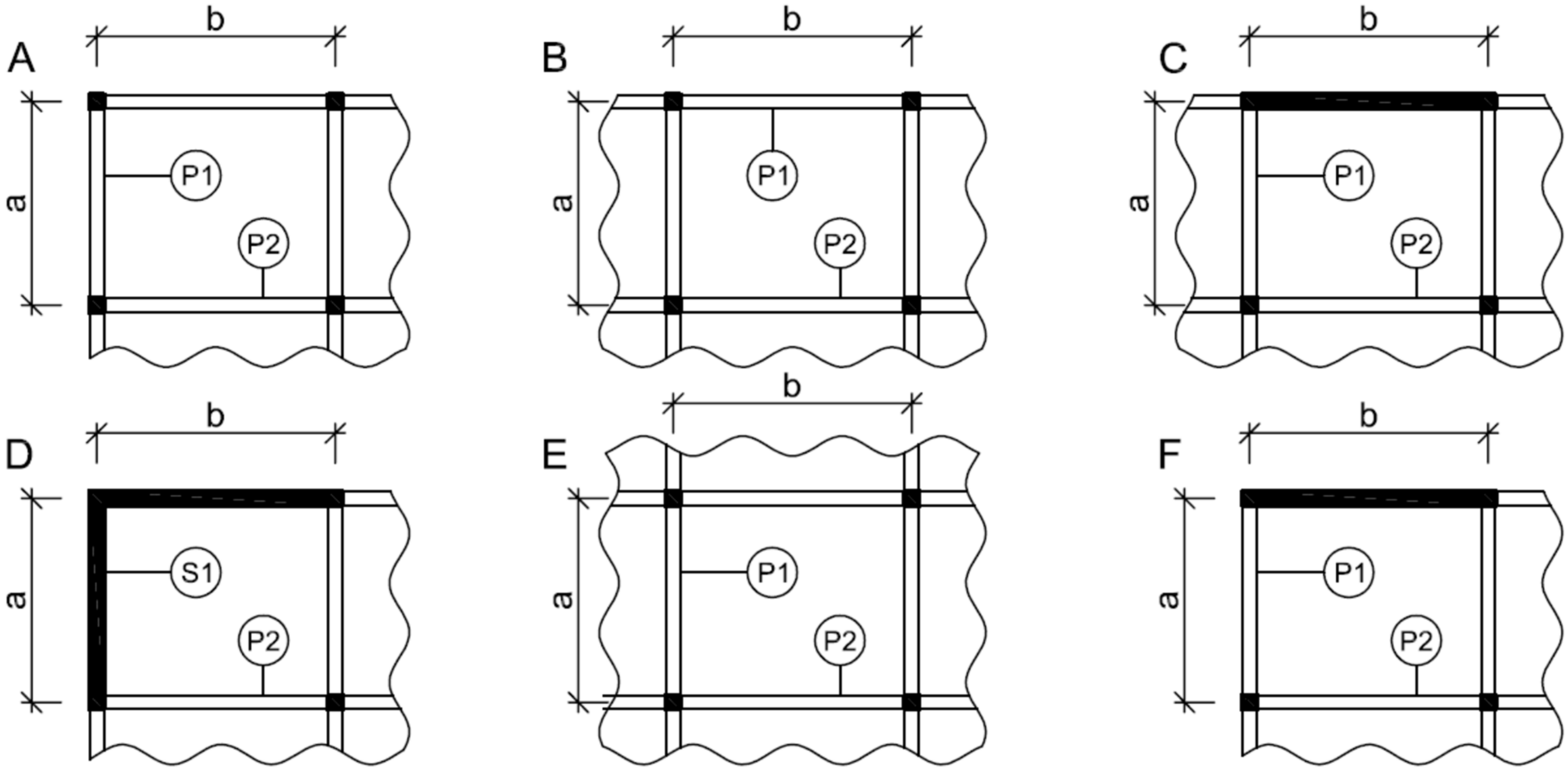
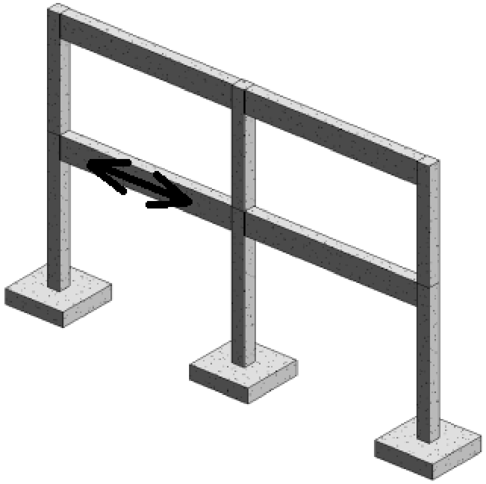
2nd task:

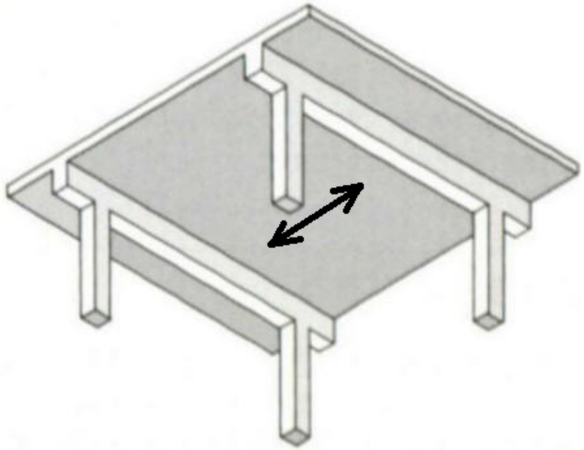
Two-way slab supported on 4 sides



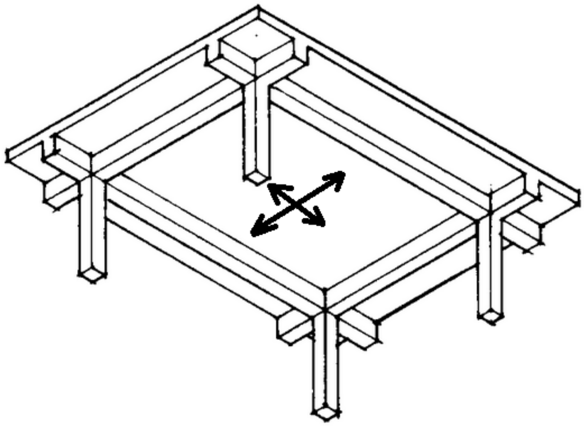
Elements subjected to bending



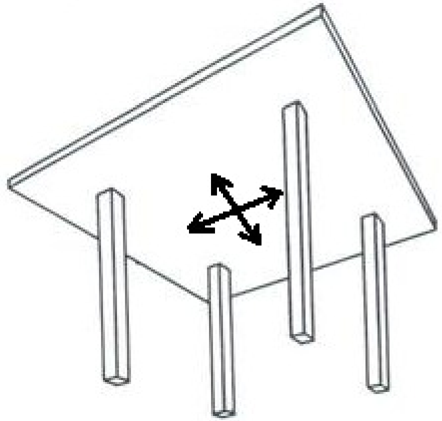
Beam (frame) – **1st task**



One-way slab – *(1st task)*

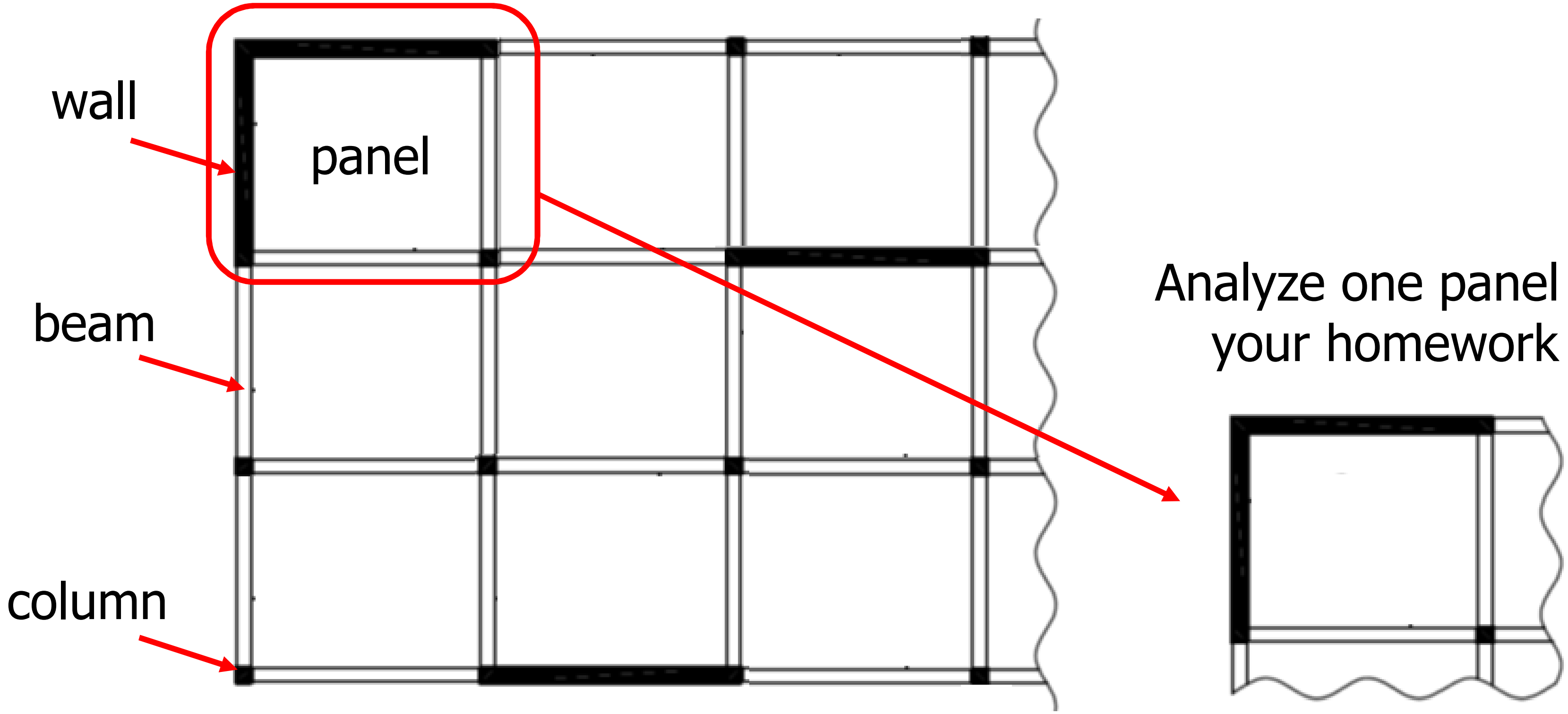


Two-way slab supported on 4 sides – **2nd task**



Two-way flat slab – **3rd task**

Two-way slab supported on 4 sides



Two-way slab supported on 4 sides

- The panel is given by the assignment
- Depth h_s is given
- Calculate bending moments using linear analysis
- Calculate bending moments using precalculated tables based on the theory of plasticity
- For „plastic“ moments, check h_S
- Calculate load of given supporting element (beam or wall)

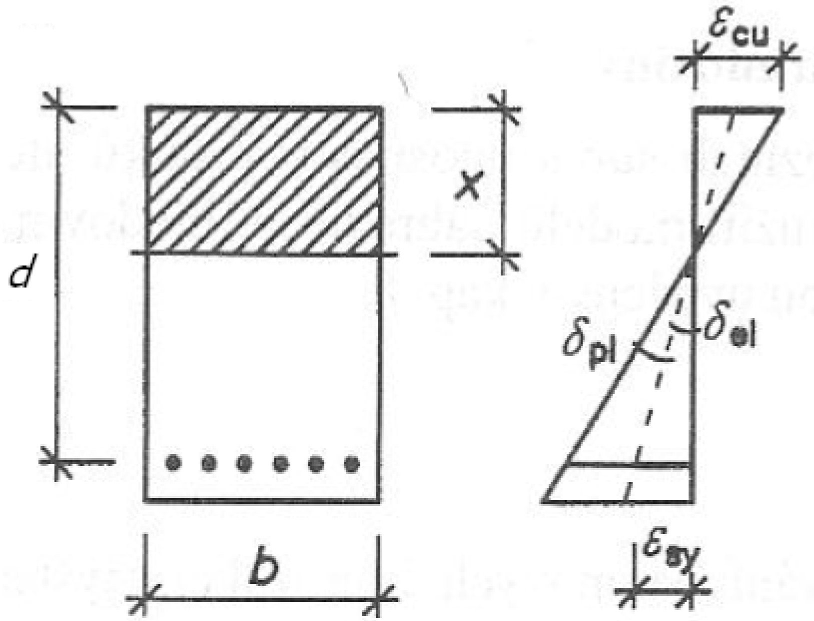
Introduction

- **Elastic theory** – always applicable, but usually less fitting; no cracks in the structure
- **Plastic theory** – closer to real behaviour of RC structures, but sufficient plastic hinge rotational capacity is necessary; the structure is cracked

$$\xi = \frac{x}{d} \leq 0.45$$

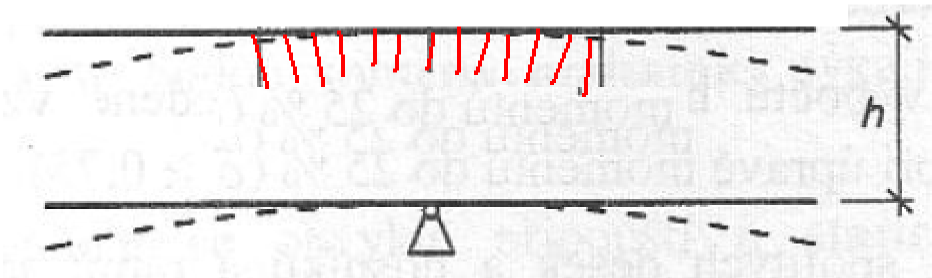
$$\xi = \frac{x}{d} \leq 0.25$$

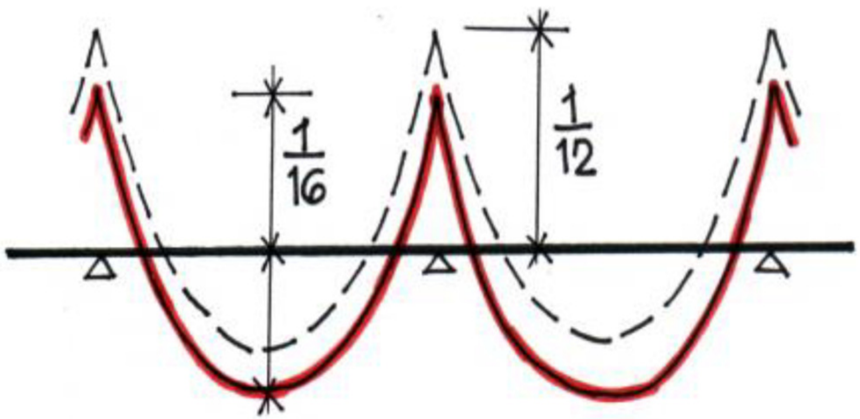
Rotational capacity does not have to be checked



Introduction

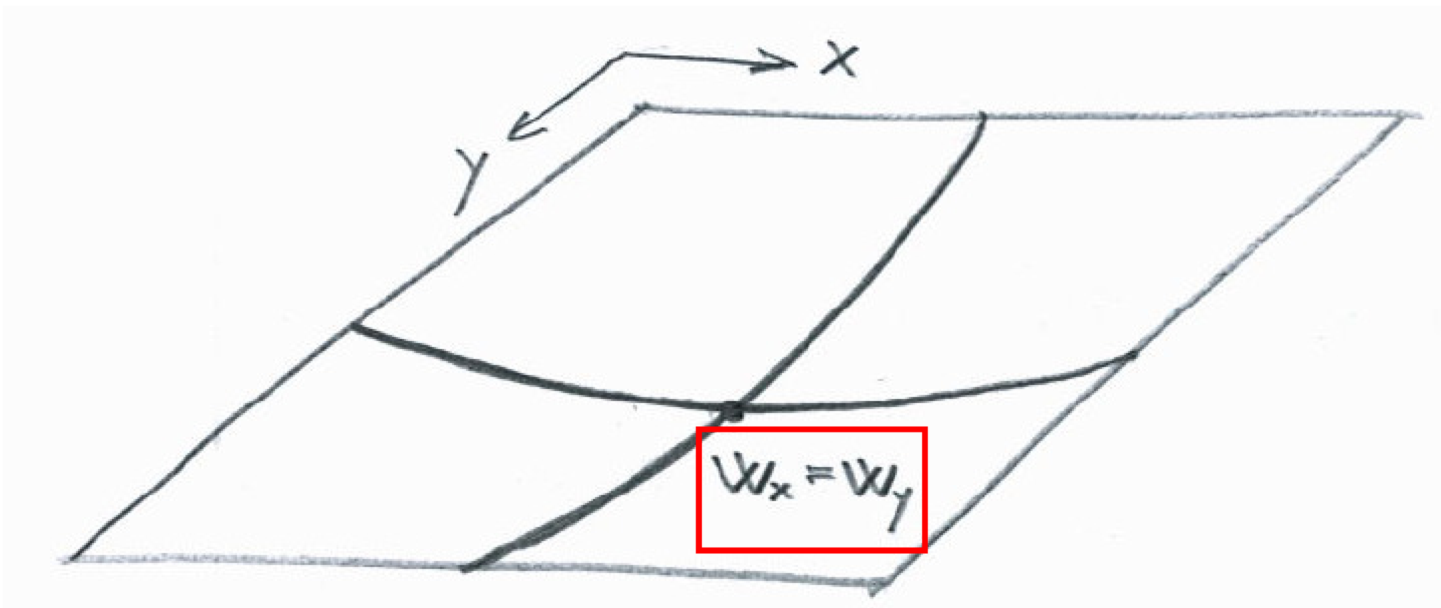
- Plastic behaviour enables redistribution of internal forces => better utilization of material





Linear analysis

- Less fitting, but very simple
- Good for **quick check** of bending moments calculated by more complex theories
- *Idea:* Deflection of the slab in both directions must be the same



Linear analysis

- Calculate total design load of the slab f_d
- The load will be **divided in two directions**:

$$f_d = f_{d,x} + f_{d,y}$$

- We can model the behavior of the slab in each direction as one of the following **beam types**:

- **Fixed end** = continuity or outer edge with wall
- **Pinned end** (simply supported) = outer edge with beam
- Select the correct beam types for your structure

Linear analysis

- The deflection in the middle of a beam is:

$$w = k \cdot \frac{fl^4}{EI}$$

Coefficient according to beam type (k)

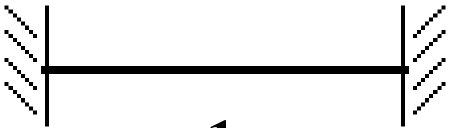
Load of the beam in the given direction (f)

Span of the beam (l)

Elastic modulus of concrete (E)

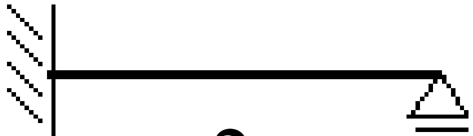
Moment of inertia of the cross-section (I)

- Values of *k*:



$$\frac{1}{384}$$

$$\frac{5}{384}$$



$$\frac{2}{384}$$

Linear analysis

- As $w_x = w_y$, we can say that:

$$k_x \cdot \frac{f_{d,x} l_x^4}{EI} = k_y \cdot \frac{f_{d,y} l_y^4}{EI}$$

- $E_x = E_y$ and $I_x = I_y$ (in both directions, we consider that the cross-section is h_s*1 m)
- After rearranging:

$$\frac{f_{d,x}}{f_{d,y}} = \frac{k_y}{k_x} \cdot \left(\frac{l_y}{l_x} \right)^4$$

Linear analysis

- As $f_d = f_{d,x} + f_{d,y}$, we can say that:

$$\frac{f_{d,x}}{f_d - f_{d,x}} = \frac{k_y}{k_x} \cdot \left(\frac{l_y}{l_x} \right)^4$$

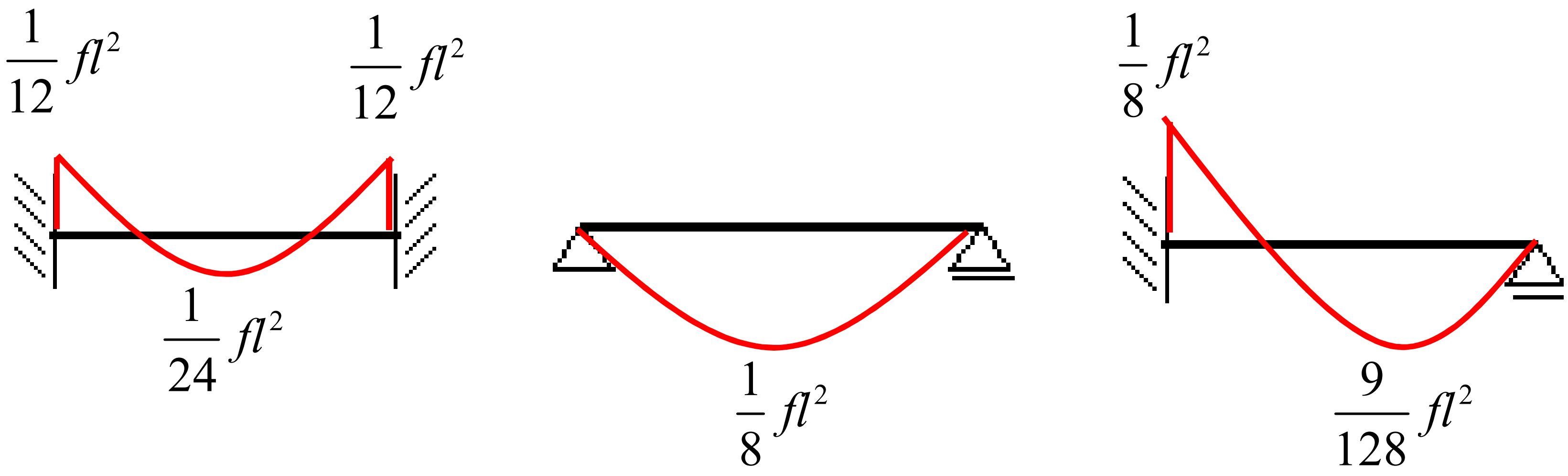
- And after rearranging, we receive **formulas for the loads** in *x* and *y* directions:

$$f_{d,x} = \frac{f_d \cdot \frac{k_y}{k_x} \cdot \left(\frac{l_y}{l_x} \right)^4}{1 + \frac{k_y}{k_x} \cdot \left(\frac{l_y}{l_x} \right)^4}$$

$$f_{d,y} = f_d - f_{d,x}$$

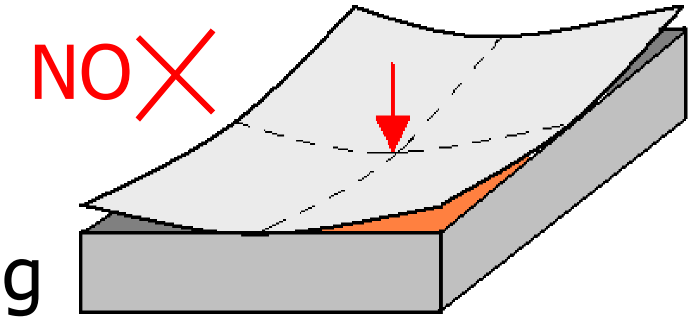
Linear analysis

- Now we can calculate „linear" bending moments in the slab in each direction



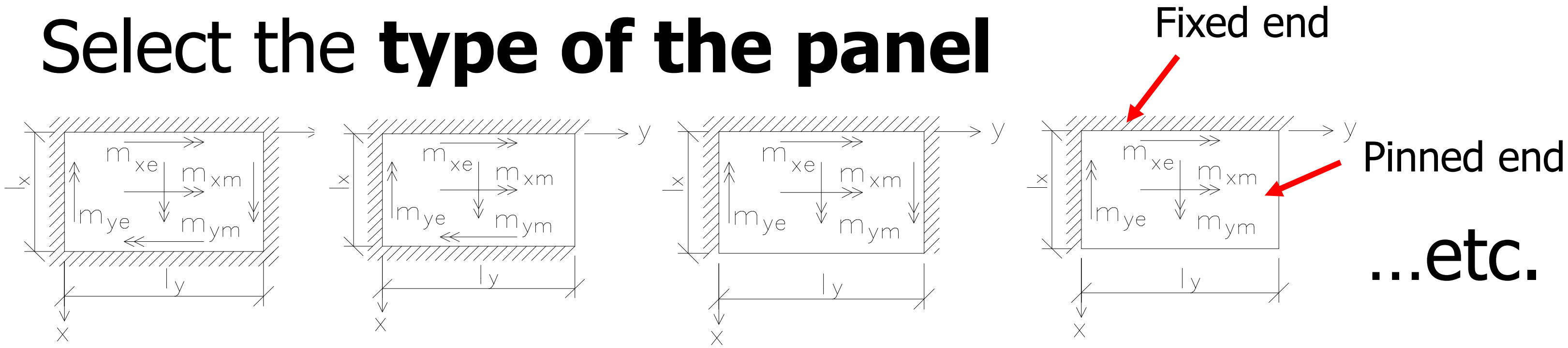
Assumptions for use of the tables based on the theory of plasticity

- Constant depth of the slab
- Rigid supports
- Corners prevented from lifting
- Approximately same load of adjacent panels
- Approximately same spans of adjacent panels
- Sufficient ductility of reinforcement (steel class B, C)
- Sufficient rotational capacity ($x/d \leq 0.25$)



Plastic analysis

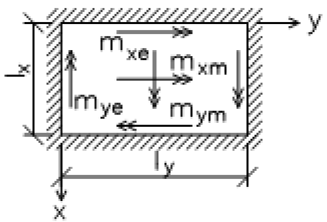
- Select the **type of the panel**



- Calculate ratio of spans
- Look up β coefficients in the table

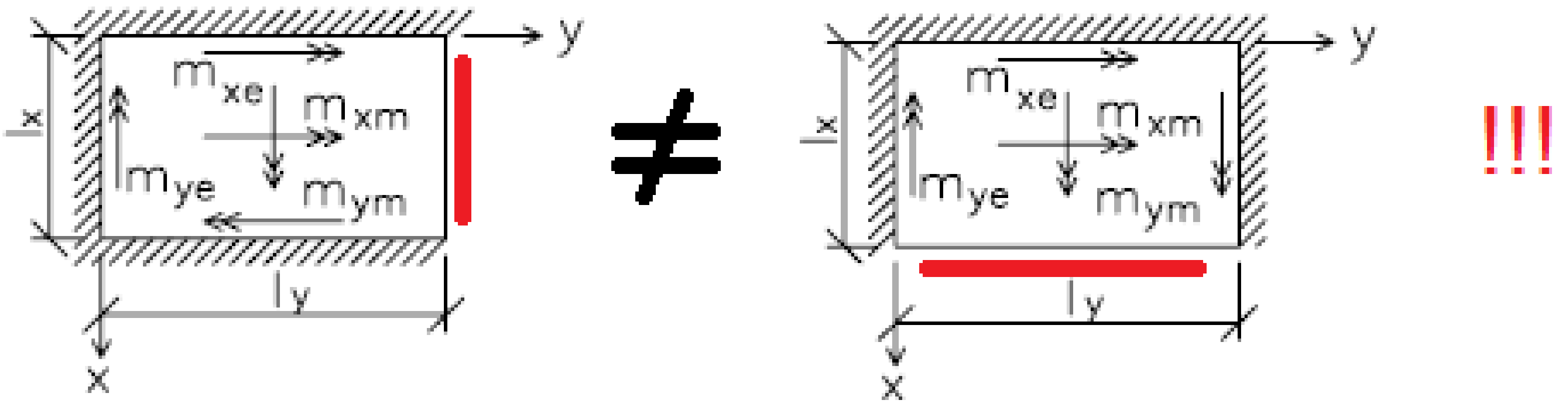
$$\frac{l_y}{l_x} \rightarrow \beta$$

!!! For all panel types, l_x is the <u>shorter</u> span

Support condition	Factor	Coefficients β_{xe}, β_{xm}, β_{ye}, β_{ym}, for l_y/l_x =							
		1,0	1,1	1,2	1,3	1,4	1,5	1,75	≥2,0
	β_{xe}	-0,031	-0,037	-0,042	-0,046	-0,050	-0,053	-0,059	-0,063
	β_{xm}	0,024	0,028	0,032	0,035	0,037	0,040	0,044	0,048
	β_{ye}	-0,032							-0,032
	β_{ym}	0,024							0,024

Plastic analysis

- Use linear interpolation to calculate β coefficients
- β_y coefficients are constant for all values of l_y/l_x
- Be careful when selecting the type of the panel:



Plastic analysis

- Calculation of bending moments

$$m_{xe} = \beta_{xe} m_0$$

$$m_{xm} = \beta_{xm} m_0$$

$$m_{ye} = \beta_{ye} m_0$$

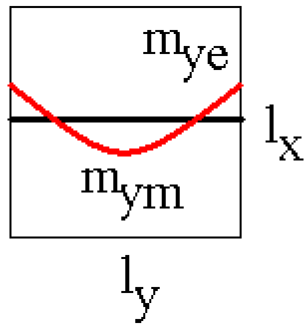
$$m_{ym} = \beta_{ym} m_0$$

$$m_0 = f_d \cdot l_x^2$$

Basic value of bending moment, constant for all moments!!!

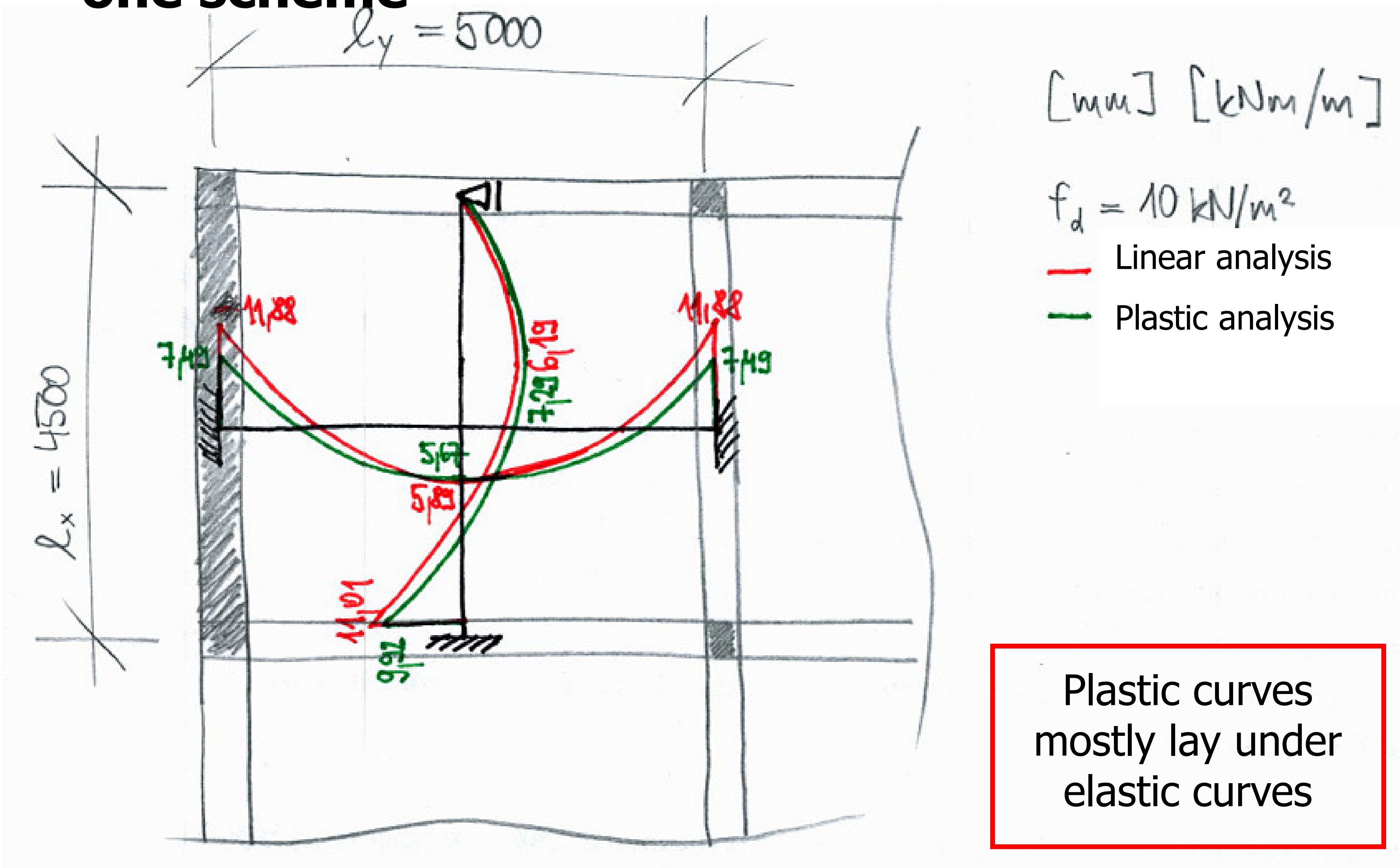
!!! Be careful about directions – m_x is the moment in the direction of l_x, the same applies to *y* index.

- Indices:
  - x, y – direction of a moment
  - m – midspan moment
  - e – support moment



Scheme of bending moments

- **Compare** linear elastic and plastic moments in **one scheme**



Plastic curves mostly lay under elastic curves

Design of bending reinforcement

- In the homework, you DO NOT HAVE TO design the reinforcement
- BUT remember, that the procedure of design of bending reinforcement for two-way slabs is IDENTICAL to beams (see 3rd seminar)
- You should use the plastic moments for design of reinforcement (closer to real behaviour of RC structure)
- The only difference is that you design the reinforcement in **2 directions** and that the width of the cross-section is taken as b = 1 m

Check of h_s

- Check the given value of h_s for the biggest moment from **plastic** analysis ($m_{Ed,max}$)
- Calculate the required cross-sectional area of reinforcement:

$$a_{s,rqd} = \frac{m_{Ed,\max}}{0.9 d f_{yd}}$$

Calculation of effective depth – see 3rd seminar.

Estimate 10 mm rebars and take cover depth from the frame structure

- Estimate the depth of the compressed zone:

$$x = \frac{1.2 a_{s,rqd} f_{yd}}{0.8 b f_{cd}}$$

Estimation of $a_{s,prov}$

For slabs, you generally consider 1 m wide strip => b = 1000 mm

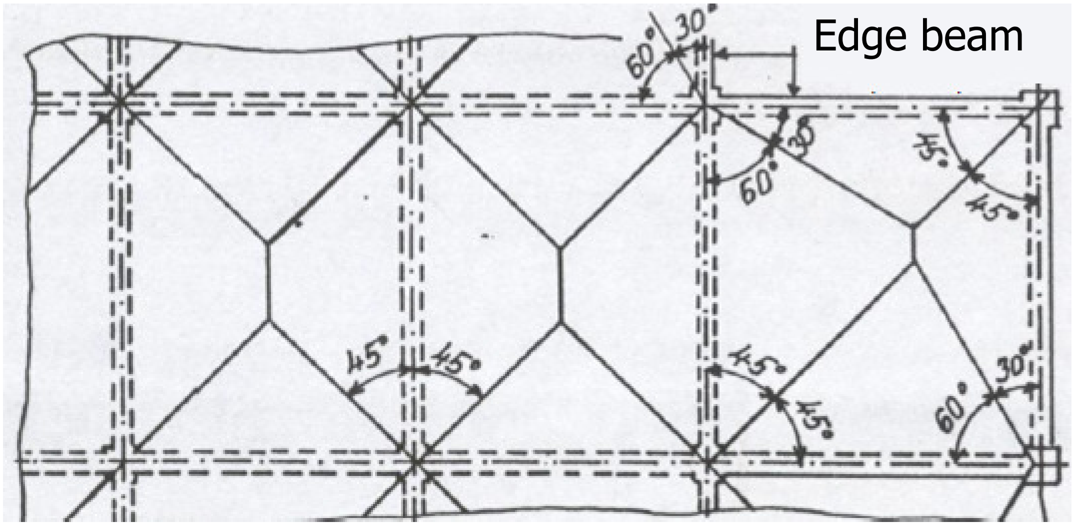
Check of h_s

- Check the span/depth ratio (deflection control) – see 1st seminar
- **If**:
  1. $a_{s,rqd} \geq a_{s,min}$ (calculation of $a_{s,min}$ – see 3rd seminar)
  2. $\xi = \frac{x}{d} \leq 0.25$
  3. Span to depth ratio is checked

  then the original h_s is **correct.**
- If some of the conditions are not checked, **propose a solution** (just describe it, don´t calculate anything)

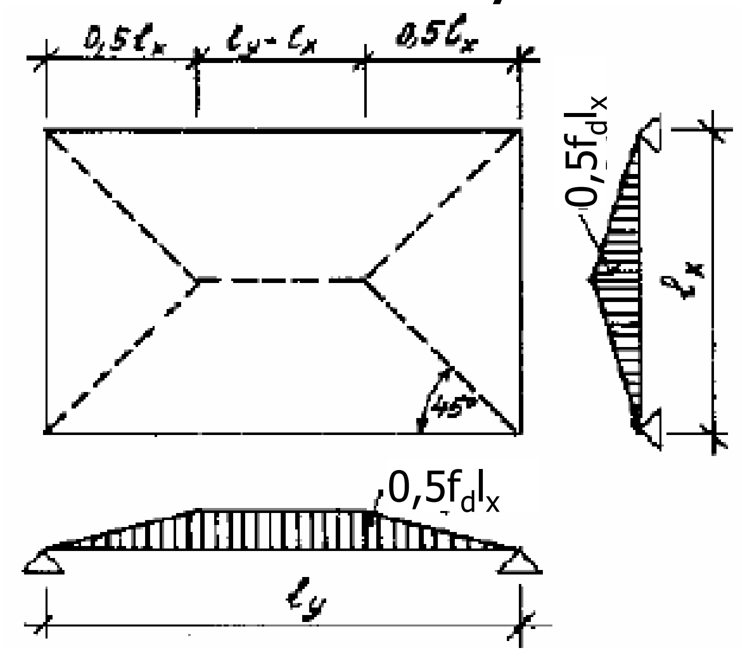
Load of beam/wall

- Draw **tributary areas** of all the supporting elements
- The angle between identical supports (fixed/fixed, pinned/pinned) is **45°**
- Between fixed and pinned, **60°** go to fixed
- **Calculate the load** of one given supporting element (wall or beam)



Load of beam/wall

- For your given element, draw **load diagram** (with values, not just the shape!)
- The load in each point is: total load of the slab (f_d) * width of the tributary area



- Be careful – inner walls and beams are loaded by 2 adjacent panels!