## 3.13 Nonsmooth - nonconvex analysis, optimalization and mechanics (short version)

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# Nonsmooth-nonconvex analysis, optimization and mechanics

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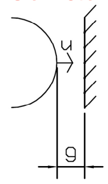
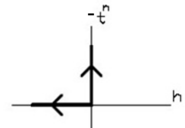
1

### Motivation: unilateral contact problem




Bodies in contact




Graphical representation of the complementarity relation

- $u - g \le 0 \Longrightarrow h \le 0$ → Nonpenetration relation
- $-t^n \ge 0$ → Only compressive stresses are allowed (contact pressure)
- $t^n(u-g) = 0.$ → Complementarity relation



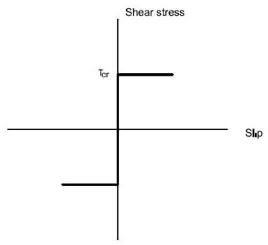
### Motivation: Frictional Stick-Slip problem

(Tangential direction of the interfaces)

Coulomb friction model

- Two contacting surfaces start sliding when $\tau_{cr} = \mu \cdot t^n$
- Stick conditions: No sliding when $\tau < \tau_{cr}$
- $\tau_{cr}$: Critical shear stress
- μ: Friction coefficient
- $t^n$: Contact pressure

- Graphical representation






### Complementarity Problems

Linear Complementarity

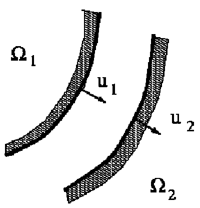
$$x \ge 0,\ y = Mx + b \ge 0,\ x^T y = 0$$

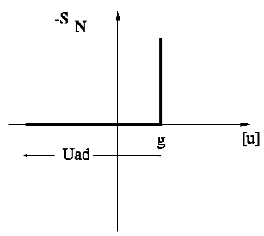
Nonlinear Complementarity

$$x \ge 0,\ F(x) \ge 0,\ x^T F(x) = 0$$



### Unilateral Contact Interface




$$[u]_N - g \le 0,\ -S_N \ge 0,\ -S_N([u]_N - g) = 0$$

Linear Complementarity Interface, Variational Inequalities, Nonsmooth Mechanics



### Complementarity Problems (other applications)

Flow in Networks (e.g. transportation)

Equilibrium in the sense of Nash – Walrasch

Financial Mathematics

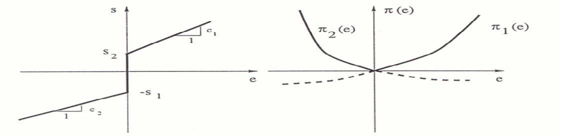
Price analysis of derivatives and options

Flow in porous media

Multiphase flow, salt water front analysis etc

# Monotone Laws and Convex Nonsmooth Superpotentials

**Locking spring**




nonlinear, monotone constitutive law

$$s = \begin{cases} s_2 + c_1 e, & \text{for } e \geq 0, \\ [-s_1, s_2], & \text{for } e = 0, \\ -s_1 + c_2 e, & \text{for } e \leq 0, \end{cases}$$

a nondifferentiable, convex *superpotential*

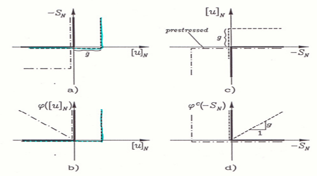
$$\pi(e) = \max\{\pi_1(e), \pi_2(e)\}$$
$$= \max\left\{s_2 + \frac{1}{2}c_1 e^2, -s_1 + \frac{1}{2}c_2 e^2\right\}$$



# Monotone Laws

CONTACT RELATIONS

$$-S_N \geq 0,\ [u]_N - g \leq 0,\ -S_N([u]_N - g) = 0,$$



$$-S_N \in \partial \mathbf{I}_{U_{ad}^N}([u]_N) = \mathcal{N}_{U_{ad}^N}([u]_N)$$

or $-S_N \in \partial\phi([u]_N)$
with

$$U_{ad}^N = \{[u]_N \mid [u]_N - g \leq 0\}$$

variational inequality

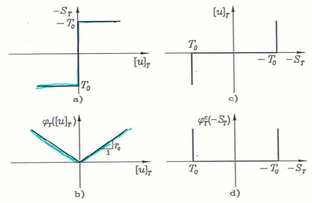
$$-S_N([u]_N)([u]_N^* - [u]_N) \leq 0,\ \forall [u]_N^* \in U_{ad}^N$$



# Monotone Laws

FRICTION PROBLEM

$$-S_T = \begin{cases} -T_0 & \text{if } [u]_T \leq 0 \\ [-T_0\ ,\ T_0] & \text{if } [u]_T = 0. \\ T_0 & \text{if } [u]_T \geq 0 \end{cases}$$



$$-S_T \in \partial\phi_T([u]_T).$$

variational inequality

$$-S_T([u]_T)([v]_T - [u]_T) \leq \phi_T([v]_T) - \phi_T([u]_T),\ \forall [u]_T \in \mathbb{R},$$

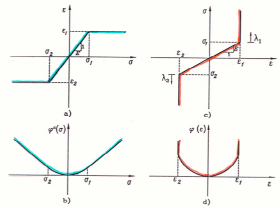


# Monotone Laws

*Elastic locking spring*
$\sigma \in \partial\phi(\varepsilon), \varepsilon \in \partial\phi^c(\sigma)$

$$\phi^c(\sigma) = \frac{1}{2}E^{-1}\sigma^2 - \frac{1}{2}E^{-1}(\sigma - \sigma_1)_+^2 - \frac{1}{2}E^{-1}(\sigma_2 - \sigma)_+^2.$$

$$\phi(\varepsilon) = \frac{1}{2}E\varepsilon^2 + \mathbf{I}_{[\varepsilon_2, \varepsilon_1]}(\varepsilon).$$



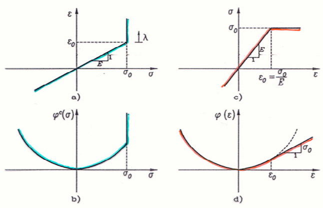


# Monotone Laws

*Elastic perfectly plastic spring* $\sigma \in \partial\phi(\varepsilon) = \nabla\phi(\varepsilon), \varepsilon \in \partial\phi^c(\sigma)$,

$$\phi^c(\sigma) = \frac{1}{2}E^{-1}\sigma^2 + \mathbf{I}_{(-\infty, \sigma_0]}(\sigma).$$

$$\phi(\varepsilon) = \frac{1}{2}E\varepsilon^2 - \frac{1}{2}E(\varepsilon - \varepsilon_0)_+^2.$$





# Conclusions

- Nonsmooth-nonconvex potentials in mechanics and thermomechanics allow us extend classical approaches
- Algorithms are expanded with the help of nonsmooth – nonconvex optimization
- Inverse and parameter identification problems can be solved by classical optimization or soft computing.