Finite Element Modelling of Beams in Elevated Temperature – Benchmark Problems

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Why do we need benchmarks?

- Verification of developed models
- Learning how to use FE programms
- Checking new FE codes and their components
- Cross-checking of different FE codes

Benchmark developement

- Choice of proper problem
- Development of FE model
- Sensitivity study
  - GCI
  - FE formulation
  - Material models
- Comparison with other solutions (analytical, different FE codes)
- Proper description of problem and results (report, input and output spreadsheets)

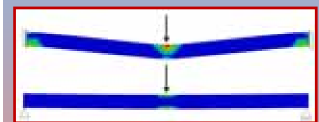
Exemplary benchmark problems

Sawicki B., Pełczyński J., Kwaśniewski L.
BENCHMARK EXAMPLE PROBLEMS FOR BEAMS At Elevated Temperatures
APPLICATIONS OF STRUCTURAL FIRE ENGINEERING, pp.29-35, CTU Publishing House, Prague 2013

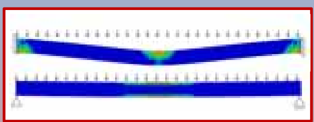
Exemplary benchmark problems - geometry and load

- Rectangular 30x50mm section
- Three loading cases
- Two types of boundary conditions

Pure bending of simply supported beam



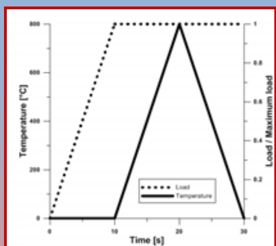
Fixed and simply supported beams under point load



Fixed and simply supported beams under uniformly distributed load

Exemplary benchmark problems - loading

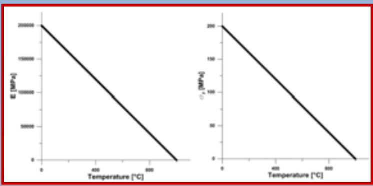
- Fixed load during thermal analysis
- Varying temperature



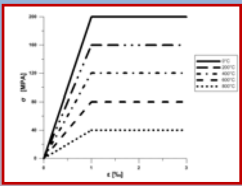
Temperature and load variation during implicit analyses

Exemplary benchmark problems - material

Yield stress and Young modulus are linearly temperature dependent



Change of material properties under elevated temperature

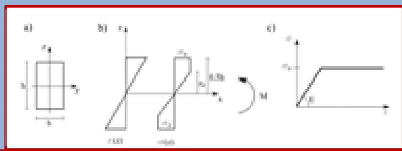


Stress-strain curves at chosen temperatures

Exemplary benchmark problems - analytical solutions

Assumptions:

- cross sections stay planar,
- the effect of shear is neglected,
- the approximate formula (second derivative) for the curvature can be applied to find beam deflection.

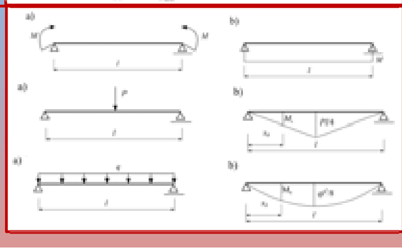


$$f = \frac{1}{8\sqrt{3}}\frac{\sigma_y l^2}{Eh}\left(\frac{1}{4}-\frac{1}{6}\mu\right)^{-1/2}$$

$$\mu = \frac{M}{\sigma_y \frac{bh^2}{6}}$$

$$f = \frac{1}{6}\frac{\sigma_y l^2}{Eh}\frac{3\sqrt{3-2\psi}+\psi\sqrt{3-2\psi}-5}{\psi^2}$$

$$\psi = \frac{P}{\frac{2}{3}\frac{\sigma_y bh^2}{l}}$$



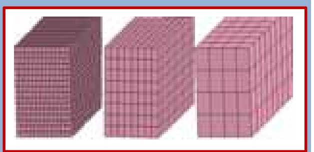
Exemplary benchmark problems – mesh density study; GCI

Convergence rate $p = \frac{\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right)}{\ln(r)}$ $r = 2$

Asymptotic solution $f_{h=0} \cong f_1 + \frac{f_1 - f_2}{r^p - 1}$

Relative difference $\varepsilon = \frac{f_1 - f_2}{f_1}$

Grid Convergence Index $GCI = \frac{F_s|\varepsilon|}{r^p - 1}100\%$ $F_s = 1$

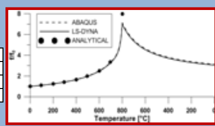


Solver	Temperature [°C]	Result [mm]			p	$f_{h=0}$ [mm]	GCI_{12}	GCI_{23}	$\frac{GCI_{23}}{r^p GCI_{12}}$
		f_3	f_2	f_1					
ABAQUS	0	1.367	1.391	1.398	1.778	1.401	0.206	0.710	1.005
	800	9.256	10.490	10.970	1.362	11.276	2.786	7.489	1.046
	varying	8.478	9.636	9.963	1.824	10.092	1.292	4.729	1.034
LS-DYNA	0	1.369	1.393	1.400	1.778	1.403	0.206	0.709	1.005
	800	9.640	11.023	11.818	0.799	12.893	9.095	16.963	1.072
	varying	8.384	9.228	9.814	0.526	11.145	13.562	20.774	1.064

Exemplary benchmark problems – results for solid elements

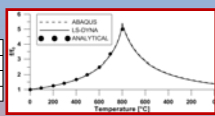
Deflection of simply supported beam with pure bending [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.400	1.750	2.800	3.500	4.667	11.180	-	-	-	-	-
ABAQUS	1.398	1.747	2.796	3.495	4.569	9.963	7.635	6.469	5.770	4.727	4.372
LS-DYNA	1.400	1.743	2.773	3.461	4.609	9.814	7.512	6.358	5.667	4.626	4.275



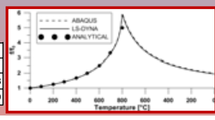
Deflection of simply supported beam with concentrated force [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	0.933	1.167	1.866	2.333	3.111	4.667	-	-	-	-	-
ABAQUS	0.940	1.174	1.882	2.352	3.136	5.074	3.507	2.723	2.253	1.548	1.312
LS-DYNA	0.940	1.172	1.869	2.333	3.108	5.008	3.462	2.688	2.223	1.526	1.293



Deflection of simply supported beam with distributed loading [mm]

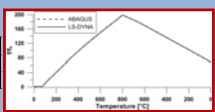
	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.167	1.458	2.333	2.917	3.889	5.833	-	-	-	-	-
ABAQUS	1.172	1.466	2.347	2.934	3.912	7.019	5.065	4.088	3.501	2.621	2.328
LS-DYNA	1.173	1.462	2.331	2.911	3.877	6.920	4.991	4.024	3.444	2.571	2.279



Exemplary benchmark problems – results for solid elements cont'd

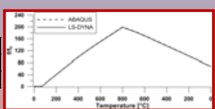
Deflection of fixed beam with concentrated force under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
ABAQUS	0.239	9.649	29.946	35.996	41.839	47.714	44.449	40.734	36.669	24.386	16.060
LS-DYNA	0.239	9.607	29.810	35.840	41.664	47.514	44.261	40.555	36.503	24.311	16.110

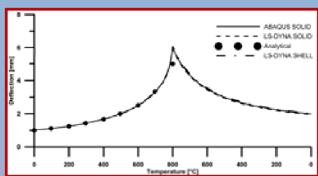


Deflection of fixed beam with distributed loading under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
ABAQUS	0.239	9.649	29.946	35.996	41.839	47.714	44.449	40.734	36.669	24.386	16.060
LS-DYNA	0.239	9.607	29.810	35.840	41.664	47.514	44.261	40.555	36.503	24.311	16.110



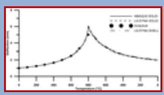
Exemplary benchmark problems – solid elements vs. shell elements



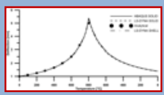
Deflection of simply supported beam with distributed loading [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.167	1.458	2.333	2.917	3.889	5.833	-	-	-	-	-
ABAQUS SOLID	1.172	1.466	2.347	2.934	3.912	7.019	5.065	4.088	3.501	2.621	2.328
LS-DYNA SOLID	1.173	1.462	2.331	2.911	3.877	6.920	4.991	4.024	3.444	2.571	2.279
LS-DYNA SHELL	1.173	1.469	2.344	2.959	3.904	7.141	5.115	4.170	3.592	2.674	2.383

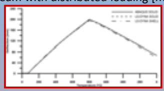
Exemplary benchmark problems – solid elements vs. shell elements



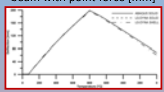
Deflection of simply supported beam with distributed loading [mm]



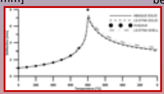
Deflection of simply supported beam with point force [mm]



Deflection of fixed beam with distributed loading [mm]

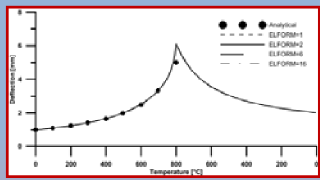


Deflection of fixed beam with point force [mm]



Deflection of simply supported beam with pure bending [mm]

Exemplary benchmark problems – different formulation of shell elements



Deflection of simply supported beam with distributed loading [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
ANALYTICAL	1.167	1.458	2.333	2.917	3.889	5.833	-	-	-	-	-
ELFORM=1	1.173	1.469	2.344	2.961	3.904	7.147	5.117	4.172	3.595	2.676	2.386
ELFORM=2	1.173	1.470	2.344	2.961	3.904	7.144	5.116	4.171	3.594	2.674	2.384
ELFORM=6	1.173	1.473	2.354	2.973	3.920	7.172	5.132	4.182	3.598	2.670	2.373
ELFORM=16	1.173	1.469	2.344	2.959	3.904	7.141	5.115	4.170	3.592	2.674	2.383

Example of use of benchmarks

- Problem by Tomaž Hozjan, F. of Civil and Geodetic Engineering, Univ. of Ljubljana, Slovenia
- Aim: check of new implementations in Fire software

Summary

- Benchmark problem should be checked by all possible means to proove it's correctness
- All possible factors influencing final results should be recognized and desribed in report
- Clear input and output data
- Eery single use of benchmark is it's additional check

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	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.167	1.458	2.333	2.917	3.889	5.833	-	-	-	-	-
ABAQUS SOLID	1.172	1.466	2.347	2.934	3.912	7.019	5.065	4.088	3.501	2.621	2.328
LS-DYNA SOLID	1.173	1.462	2.331	2.911	3.877	6.920	4.991	4.024	3.444	2.571	2.279
LS-DYNA SHELL	1.173	1.469	2.344	2.959	3.904	7.141	5.115	4.170	3.592	2.674	2.383
FIRE	1.167	1.465	2.361	2.959	3.954	7.841	-	-	-	-	-

Future tasks; challenges

- Proper EC material model incorporation into LS-DYNA
- Different load cases, boundary conditions, cross sections

Future tasks; challenges

- Proper EC material model incorporation into LS-DYNA
- Different load cases, boundary conditions, cross sections
- Description of factors affecting model
- Proper description with all data included
- RC beams???