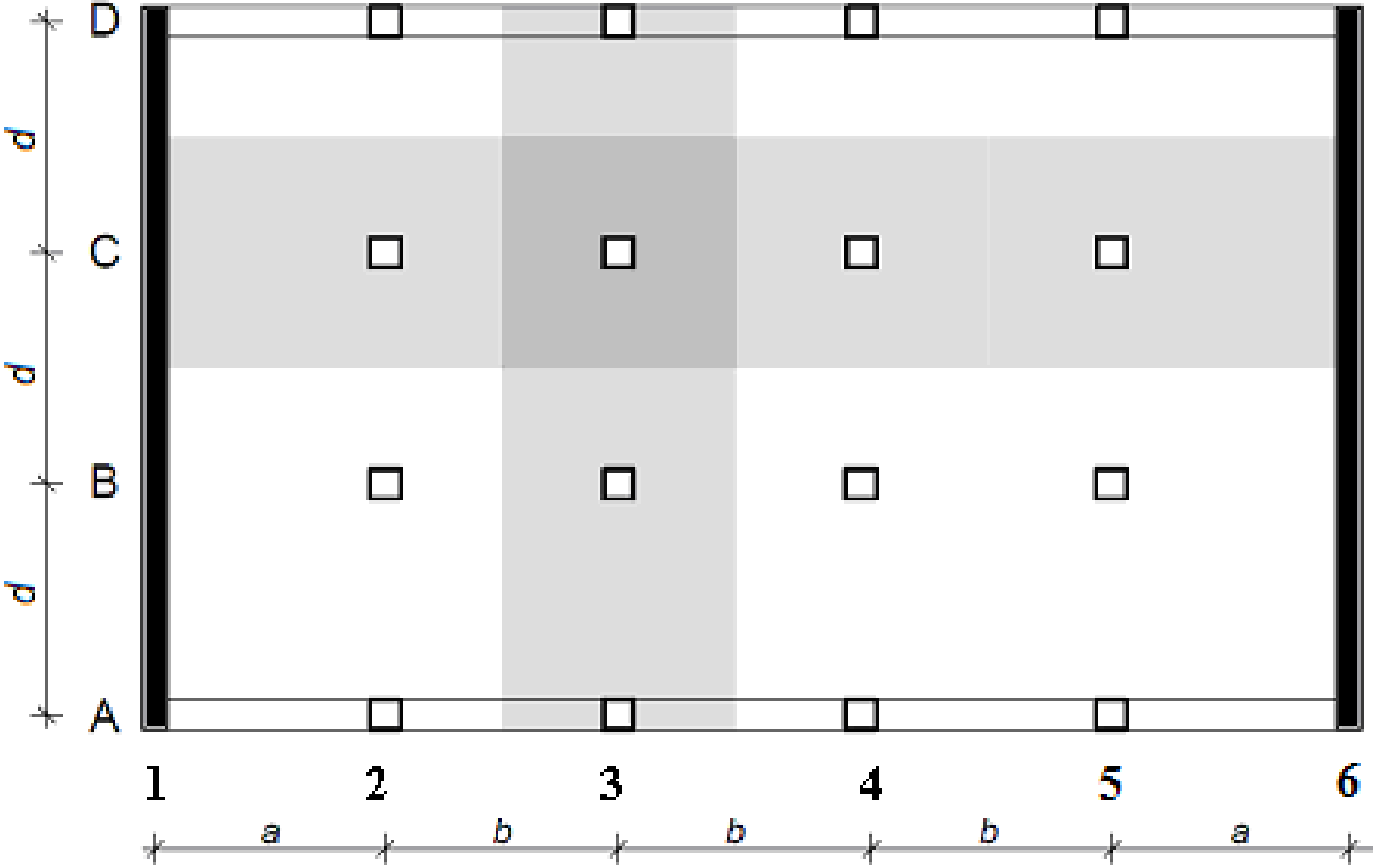
3rd task:

Two-way slab supported by columns (flat slab)



Reinforcement of the slab

- Design of bending reinforcement
- Drawing of bending reinforcement
- Detailed check of punching
- Design of punching reinforcement (if necessary)
- Sketch of punching reinforcement

Design of bending reinforcement

- Generally, it is the same as for beams
- Do the calculation for the moments calculated in 7th seminar
- Do the calculation in a table like this:

Design and check of bending reinforcement of the slab																	
Panel	Cross-section	Strip	Design							Check							
			m_{Ed}	d	z	$a_{s,rqd}$	$a_{s,min}$	Design	$a_{s,prov}$	x	ξ	z	m_{Rd}	$m_{Rd} > m_{Ed}$	$\xi < 0{,}45$	spacing	
			[kNm/m]	mm	mm	[mm^2]	[mm^2]		[mm^2]	[mm]		[mm]	[kNm/m]			of bars	
C_o	1 (left support)	no division	31,02	169	152	469	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK	
	2 (midspan)	Column	34,46	169	152	521	220	Ø12 á 200 mm	566	23,07	0,137	160	39,32	OK	OK	OK	
		Middle	22,97	169	152	347	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK	
	3 (right support)	Column	60,31	169	152	912	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK	
		Middle	20,10	169	152	304	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK	
C_{in}	1 (left support)	Column	70,08	169	152	1060	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK	
		Middle	23,36	169	152	353	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK	
	2 (midspan)	Column	30,19	169	152	456	220	Ø12 á 250 mm	452	18,42	0,109	162	31,76	OK	OK	OK	
		Middle	20,12	169	152	304	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK	
	3 (right support)	Column	70,08	169	152	1060	220	Ø12 á 100 mm	1131	46,10	0,273	151	74,04	OK	OK	OK	
		Middle	23,36	169	152	353	220	Ø8 á 150 mm	335	13,65	0,081	164	23,82	OK	OK	OK	
3_o	1 (left support)	Column	36,14	156	140	592	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK	
		Middle	12,65	156	140	207	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	
	2 (midspan)	Column	29,78	156	140	488	204	Ø12 á 200 mm	566	23,07	0,147	147	36,36	OK	OK	OK	
		Middle	10,69	156	140	175	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	
	3 (right support)	Column	50,12	156	140	821	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK	
		Middle	9,00	156	140	147	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	
3_{in}	1 (left support)	Column	56,62	156	140	927	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK	
		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	
	2 (midspan)	Column	24,39	156	140	400	204	Ø12 á 250 mm	452	18,42	0,117	149	29,41	OK	OK	OK	
		Middle	11,50	156	140	188	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	
	3 (right support)	Column	56,62	156	140	927	204	Ø12 á 100 mm	1131	46,10	0,294	138	68,14	OK	OK	OK	
		Middle	13,35	156	140	219	204	Ø8 á 200 mm	251	10,23	0,065	152	16,69	OK	OK	OK	

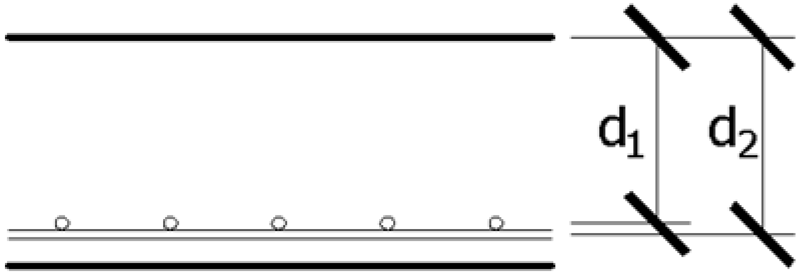
Design of bending reinforcement

- Cross-sectional area of reinforcement:

$$a_{\mathrm{s,prov}} \geq a_{\mathrm{s,rqd}} = \frac{m_{\mathrm{Ed}}}{z f_{\mathrm{yd}}}$$

$f_{yd} = f_{yk}/1.15$

Lever arm of internal forces, estimation: $z = 0.9\,d$



- Two-way slab = two values of *d* !!!
- Use higher *d* in the direction of higher m_{Ed}
- Use 8 – 14 mm bars
- Use cover depth from the 1st task

Minimum reinforcement

- Brittle failure precaution:

$$a_{s,prov} \geq a_{s,min,1} = \max\left(0.26\frac{f_{ctm}}{f_{yk}}bd; 0.0013bd\right)$$

Mean tensile strength of concrete, see table

Width of the slab, 1 m

- Excessive cracking precaution:

In our case, $f_{ct,eff} = f_{ctm}$

$$a_{s,prov} \geq a_{s,min,2} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s}$$

Coefficients describing stress distribution in the cross-section, $k_c = 0.4$ and $k = 1.0$

Area of concrete within tensile zone at the first crack, $A_{ct} = 0.5bd$

Mean value of the tensile strength of the concrete effective at the time when the first cracks may be expected to occur; $f_{ct,eff} = f_{ctm}$

Maximum stress permitted in the reinforcement immediately after formation of the crack, $\sigma_s = f_{yk} =$ 500 MPa

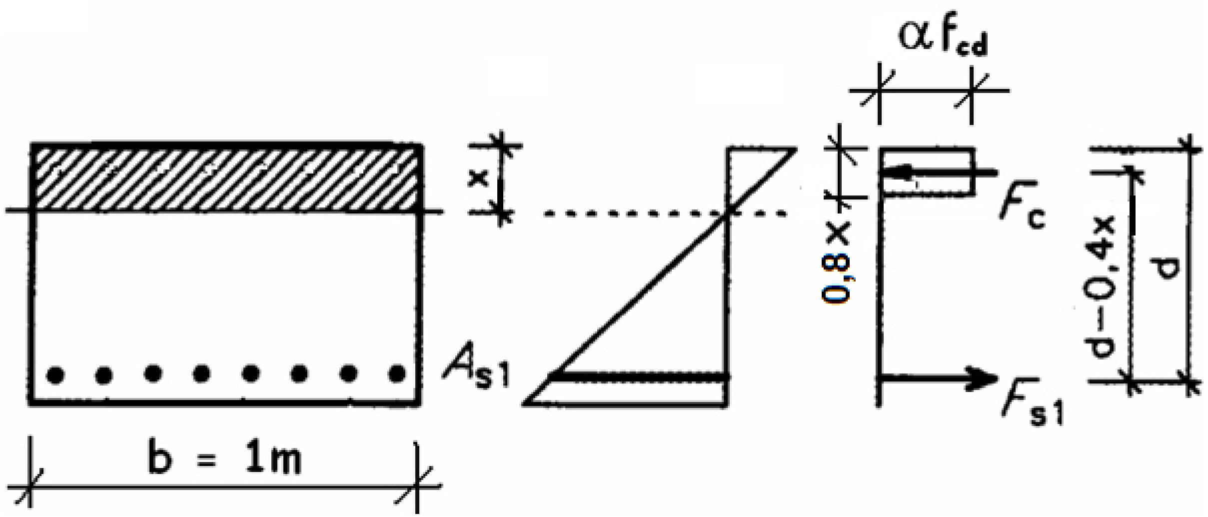
Check of the design

Height of compressed zone of concrete cross-section → $$x = \frac{a_{\mathrm{s,prov}} f_{\mathrm{yd}}}{0.8 b f_{\mathrm{cd}}}$$ → $f_{cd} = f_{ck}/1.5$

Actual value of lever arm of internal forces → $$z = d - 0.4x$$

$$m_{\mathrm{Rd}} = a_{\mathrm{s,prov}} f_{\mathrm{yd}} z$$

$$m_{\mathrm{Rd}} \geq m_{\mathrm{Ed}}$$



Detailing rules

- Relative height of compressed zone:

$$\xi = \frac{x}{d} \leq 0.45$$

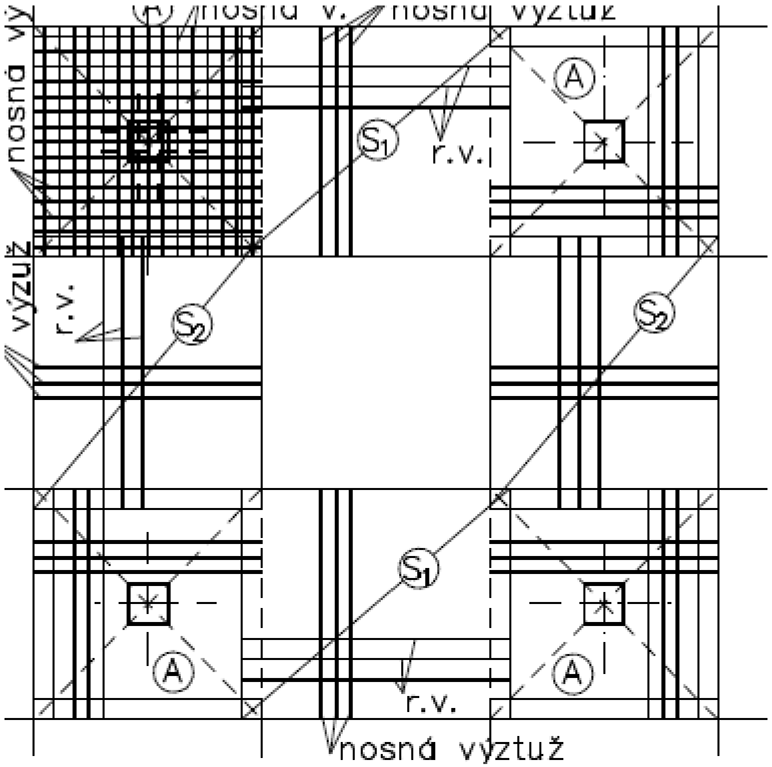
- Spacing of rebars:

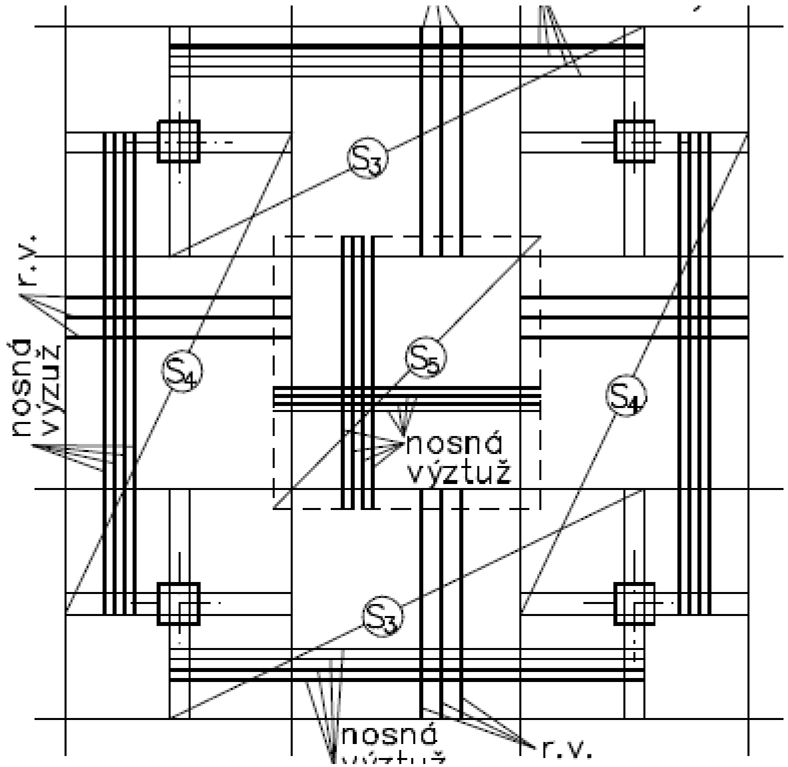
$$s \leq \min\left(2h_\mathrm{S};\ 250\ \mathrm{mm}\right)$$

- Recommendation: Spacing not less than 100 mm
- RULE: It is always better to use higher number of smaller bars than lower number of bigger bars (deflections, stress distribution…)

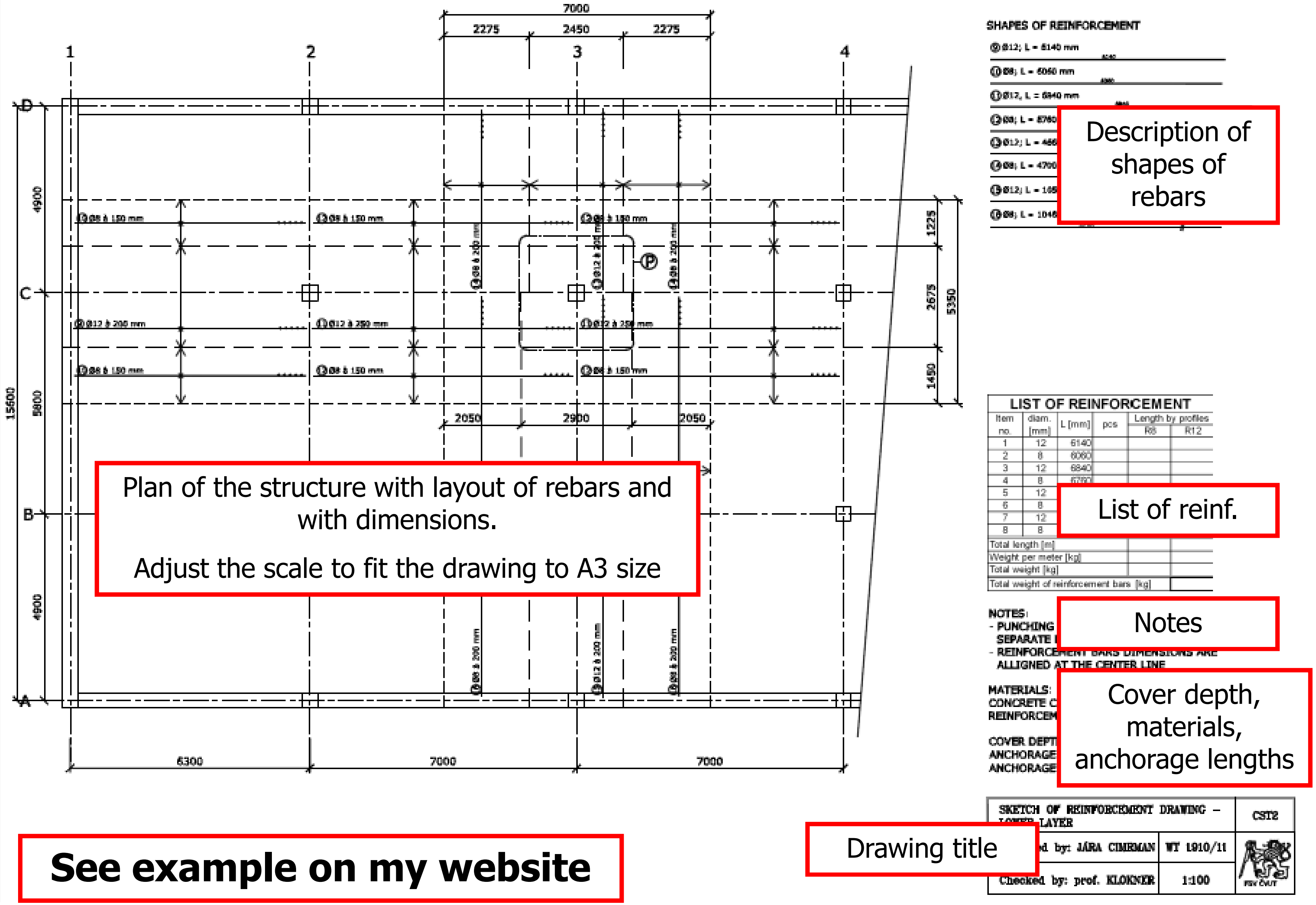
Drawing of bending reinforcement

- 1 drawing for upper reinforcement
- 1 drawing for lower reinforcement
- Calculation of anchorage length – see 5th seminar



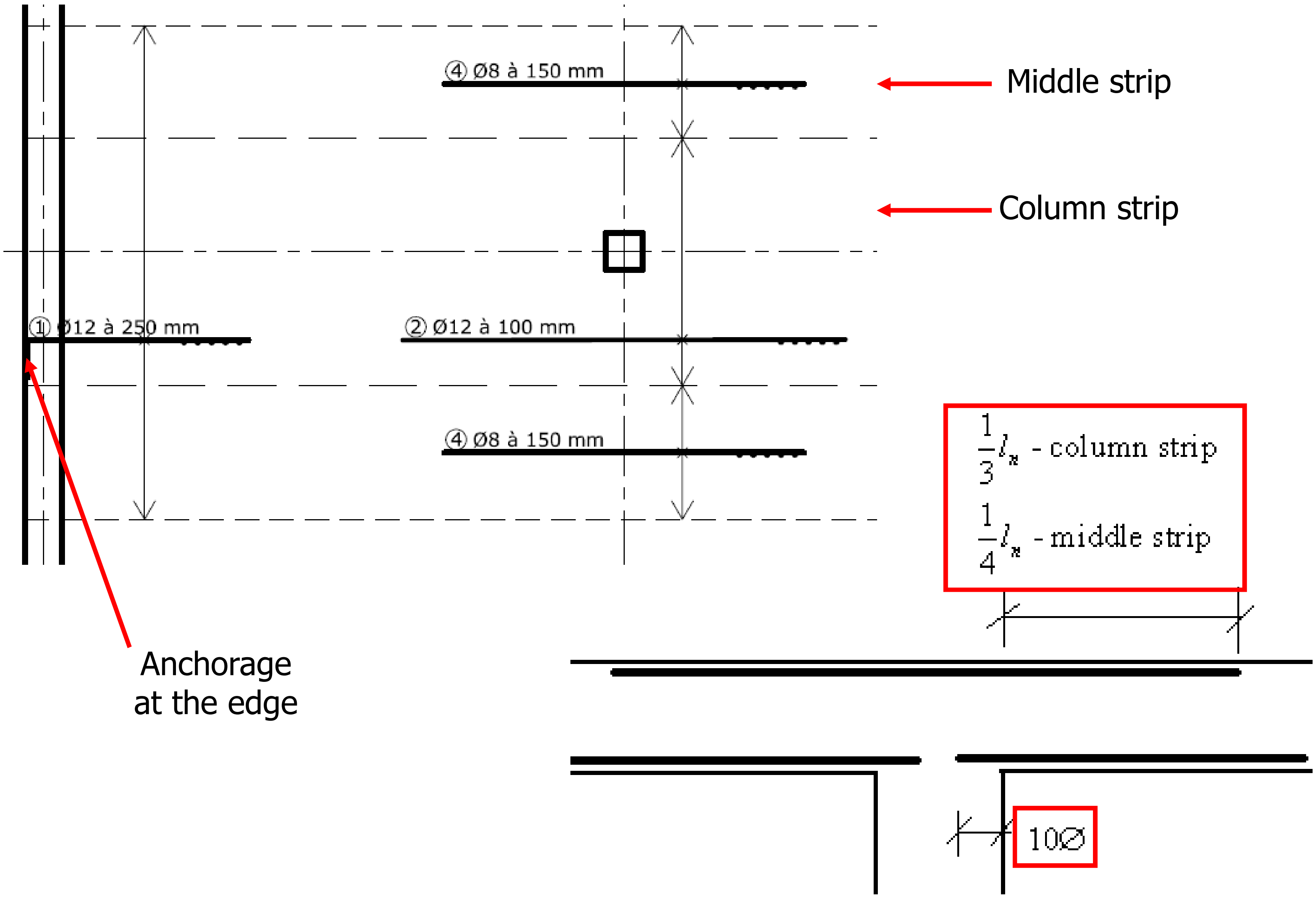


Drawing of bending reinforcement

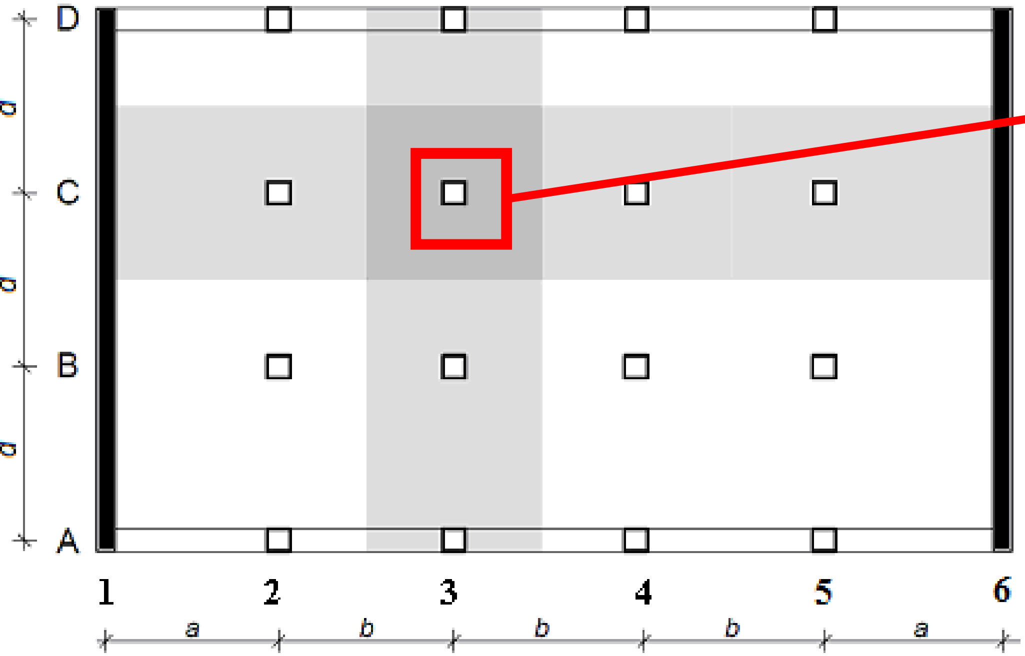


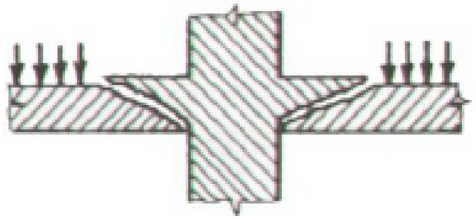
See example on my website

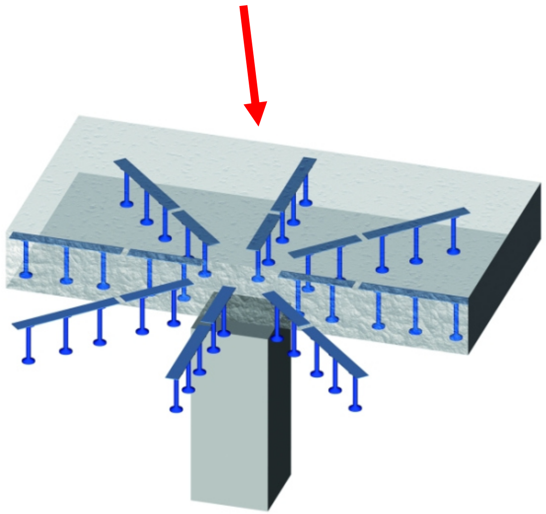
Layout of bending reinforcement



Punching in column C3







Double-headed studs
connected to a spacer bar

Detailed check of punching

- 3 steps
- *Is the resistance of compressed concrete in the face of the column sufficient?* Already checked in preliminary design.

$$v_{\mathrm{Ed,max}} \leq v_{\mathrm{Rd,max}}$$

- *Is the slab able to carry the load without reinforcement?*

$$v_{\mathrm{Ed,1}} \leq v_{\mathrm{Rd,c}}$$

- *Is the designed reinforcement sufficient?*

$$v_{\mathrm{Ed,1}} \leq v_{\mathrm{Rd,cs}}$$

Resistance without reinforcement

$$v_{\mathrm{Ed,1}} = \frac{\beta V_{\mathrm{Ed}}}{u_1 d} \leq v_{\mathrm{Rd,c}} = \max\left[C_{\mathrm{Rd,c}} \cdot k \cdot \sqrt[3]{\left(100 \rho_l \cdot f_{\mathrm{ck}}\right)}; \quad 0{,}035\sqrt{k^3 f_{\mathrm{ck}}} \right]$$

Stress in control perimeter u_1 - see preliminary design

Punching shear resistance of a slab <u>without</u> punching reinforcement

Reinforcement ratio for tensile reinforcement

Minimum value of $v_{\mathrm{Rd,c}}$ called v_{min}

$$\rho_l = \sqrt{\rho_{lC} \cdot \rho_{l3}} \leq 0{,}02$$

$$d = \frac{d_{\mathrm{C}} + d_3}{2}$$

$$\rho_{l\mathrm{C}} = \frac{a_{\mathrm{sC}}}{1000 d_{\mathrm{C}}}, \quad \rho_{l3} = \frac{a_{\mathrm{s3}}}{1000 d_3}$$

If you used caps (steel flanged collars), use u_1 calculated with the cap

a_{sC} and a_{s3} are cross-sectional areas of upper reinforcement per 1 m in belt C and belt 3 (in the distance of 3d from perimeter u_0)

Remaining coefficients – see preliminary design (7th seminar)

$v_{\mathrm{Ed,1}} \leq v_{\mathrm{Rd,c}}$ ☺ No punching reinforcement needed

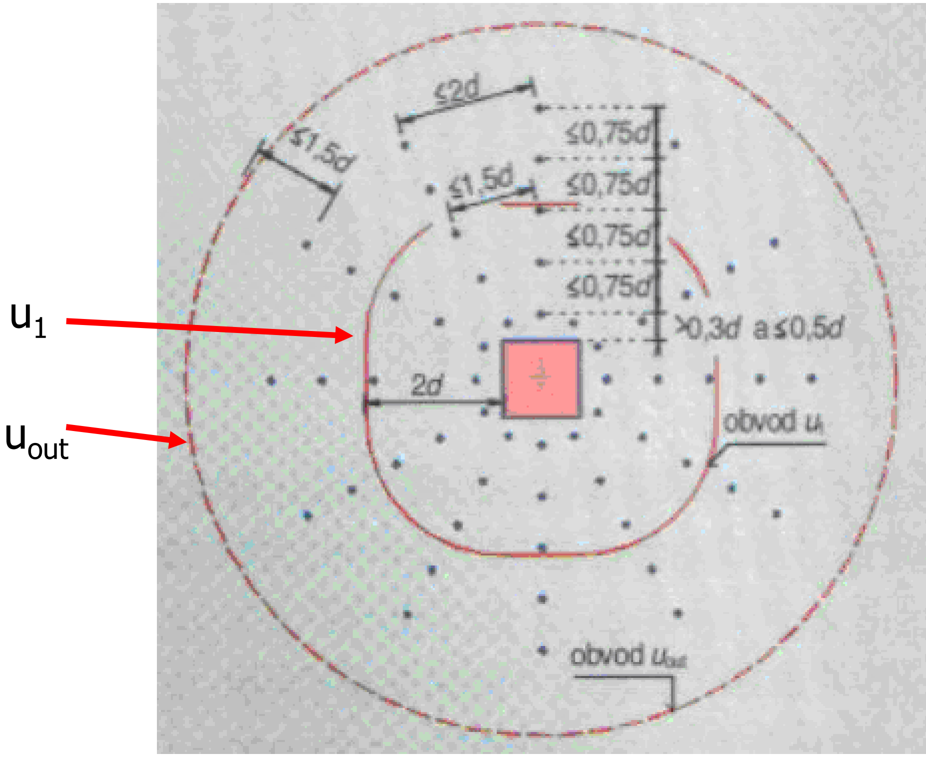
$v_{\mathrm{Ed,1}} > v_{\mathrm{Rd,c}}$ ☹ Design of punching reinforcement

Design of punching reinforcement

- Control perimeter where the reinforcement is not necessary ($v_{Ed,out} = v_{Rd,c}$) and its radius:

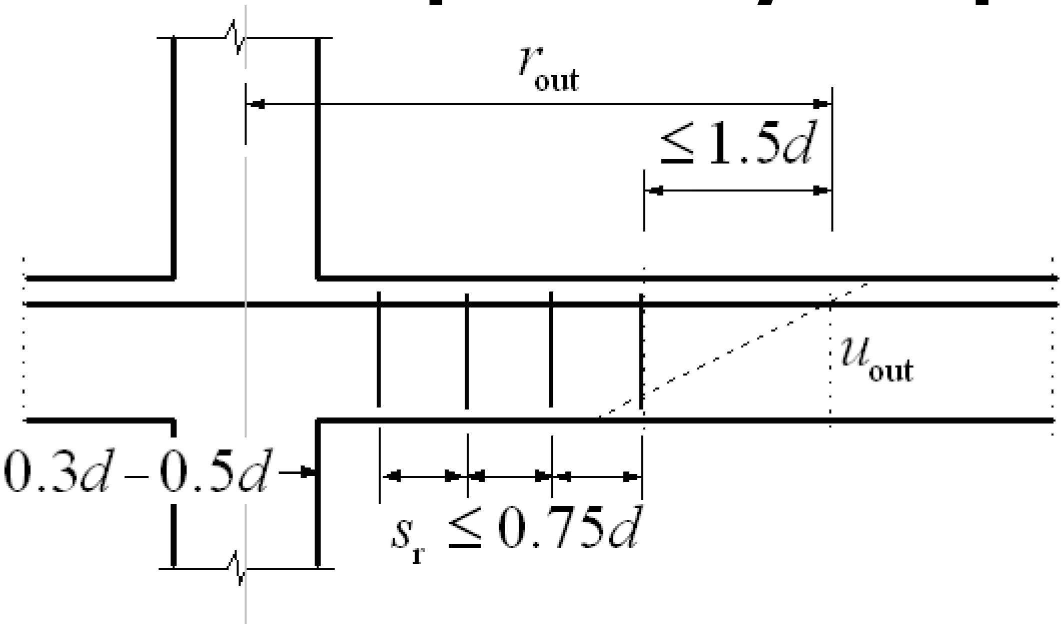
$$u_{out} = \frac{\beta\, V_{Ed}}{v_{Rd,c}\, d}$$

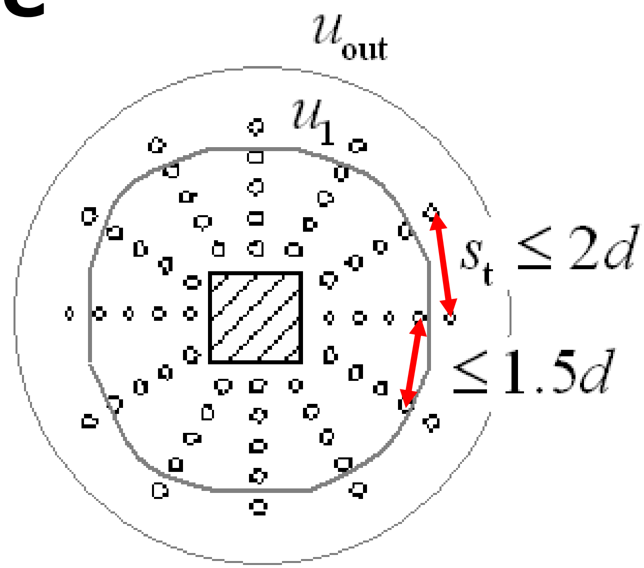
$$r_{out} = \frac{u_{out}}{2\pi}$$



Layout of punching reinforcement

- First stud (0.3–0.5)*d* behind the face of column
- Last stud not more than 1.5*d* from u_{out}
- Spacing of intermediate studs $s_r \leq 0.75d$
- Spacing of rails $s_t \leq 2d$
- Spacing of rails in perimeter u_1 less than 1.5*d*
- **See example on my webpage**





Design of punching reinforcement

- Number of rails („sun rays")

$$n \geq \max\left(\frac{2\pi(r_{out} - 1,5d)}{2d}; \frac{u_1}{1,5d}\right)$$

- Diameter of studs: 10 – 14 mm (up to 25 mm if necessary) => cross-sectional area of one stud $A_{sw,1}$
- Cross-sectional area of studs in one perimeter:

$$A_{sw} = n.A_{sw,1}$$

- If you used caps (steel flanged collars), use u_1 calculated with the cap

Check of punching reinforcement

$$v_{\mathrm{Ed},1} = \frac{\beta V_{\mathrm{Ed}}}{u_1 d} \le v_{\mathrm{Rd,cs}}$$

Stress in control perimeter u_1

Punching shear resistance of a slab with punching reinforcement

$$v_{\mathrm{Rd,cs}} = 0,75 v_{\mathrm{Rd,c}} + 1,5 \frac{d}{s_r} A_{sw} f_{\mathrm{ywd,ef}} \frac{1}{u_1 d} \sin\alpha \le k_{\max} v_{\mathrm{Rd,c}}$$

Punching shear resistance of a slab without punching reinforcement

Spacing of studs

Effective design strength of punching reinforcement

Angle between studs and slab, 90°

$$f_{\mathrm{ywd,ef}} = 250 + 0,25d \le f_{\mathrm{ywd}}$$

Design strength of reinforcement steel, ask the producer of studs or take 435 MPa

$v_{\mathrm{Ed},1} \le v_{\mathrm{Rd,cs}}$ ☺ Checked

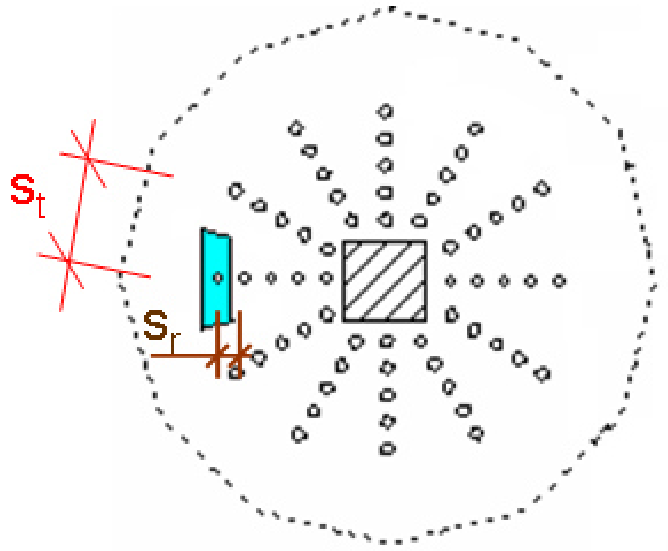
$v_{\mathrm{Ed},1} > v_{\mathrm{Rd,cs}}$ ☹ Redesign: Decrease s_r or increase $A_{sw,1}$

Punching reinforcement ratio

$$\rho_{sw} = 1,5\frac{A_{sw,1}}{s_r s_t} \geq \rho_{sw,min} = 0,08\frac{\sqrt{f_{ck}}}{f_{yk}}$$

Punching reinforcement ratio of your slab

Minimum punching reinforcement ratio



$\rho_{sw} \geq \rho_{sw,min}$ ☺ Checked

$\rho_{sw} < \rho_{sw,min}$ ☹ Redesign: Decrease s_r or increase $A_{sw,1}$

Sketch of punching reinforcement

- Plan and section in scale 1:20
- See example on my website