $t = \propto$ with the eccentricity e_p; $t = \propto \cong$ 500 000 hours $\cong$ 57 years

  b) service load in prescribed combination g + q

in the midle of the beam





$\sigma_{ct\propto}$

$|\sigma_{ct\propto}| \leq 0{,}6\ f_{ck}$

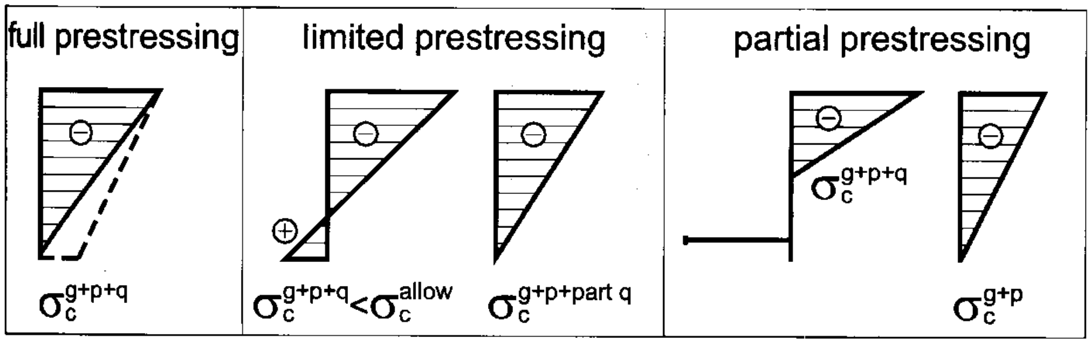
$\sigma_{cb\propto}$

by pre-tensioning:

full $|\sigma_{cb\propto}| \leq 0$

limited $\sigma_{cb\propto} \leq f_{ctm}$

partial $\sigma_{cb\propto} > f_{ctm}$



Prestressing force

- **The initial prestress force $P_{m0}(x)$ at time $t = t_0$** **immediately after transfer**

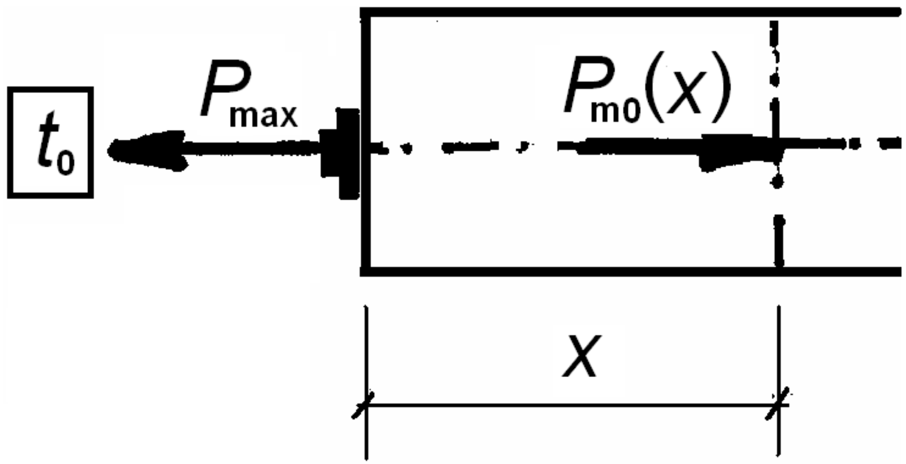
$$P_{m0}(x) = P_{max} - \sum \Delta P_i(x) \le A_p \cdot \sigma_{pm0}(x)$$

where $P_{max} \le A_p \cdot \sigma_{pmax}$ **is the force at the active end during tensioning**

$$\sigma_{pm,\max} \le \min(0{,}8\, f_{pk} ; 0{,}9\, f_{p0{,}1k})$$

$\Delta P_i(x)$ **are the short-term losses**

$$\sigma_{pm0,\max} \le \min(0{,}75\, f_{pk} ; 0{,}85\, f_{p0{,}1k})$$



- **Mean value of prestressing force at time *t***

$$P_{m,t}(x) = P_{m0}(x) - A_p \, \Sigma \, |\sigma_{pi}(x)|$$

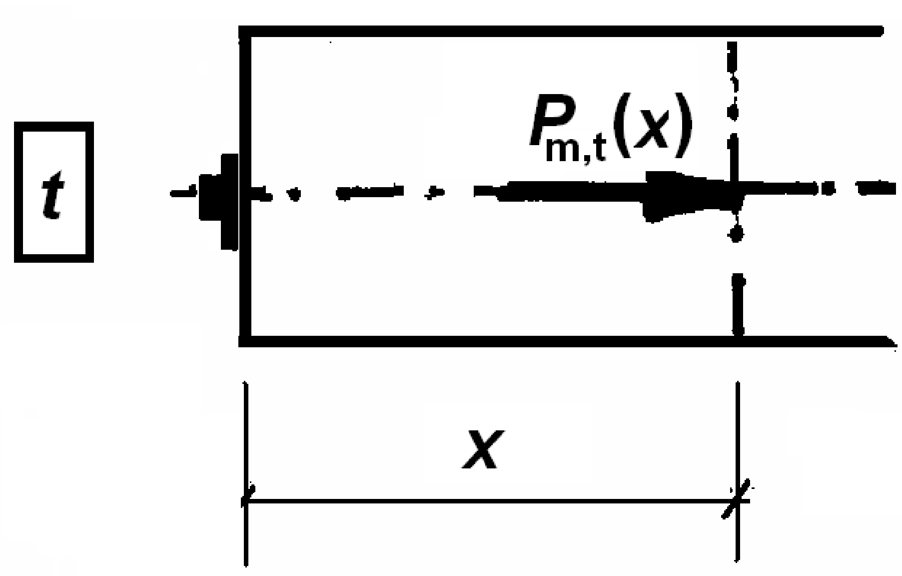
where

$P_{m0}(x)$ **is the initial prestress force at time t_0 of prestress**

A_p **the sectional area of tendon**

$\Sigma \, |\sigma_{pi}(x)|$ **the sum of long-term losses of prestress**

$P_{m,t}(x)$



- **Effect of prestressing at SLS**

Allowance shall be made for possible variations in prestress

Two characteristic values of prestressing force are estimated:

- upper characteristic value $P_{k,sub} = r_{sub} P_{m,t}(x)$

- lower characteristic value $P_{k,inf} = r_{inf} P_{m,t}(x)$

for pre-tensioning or unbonded tendon:

$$r_{sub}=1{,}05,\ r_{inf}=0{,}95$$

for post-tensioning with bonded tendon:

$$r_{sub}=1{,}10,\ r_{inf}=0{,}90$$

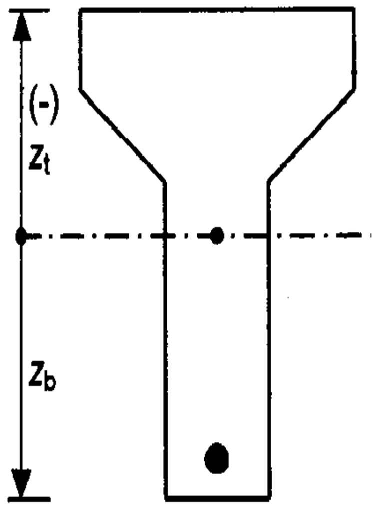
Design of prestress

- **Prestressing force**

Most effective - the prestressing force with the eccentricity

SLS - the stress in the cross section in concrete fibres of statically determinate structures

$$\sigma_c = \frac{-P}{A} + \frac{-Pe + M}{I} z$$



P **- the force in tendon (compression +)**

e **- the eccentricity of the tendon (as *z*)**

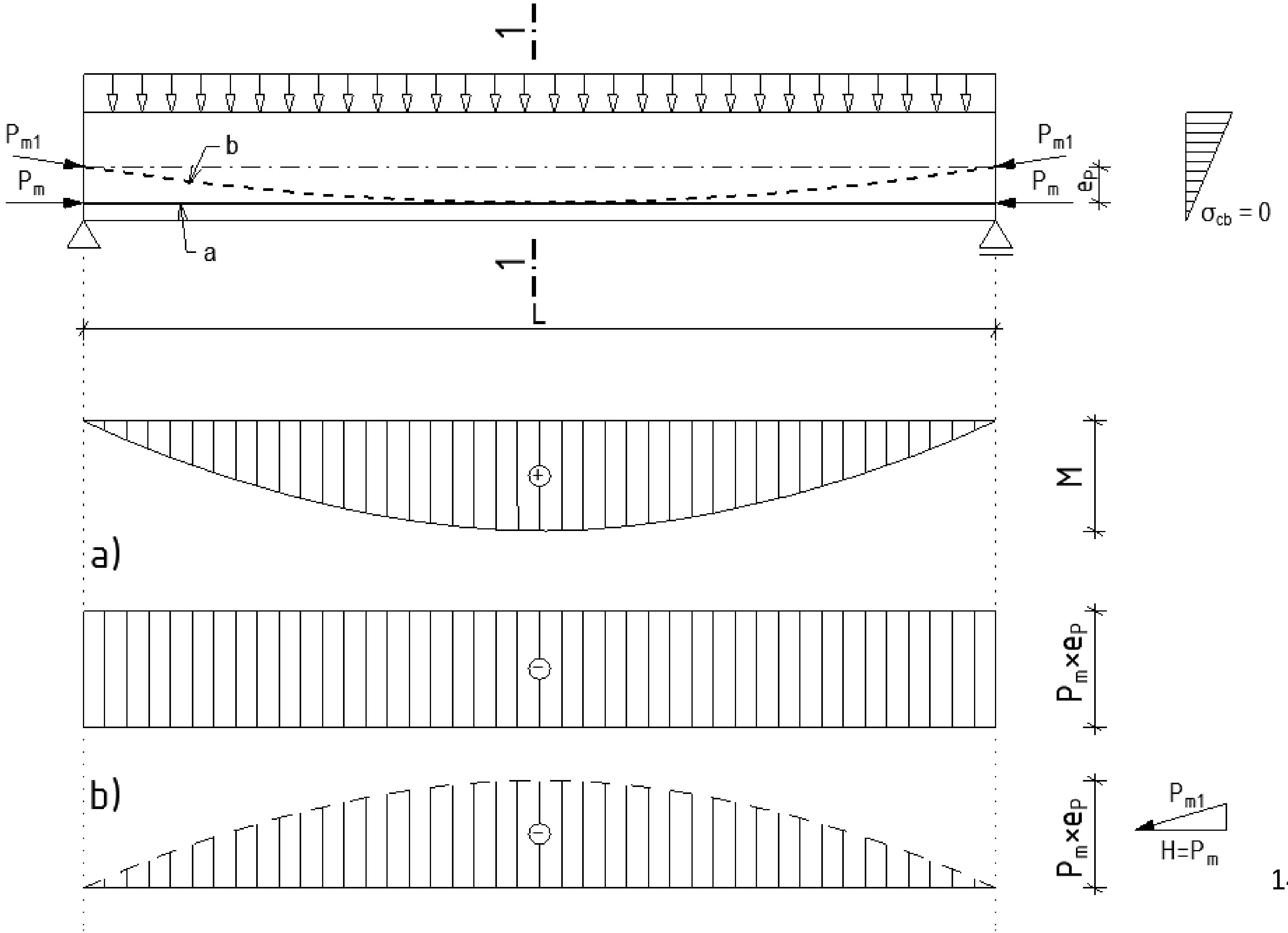
A **- the area of cross-section**

I **- the moment of the inertia**

M **- the bending moment at SLS in the cross section**

z **- the distance of the concrete fibre from the centre of gravity, + in direction to the bottom of cross section**

Design of the prestressing force *P*

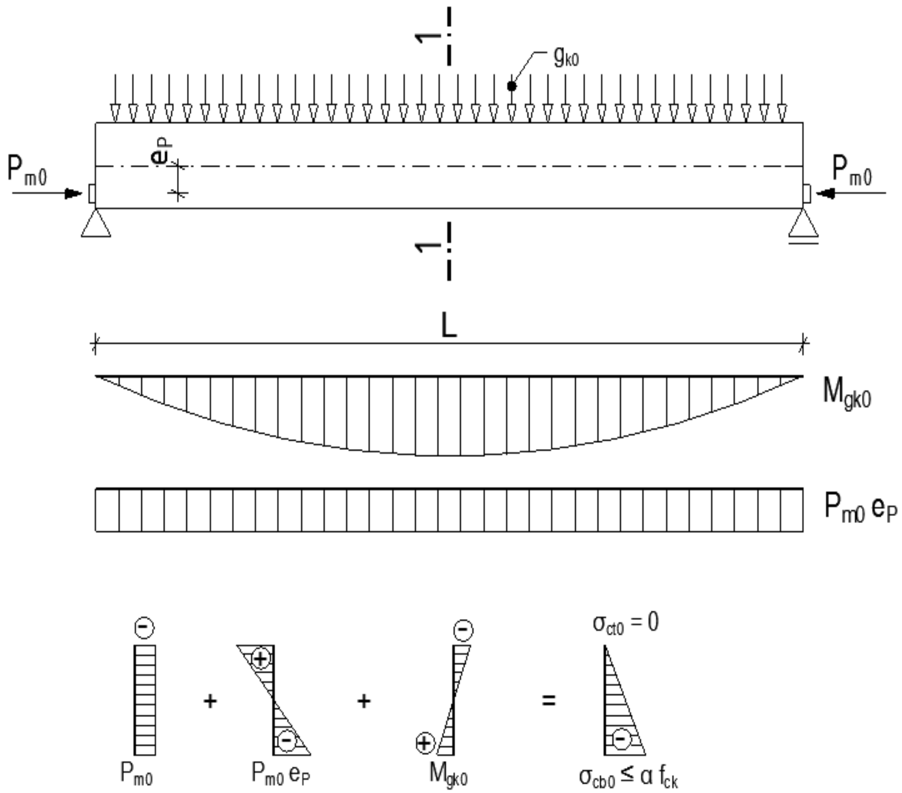


Design of the prestressing force *P* and eccentricity e_p

Simplified design, full prestressing – section 1-1

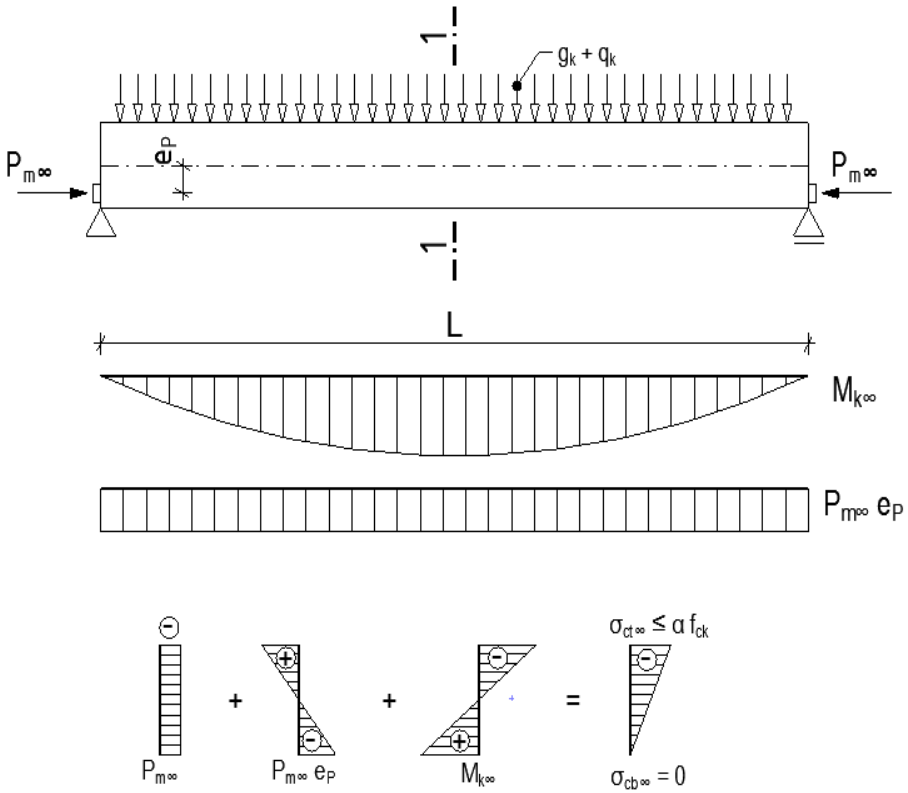
Stage of prestressing P_{m0}

$$M_{gk0} = \frac{g_{k0} \times L^2}{8}$$



Stage of service $P_{m\infty}$

$$M_{gk0} = \frac{g_{k0} \times L^2}{8}$$



Design of prestressing force P and eccentricity e_p

Conditions for stress in concrete:

Stage of prestressing P_{m0}

$$(B)\ \sigma_{ct0} = \frac{P_{m0}}{W_t}(r_t + e_p) + \frac{M_{gk0}}{W_t} = 0$$

$$\sigma_{cb0} = \left|\frac{P_{m0}}{W_b}(r_b + e_p) + \frac{M_{gk0}}{W_b}\right| \leq \alpha_P\, f_{ck,P}$$

Stage of service $P_{m\infty}$

$$\sigma_{ct\infty} = \left|\frac{P_{m\infty}}{W_t}(r_t + e_p) + \frac{M_{k\infty}}{W_t}\right| \leq \alpha_S\, f_{ck}$$

$$(A)\ \sigma_{cb\infty} = \frac{P_{m\infty}(r_b + e_p)}{W_b} + \frac{M_{k\infty}}{W_b} = 0$$

from (A):

$$P_{m\infty} = -\frac{M_{k\infty}}{r_b + e_P}; \qquad P_{m\infty} \cong 0{,}8\, P_{m0}$$

$$r_b + e_p = -\frac{M_{k\infty}}{0{,}8\, P_{m0}} \quad (1)$$

from (B):

$$r_t + e_p = -\frac{M_{gk0}}{P_{m0}} \quad (2)$$

Design of prestressing force P and eccentricity e_p

from (A):

$$r_b + e_p = -\frac{M_{k\infty}}{0{,}8\,P_{m0}} \quad (1)$$

from (B):

$$r_t + e_p = -\frac{M_{gk0}}{P_{m0}} \quad (2)$$

$$\frac{r_t + e_P}{r_b + e_P} = \frac{\dfrac{M_{gk0}}{P_{m0}}}{\dfrac{M_{k\infty}}{0{,}8P_{m0}}} = 0{,}8\,\frac{M_{gk0}}{M_{k\infty}}$$

$$K = 0{,}8\,\frac{M_{gk0}}{M_{k\infty}} \quad (3)$$

$$\boldsymbol{e_P = \frac{-r_t + K\,r_b}{1 - K}}$$

from (1):

$$r_b + e_P = -\frac{M_{k\infty}}{0{,}8\,P_{m0}} \rightarrow \boldsymbol{P_{m0} = -\frac{M_{k\infty}}{0,8\,(r_b + e_P)}}$$

$$P_{m0} = 0{,}95\,P_{max}$$

$$A_P \cong \frac{P_{m0}}{0{,}95\sigma_{p,max}}$$

$P_{m0} - force\ by\ prestressing$

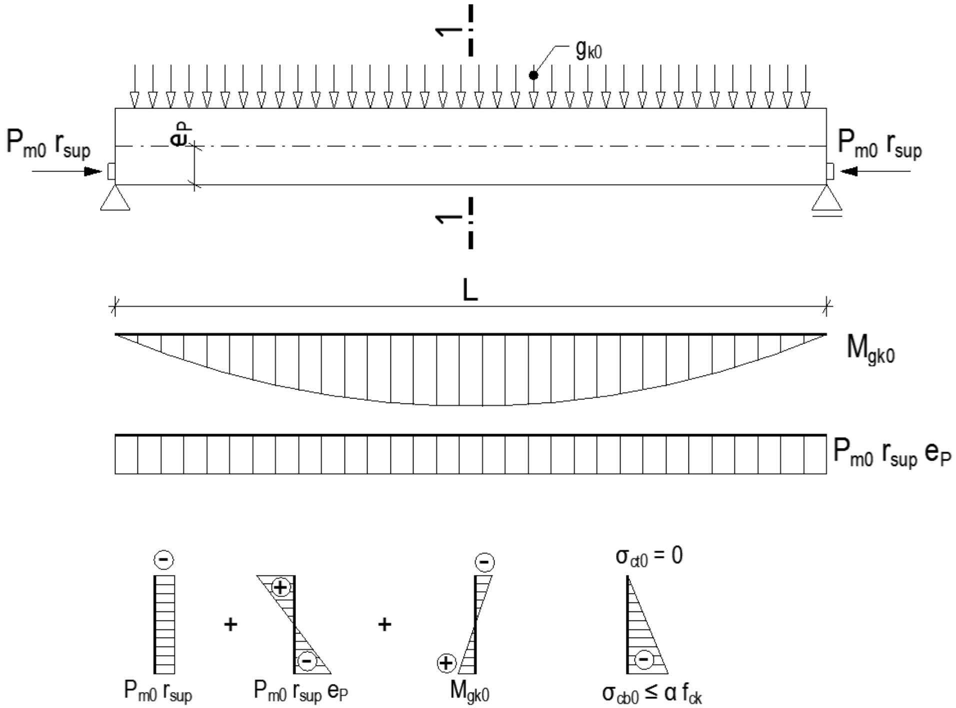
Design of the prestressing force *P* and eccentricity e_p

Simplified design, full prestressing – section 1-1- with consideration to r_i (r_{sup}, r_{inf}) – creficient representing the accurancy of prestressing

Stage of prestressing

$$P_{k,sup} = r_{sup} \times P_{m0}$$

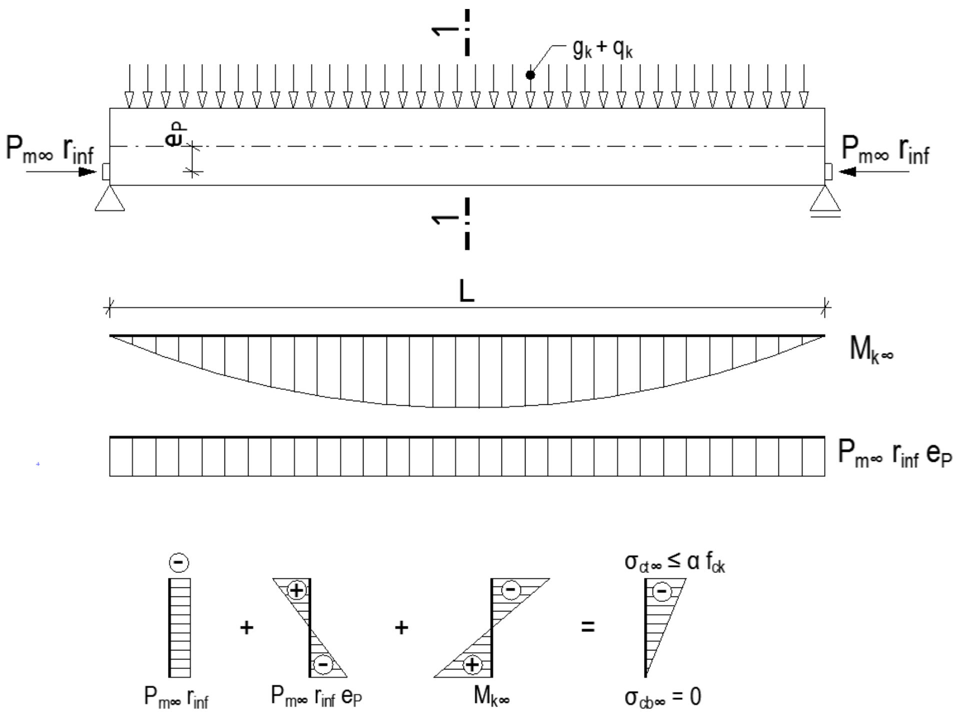
$$M_{gk0} = \frac{g_{k0} \times L^2}{8}$$



Stage of service

$$P_{k,inf} = r_{inf} \times P_{m\infty}$$

$$M_{gk} = \frac{(g_k + q_k) \times L^2}{8}$$



Design of prestressing force P and eccentricity e_p

Conditions for stress in concrete:

Stage of prestressing

$$P_{k,sup} = r_{sup} \times P_{m0}$$

$$(B)\ \sigma_{ct0} = \frac{P_{m0}}{W_t}(r_t + e_p) + \frac{M_{gk0}}{W_t} = 0$$

$$\sigma_{ct0} = \left|\frac{P_{m0} \times r_{sup}}{W_b}(r_b + e_p) + \frac{M_{gk0}}{W_b}\right| \leq \alpha_P f_{ck,P}$$

Stage of service

$$P_{k,inf} = r_{inf} \times P_{m\infty}$$

$$\sigma_{ct\infty} = \left|\frac{P_{m\infty} \times r_{inf}}{W_t}(r_t + e_p) + \frac{M_{k\infty}}{W_t}\right| \leq \alpha_S f_{ck}$$

$$(A)\ \sigma_{cb\infty} = \frac{P_{m\infty} \times r_{inf}}{W_b}(r_b + e_p) + \frac{M_{k\infty}}{W_b} = 0$$

from (A):

$$P_{m\infty} \times r_{inf} = -\frac{M_{k\infty}}{r_b + e_P}; \qquad P_{m\infty} \cong 0{,}8\, P_{m0}$$

$$r_b + e_p = -\frac{M_{k\infty}}{0{,}8\, P_{m0} \times r_{inf}} \qquad (1)$$

from (B):

$$r_t + e_p = -\frac{M_{gk0}}{P_{m0} \times r_{sup}} \qquad (2)$$

Design of prestressing force P and eccentricity e_p

from (A):

$$r_b + e_p = -\frac{M_{k\infty}}{0{,}8\, P_{m0} \times r_{inf}} \quad (1)$$

from (B):

$$r_t + e_p = -\frac{M_{gk0}}{P_{m0} \times r_{sup}} \quad (2)$$

$$\frac{r_t + e_P}{r_b + e_P} = \frac{\dfrac{M_{gk0}}{P_{m0} \times r_{sup}}}{\dfrac{M_{k\infty}}{0{,}8 P_{m0} \times r_{inf}}} = 0{,}8\frac{M_{gk0} \times r_{inf}}{M_{k\infty} \times r_{sup}}$$

$$K = 0{,}8\frac{M_{gk0} \times r_{inf}}{M_{k\infty} \times r_{sup}} \quad (3)$$

$$\boldsymbol{e_P = \frac{-r_t + K\, r_b}{1 - K}}$$

from (1):

$$r_b + e_P = -\frac{M_{k\infty}}{0{,}8\, P_{m0} \times r_{inf}} \rightarrow \boldsymbol{P_{m0} = -\frac{M_{k\infty}}{0{,}8\,(r_b + e_P) \times r_{inf}}}$$

$$P_{m0} = 0{,}95\, P_{max}$$

$$A_P \cong \frac{P_{m0}}{0{,}95\sigma_{p,max}}$$

$$P_{m0} - force\ by\ prestressing$$

- **Design of the prestressing force *P* and the eccentricity *e***

We assume:

a) $P_{m\infty} = 0{,}8\, P_{m0}$ (20 % losses)

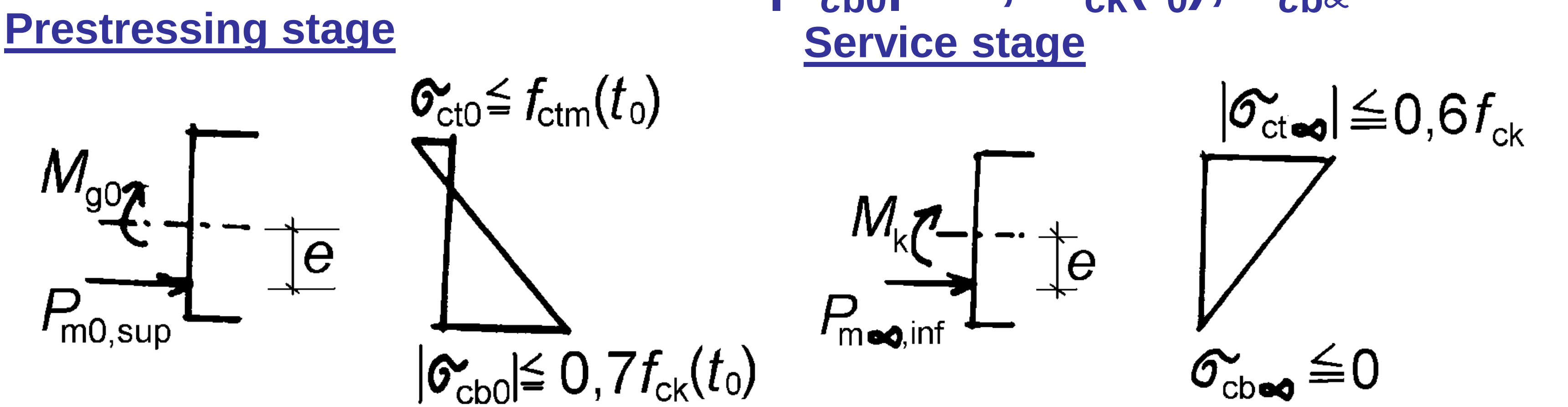
b) SLS: limit values of concrete stresses

eg. for pre-tensioned member – cross section M_{max}

prestressing stage: limited prestressing;

service stage: full prestressing

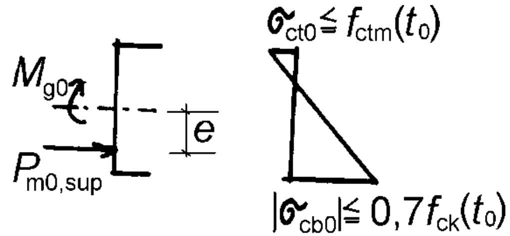
- top concrete fibres $\sigma_{ct0} \leq f_{ctm}(t_0)$; $|\sigma_{ct\infty}| \leq 0{,}6\ f_{ck}$
- bottom concrete fibre $|\sigma_{cb0}| \leq 0{,}7\ f_{ck}(t_0)$; $\sigma_{cb\infty} \leq 0$



c) SLS: requested combinations of load effects

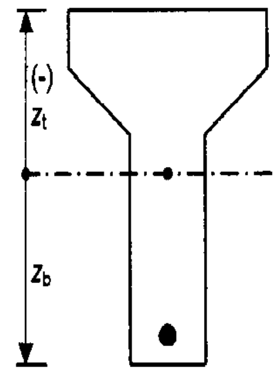
- ## The conditions for the limit values of concrete stresses in extreme fibres

prestressing stage: limited prestressing



$$\sigma_{ct0} = \frac{-P_{m0,\sup}}{A} + \frac{-P_{m0,sup}.e + M_{g0}}{I} z_t \leq f_{ctm}(t_0)............(1)$$

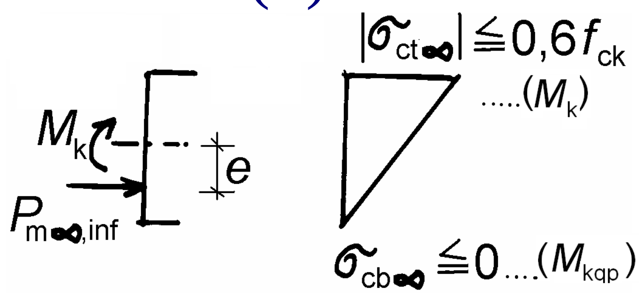
$$\sigma_{cb0} = \frac{-P_{m0,\sup}}{A} + \frac{-P_{m0,sup}.e + M_{g0}}{I} z_b \geq -0,7 f_{ck}(t_0)....(2)$$



service stage: full prestressing

$$\sigma_{ct\infty} = \frac{-P_{m\infty,\inf}}{A} + \frac{-P_{m\infty,\inf}.e + M_k}{I} z_t \geq -0,6 f_{ck}............(3)$$

$$\sigma_{cb\infty} = \frac{-P_{m\infty,\inf}}{A} + \frac{-P_{m\infty,\inf}.e + M_{kpq}}{I} z_b \leq 0......................(4)$$



where $P_{m\infty,\sup} = r_{\sup} P_{m0}$

$$P_{m\infty,\inf} = P_{m\infty} = r_{\inf}.0,8 P_{m0}$$

From the unevenness (1) we receive

$$\frac{r_{\sup}}{A}+\frac{r_{\sup}ez_t}{I}\geq(-f_{ctm}\left(t_0\right)+\frac{M_{g0}z_t}{I})\frac{1}{P_{mo}}$$

Points where the lines cut the coordinate *e* we receive assuming that (1/P_{m0}) = 0.

$$e_1=-\frac{I}{Az_t};e_2=-\frac{I}{Az_b};e_3=-\frac{I}{Az_t};e_4=-\frac{I}{Az_b};e_1=e_3;e_2=e_4$$

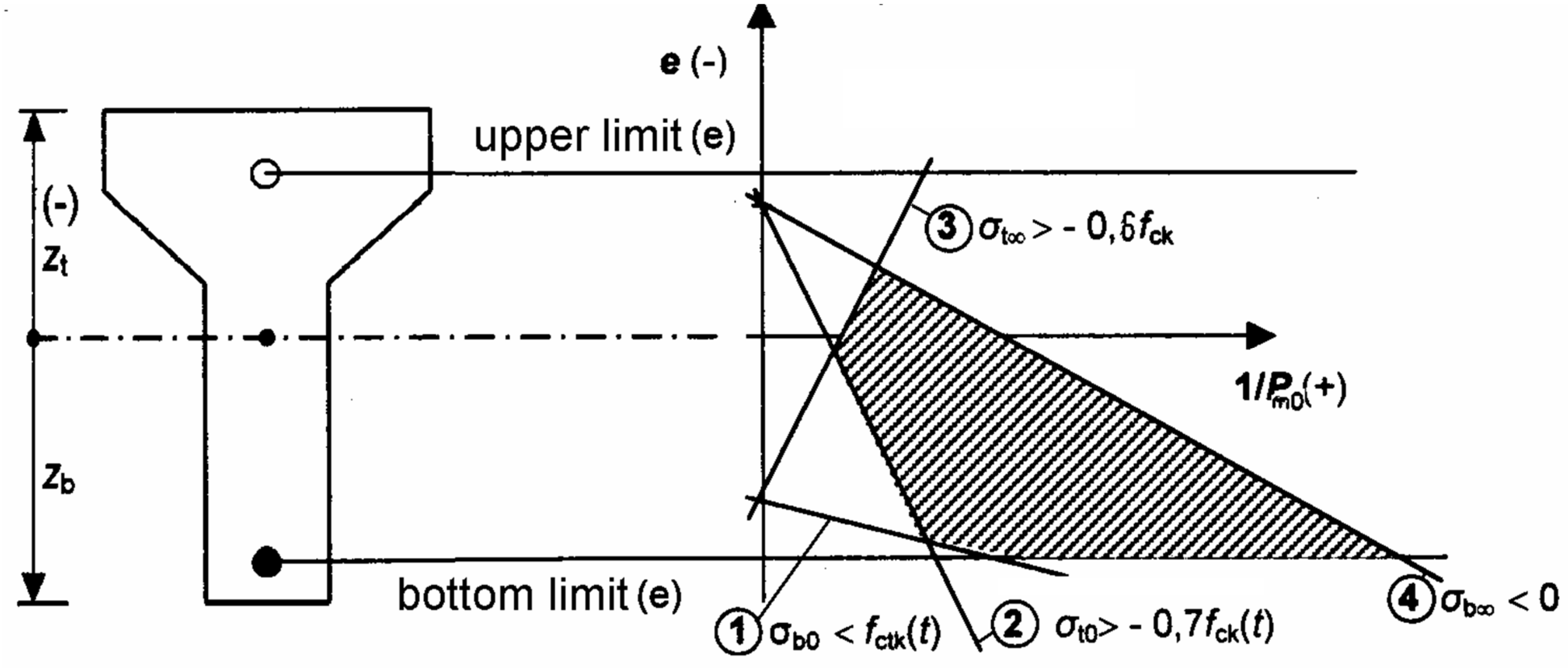
Assuming that *e* = 0 we receive the points where lines cut the coordinate (1/P_{m0}).

$$\frac{1}{P_{m01}}=\frac{r_{\sup}}{A}\left(\frac{1}{-f_{ctm}\left(t_0\right)+\frac{M_{g0}z_t}{I}}\right);\frac{1}{P_{m02}}=\frac{r_{\sup}}{A}\left(\frac{1}{0,7f_{ck}\left(t_0\right)+\frac{M_{g0}z_b}{I}}\right)$$

$$\frac{1}{P_{m03}}=\frac{0,8r_{\inf}}{A}\left(\frac{1}{0,6f_{ck}+\frac{M_kz_t}{I}}\right);\frac{1}{P_{m04}}=\frac{0,8r_{\inf}}{A}\left(\frac{1}{\frac{M_kz_b}{I}}\right)$$

From the unevenness (1) to (4) we can receive the set of permissible solutions for the prestressing force and its eccentricity.

Each unevenness presents linear relation for *e* and $1/P_{mo}$.



The diagram ($1/P_{m0}$, *e*) with four lines determinative the set of points satisfactory the limit stresses in extreme concrete fibres.

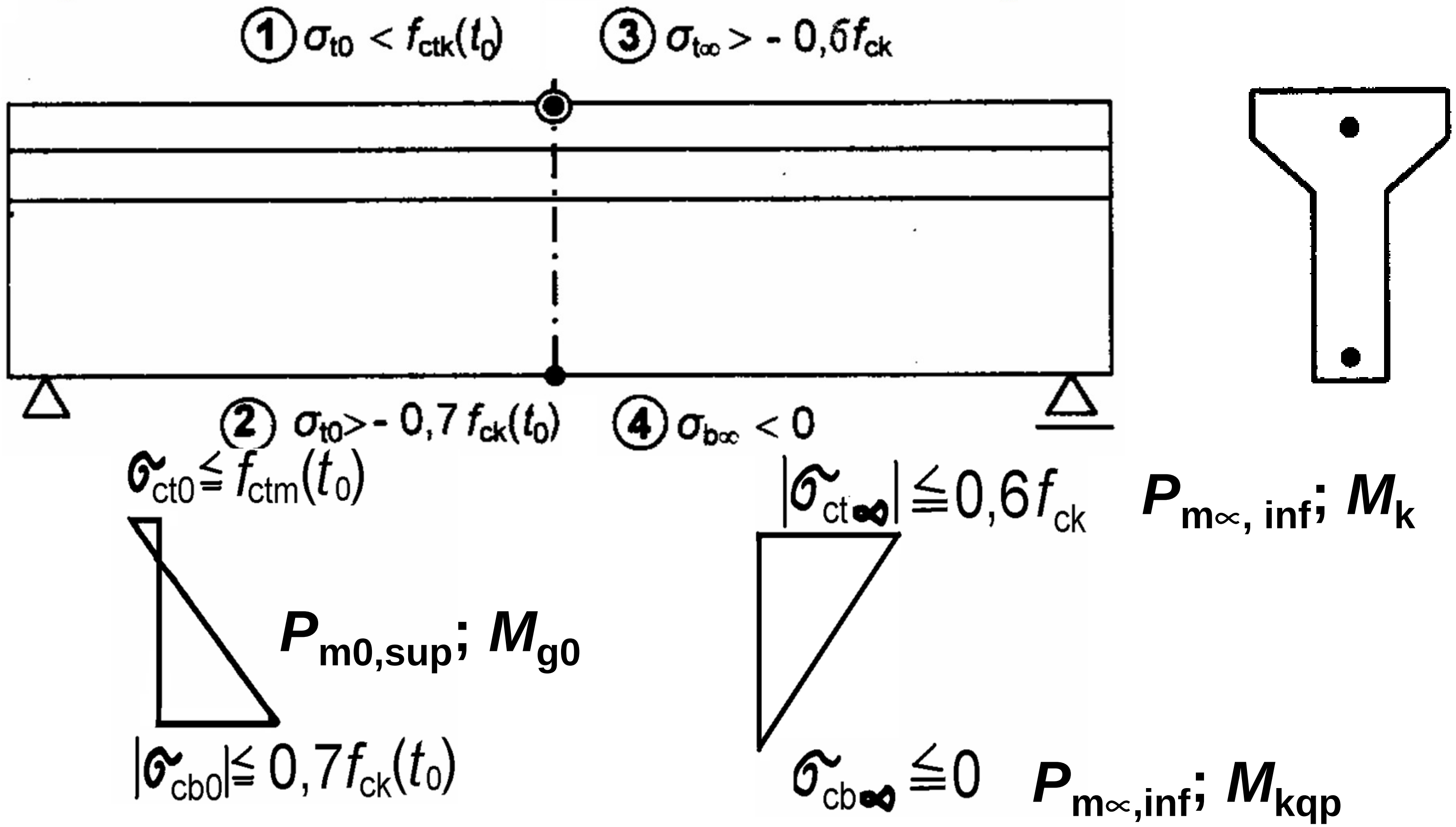
- **Example**

Pre-tensioned precast panel with TT cross section

C40/50; A=0,325 m^2; I= 0,0173 m^4; z_t=-0,229 m; z_b=0,511 m

prestressing: C35/45; 0,7 $f_{ck}(t_0)$≅ 25 Mpa, $f_{ctm}(t_0)$ ≅ 3,1 Mpa

M_{g0}=0,329 MNm; M_k=0,565 MNm; M_{kqp}=0,413 MNm (f_{ctm}=0)



- **Points where the lines cut the coordinate *e* we receive assuming that (1/P_{m0}) = 0**

$$e_1 \le -\frac{I}{Az_t} = -\frac{0,0173}{0,325.(-0,229)} = 0,232$$

$$e_2 \le -\frac{I}{Az_b} = -\frac{0,0173}{0,325.0,511} = -0,104$$

$$e_3 \ge -\frac{I}{Az_t} = -\frac{0,0173}{0,325.(-0,229)} = 0,232$$

$$e_4 \ge -\frac{I}{Az_b} = -\frac{0,0173}{0,325.0,511} = -0,104$$

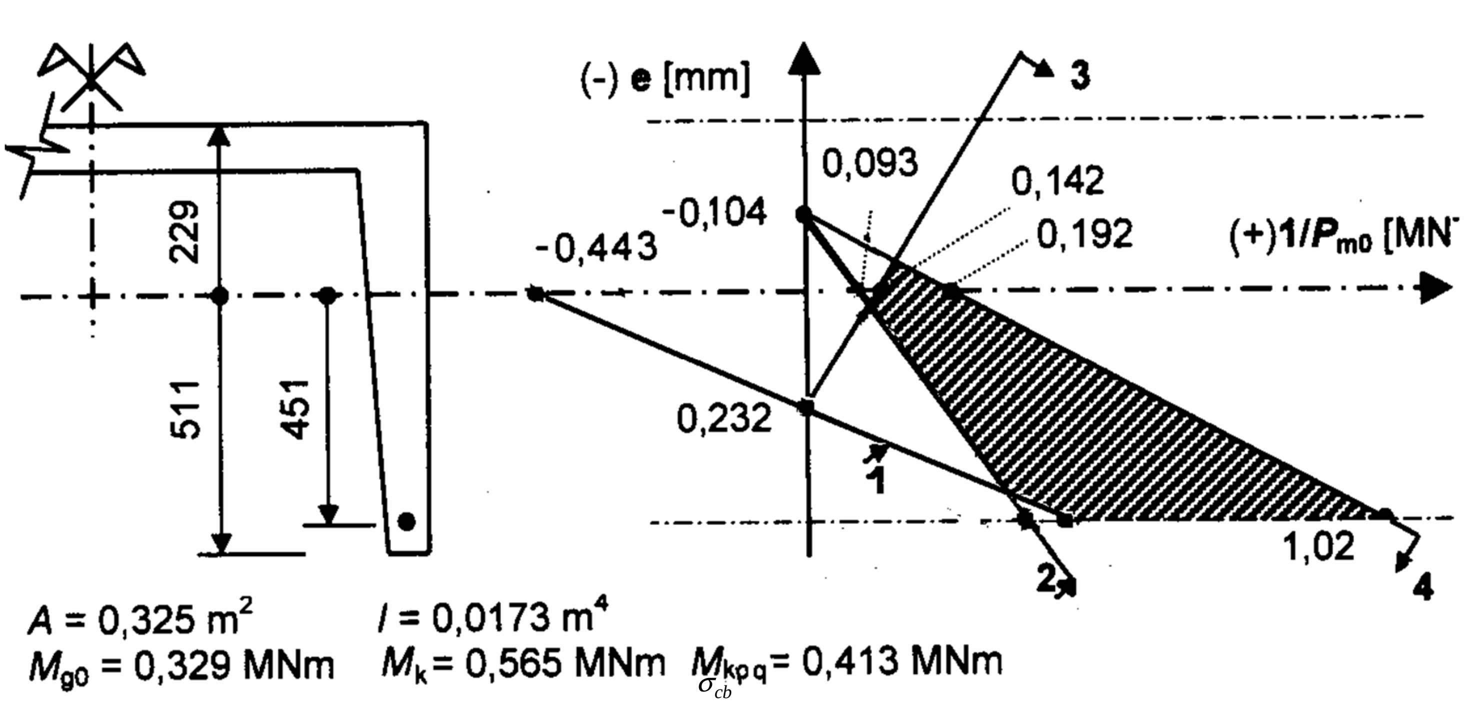
Assuming that *e* = 0 we receive the points where lines cut the coordinate (1/P_{m0})

$$\frac{1}{P_{m01}}=\frac{r_{\sup}}{A}\left(\frac{1}{-f_{ctm}\left(t_0\right)+\frac{M_{g0}z_t}{I}}\right)=\frac{1,05}{0,325}\left(\frac{1}{-3,1+\frac{0,329.(-0,229))}{0,0173}}\right)=-0,433MN$$

$$\frac{1}{P_{m02}}=\frac{r_{\sup}}{A}\left(\frac{1}{0,7\,f_{ck}\left(t_0\right)+\frac{M_{g0}z_b}{I}}\right)=\frac{1,05}{0,325}\left(\frac{1}{25+\frac{0,329.0,511}{0,0173}}\right)=0,093MN$$

$$\frac{1}{P_{m03}}=\frac{0,8r_{\inf}}{A}\left(\frac{1}{0,6\,f_{ck}+\frac{M_k z_t}{I}}\right)=\frac{0,8.0,95}{0,325}\left(\frac{1}{24+\frac{0,565.(-0,229)}{0,0173}}\right)=0,142MN$$

$$\frac{1}{P_{m04}}=\frac{0,8r_{\inf}}{A}\left(\frac{1}{\frac{M_{kqp}z_b}{I}}\right)=\frac{0,8.0,95}{0,325}\left(\frac{1}{\frac{0,413.0,511}{0,0173}}\right)=0,192MN$$



For the selected eccentricity *e* = 451 mm we receive min. force 1/P_{m0}=1,02 ⇒P_{m0} =0,98 MN. Assuming the strand Ø Lp 15,5: A_{p1}=0,000141 m², σ_{pm0} 1350 MPa; P_{m01}= 190 kPa, n = 0,98/0,19= 5,1; ⇒ 6 Ø Lp 15,5;

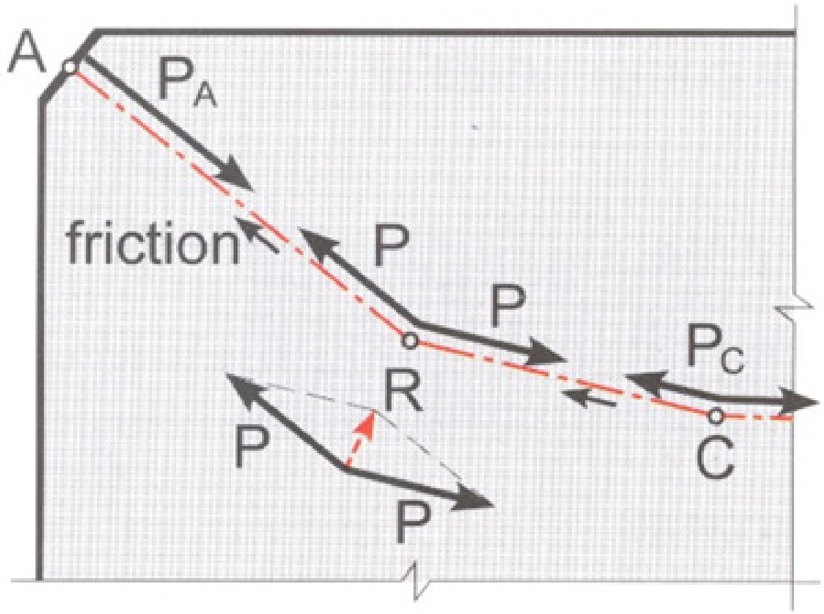
P_{m0}= 6.0,000141.1350=1,142 MN, 1/P_{m0}=1/1,142= 0,876 it is in the interval <0,496; 1,02> ⇒ OK!

Check of bottom stress:

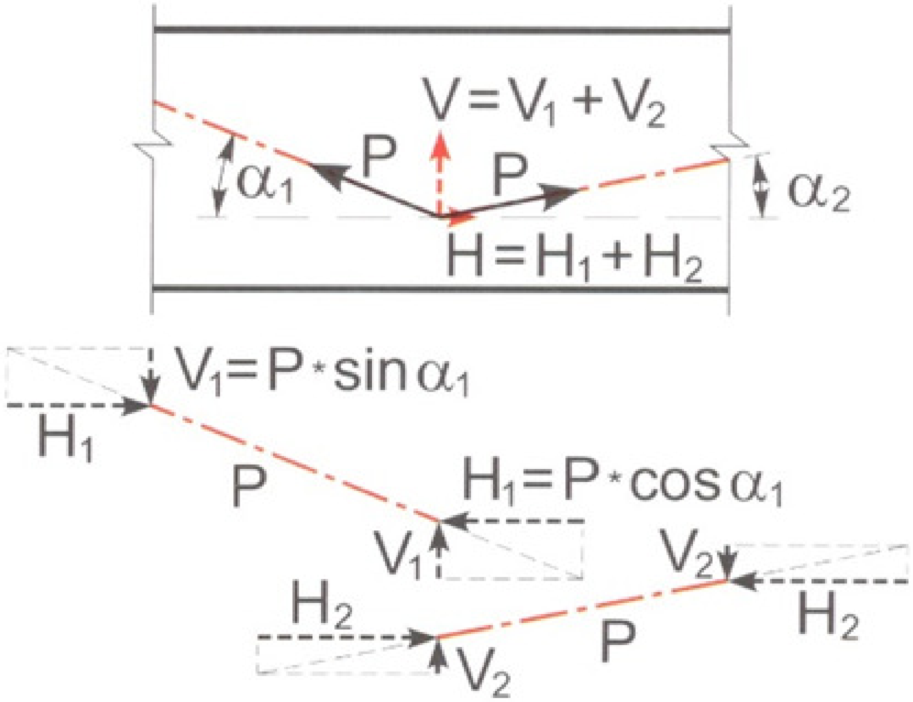
$$\sigma_{cb\infty} = \frac{-P_{m\infty,\inf}}{A} + \frac{-P_{m\infty,\inf}.e + M_{kpq}}{I} z_b = \frac{-1{,}142.0{,}8.0{,}95}{0{,}325} +$$

$$+ \frac{-1{,}142.0{,}8.0{,}95.0{,}451 + 0{,}413}{0{,}173} 0{,}511 = -2 \le 0$$

Force action of curved tendon on the concrete

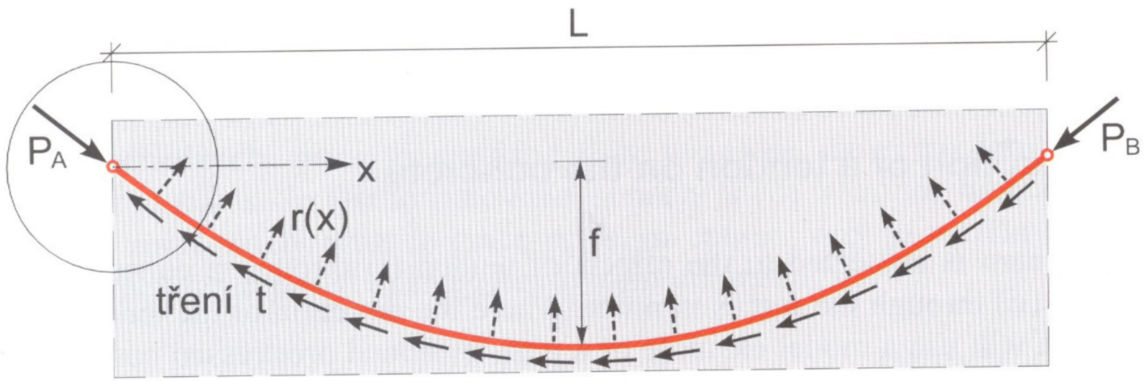


Source of radial forces

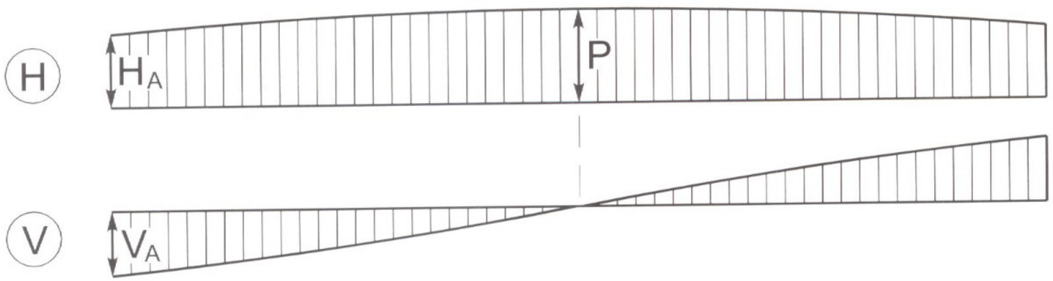


Forces caused by prestressing acting on concrete

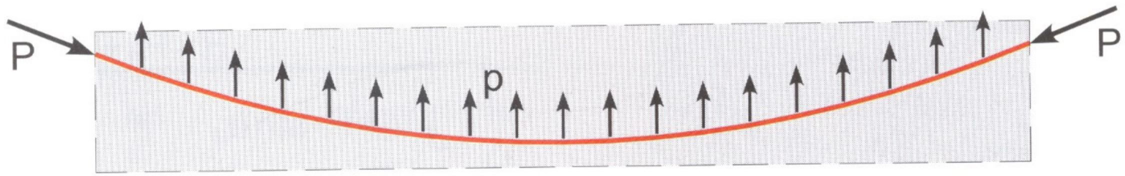
Equivalent load for a parabolic tendon



(a) Real force action of tendon on concrete

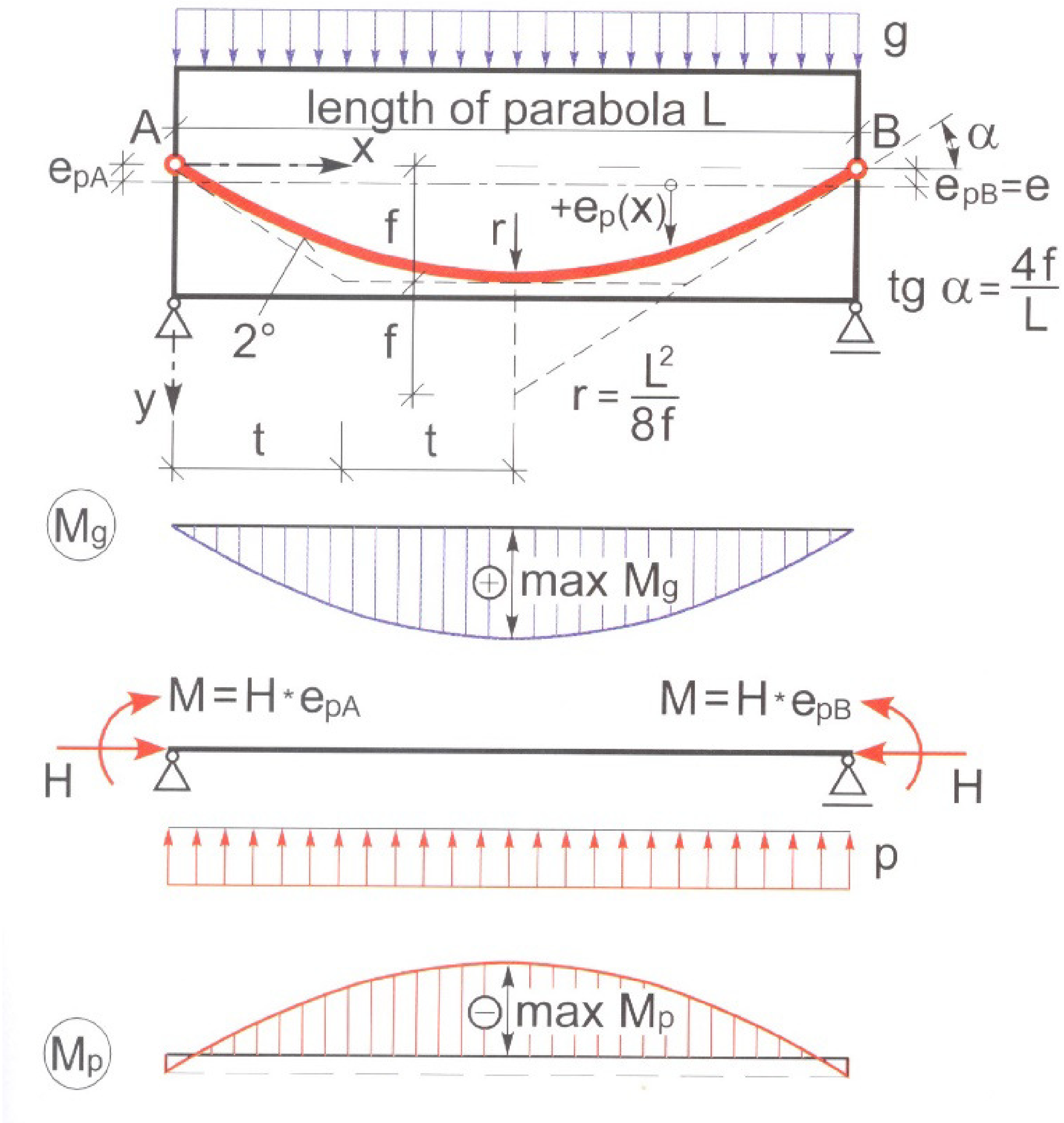


(b) Distribution of horizontal and vertical components of prestressing force when P = const

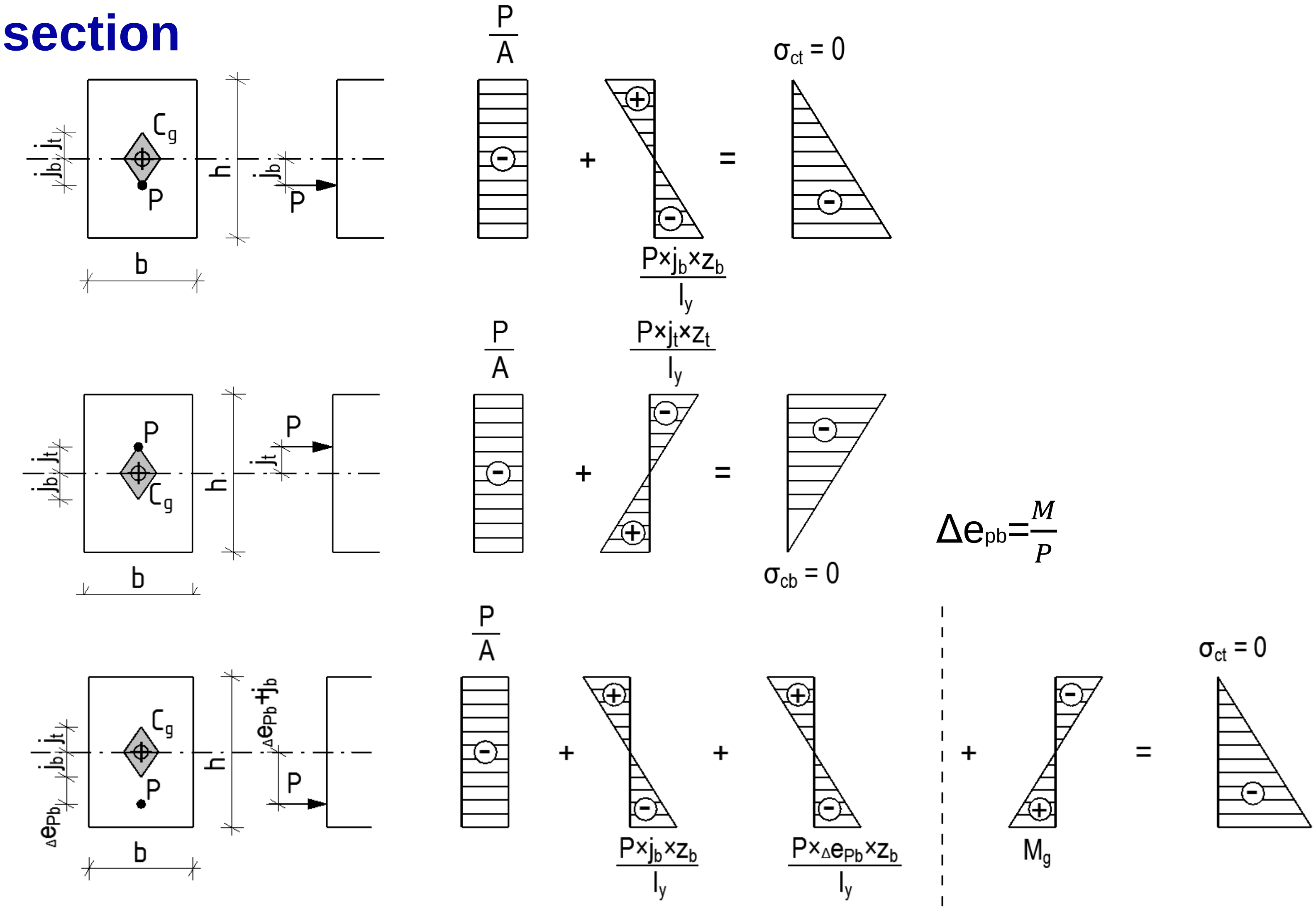


(c) Equivalent load for $\frac{f}{L} \leq \frac{1}{15}$ (H = P = const)

Beam with parabolic tendon



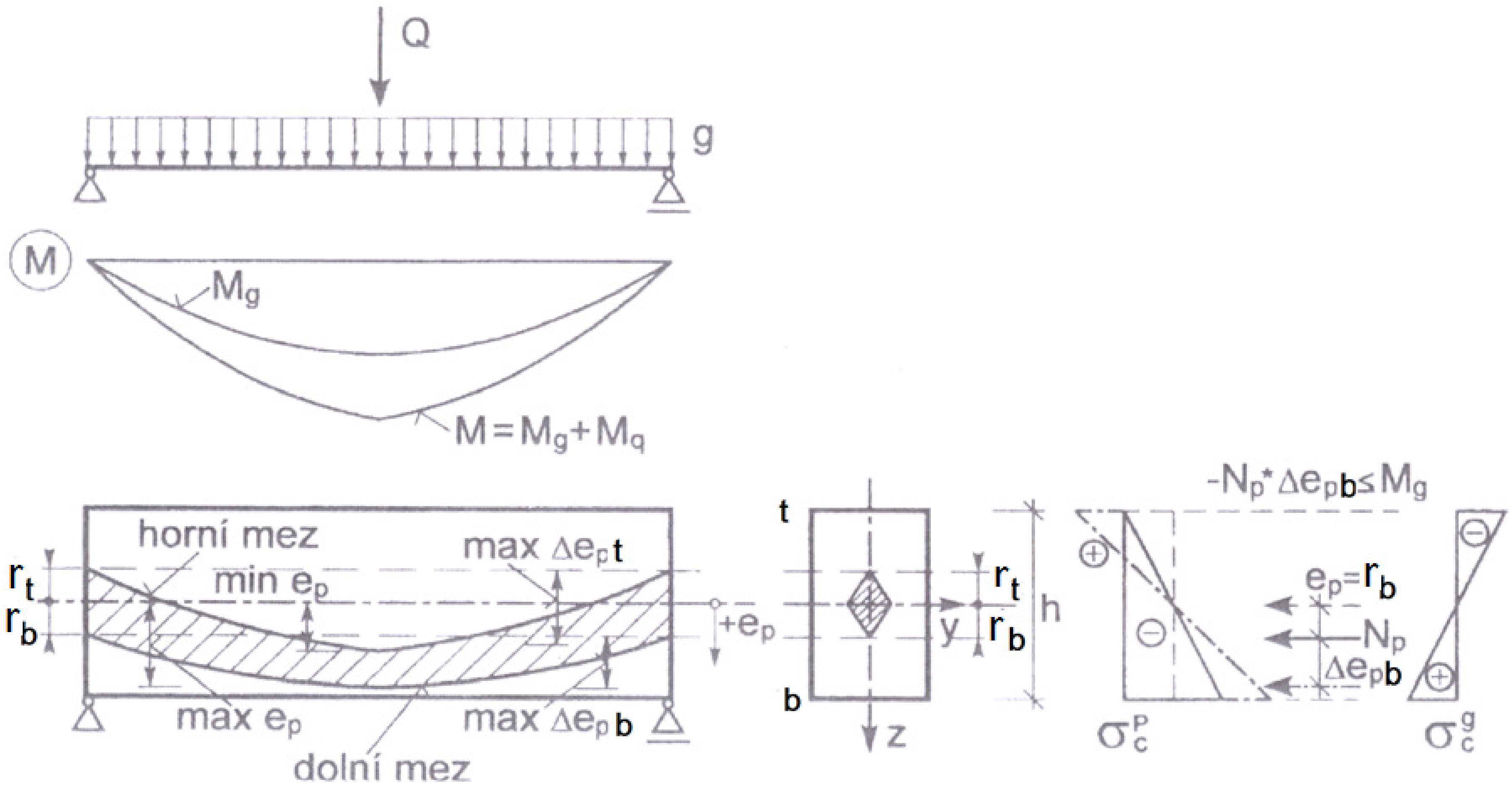
Core of section



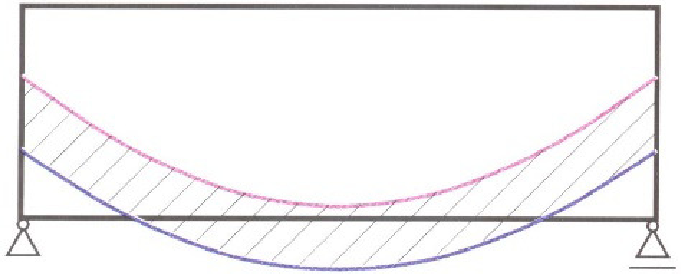
Part of the bending moment(for instance M_g, ev. M_g + αM_q) can be eliminated by the curved tendon (equivalent load). Centre of the gravity of tendon is necessary placed in the cross section to avoid the tensioned stresses.

Location of limiting zone for centre of gravity of tendon

Design of approximate value of N_p by equivalent load

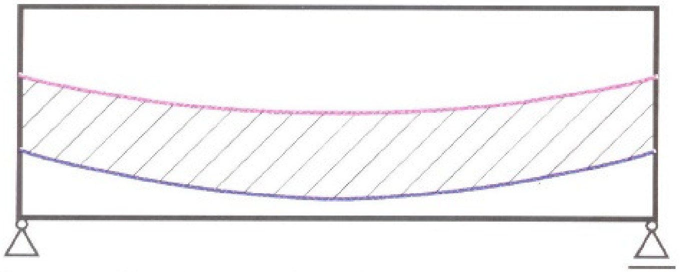


Undesirable locations of limiting zone for tendon centroid



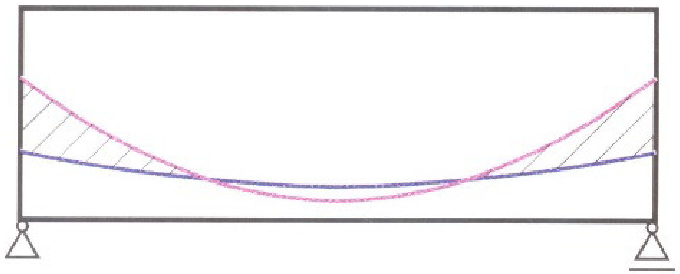
(a) Upper limit too near bottom fibre

a) - increase the prestressing force absolute value), or
- increase the height of beam



(b) Upper limit too far from bottom fibre

b) - reduce the prestressing force, or
- reduce the height of beam



(c) Upper and lower limits cross

c) - in the middle part no limiting part for the location of the tendon;
- increase the prestressing force, or
- increase the height of beam

Limiting zone for tendon location for full and limited prestressing