CONSTITUTIVE EQUATIONS FOR STRUCTURAL STEEL SUBJECTED TO FIRE - SOME REMARKS

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INTRODUCTION

Although identified on the basis of so-called instationary creep-tests the constitutive model of Eurocode 3 (EC3) represents a non-linear rate-independent relationship between stress and mechanical strain. As a consequence some important phenomena cannot properly be described: E.g. creep or relaxation at constant temperature, creep or relaxation at non-monotonic temperature rates or sensitivity of the transient creep process on the temperature rate.

1 CONSTITUTIVE EQUATION AND TESTING MACHINE AS AN OPERATOR

Based on the capabilities of a servocontrolled material testing system (s. Fig. 1) the material as well as the model is looked upon as an operator (Krempl, 1974). It maps a time-dependent *input* (loading function) into a corresponding time-dependent *output* (response function). The experimental input-output relationship corresponds to a mathematical operator (Onat, 1972), i.e.

$$\sigma(t) = \underset{0\le\tau\le t}{\mathfrak{F}}\left(\varepsilon(\tau),\theta(\tau)\right) \qquad (1)$$

Eq. (1) defines an operator, which assigns with every strain *input* $\varepsilon(\tau)$ and temperature *input* $\theta(\tau)$ on [0,t] a stress *output* $\sigma(t)$. It is a challenging task for the experimenter to identify the properties of this operator, which represents the material and is referred to as a *constitutive equation*. This equation is never obtained directly because in an experiment it is only possible to obtain a response function. Therefore, a constitutive equation must be constructed in such a way that it gives for a certain input the corresponding output like the tested material.

2 EC3 MODEL

The constitutive equation of EC3, i.e.

$$\sigma = f\left(\varepsilon^m, \theta\right) \qquad (2)$$

defines a one-dimensional non-linear algebraic relation between stress σ, infinitesimal mechanical strain ε_m, and temperature θ. It looks like a non-linear thermo-elastic constitutive model. Therefore by definition eq. (2) is rate-independent. Another point of view is achieved through the well known fact, that the experimental basis of eq. (2) are so-called instationary creep tests. It can be shown that eq. (2) is derived through the usage of response functions as constitutive equations. As a result the phenomenon of creep at constant temperature and the influence of the temperature rate on creep cannot be described.

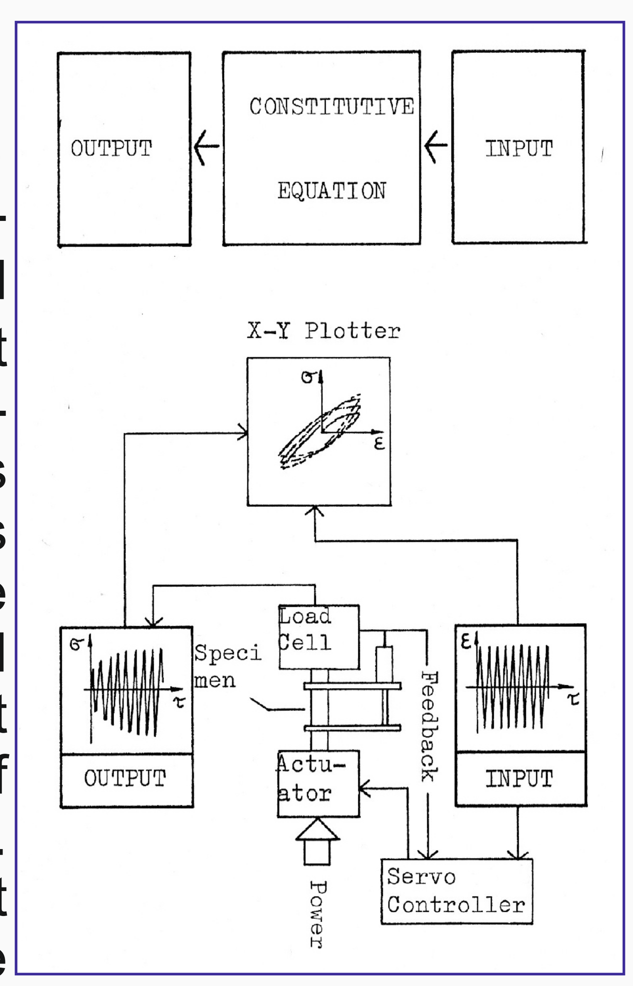

Fig. 1: Constitutive Equation and Material as an Operator (Korzen, 1987)

3 CONCEPT OF A CONSTITUTIVE MODEL

Our model is based on the very classical 3-parameter solid of visco-elasticity (Flügge, 1975). Only two main modifications are introduced: (i) the strain is replaced by the mechanical strain and (ii) the static stress is a rate-independent functional of the deformation variable:

$$\dot{\sigma} = -\frac{1}{\lambda\left(\theta,\sigma-\sigma^{stat}\right)}\left[\sigma-\sigma^{stat}\right]+E_0\left(\theta\right)\dot{\varepsilon}^{mech}, \sigma\left(0\right)=0 \qquad (3)$$

$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{m}, \sigma^{stat}\right)\dot{\varepsilon}^{mech}, \sigma^{stat}\left(0\right)=0 \qquad (4)$$

$$g\left(.\right) := E_e(\theta)\frac{\beta(\theta)E_e(\theta)-sign(\dot{\varepsilon}^{mech})\left[\sigma^{stat}-E_p(\theta)\varepsilon^{mech}\right]}{\beta(\theta)E_e(\theta)-\kappa(\theta)sign(\dot{\varepsilon}^{mech})\left[\sigma^{stat}-E_p(\theta)\varepsilon^{mech}\right]} \qquad (5)$$

$$\varepsilon^{mech} := \varepsilon-\alpha(\theta)\left[\theta-\theta_0\right] \qquad (6)$$

Eqs. (3) and (4) define a system of non-linear non-autonomous ordinary differential equations of type

$$\dot{\underline{y}} = \underline{h}\left(\underline{y}(t),t\right), \underline{y}(0)=\underline{y}_0 \qquad (7)$$

including initial conditions. Eq. (4) defines implicitly the aforementioned rate-independent functional for the static stress σ_{stat} (Valanis, 1980). Eqs. (3) and (4) belong to the class of so-called *unified theories* which do not separate plastic and creep strains, and represent inelastic deformation by a set of equations, employing a number of internal variables. In our case one internal variable is used and corresponds to σ_{stat}. Numerical simulations are realized with the software package RAUDAU5 (Hairer, 1996).

4 STRESS AND TEMPERATURE CONTROLLED INPUT FUNCTION - CREEP (s. Fig. 3, 4, 5 and 6)

$$\dot{\varepsilon}^{mech} = \frac{\sigma^{*}-\sigma^{stat}}{\lambda\left(\theta,\sigma^{*}-\sigma^{stat}\right)E_0\left(\theta\right)}, \varepsilon^{mech}\left(0\right)=0 \qquad (8)$$

$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{mech}, \sigma^{stat}\right)\dot{\varepsilon}^{mech}, \sigma^{stat}\left(0\right)=0 \qquad (9)$$

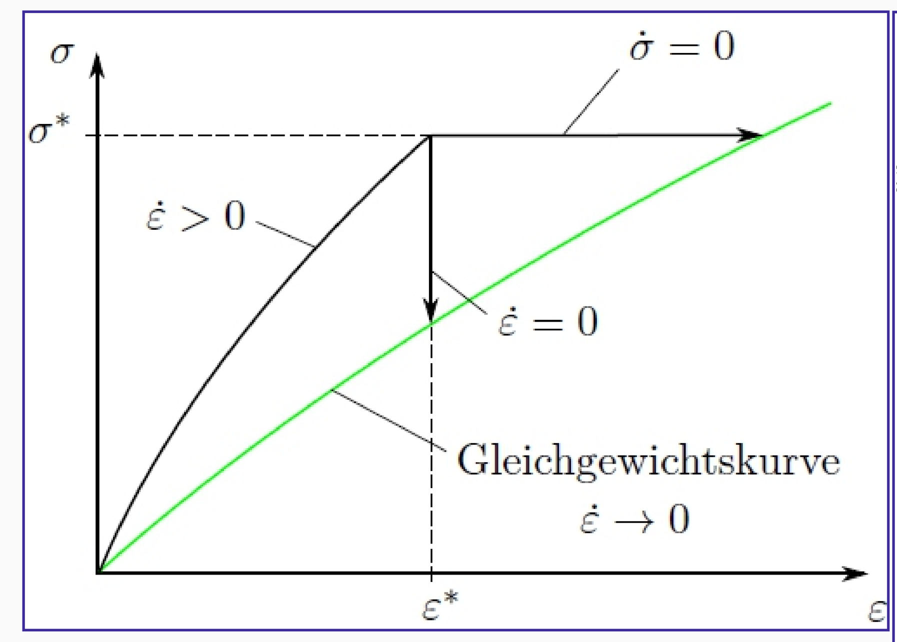

Fig. 3: Qualitative Material Response in Stress-Strain Diagram for Creep and Relaxation at Constant Temperature

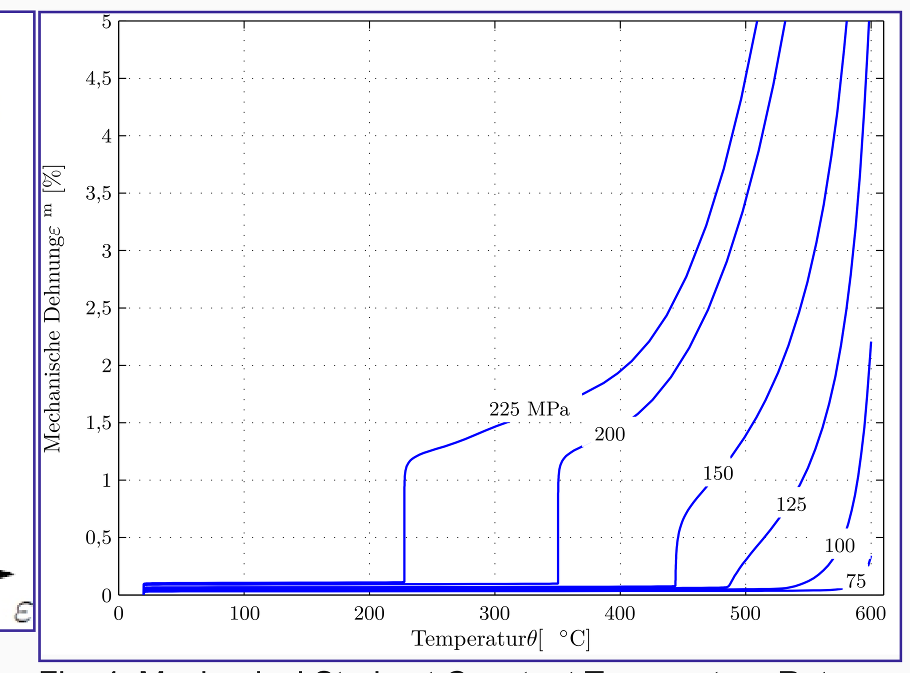

Fig. 4: Mechanical Strain at Constant Temperature Rate (7 K/min) at Different Constant Stress Levels

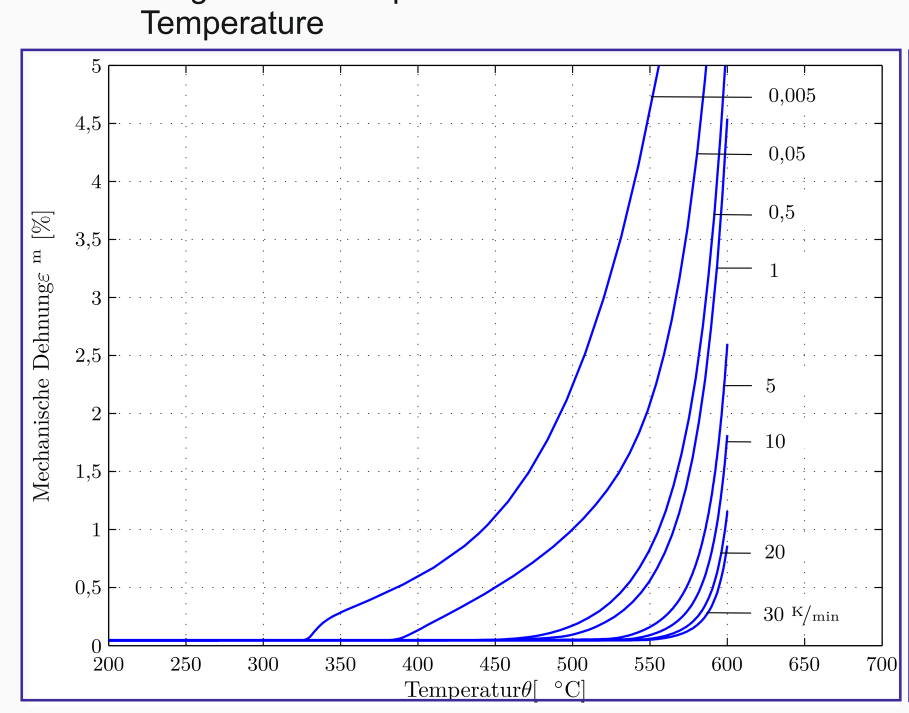

Fig. 5: Mechanical Strain at Constant Stress (100 MPa) at Different Temperature Rates

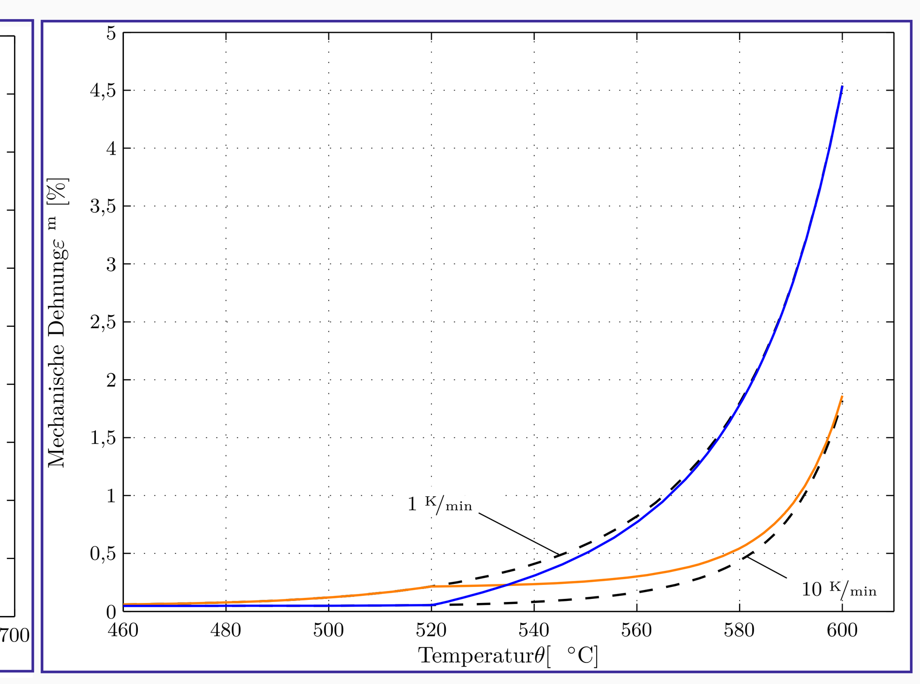

Fig. 6: Mechanical Strain at Constant Stress (100 MPa) at Different Piecewise Constant Temperature Rates

5 COMPARISON BETWEEN MODEL AND EXPERIMENT IN NON-MONOTONIC TRANSIENT CREEP TEST (s. Fig. 7)

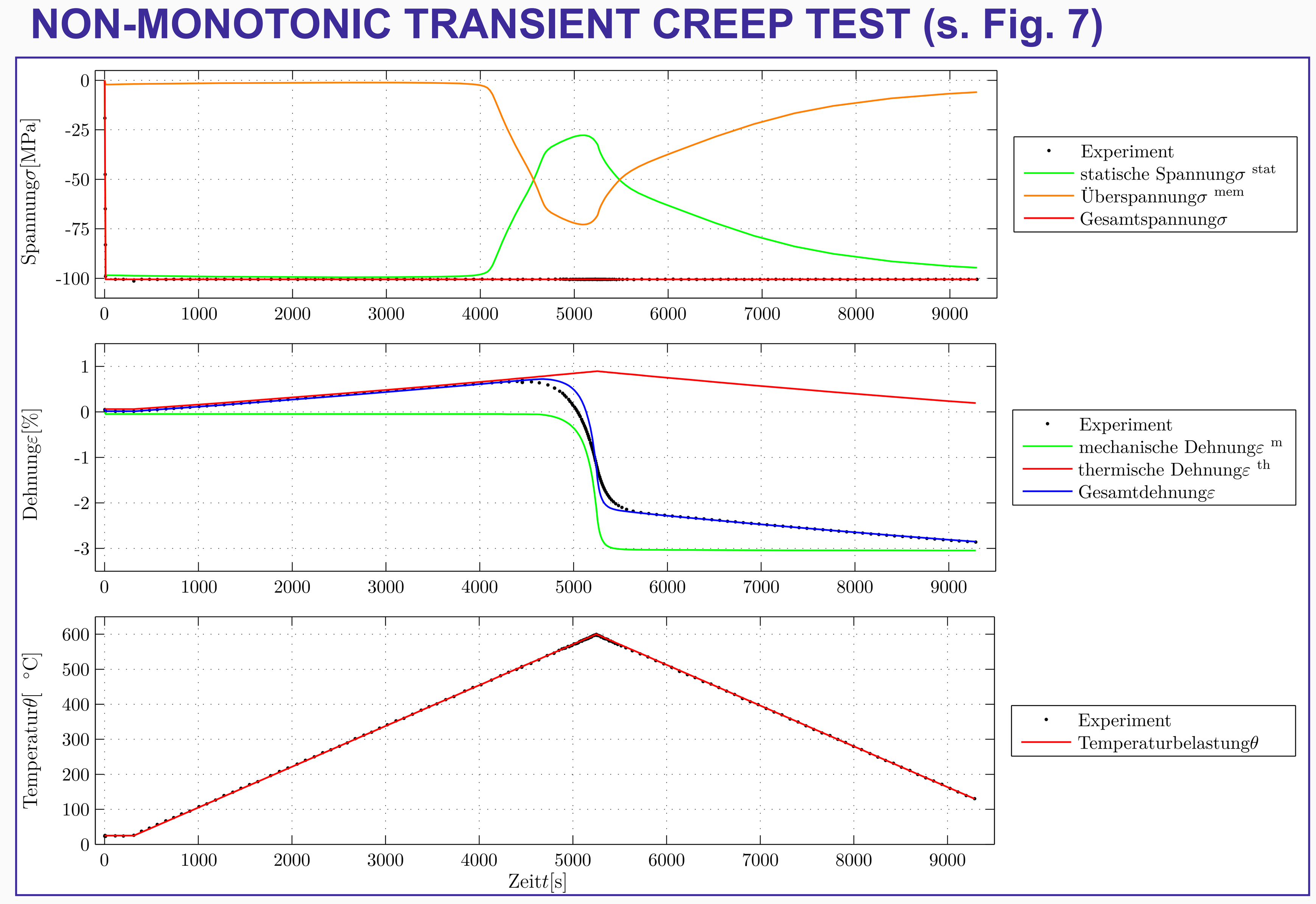

Fig. 7: Input (Temperature, Stress) and Output (Strain) of Non-Monotonic Transient Creep Test at Constant Stress (100 MPa) and Constant Absolute Value of Temperature Rate (7 K/min)

6 CONCLUSIONS

An alternative constitutive equation approach within the operator terminology is presented together with its basic capabilities. Finally the behaviour of the proposed model is compared with the results of a challenging experiment related to transient creep, and demonstrates a quite good correlation.

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