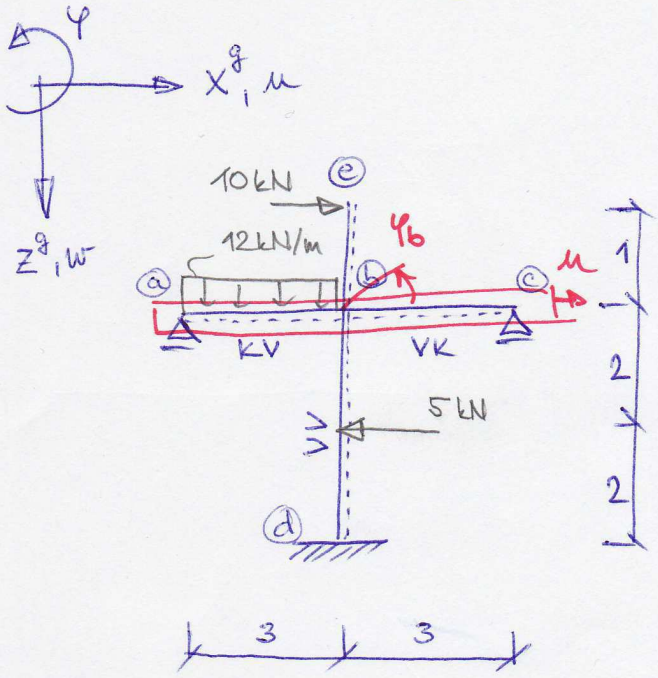
ZDM

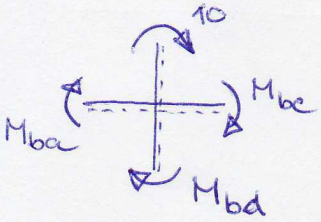
PŘ: Vyřešte a vykreslete průběhy vnitřních sil (N,V,M)

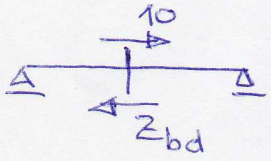


$EI = 20\ MNm^2 = 20\,000\ kNm^2$

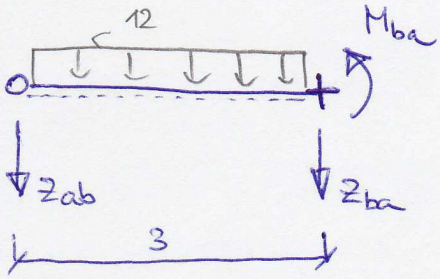
Neznámé: φ_b, u

Podmínky rovnováhy:





Prut (a,b): K-V



$$M_{ba} = \overline{M_{ba}} + \frac{3}{2} k\left(\varphi_b + \frac{w_b^l - w_a^l}{L}\right) =$$

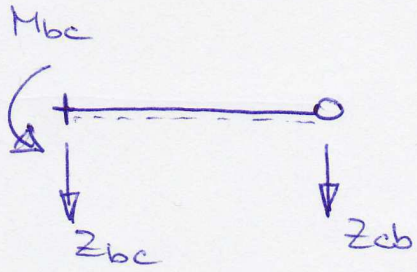
$$= -\frac{fL^2}{8} + \frac{3}{2} \cdot \frac{2EI}{L}(\varphi_b) =$$

$$= -13{,}5 + 20000\,\varphi_b \quad (-19{,}72\ kNm)$$

$$Z_{ab} = \overline{Z_{ab}} - \frac{3}{2L} k(\varphi_b) = -\frac{3fL}{8} - \frac{3}{2L} \cdot \frac{2EI}{L} \varphi_b = -13{,}5 - 6666{,}6\,\varphi_b \quad (-11{,}43\ kN)$$

$$Z_{ba} = \overline{Z_{ba}} + \frac{3}{2L} k(\varphi_b) = -\frac{5fL}{8} + \frac{3}{2L} \cdot \frac{2EI}{L} \varphi_b = -22{,}5 + 6666{,}6\,\varphi_b \quad (-24{,}57\ kN)$$

Prut (b,c) : V-K



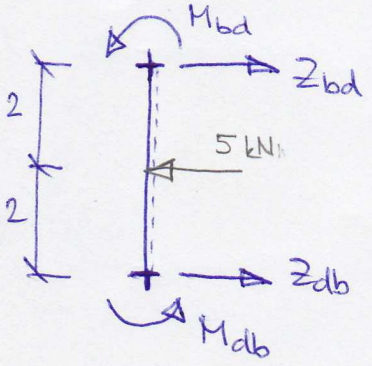
$$M_{bc} = \overline{M_{bc}} + \frac{3}{2} k \left(\varphi_b + \frac{w_c^l - w_b^l}{L} \right) =$$

$$= 0 + \frac{3}{2} \frac{2EI}{L} (\varphi_b) = 20000 \varphi_b \quad (-6{,}2 \text{ kNm})$$

$$Z_{bc} = \overline{Z_{bc}} - \frac{3}{2L} k \left(\varphi_b + \frac{w_c^l - w_b^l}{L} \right) = -\frac{3}{2L} \frac{2EI}{L} \varphi_b = -6666{,}6 \varphi_b \quad (2{,}07 \text{ kN})$$

$$Z_{cb} = \overline{Z_{cb}} + \frac{3}{2L} \cdot k \left(\varphi_b + \frac{w_c^l - w_b^l}{L} \right) = \frac{3}{2L} \frac{2EI}{L} \varphi_b = 6666{,}6 \varphi_b \quad (-2{,}07 \text{ kN})$$

Prut (d,b) : V-V



$$M_{bd} = \overline{M_{bd}} + k \left(\varphi_d + 2\varphi_b + \frac{3}{L} \left(w_b^l - w_d^l \right) \right) =$$

$$= -\frac{(-F) \cdot L}{8} + \frac{2EI}{L} \left(2\varphi_b + \frac{3}{L} \cdot (u - \emptyset) \right) =$$

$$= 2{,}5 + 20000 \varphi_b + 7500 u \quad (15{,}94 \text{ kNm})$$

$$M_{db} = \overline{M_{db}} + k \left(2\varphi_d + \varphi_b + \frac{3}{L} \left(w_b^l - w_d^l \right) \right) = \frac{(-F) \cdot L}{8} + \frac{2EI}{L} \left(\varphi_b + \frac{3}{L} (u - \emptyset) \right) =$$

$$= -2{,}5 + 10000 \varphi_b + 7500 u \quad (19{,}06 \text{ kNm})$$

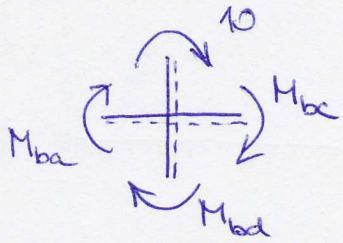
$$Z_{bd} = \overline{Z_{bd}} + \frac{3}{L} k \left(\varphi_d + \varphi_b + \frac{2}{L} \left(w_b^l - w_d^l \right) \right) = -\frac{(-F)}{2} + \frac{3}{L} \cdot \frac{2EI}{L} \left(\varphi_b + \frac{2}{L} (u - \emptyset) \right) =$$

$$= 2{,}5 + 7500 \varphi_b + 3750 u \quad (10 \text{ kN})$$

$$Z_{db} = \overline{Z_{db}} - \frac{3}{L} k \left(\varphi_d + \varphi_b + \frac{2}{L} \left(w_b^l - w_d^l \right) \right) = -\frac{(-F)}{2} - \frac{3}{L} \cdot \frac{2EI}{L} \left(\varphi_b + \frac{2}{L} \cdot u \right) =$$

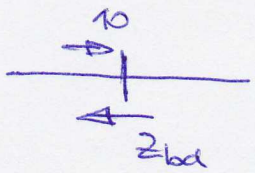
$$= +2{,}5 - 7500 \varphi_b - 3750 u \quad (-5 \text{ kN})$$

VÝPOČET NEZNÁMÝCH Z PODMÍNEK ROVNOVÁHY



$$M_{bc} + M_{bd} + M_{ba} + 10 = 0$$

$$Z_{bd} - 10 = 0$$



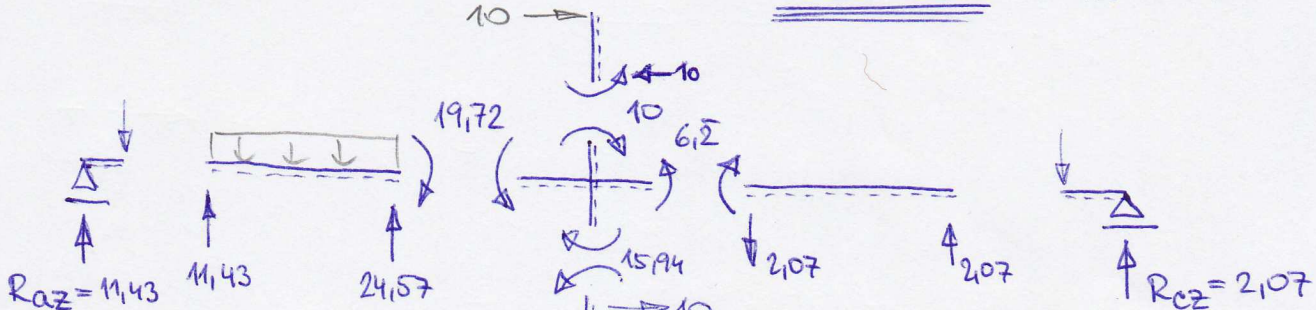
$$10 + 20000\varphi_b + 2{,}5 + 20000\varphi_b + 7500u - 13{,}5 + 20000\varphi_b = 0$$

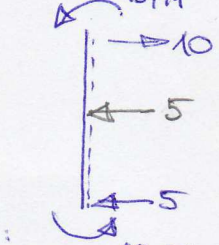
$$-10 + 2{,}5 + 7500\varphi_b + 3750u = 0$$

$$\begin{bmatrix} 60\,000 & 7500 \\ 7500 & 3750 \end{bmatrix} \begin{Bmatrix} \varphi_b \\ u \end{Bmatrix} = \begin{bmatrix} 1 \\ +7{,}5 \end{bmatrix} \Rightarrow \begin{array}{l} \varphi_b = -3{,}\overline{1} \cdot 10^{-4}\ \text{rad} \\ u = 2{,}6\overline{2} \cdot 10^{-3}\ \text{m} \end{array}$$

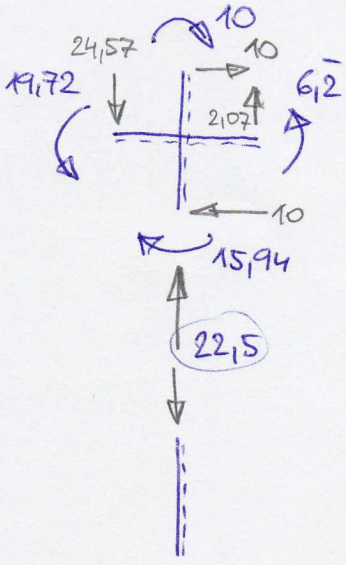
– dosadit a zpětně dopočítat + koncové síly

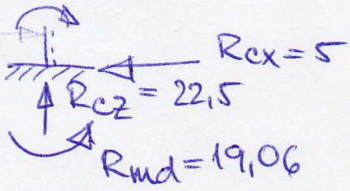
VYKRESLENÍ KONCOVÝCH SIL PODLE SKUTEČNÉ ORIENTACE



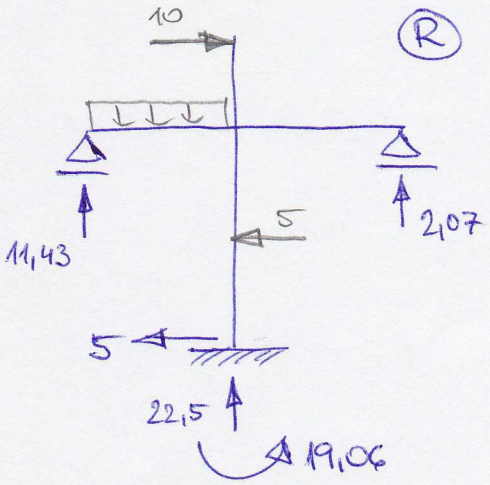


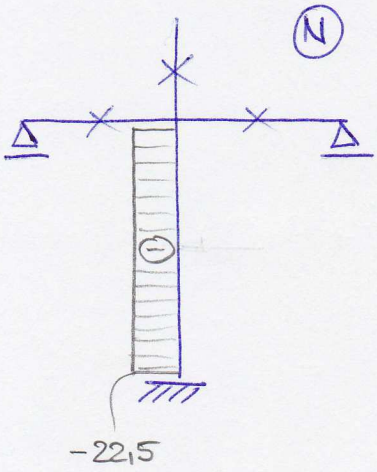
rovnováha ve styčníku "b":

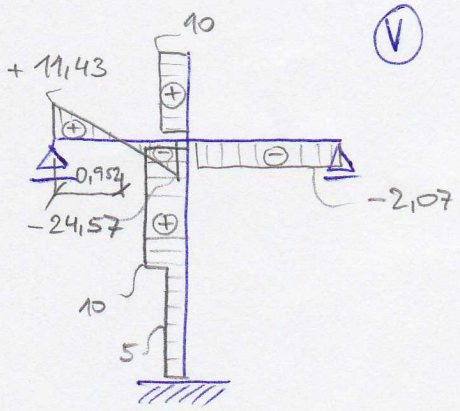


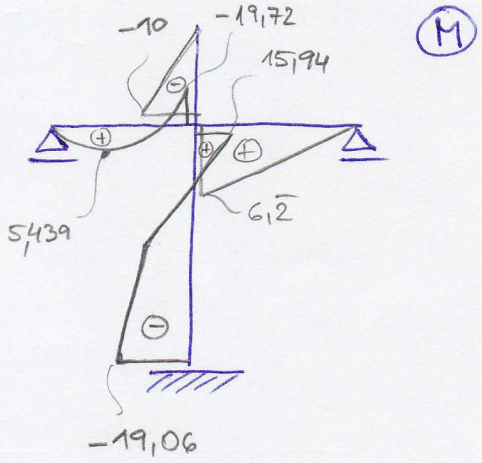


VÝSLEDNÉ VYKRESLENÍ



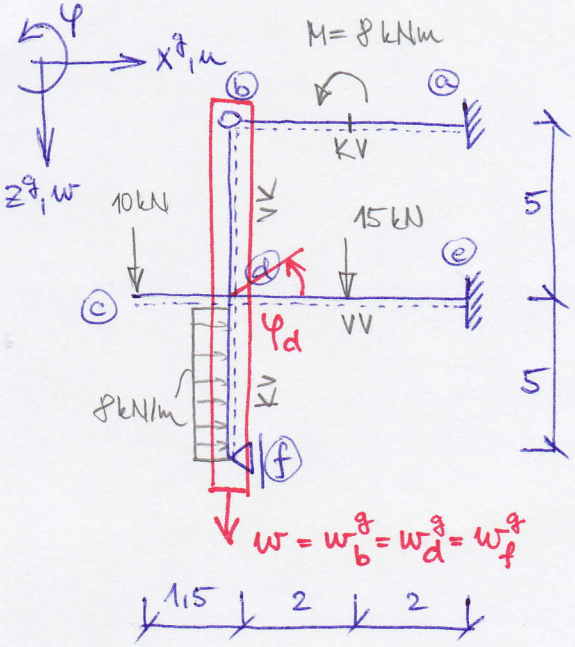






ZDM

Př: Vyřešte a vykreslete průběhy vnitřních sil N, M, V



$I_y = \frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3 = 0{,}003125\ m^4$

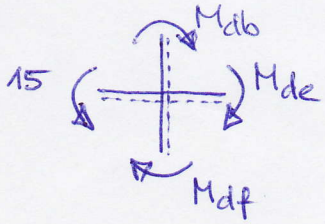
$E = 30\ GPa$

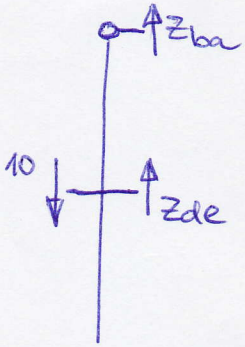
Použít "kompatibilní" jednotky!

$EI = 93750\ kNm^2$

Neznáme: φ_d, w (složkový posun)

Podmínky rovnováhy:

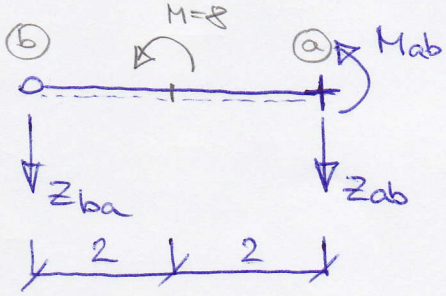




$M_{db} + M_{de} + M_{df} - 15 = 0$

$Z_{ba} + Z_{de} - 10 = 0$

Prut (b, a): K-V



$$Z_{ba} = \overline{Z_{ba}} - \frac{3}{2L} \cdot k \left(\varphi_a + \frac{w_a^l - w_b^l}{L} \right) =$$

$$= -\frac{M}{L} \left(1 + \frac{L^2 - 3a^2}{2L^2} \right) - \frac{3}{2L} \cdot \frac{2EI}{L} \cdot \left(0 + \frac{0 - w}{L} \right) =$$

$$= \underbrace{-\frac{\quad}{2{,}25}} + 4394{,}53 \cdot w \quad (4{,}71)\ kN$$

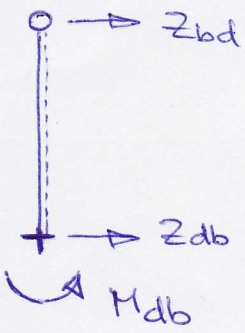
$$Z_{ab} = \overline{Z_{ab}} + \frac{3}{2L} k \left(\varphi_a + \frac{w_a^l - w_b^l}{L} \right) = \frac{M}{L} \left(1 + \frac{L^2 - 3a^2}{2L^2} \right) +$$

$$+ \frac{3}{2L} \cdot \frac{2EI}{L} \left(\cancel{\varphi} + \frac{0 - w_b}{L} \right) = \underline{2{,}25 - 4394{,}53\, w} \quad (-4{,}71\ kN)$$

$$M_{ab} = \overline{M_{ab}} + \frac{3}{2} k \left(\varphi_a + \frac{w_a^l - w_b^l}{L} \right) = \frac{M}{2L^2} \left(L^2 - 3a^2 \right) +$$

$$+ \frac{3}{2} \cdot \frac{2EI}{L} \left(\cancel{\varphi} + \frac{0 - w_b}{L} \right) = \underline{1 - 17578{,}13\, w} \quad (-26{,}83\ kNm)$$

Prut (d,b) : V-K



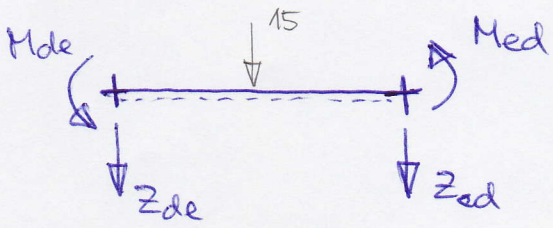
$$Z_{bd} = \overline{Z_{bd}} + \frac{3}{2L} k \left(\varphi_d + \frac{w_b^l - w_d^l}{L} \right) =$$

$$= \cancel{\varphi} + \frac{3}{2L} \frac{2EI}{L} \left(\varphi_d + \frac{0-0}{L} \right) = \underline{11250\, \varphi_d} \quad (4{,}81\ kN)$$

$$Z_{db} = \overline{Z_{db}} - \frac{3}{2L} k \left(\varphi_d + \frac{w_b^l - w_d^l}{L} \right) = \underline{-11250\, \varphi_d} \quad (-4{,}81\ kN)$$

$$M_{db} = \overline{M_{db}} + \frac{3}{2} k \left(\varphi_d + \frac{w_b^l - w_d^l}{L} \right) = \underline{56250 \cdot \varphi_d} \quad (24{,}04\ kNm)$$

Prut (d,e) : V-V



$$M_{de} = \overline{M_{de}} + k \left(2\varphi_d + \varphi_e + \frac{3}{L} \left(w_e^l - w_d^l \right) \right) =$$

$$= \frac{FL}{8} + \frac{2EI}{L} \left(2\varphi_d + \cancel{\varphi} + \frac{3}{L} \left(\cancel{\varphi} - w \right) \right) =$$

$$= \underline{7{,}5 + 93750\, \varphi_d - 35156{,}3\, w} \quad (-8{,}09\ kNm)$$

$$M_{ed} = \overline{M_{ed}} + k \left(\varphi_d + 2\varphi_e + \frac{3}{L} \left(w_e^l - w_d^l \right) \right) = -\frac{FL}{8} + \frac{2EI}{L} \left(\varphi_d + \cancel{\varphi} + \frac{3}{L} \left(-w \right) \right) =$$

$$= \underline{-7{,}5 + 46875\, \varphi_d - 35156{,}3\, w} \quad (-43{,}12\ kNm)$$

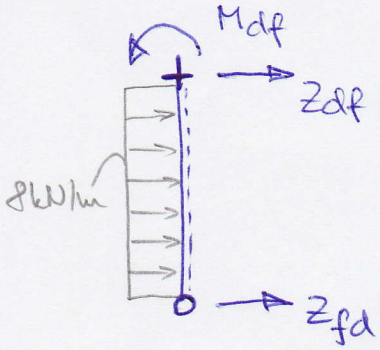
$$z_{de} = \overline{z_{de}} - \frac{3}{L} k \left(\varphi_d + \varphi_e + \frac{2}{L} \left(w_e^l - w_d^l \right) \right) = -\frac{F}{2} - \frac{3}{L} \cdot \frac{2EI}{L} \left(\varphi_d + \right.$$

$$\left. + \frac{2}{L} \left(\theta - w \right) \right) = \underline{-7{,}5 - 35156{,}3\, \varphi_d + 17578{,}13\, w} \quad (5{,}30\ \text{kN})$$

$$z_{ed} = \overline{z_{ed}} + \frac{3}{L} k \left(\varphi_d + \varphi_e + \frac{2}{L} \left(w_e^l - w_d^l \right) \right) = -\frac{F}{2} + \frac{3}{L} \frac{2EI}{L} \left(\varphi_d + \right.$$

$$\left. + \frac{2}{L} \left(\theta - w \right) \right) = \underline{-7{,}5 + 35156{,}3\, \varphi_d - 17578{,}13\, w} \quad (-20{,}3\ \text{kN})$$

Prut (f,d) : K-V



$$M_{df} = \overline{M_{df}} + \frac{3}{2} k \left(\varphi_d + \frac{w_d^l - w_f^l}{L} \right) =$$

$$= \frac{-fL^2}{8} + \frac{3}{2} \cdot \frac{2EI}{L} \left(\varphi_d + \theta \right) = \underline{-25 + 56250\, \varphi_d} \quad (-0{,}96\ \text{kNm})$$

$$z_{df} = \overline{z_{df}} + \frac{3}{2L} k \left(\varphi_d + \frac{w_d^l - w_f^l}{L} \right) =$$

$$= -\frac{5fL}{8} + \frac{3}{2L} \frac{2EI}{L} \left(\varphi_d + \theta \right) = \underline{-25 + 11250\, \varphi_d} \quad (-20{,}19\ \text{kN})$$

$$z_{fd} = \overline{z_{fd}} - \frac{3}{2L} k \left(\varphi_d + \frac{w_d^l - w_e^l}{L} \right) = -\frac{3fL}{8} - \frac{3}{2L} \frac{2EI}{L} \left(\varphi_d \right) = \underline{-15 - 11250\, \varphi_d} \quad (-19{,}81\ \text{kN})$$

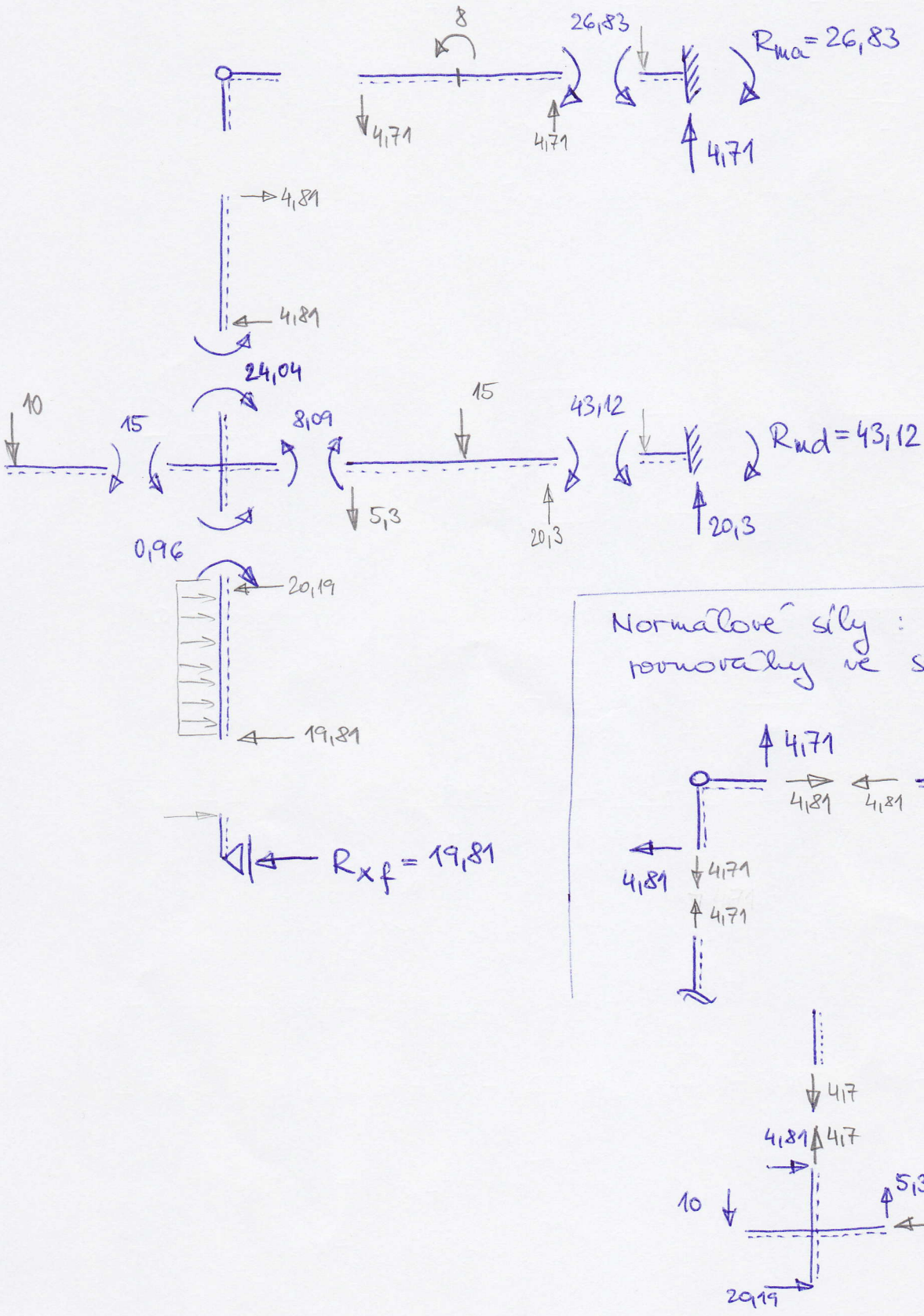
Výpočet neznámých z podmínek rovnováhy

$\curvearrowright$: $56250\, \varphi_d + 7{,}5 + 93750\, \varphi_d - 35156{,}3\, w - 25 + 56250\, \varphi_d - 15 = 0$

$\uparrow$: $-2{,}125 + 4394{,}53\, w - 7{,}5 - 35156{,}3\, \varphi_d + 17578{,}13\, w - 10 = 0$

$$\boxed{\begin{aligned} \varphi_d &= 4{,}273 \cdot 10^{-4}\ \text{rad} \\ w &= 1{,}583 \cdot 10^{-3}\ \text{m} \end{aligned}}$$

Vykreslení koncových sil podle skut. orientace



Normálové síly: z podmínky rovnováhy ve styčníku

Výsledné vykreslení: