Prestressed Concrete

Part 1

(Introduction; Materials)

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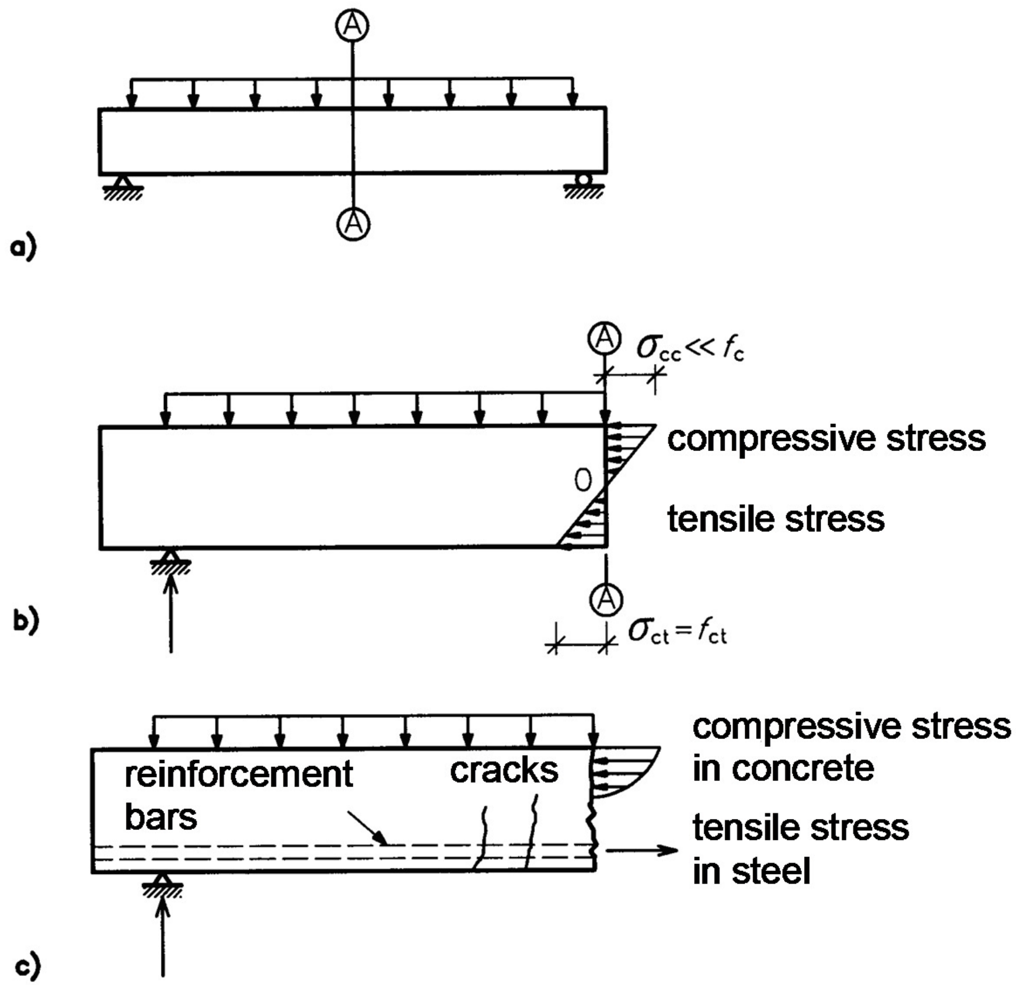
Literature:

Standards:

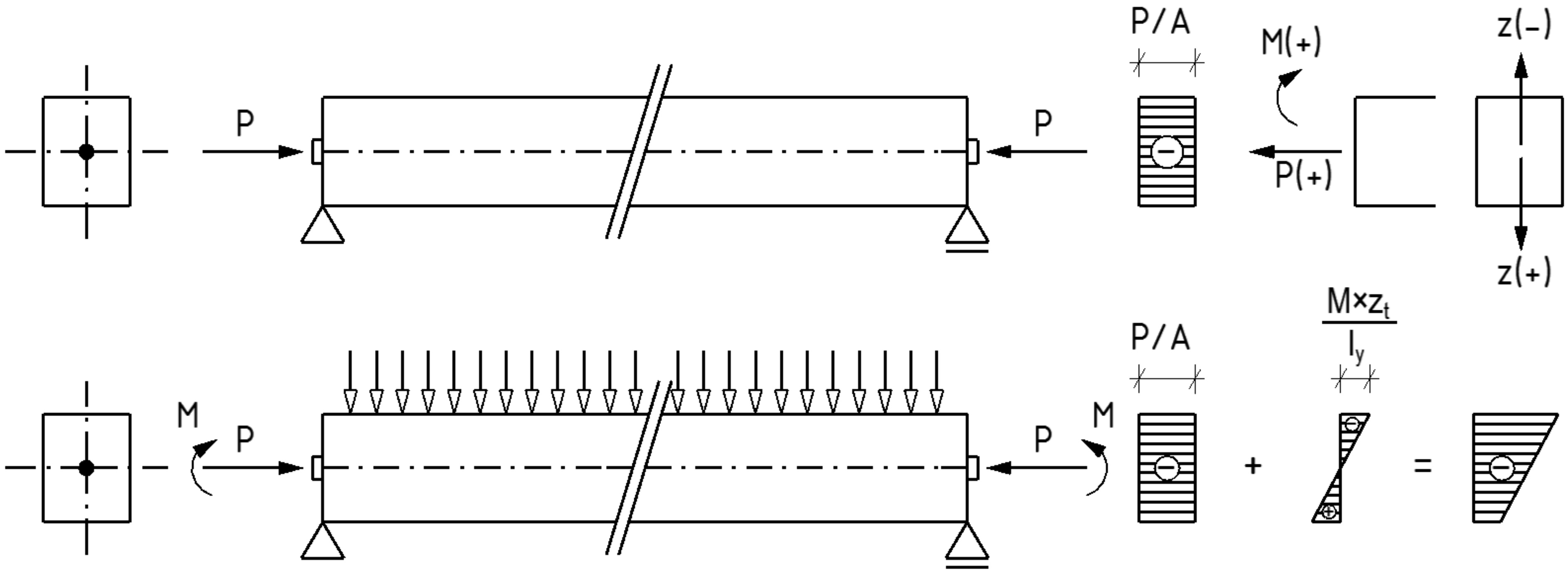
- EN 1992-1-1: Design of concrete structures - Part 1-1: General rules and rules for buildings
- EN 1992-1-2: Design of concrete structures - Part 1-2: Fire design

Monograph, textbook:

- Navrátil,J.: Prestressed concrete structures, CERM Brno, ISBN 80-7204-462-1, 2006
- Procházka, J., Štemberk, P.: Concrete Structures 1, ČVUT, ISBN 978-80-01-03607-5, 2009
- Procházka, J., Štemberk, P.: Design Procedures for Reinforced Concrete Structures, ČVUT, ISBN 978-80-01-04240-3, 2009



Plain and reinforced concrete beams



$$\sigma_t = \frac{P}{A} + \frac{M \times z_t}{I}$$

$$\sigma_b = \frac{P}{A} + \frac{M \times z_b}{I}$$

t – top

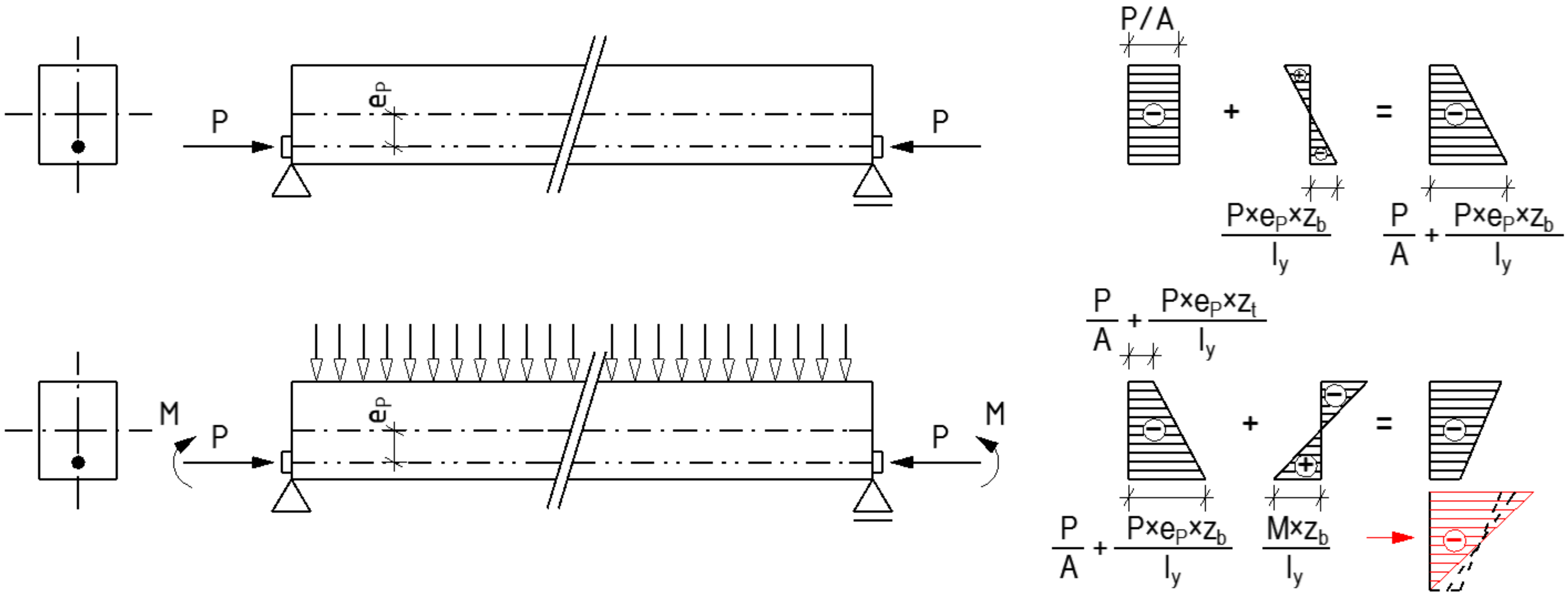
b – bottom

z – distance to fiber

I – moment of inertia

Prestressed concrete beam

prestressing in centre of gravity of cross section

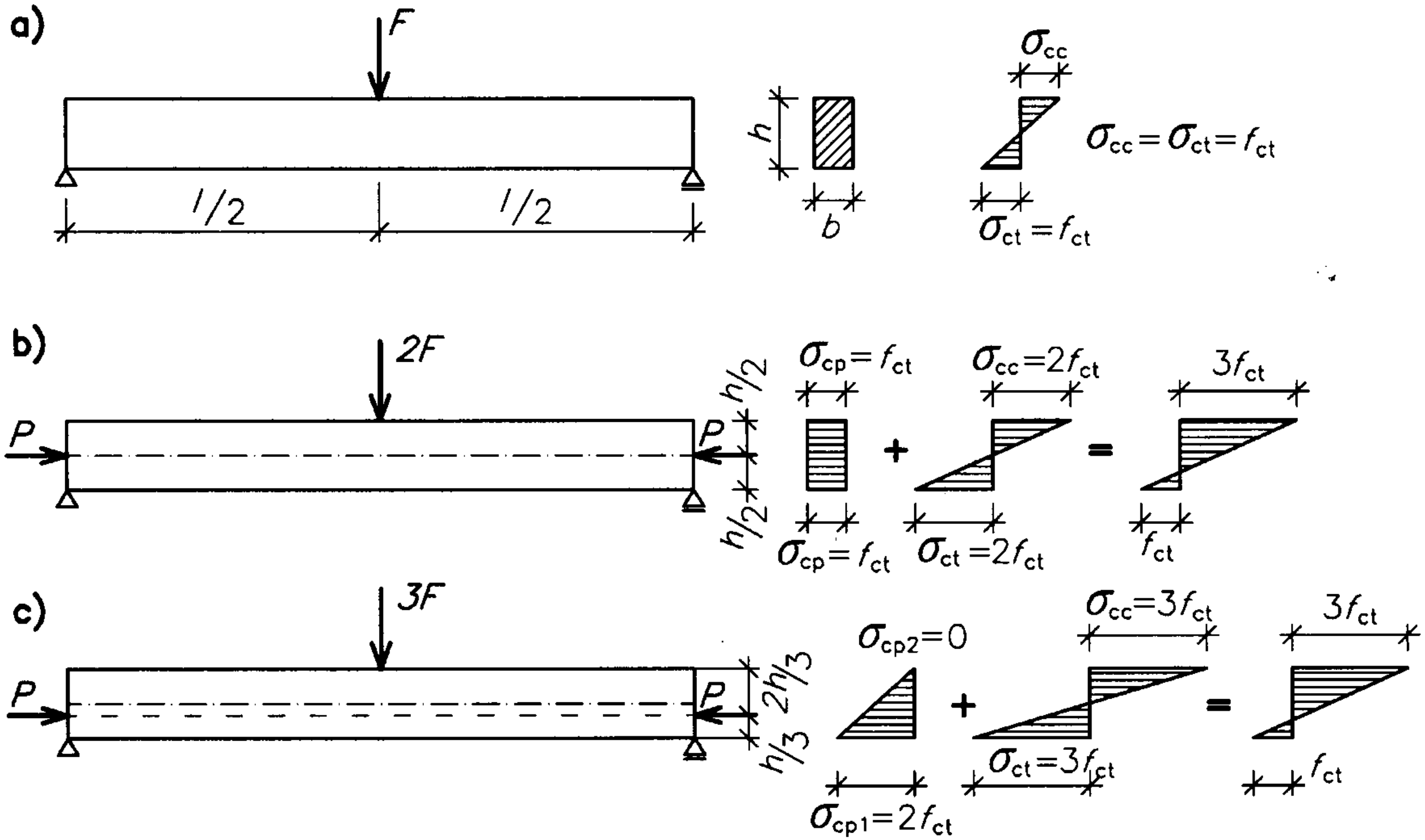


$$\sigma_t = \frac{P}{A} + \frac{P \times e_p \times z_t}{I} + \frac{M \times z_t}{I}$$

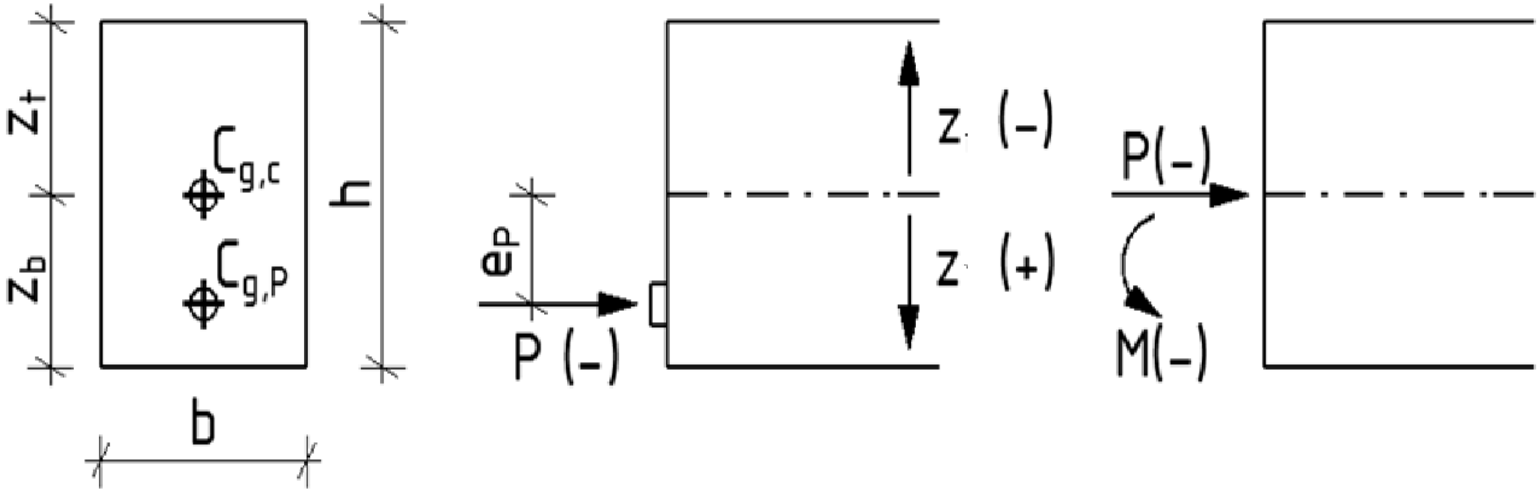
$$\sigma_b = \frac{P}{A} + \frac{P \times e_p \times z_b}{I} + \frac{M \times z_b}{I}$$

Prestressed concrete beam

prestressing with eccentricity *ep*



Plain and prestressed concrete beams



Elastic section modulus:

$$\sigma_t = \frac{P}{A} + \frac{P \times e_p \times z_t}{I}$$

$$W_t = \frac{I}{z_t}\ ;\ W_b = \frac{I}{z_b}$$

$$\sigma_b = \frac{P}{A} + \frac{P \times e_p \times z_b}{I}$$

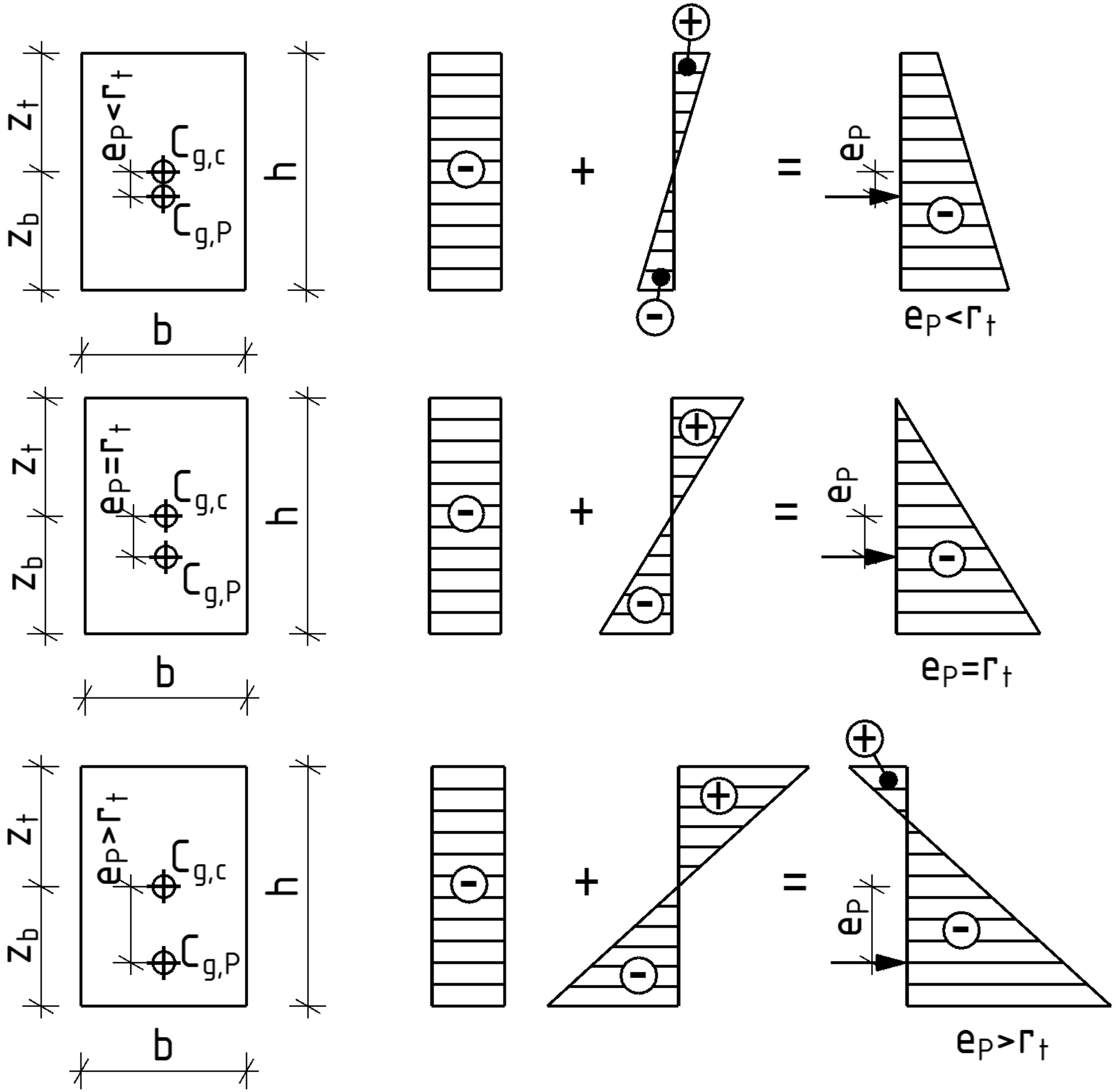
$$b, h:\ W_t = \frac{\frac{1}{12} \times b \times h^3}{-\frac{h}{2}} = -\frac{b \times h^2}{6}$$

$$W_b = \frac{b \times h^2}{6}$$

Prestressed concrete beam

prestressing with eccentricity *ep*

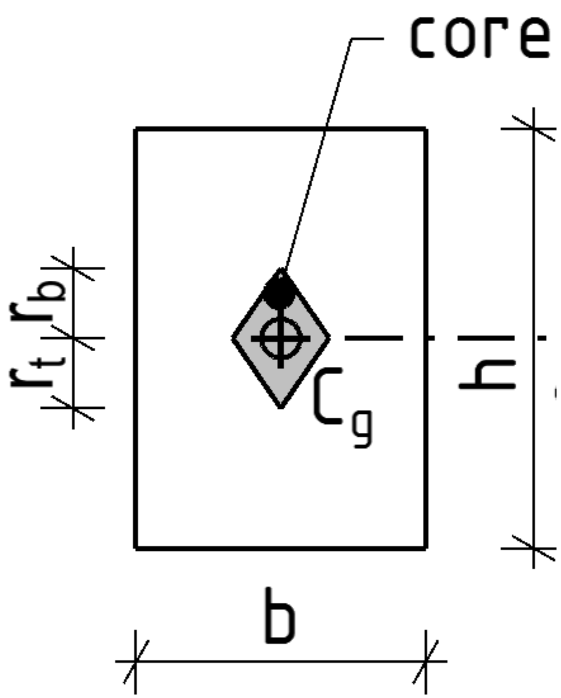
Prestressed concrete beam - prestressing with eccentricity *ep*



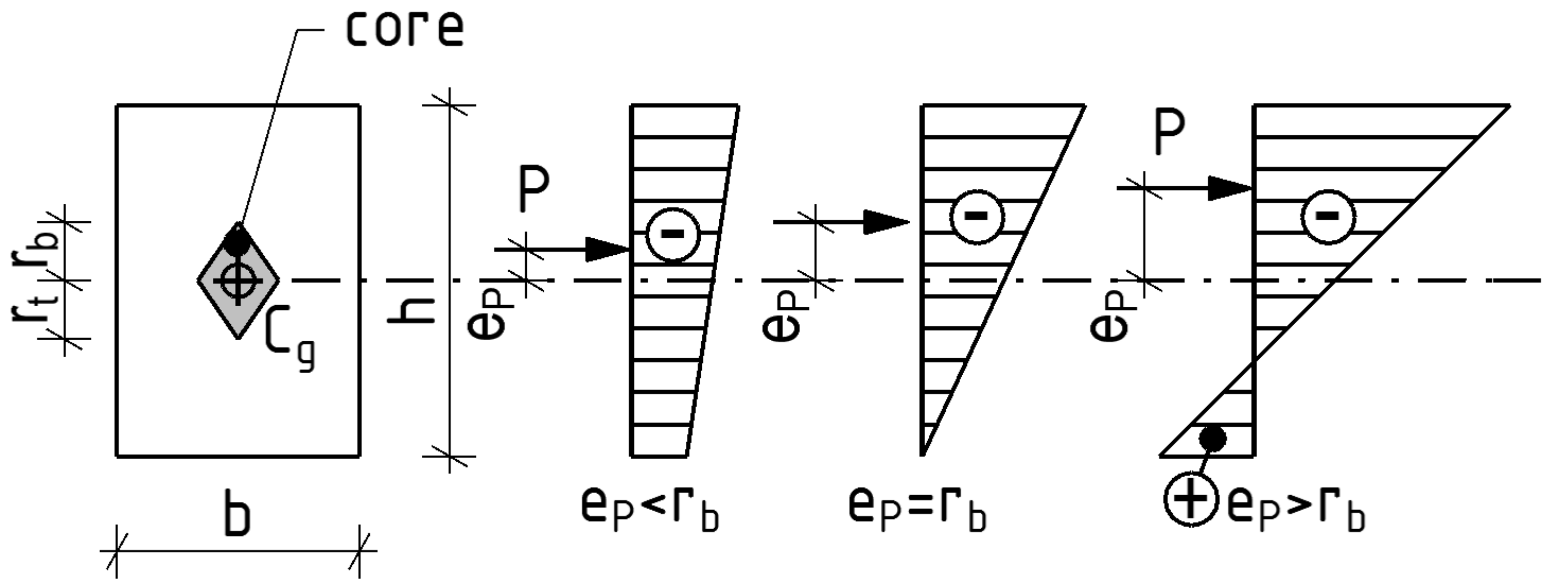
Prestressed concrete beam - prestressing with eccentricity *ep*

$$\sigma_t = \frac{P}{A} + \frac{P \times\ e_P \times z_t}{I_y} \Rightarrow 0 = \frac{P}{A} + \frac{P \times\ r_t \times z_t}{I_y} \longrightarrow \boldsymbol{r_t} = -\frac{\boldsymbol{I_y}}{\boldsymbol{A} \times \boldsymbol{z_t}}$$

$$b, h;\ r_t = -\frac{\frac{1}{12} \times b \times h^3}{b \times h \times (-\frac{h}{2})} = \frac{\boldsymbol{h}}{\boldsymbol{6}}$$



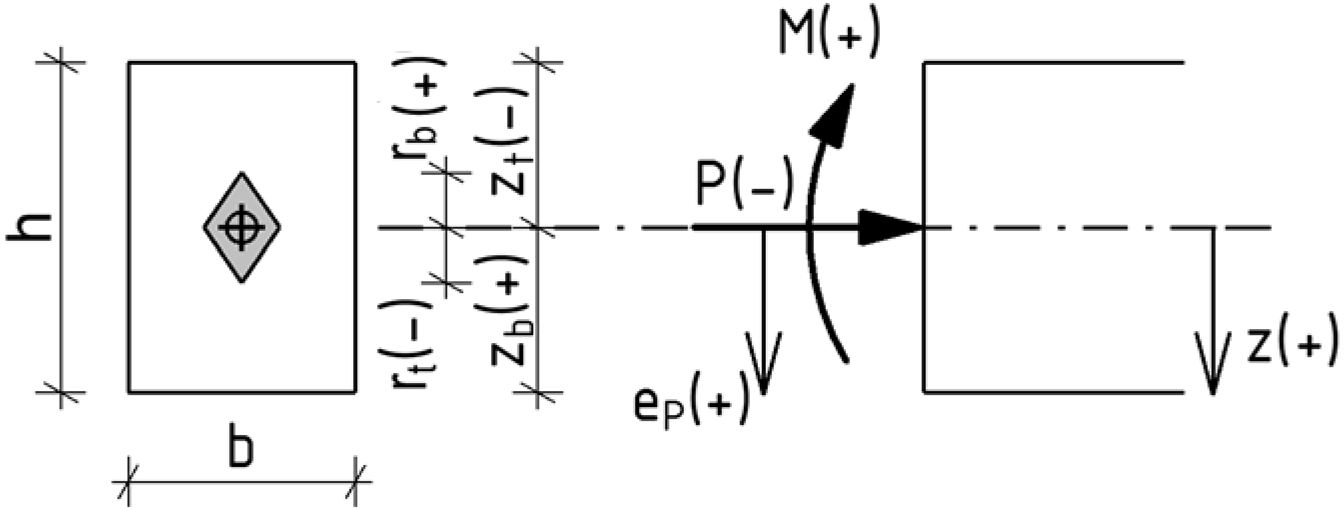
Prestressed concrete beam - prestressing with eccentricity *ep*



$$\sigma_b = \frac{P}{A} + \frac{P \times \; e_P \times z_b}{I_y} \Rightarrow 0 = \frac{P}{A} + \frac{P \times \; j_b \times z_b}{I_y} \longrightarrow \boldsymbol{r_b} = -\frac{\boldsymbol{I_y}}{\boldsymbol{A} \times \boldsymbol{z_b}}$$

$$b, h;\; r_b = -\frac{\frac{1}{12} \times b \times h^3}{b \times h \times \frac{h}{2}} = -\frac{\boldsymbol{h}}{\boldsymbol{6}}$$

Calculation of stress in bottom and top fibres



$$W_t = \frac{I}{z_t}(-) \qquad W_b = \frac{I}{z_b}(+)$$

$$r_t = \frac{I}{A \times z_t\,(-)} = \frac{W_t}{A}(-) \qquad r_b = \frac{I}{A \times z_b\,(+)} = \frac{W_b}{A}(+)$$

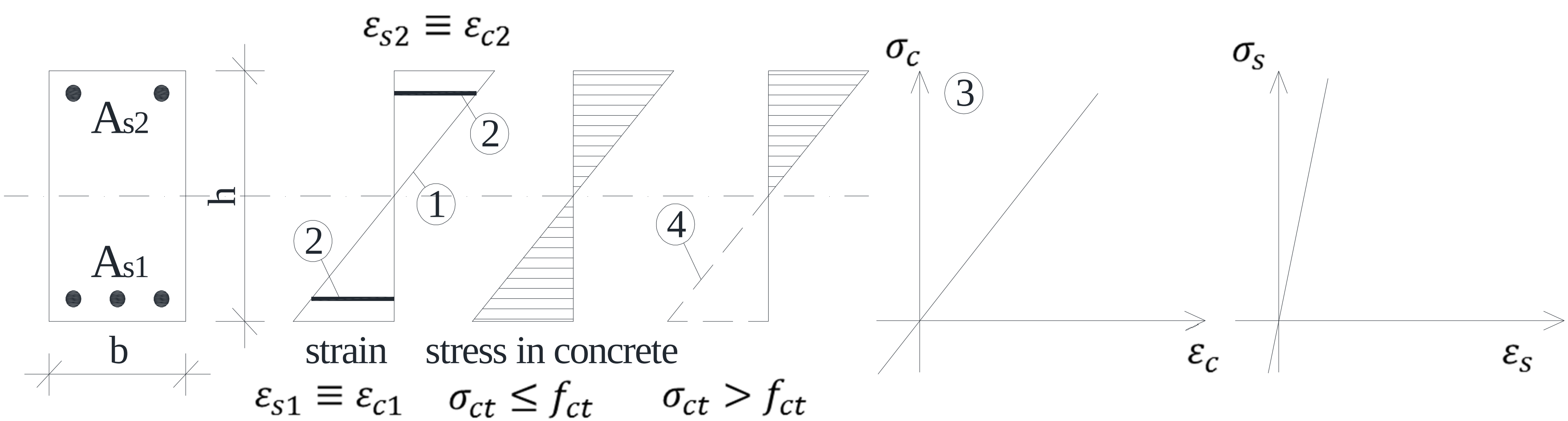
$$\sigma_b = \frac{P}{A} + \frac{P \times e_P \times z_b}{I} + \frac{M \times z_b}{I} = \frac{P}{A} + \frac{P \times e_P}{W_b} + \frac{M}{W_b} = \frac{P}{W_b} \times \left(\frac{W_b}{A} + e_P\right) + \frac{M}{W_b}$$

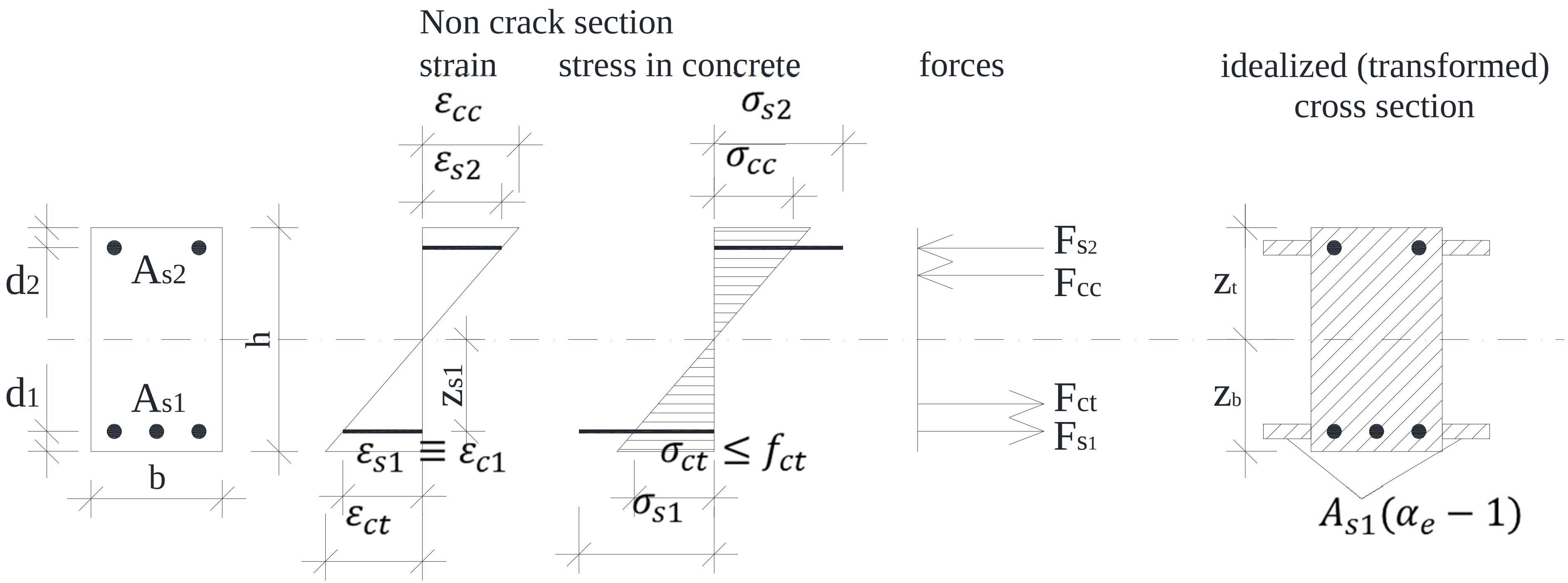
$$= \frac{P}{W_b} \times (r_b + e_P) + \frac{M}{W_b}$$

$$\sigma_t = \frac{P}{A} + \frac{P \times e_P \times z_t}{I} + \frac{M \times z_t}{I} = \frac{P}{W_t} \times (r_t + e_P) + \frac{M}{W_t}$$

Assumptions for the calculation of stresses:

1. The strain is directly proportional to the distance from neutral axis
2. The strain in bonded reinforcement is the same as that in the surrounding concrete
3. Concrete and reinforcing steel is assumed as idealy elastic material
4. The tension concrete is neglected when the tension stress reaches the strength of concrete





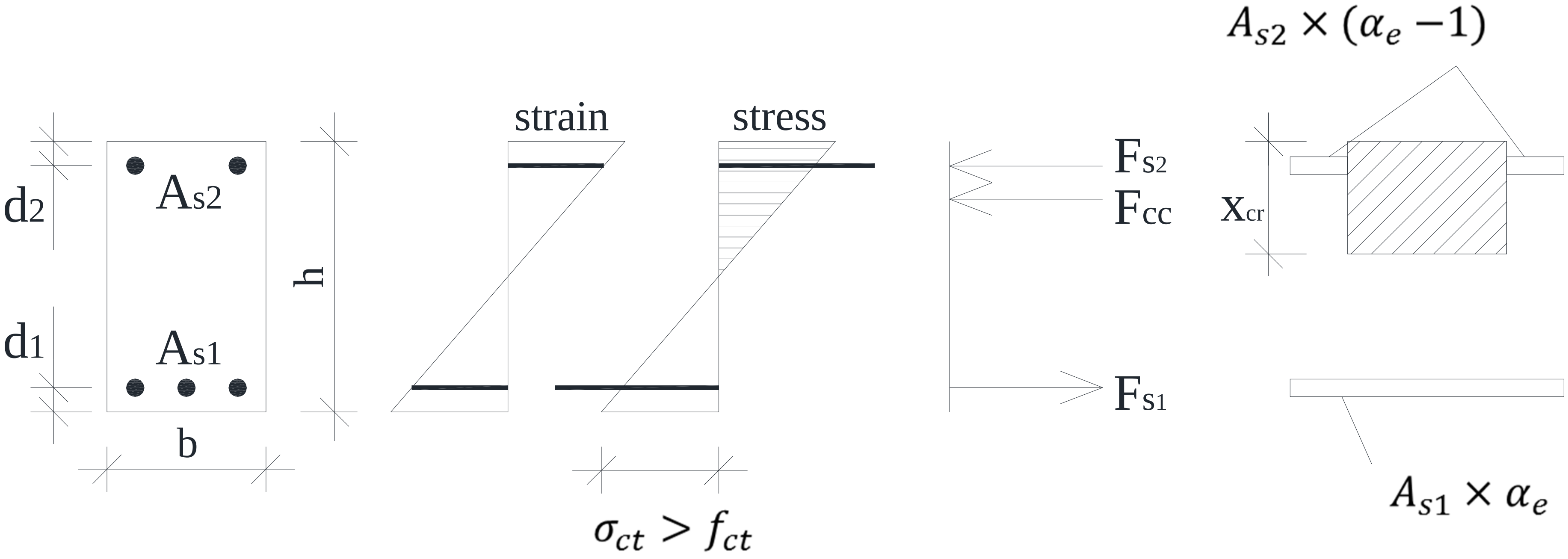
$$\varepsilon_{s1} = \frac{\sigma_{s1}}{E_s} \qquad \varepsilon_{c1} = \frac{\sigma_{c1}}{E_c}$$

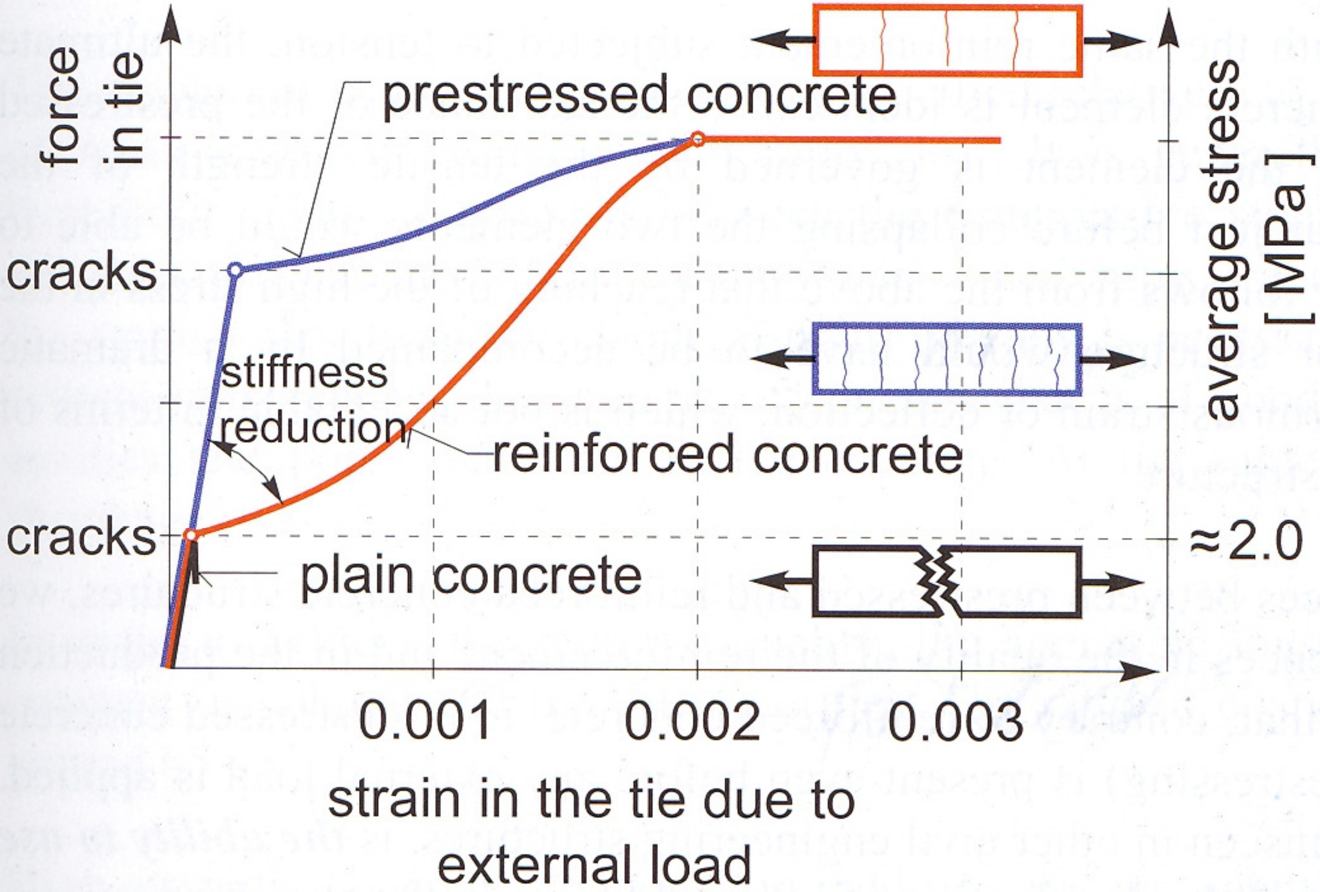
$$\varepsilon_{s1} = \varepsilon_{c1} \quad \rightarrow \quad \frac{\sigma_{s1}}{E_s} = \frac{\sigma_{c1}}{E_c}$$

$$\sigma_{s1} = \sigma_{c1}\frac{E_s}{E_c} = \sigma_{c1}\,\alpha_e \qquad \boldsymbol{\alpha_e} = \frac{\boldsymbol{E_s}}{\boldsymbol{E_c}}$$

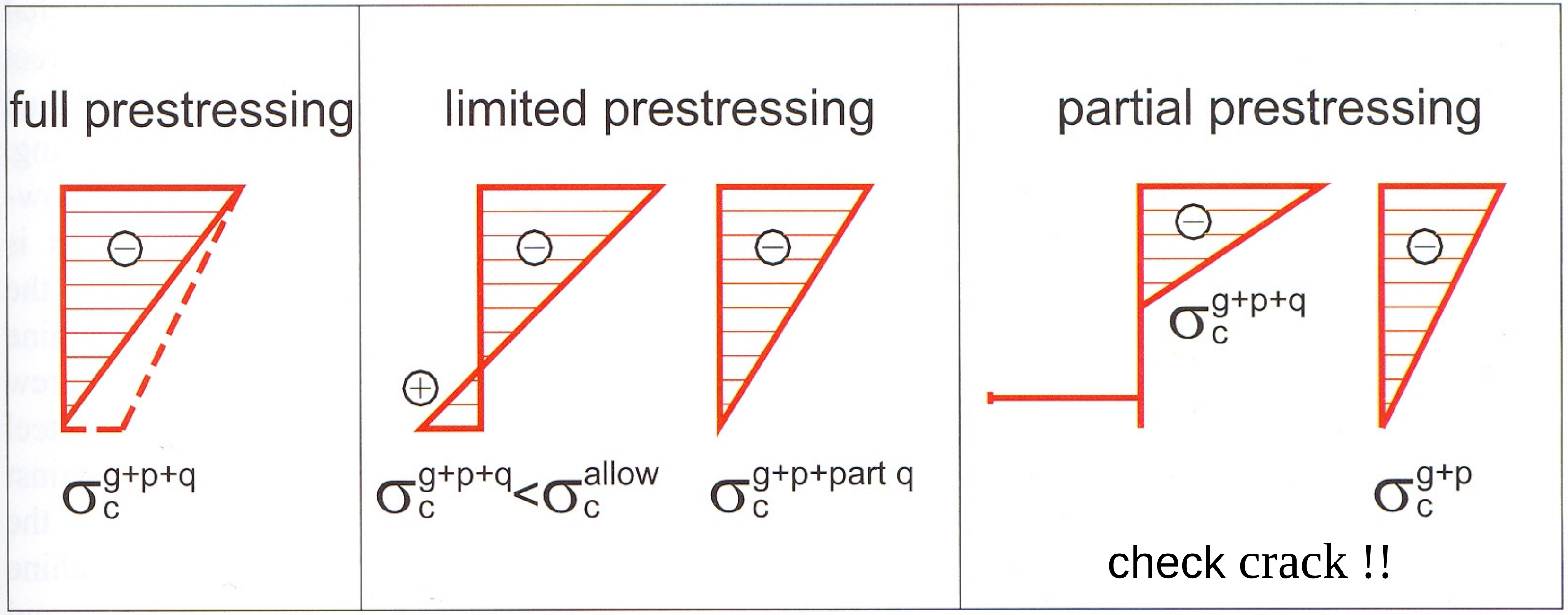
$$F_{s1} = A_{s1} \times \sigma_{c1} \times \alpha_e$$

Cracked section

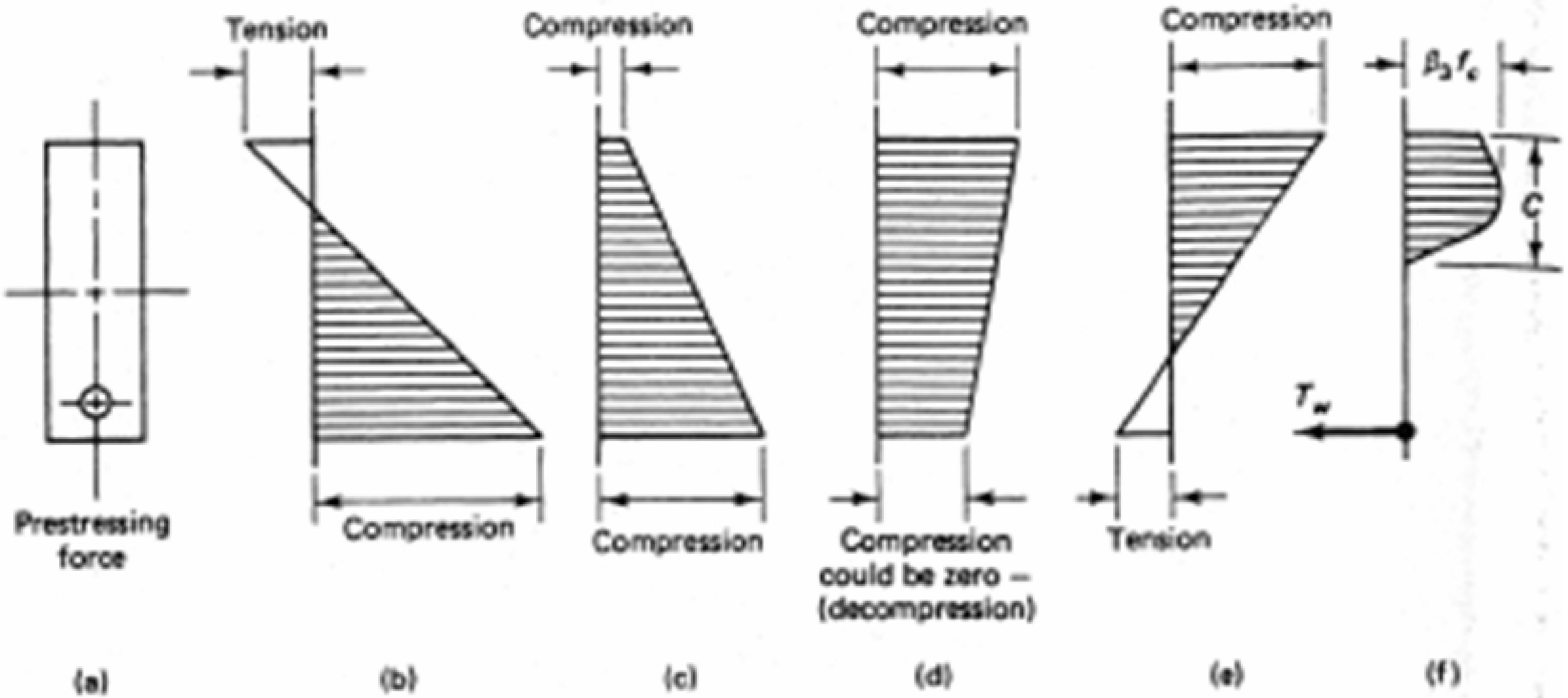




Response of plain, reinforced and prestressed concrete in tension

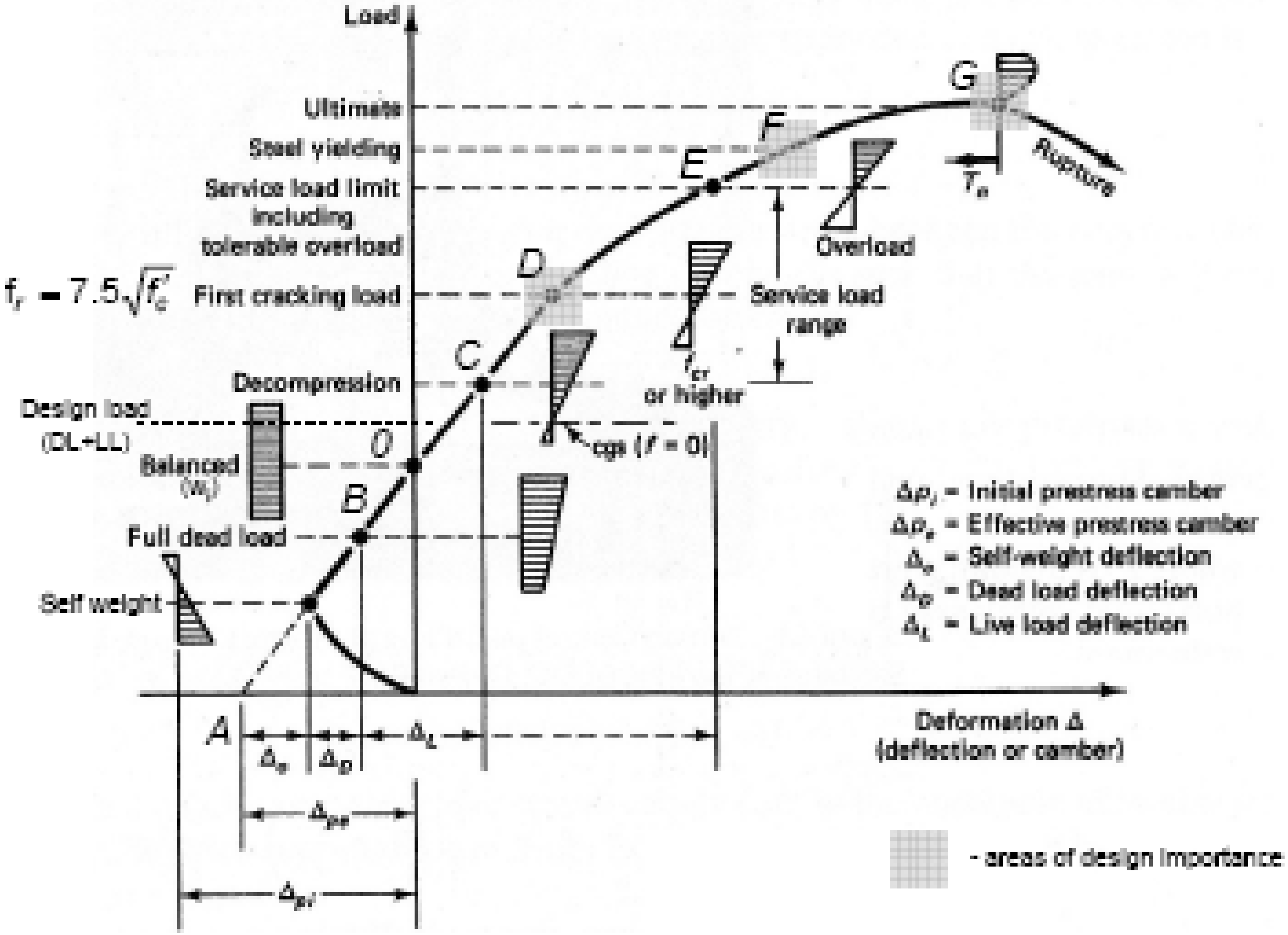


Classification of prestressed concrete by the level of prestressing



(a) Beam section.
(b) Initial prestressing stage.
(c) Self-weight and effective prestress.
(d) Full dead load plus effective prestress.
(e) Full service load plus effective prestress.
(f) Limit state of stress at ultimate load for underreinforced beam.

Flexural stress distribution throughout loading history.



Load
Ultimate
Steel yielding
Service load limit
including
tolerable overload
$f_r = 7.5\sqrt{f'_c}$
First cracking load
Decompression
Design load
(DL+LL)
Balanced
Full dead load
Self weight
G
F
E
D
C
O
B
A
Rupture
Overload
Service load
range
or higher
Deformation Δ
(deflection or camber)
Δp_i = Initial prestress camber
Δp_e = Effective prestress camber
Δ_o = Self-weight deflection
Δ_D = Dead load deflection
Δ_L = Live load deflection
- areas of design importance

Materials

- **Concrete**
- **Steel - reinforcing**

  - prestressing

Concrete (EN 1992-1-1)

Concrete strength classes

Concrete strength class C8/10 to C100/115.
(Characteristic cylinder strength / char. cube strength)

≥ C100/115

very brittle behaviour

≥ C30/37

prestressed concrete

Concrete strength classes and properties

	Strength classes for concrete													
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0
$f_{ctk,0,05}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5
$f_{ctk,0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6
E_{cm} (Gpa)	27	29	30	31	32	34	35	36	37	38	39	41	42	44
ε_{c1} (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8
ε_{cu1} (‰)	3,5									3,2	3,0	2,8	2,8	2,8
ε_{c2} (‰)	2,0									2,2	2,3	2,4	2,5	2,6
ε_{cu2} (‰)	3,5									3,1	2,9	2,7	2,6	2,6
n	2,0									1,75	1,6	1,45	1,4	1,4
ε_{c3} (‰)	1,75									1,8	1,9	2,0	2,2	2,3
ε_{cu3} (‰)	3,5									3,1	2,9	2,7	2,6	2,6

Design Strength Values

(3.1.6)

- Design compressive strength, f_{cd}

$$f_{cd} = \alpha_{cc} \, f_{ck} / \gamma_c$$

- Design tensile strength, f_{ctd}

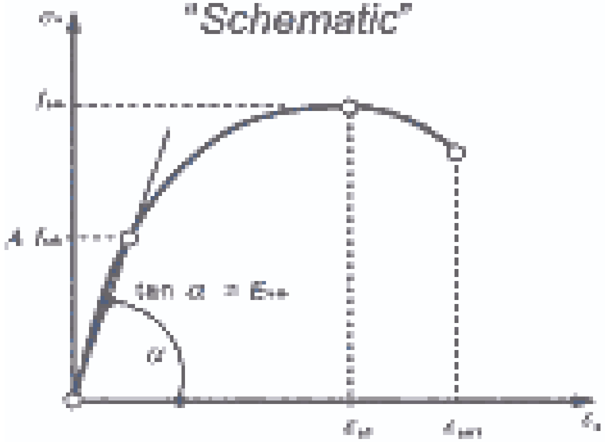
$$f_{ctd} = \alpha_{ct} \, f_{ctk,0.05} / \gamma_c$$

α_{cc} (= 1,0) and α_{ct} (= 1,0) are coefficients to take account of long term effects on the compressive and tensile strengths and of unfavourable effects resulting from the way the load is applied (national choice)

Concrete stress - strain relations (3.1.5 and 3.1.7)

For structural analysis

"Schematic"



ε_{c1} (‰) = 0,7 $f_{cm}^{0,31}$

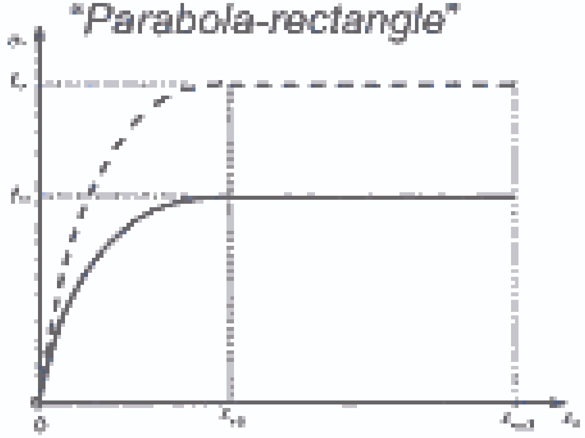
ε_{cu1} (‰) =

2,8 + 27[(98-f_{cm})/100]^4 f_{cm})/100)^4

for $f_{ck} \geq$ 50 MPa otherwise 3.5

For section analysis

"Parabola-rectangle"



$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \text{ for } 0 \leq \varepsilon_c < \varepsilon_{c2}$$

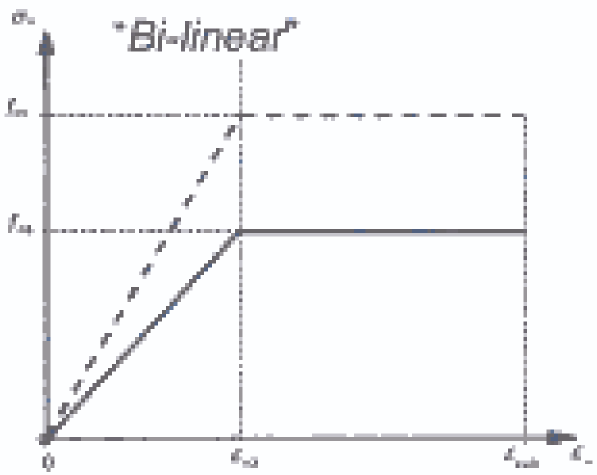
$$\sigma_c = f_{cd} \text{ for } \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2}$$

n = 1,4 + 23,4 [(90- f_{ck})/100]^4
for $f_{ck} \geq$ 50 MPa otherwise 2,0

ε_{c2} (‰) = 2,0 + 0,085(f_{ck}-50)^{0,53}

for $f_{ck} \geq$ 50 MPa otherwise 2,0

"Bi-linear"

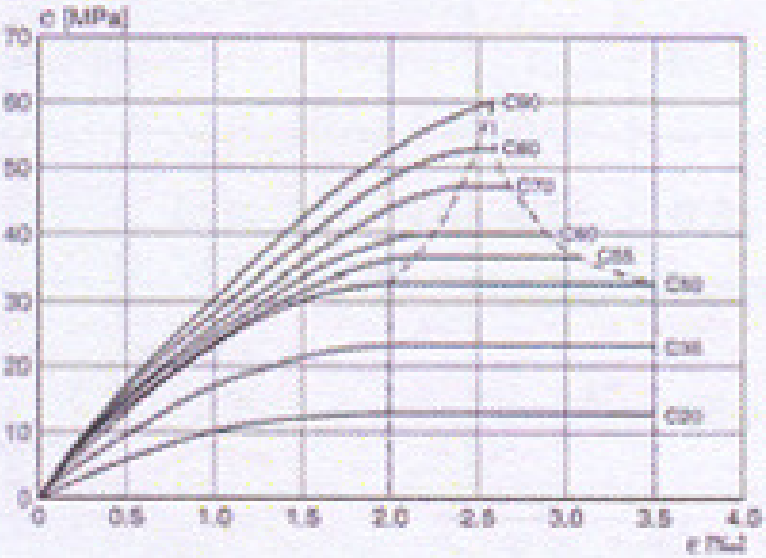


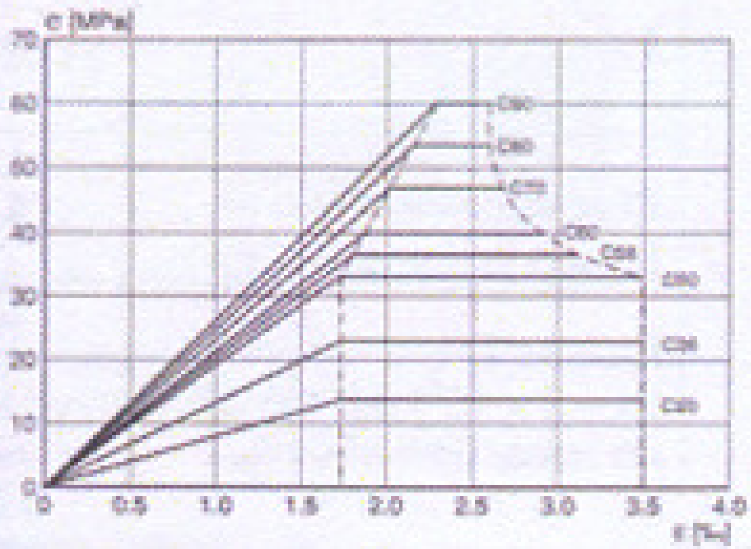
ε_{c3} (‰) = 1,75 + 0,55 [(f_{ck}-50)/40]
for $f_{ck} \geq$ 50 MPa otherwise 1,75

ε_{cu3} (‰) =2,6+35[(90-f_{ck})/100]^4
for $f_{ck} \geq$ 50 MPa otherwise 3,5

Concrete stress-strain relations

- Higher concrete strength show more brittle behaviour, reflected by shorter horizontal branche





Shrinkage (3.1.4)

- The shrinkage strain ε_{cs} is composed of two components:

  $\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$

where

- drying shrinkage strain

  $\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}$ where $\varepsilon_{cd,0}$ is the basic drying shrinkage strain

- autogenous shrinkage strain

  $\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$

Shrinkage (3.1.4)

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0{,}04\sqrt{h_0^3}}$$

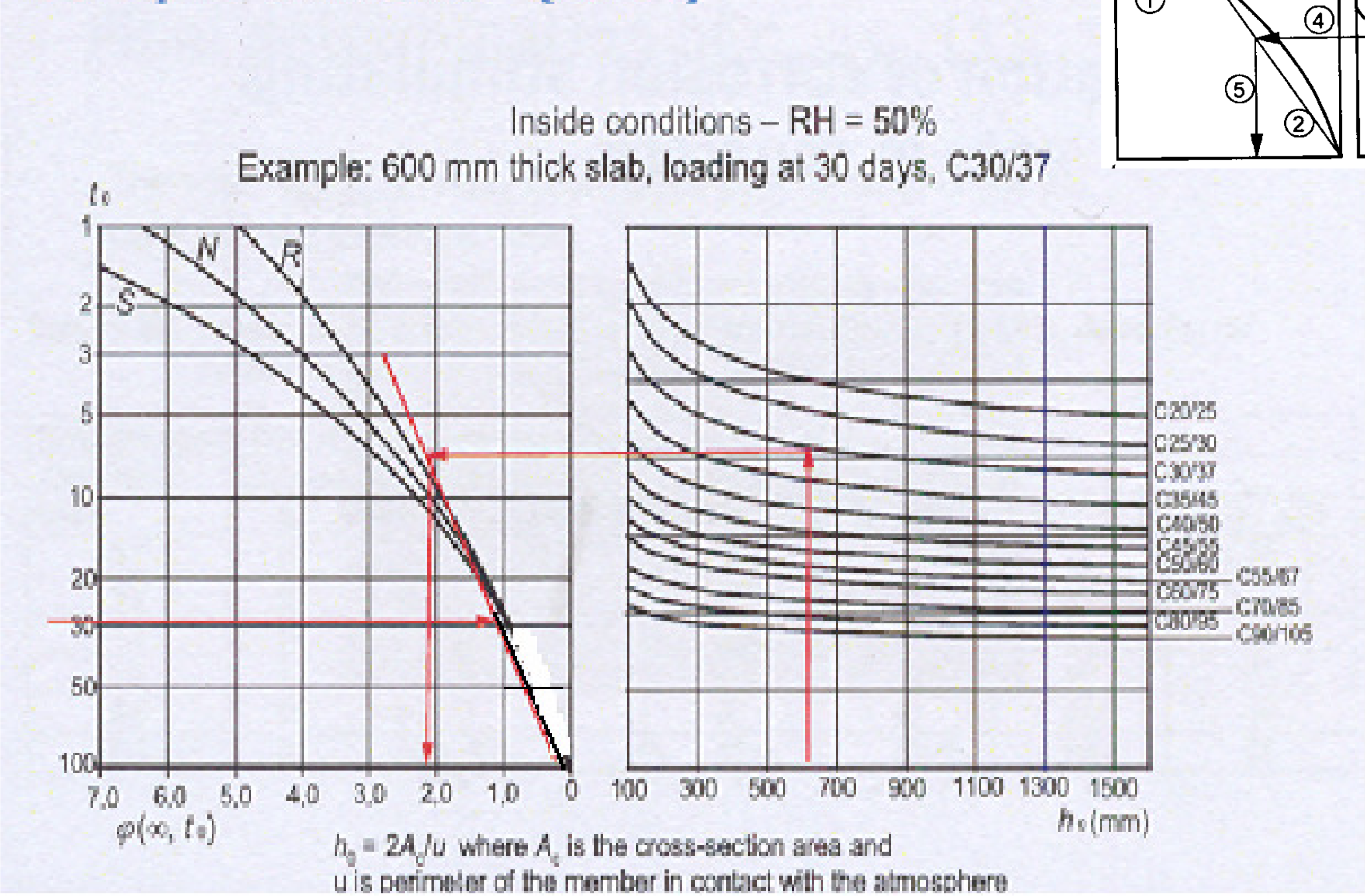
where t = age of concrete at time considered, t_s= age at beginning of drying shrinkage (mostly end of curing)

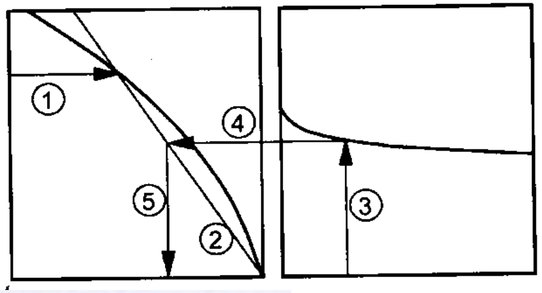
$$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty)$$

where

$\varepsilon_{ca}(\infty) = 2{,}5(f_{ck}-10)\cdot 10^{-6}$ and $\beta_{as}(t) = 1-\exp(-0{,}2t^{0,5})$

Creep of concrete (3.1.4)





R, N, S - depends on cement strength classes: R – CEM 42,5 R, CEM 52,5 N, CEM 52,5 R; N – CEM 32,5R, CEM 42,5N; S - CEM 32,5N

Reinforcing steel (EN 10 080)

Product forms:

- **Bar -** rebar plain, ribbed, $\varnothing > 8$ mm
- **Wire –** decoiled rods, $\varnothing \leq 14$ mm
- **Wire fabrics**

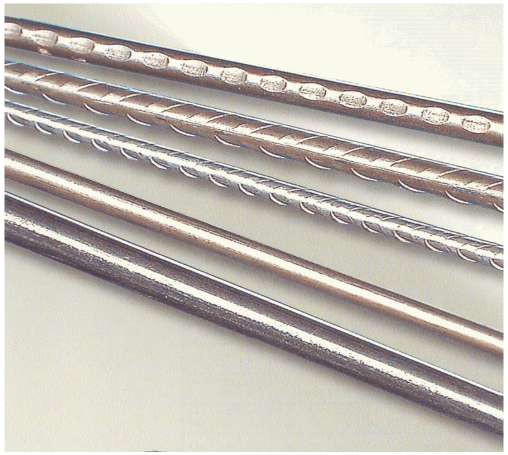
Mark of steel

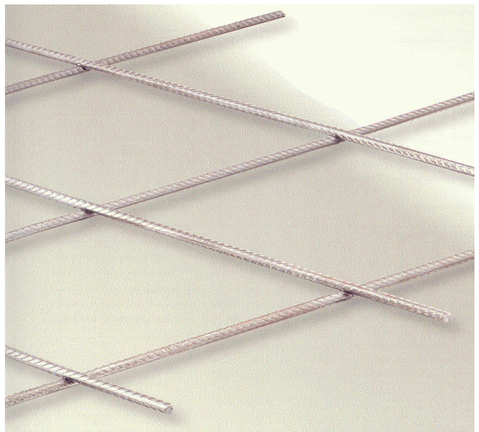
B reinforcing steel

f_{yk} ($f_{0,2k}$) in MPa

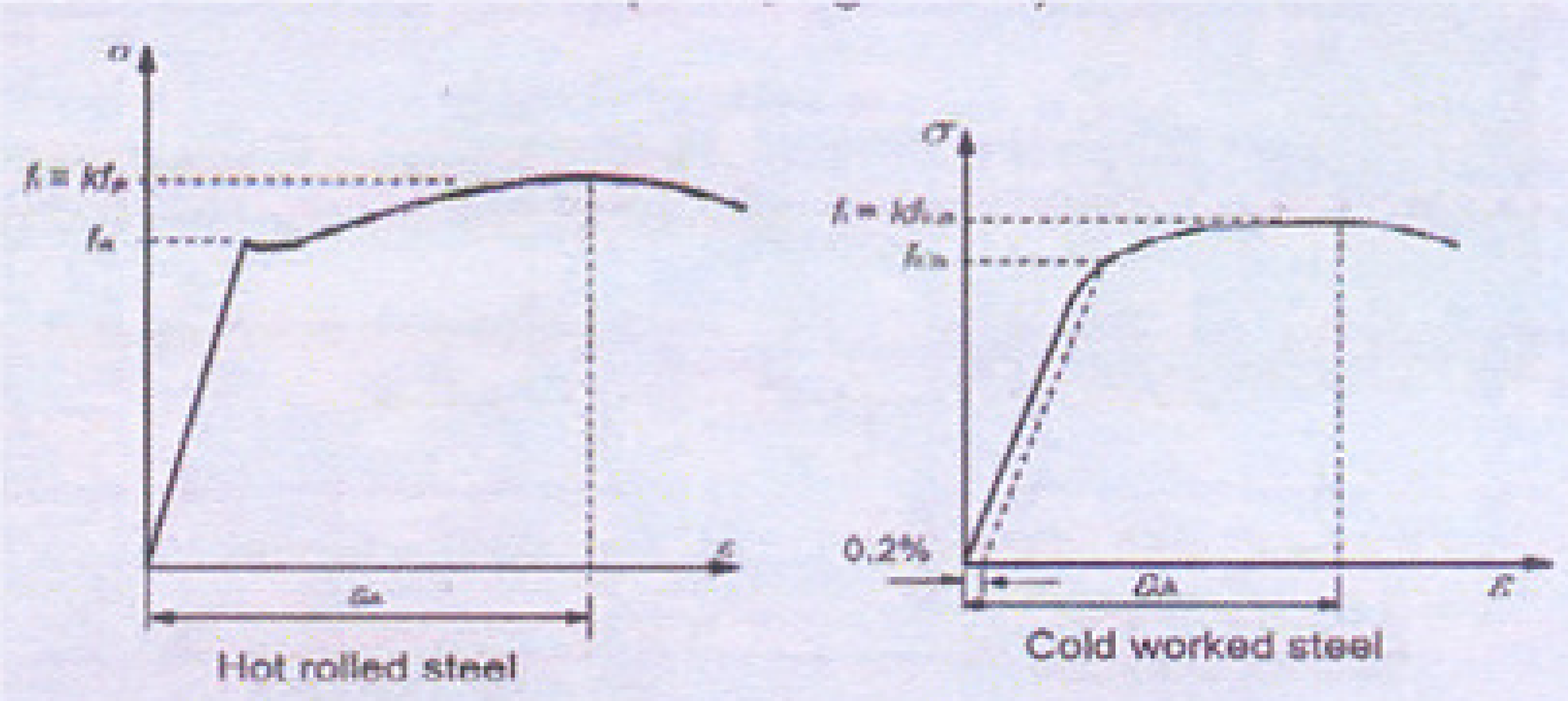
A, B, C ductility class

e.g. B 500 B





Stress-strain relations for reinforcing steel



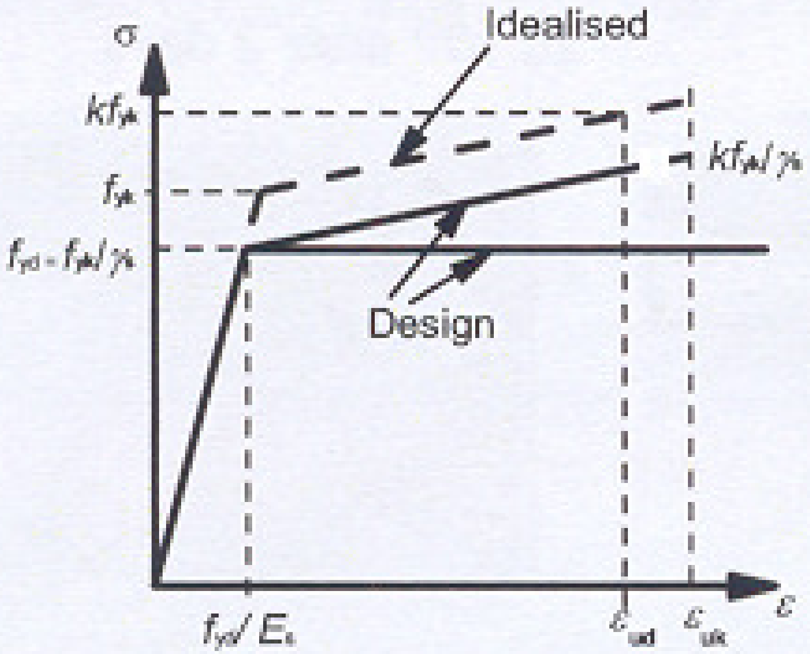
Reinforcement (2) – From Annex C

Product form	Bars and de-coiled rods			Wire Fabrics		
Class	A	B	C	A	B	C
Characteristic yield strength f_{yk} or $f_{0,2k}$ (MPa)	400 to 600					
$k = (f_t/f_y)_k$	≥1,05	≥1,08	≥1,15 <1,35	≥1,05	≥1,08	≥1,15 <1,35
Characteristic strain at maximum force, ε_{uk} (%)	≥2,5	≥5,0	≥7,5	≥2,5	≥5,0	≥7,5
Fatigue stress range ($N = 2 \times 10^6$) (MPa) with an upper limit of $0.6f_{yk}$	150			100		

cold worked (Class A) — hot rolled (Class B) — seismic (Class C)

Idealized and design stress strain relations for reinforcing steel

Alternative design stress/strain relationships are permitted:
- inclined top branch with a limit to the ultimate strain horizontal
- horizontal top branch with no strain limit



$k = (f_t/f_y)_k$

$\varepsilon_{ud} = 0.9\ \varepsilon_{uk}$

Prestressing steel (EN 10 138)

Product forms:

- **Wires** - prestressing elements delivered in coil form, ∅ ≤ 10 mm
- **Strands**
- **Bars**

Mark:

Y prestressing steel
nominal tensile strength
e. g. Y 1860 (f_{pk})

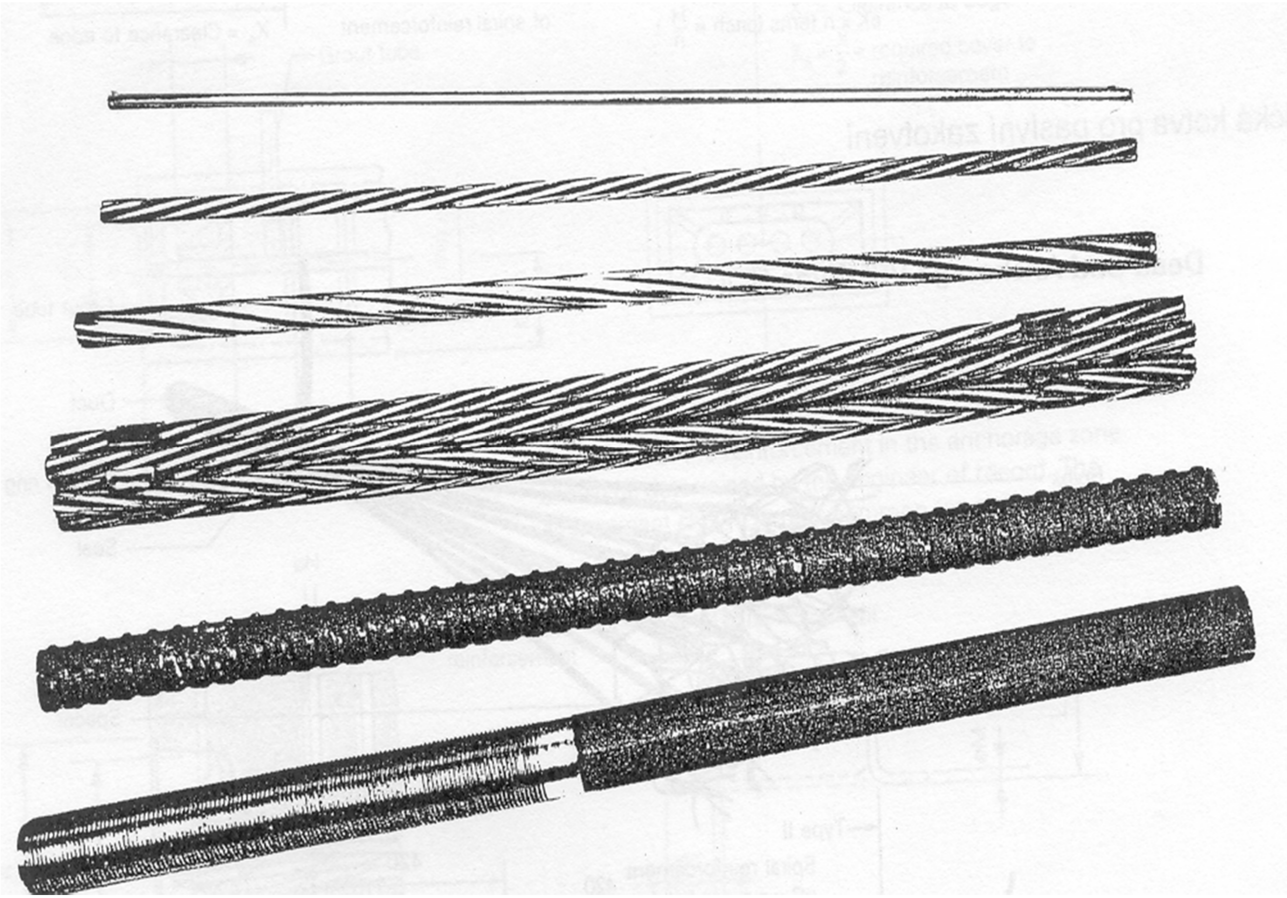
Connection with concrete:

- tendons with bond
- tendons without bond

Location in structure:

- Internal tendons
- External tendons
- Combined tendons

Types of tendons



Wire

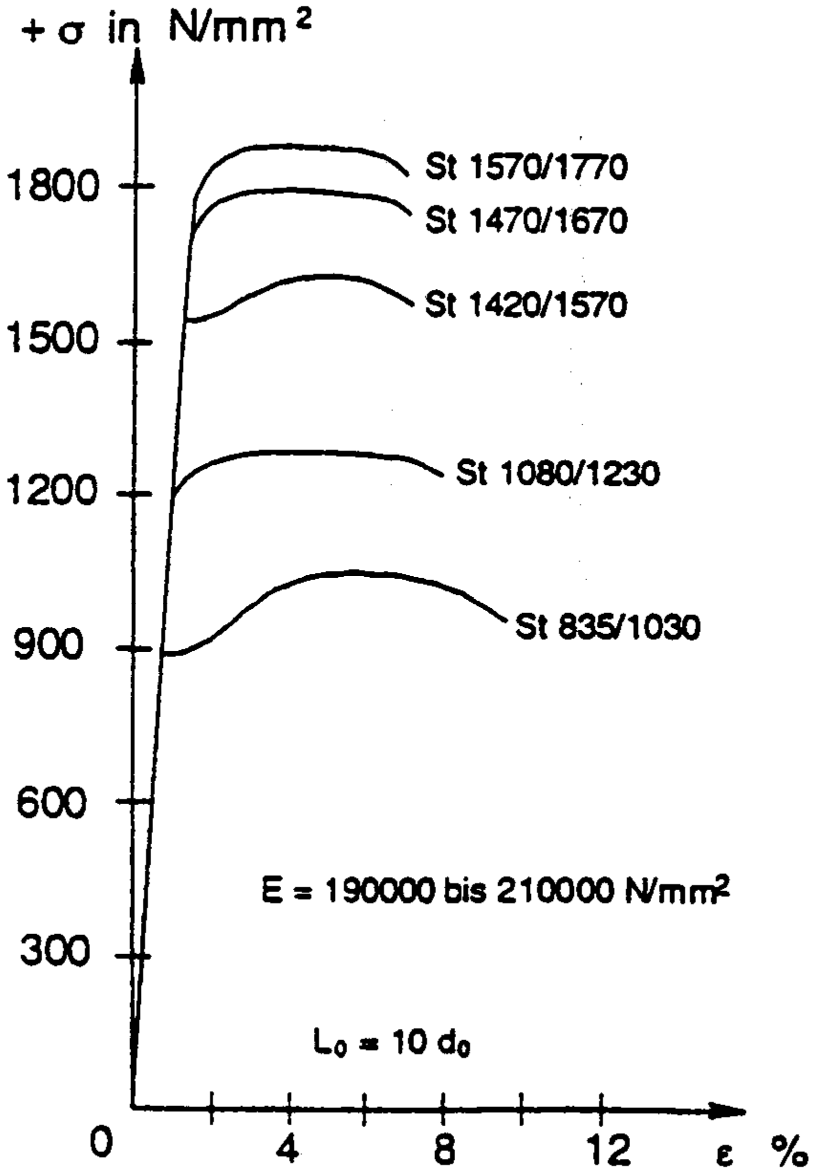
Standard strand

Drawn strand

Cable of 7 strands

Dividag bar (ribbed)

Macalloy bar



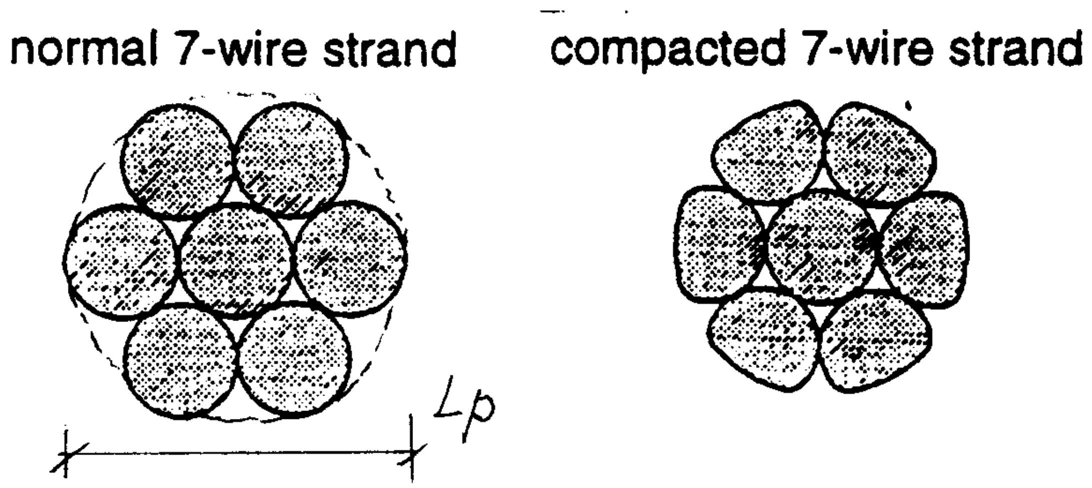
Wire: Ø

3,0; 3,5; 4,0; 4,5; 4,7; 5,0

6,0; 6,3; 6,5; 7,0; 7,5; 8,0;

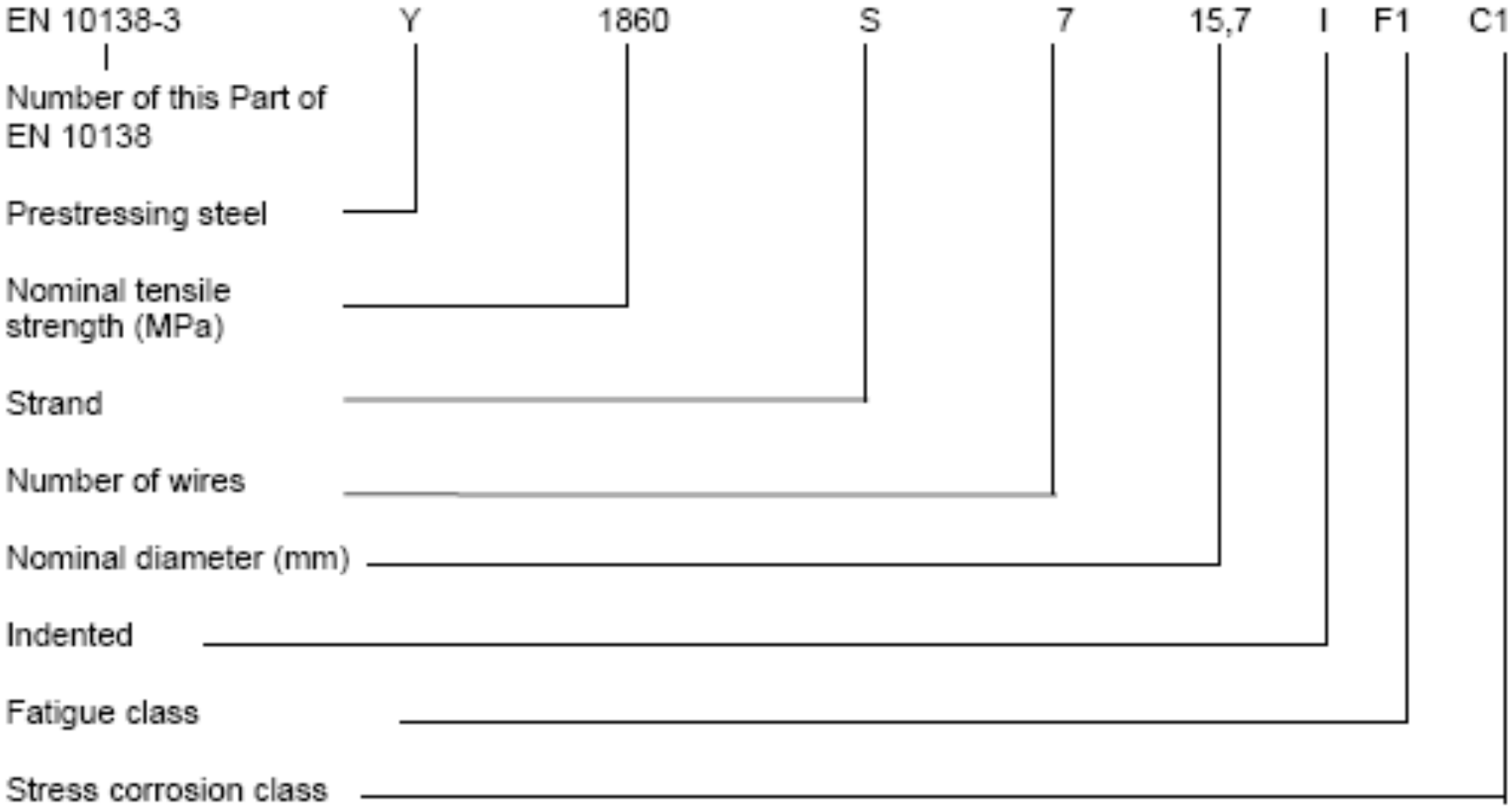
9,0; 10,0

Mark: DIN



Strand: e.g. Y 1770S7 15,2
core Ø P 5,2 + 6 x Ø P 5,0

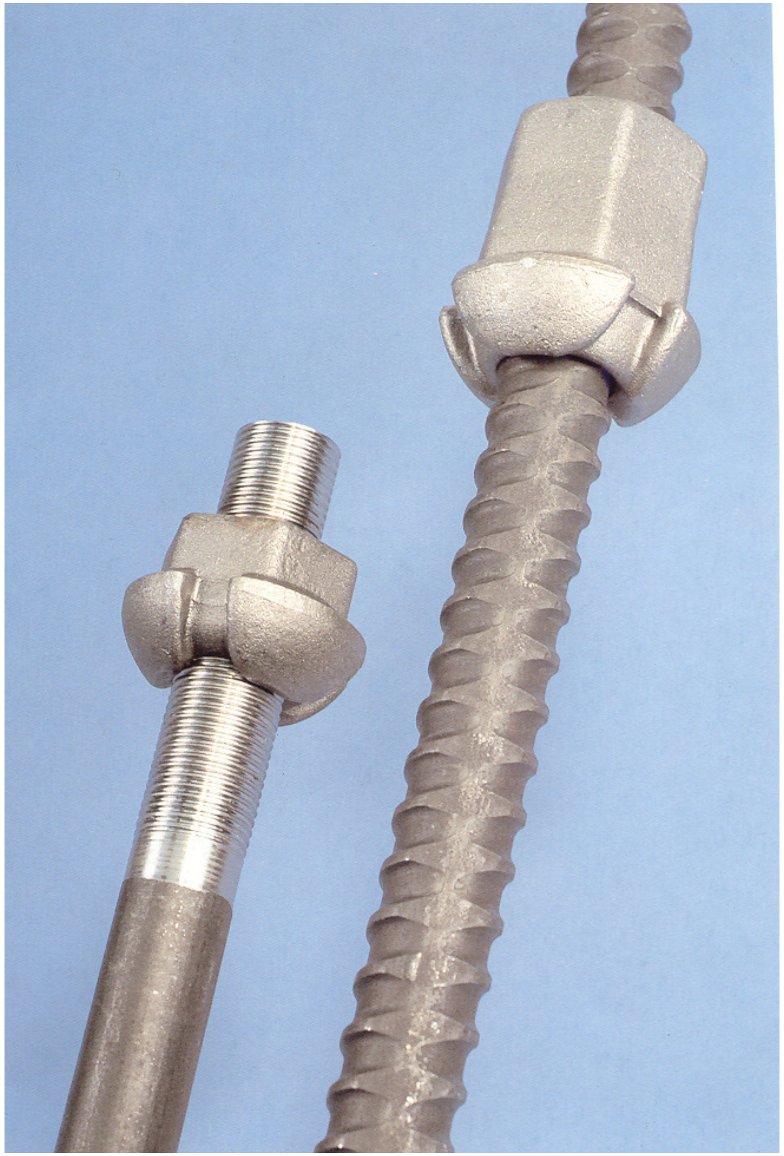
EXAMPLE

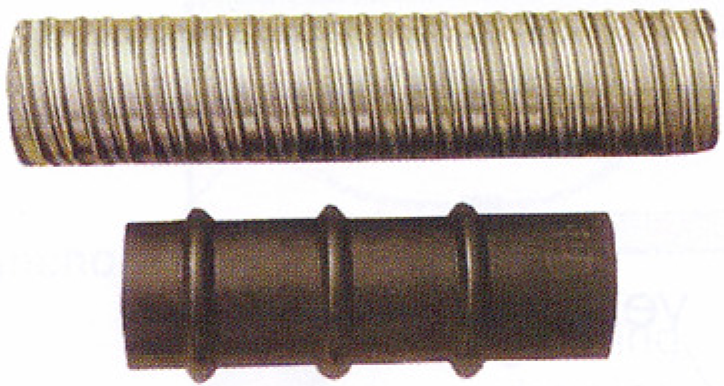


i.e. EN 10138-3-Y1860S7-15,7-I-F1-C1

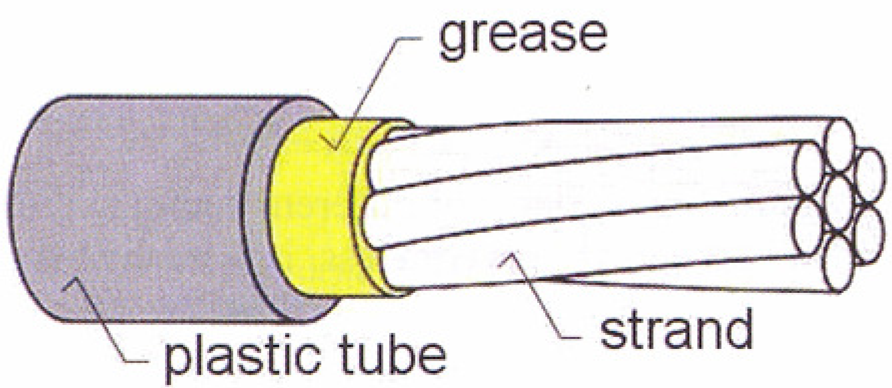
Bar:

e.g. Ø 20, 32 etc.

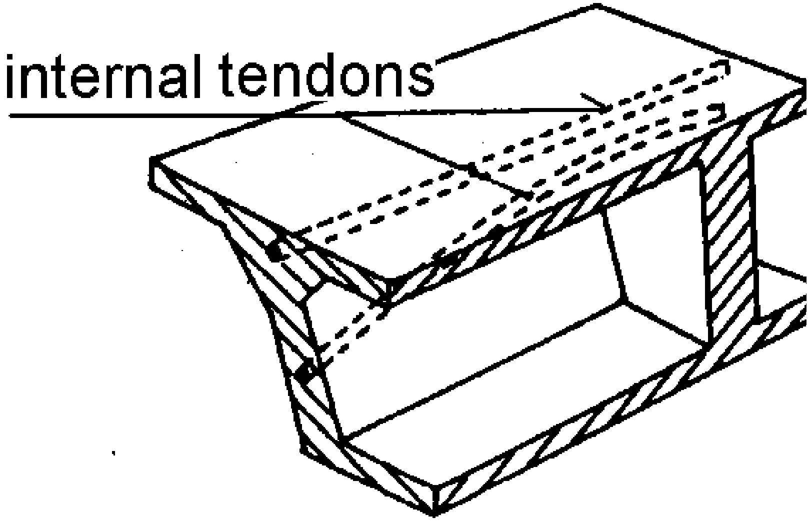


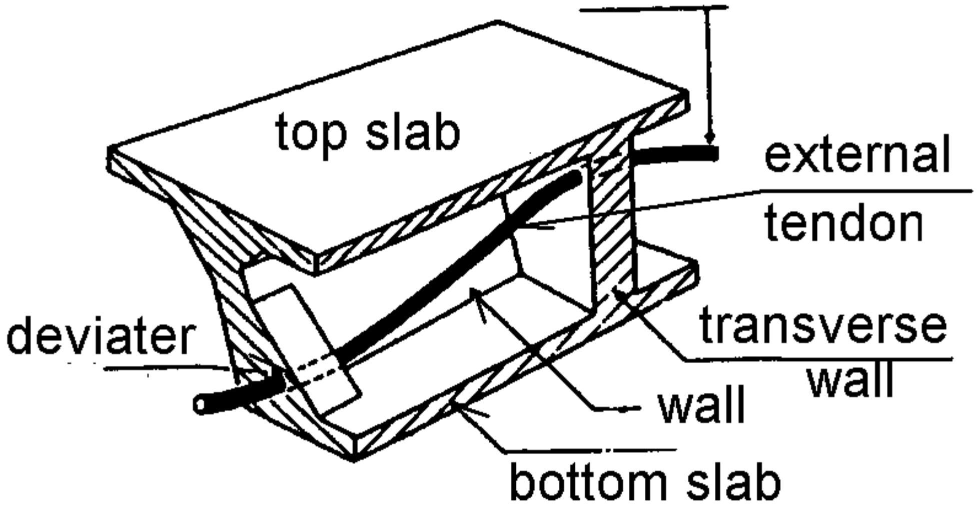


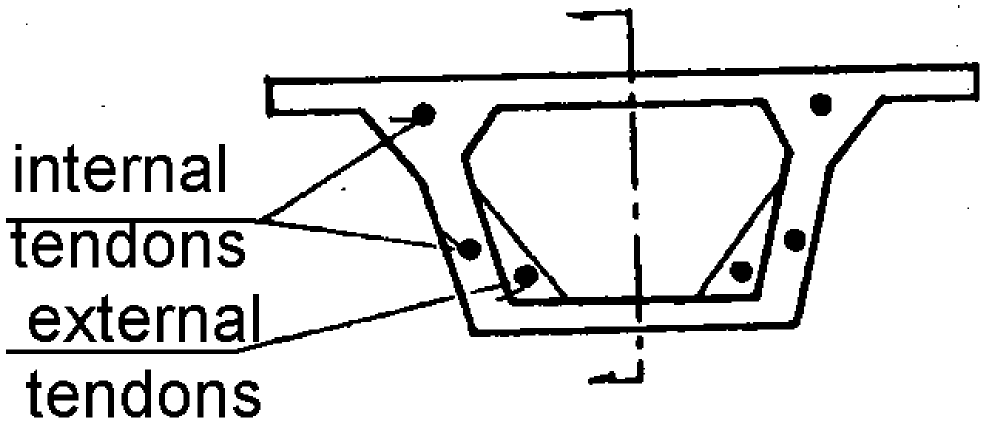
Tendon ducts



Unbonded prestressing strand

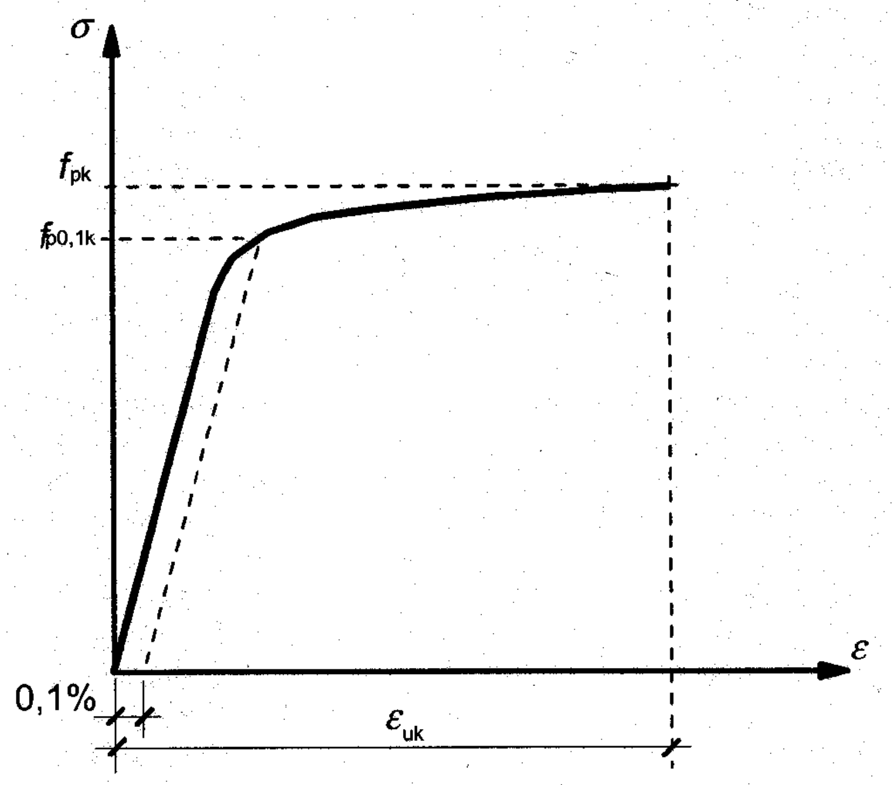

internal tendons


top slab
external
tendon
deviater
transverse
wall
wall
bottom slab


internal
tendons
external
tendons

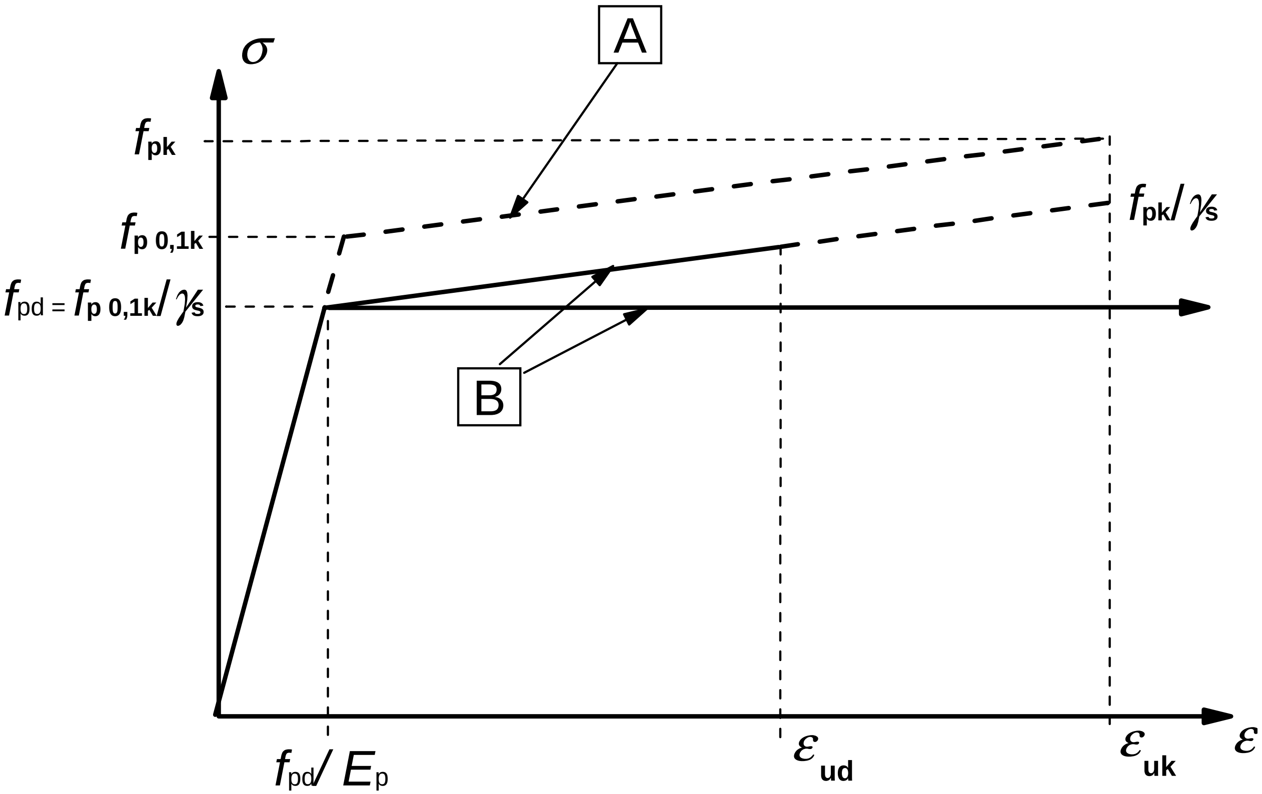
External tendons

Stress strain diagram – prestressing steel



Prestressing tendons

- Strength, denoting the value of the 0,1proof stress ($f_{p0,1k}$)
- Tensile strength to proof strength $(f_{pk}/f_{p0,1})_k$
- Elongation at maximum load (ε_{uk})



A Idealised

B Design

Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)