EFFECTS OF PRESTRESSING ON CONCRETE ELEMENTS AND STRUCTURES, DESIGN OF PRESTRESSING, ULS CHECK

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Equivalent load method

- Previous lectures => losses of prestress
- Knowing the distribution of the prestressing force along the tendon => we can examine the effect of prestressing on the structure
- We are looking for the load equivalent to the prestress
- This applied to the structure, we know the action of the prestressing on the structure
- Applying prestressing = actively changing the distribution of internal forces
Force action of a tendon

Action of the tendon via:

- Force transmitted at the anchorage
- At points where the direction of the tendon changes

**Simple case:** pre-tensioned beam, straight tendon, no losses
- Only the forces at the anchorage

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Force action of a tendon

- Anchorage force $P_A$
- It changes due to friction over the length of the tendon
- Over $dx$, it changes to $P$
- Direction change = the force $P$ is deviated
- Resultant force $R$ acts on the concrete
Force action of a tendon

• Resultant force $R$ acts on the concrete, can be decomposed to horizontal force $H$ and vertical force $V$
• Or can be obtained as the sum of horizontal and vertical components at the right(left)/hand side of the point

\[ V = V_1 + V_2 \]
\[ H = H_1 + H_2 \]

\[ V_1 = P \cdot \sin \alpha_1 \]
\[ H_1 = P \cdot \cos \alpha_1 \]

\[ V_2 \]
\[ H_2 \]

Force action of a tendon

• The system of all forces resulting from the prestressing and acting on concrete is called the **equivalent load**
• if the prestressing does not act in the centre-point of the element at the anchorage, the corresponding moment has to be added $M = H \cdot e_p$
**Force action of a tendon**

- Example of automated determination of the equivalent load
- The force $P_A$ changes to $P_B$ over the tendon due to friction
- Radial forces $r$ due to direction change

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![Diagram of tendon force action](image)

(a) Real force action of tendon on concrete

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**Force action of a tendon**

- Manual calculations = simplifying assumptions needed
- Prestressing force constant over the length of the tendon
- The $H$ and $V$ correspond to cosine and sine of the tangent to the tendon and the horizontal plane

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![Diagram of horizontal and vertical components](image)

(b) Distribution of horizontal and vertical components of prestressing force when $P = \text{const}$
Force action of a tendon

- Manual calculations = simplifying assumptions needed
- Angular change along the length of the tendon is small -> horizontal force $H$ is practically constant
- Assumed for camber $f$ to length of the parabola $L$ lower than $1/15 = \text{shallow tendons}$
- If satisfied, vertical load $p$ considered instead of radial forces $r$

\[ \frac{f}{L} \leq \frac{1}{15} \Rightarrow H = P = \text{const} \]

(c) Equivalent load for \[ \frac{f}{L} \leq \frac{1}{15} \]

Force action of a tendon

- Why parabolas???
- No change in the direction = no radial force = ineffective tendon

\[ dx \]
Equivalent load method

• Equivalent load $p$ for a parabolic tendon of a simply supported beam
• $P = H = \text{const.}$
• $e_{PA} = e_{PB}$

• The moment effect of the tendon in given section in distance $x$ from the beginning to the centre-point of the cross-section

$$M_{p}(x) = H e_{p}(x)$$

Equivalent load method

• The eccentricity is expressed from the parabola equation

$$e_{p}(x) = -\frac{4f}{L^{2}}x^{2} + \frac{4f}{L}x + e_{PA}$$

• Substituting $e_{p}(x)$ to $M_{p}(x)$

$$p = -\frac{d^{2}M_{p}(x)}{dx^{2}} = -H \frac{d^{2}e_{p}(x)}{dx^{2}} = H \frac{8f}{L^{2}}$$
Equivalent load method

- The prestressing is substituted by the uniform loading $p$ and the anchorage forces, which can be decomposed to $H$ and $V$.
- $H$ and $V$ act on eccentricities of the anchors $e_{pA} = e_{pB}$, thus produce moments to the centre-point.

\[ M = H \cdot e_{pA} \]

Equivalent load method

- Applying the equivalent forces to a simply supported beam, internal forces due to prestressing are produced.
- Due to self-equilibrium of the equivalent loads, no reactions are produced due to prestressing.

\[ M = H \cdot e_{pA} \]
The equivalent load method is general, can be used on any structure. The self-weight $g$ can be balanced by equivalent load $p$, so the moment $M_g$ is eliminated by prestressing ($e_{pa} = e_{ps} = 0$).

$\Rightarrow$ the structure is subjected only to axial compression.

The equivalent load method better expresses the fact: Applying prestressing = actively changing the distribution of internal forces.
Equivalent load method

- Determination for unsymmetrical parabolic tendon

\[ y = e \frac{L}{x^2} \]

The load balancing method

The way to effective design of prestressing:
- Understand its active role in the force action on the structure
- Use the prestressing to balance and shear forces from permanent loads

From T.Y. Lin (1963) the **load balancing method**

**Cover 80-100% of permanent loads** dependent on:
- the required level of prestressing
- Quality of calculation methods for the analysis of deflection
- Not lower than 80% (even for partial prestressing), checking non-linear behavior (cracking)
The load balancing method

Basic rules:
- Small curves of the tendons at the supports = direct transfer to the supports, no load to the span
- Distributed loads $p_i$ in the spans, moments + horizontal forces at the supports
- The formula

$$p = -\frac{d^2 M_p(x)}{dx^2} = -H \frac{d^2 e_p(x)}{dx^2} = H \frac{8f}{L^2}$$

- Can be applied for each span, H can be assumed constant
- Maximal eccentricities can not be achieved in the outer spans
Forces due to prestressing in statically determined structures

The prestressing is a self-equilibrating system = no reactions occur

Simple beam = no deformation restraint

Continuous beam = the camber of the beam is prevented in its intermediate support(s); the reaction $R$ occurs

The beam is subjected to secondary moment due to prestressing $M_{ps}$

The structure is subjected to primary (statically determinate) and secondary (statically indeterminate) effects of prestressing

The total effects of prestressing can be obtained as a sum of the primary and secondary effects

$$M_p = M_{pp} + M_{ps}$$
The equivalent load method can be used to evaluate the total effects of prestressing; just the statically indeterminate reactions must be determined.

The summation of the process:
- Structural scheme
- Equivalent load
- Reactions in statically indeterminate structures
- Internal forces due to the equivalent load and reactions
Application on complex spatial structures

Basic rules:
- Tendons inside the span transfer the load to the column strips
- The tendons strips transfer the load to the columns and edge beams
- The tendons inside the span can be fully or partially substituted by reinforcement
Application on complex spatial structures

Commercial complex Haje Prague (in desing)
Application on complex spatial structures

Commercial complex Haje Prague (in design)

Application on complex spatial structures

Commercial complex Port Shopping Mall Bratislava (in design)
Application on complex spatial structures

Commercial complex Port Shopping Mall Bratislava (in design)
Application on complex spatial structures

National Technical library (in operation)
Application on complex spatial structures

National Technical library (in operation)
The ULS check

The bending check of prestressed elements is similar to check of reinforced concrete structures performed with the **ultimate limit strain method** based on following:

a. Bernoulli hypothesis – the concrete cross-section which was plane before the loading remains plane

b. Navier hypothesis – the strain $\varepsilon$ changes linearly from the neutral axis

c. The neutral axis is the part of the cross-section where $\varepsilon = 0$, it is defined by the distance $x$ from the compressed edge of the CS

d. The stress in the compressed concrete is taken from its design stress-strain diagrams

e. The tensile resistance of concrete is neglected

f. There is an ideal bond between reinforcement (both mild and prestressing) and the concrete = the strain of concrete and reinforcement is equal in the particular position

g. The CS fails by reaching the ultimate strain in concrete $\varepsilon_{cu}$ or prestressing steel $\varepsilon_{ud}$
The ULS check

For boded reinforcement, block stress-strain diagram of concrete, constant compressive stress in concrete over the height $\lambda x$:

\[ M_{psd} + \gamma_p M_{p,eq} \leq M_{pl} = F_{psd} (d_p - 0.5\lambda x) + F_{sd} (d_s - 0.5\lambda x) \]

- $d_p$ the distance from the centre point of the prestressing units to the compressed edge
- $d_s$ the distance from the centre point of the reinforcing steel to the compressed edge
- $F_{psd}$ tensile capacity of the prestressing steel $F_{psd} = A_p \cdot f_{pd}$
- $F_{sd}$ tensile capacity of the reinforcing steel $F_{sd} = A_s \cdot f_{yd}$
- $F_{cd}$ compressive capacity of the concrete $F_{cd} = \lambda \cdot x \cdot D_{eff} \cdot \eta \cdot F_{cd}$ for $\lambda \cdot x \leq h_f$
The ULS check

The position of the neutral axis $x$ comes from

$$F_{cd} = F_{d} + F_{id} \quad \rightarrow \quad \lambda x = \frac{(F_{ud} + F_{ld})}{\eta f_{cd} b_{d}}$$

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f. There is an ideal bond between reinforcement (both mild and prestressing) and the concrete = the strain of concrete and reinforcement is equal in the particular position

THERE IS No BOND........

The effect of unbonded prestressing is always regarded as external loading
The ULS check

The effect of unbonded prestressing is always regarded as external loading with three parts:

a. **Axial**
   \[ N_{Ed,p} = -\left( \gamma_p P_{m,t} + \gamma_{AP} \Delta P \right) \cos \alpha_p \]

b. **Bending**
   \[ M_{Ed,p} = -\left( \gamma_p M_{pm,t} + \gamma_{AP} \Delta M_p \right) \]

c. **Shear**
   \[ V_{Ed,p} = -\left( \gamma_p V_{pm,t} + \gamma_{AP} \Delta V_p \right) \]

\( \gamma_p = 1 \)

\( \Delta P \) is the increase of the prestressing force in the unbonded reinforcement due to deflection of the element at ULS

\( P_{m,t} \) is the prestressing force after losses at time of the check

\( \gamma_{AP} \) is the partial coefficient of reliability for the change of prestressing, 0.8 for positive effect, 1.2 for adverse effect

The ULS check

The effect of unbonded prestressing is added to the effect of other loadings

In the case that the element is prestressed only by unbonded reinforcement:

the bending resistance is calculated as for reinforced concrete section loaded by a combination of axial force \( N_{Ed} \) and bending moment \( M_{Ed} \)
The ULS check

Example for the task 4:

This check will be performed in selected sections (sections in the columns strip will be on the depth of the strip, sections in the deck will be 1m deep).

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M_{ULS}\rightarrow M_{Rd} + \gamma_{m}M_{pm} = 245 \text{ kn.m; } N_{ULS} = 150 \text{ kN; } 

d_1 = \frac{\gamma_{m} + 3.82 + 137}{45 + 16 + 162} = 69 \text{ mm; } d_2 = 350 - 69 = 281 \text{ mm; }

x_B = d - \sqrt{\frac{M_{ULS} - N_{ULS} (0.5h - d)}{0.5b f_{cd}}} = 0.281 - \sqrt{\frac{0.245 + 0.150 (0.5 \times 0.35 - 0.053)}{0.5 \times 1.02333}} = 0.043 \text{ m; }

F_{cd} = x_B b f_{cd} = 0.043 \times 1.0 \times 23333 = 0.956 \text{ MN; }

A_{c,cal} = (F_{cal} + 1.1056 \times 19.68 \text{ cm}^2) = 0.0019678 \text{ m}^2 = 0.9 \times 0.0002011 = 0.002011 \text{ m}^2;

F_{cal} = A_{c,cal} f_{cd} = 0.002011 \times 3438.4 = 0.8742 \text{ MN.}

x_B = \frac{F_{cal} - N_{ULS}}{0.28742 + 0.59} = 0.043 \text{ m; } A_{s,14} = 0.8 \times 700 = 560 \text{ mm};

\frac{700 + f_{cd}}{F_{cal} = x_B b f_{cd} = 0.044 \times 1.0 \times 23333 = 1.0242 \text{ MN; }}

M_{ULS} = F_{cal} (d - 0.5 \times h_B) = 0.8742 \times (0.281 - 0.05) = 0.2634 \text{ MN.m; }

M_{ULS} = 0.249 \text{ MN.m.}

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Thank you for your attention

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