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# ELASTIC BUCKLING OF STEEL COLUMNS UNDER THERMAL GRADIENT

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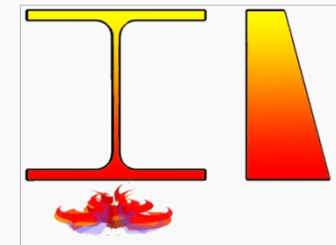
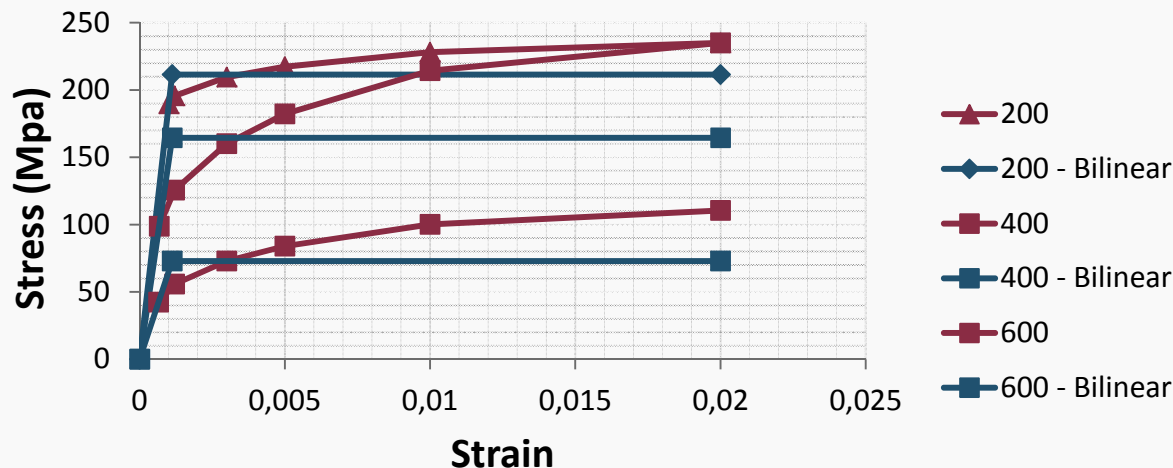
# [ Description ]

*Scope:* Analytical treatment + buckling curves

*Phenomena:* Thermal gradient - shift of the centroid  
Thermal bowing

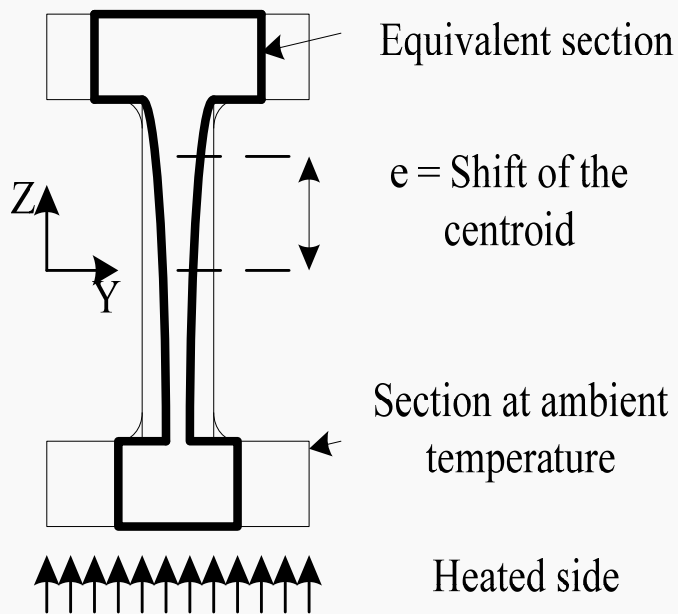
*Implementation:* IPE300 for several lengths  
Linear temperature distribution  
Bilinear material law

$$\frac{E_g}{E_{20}} = \frac{f_{y,g}}{f_{y,20}} = 2,347 \sin(0,5275T + 2,6) + 0,193 \sin(7,803T - 1,438)$$



[ Equivalent section ]

*Problem:* Arbitrary field of moduli  
*Scope:* Constant  $E_{20}$   
*Solution:* Scale the width of the section



The geometrical centroid of the equivalent section is:

$$z_g = \frac{\int E(\theta) z dA}{\int_A E(\theta) dA} \Rightarrow$$

$$z_g = \frac{\int_0^H \int_0^{B_{eq}} z dy dz}{\int_0^H \int_0^{B_{eq}} dy dz}$$



*Assumption:* No thermal expansion

*Equation:* Bending of a beam-column by couples  
Eccentrically applied axial force

*Differential equation:*  $P(e + w(x)) = -EI_{eq} w''(x)$

*Initial yield criterion:*  $\sigma_{y,\theta_{\max}} = k_{E,\max} P \left( \frac{1}{A_{eq}} + \frac{e}{I_{eq}} c \sec \left( \frac{l}{2} \sqrt{\frac{P}{EI_{eq}}} \right) \right)$

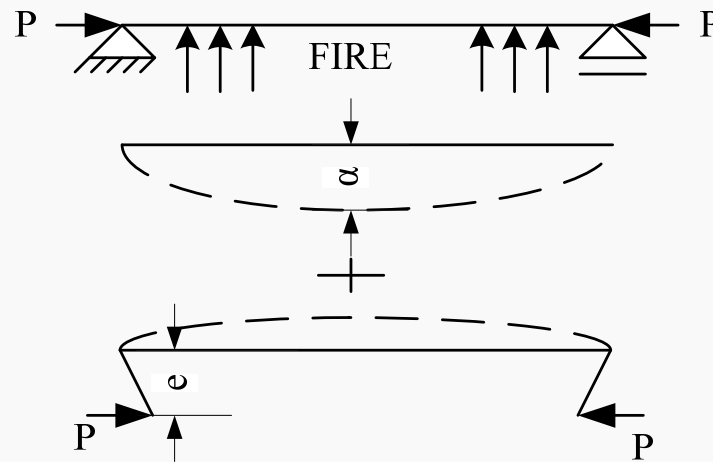
*'The eccentricity that arises from the shift of the centroid cannot be studied independently of thermal expansion effects'*

[ 2<sup>nd</sup> approach ]

*Coupled thermal gradient and thermal bowing effect*

*Assumption:* No residual stresses

*Equation:* Beam-column with initial curvature

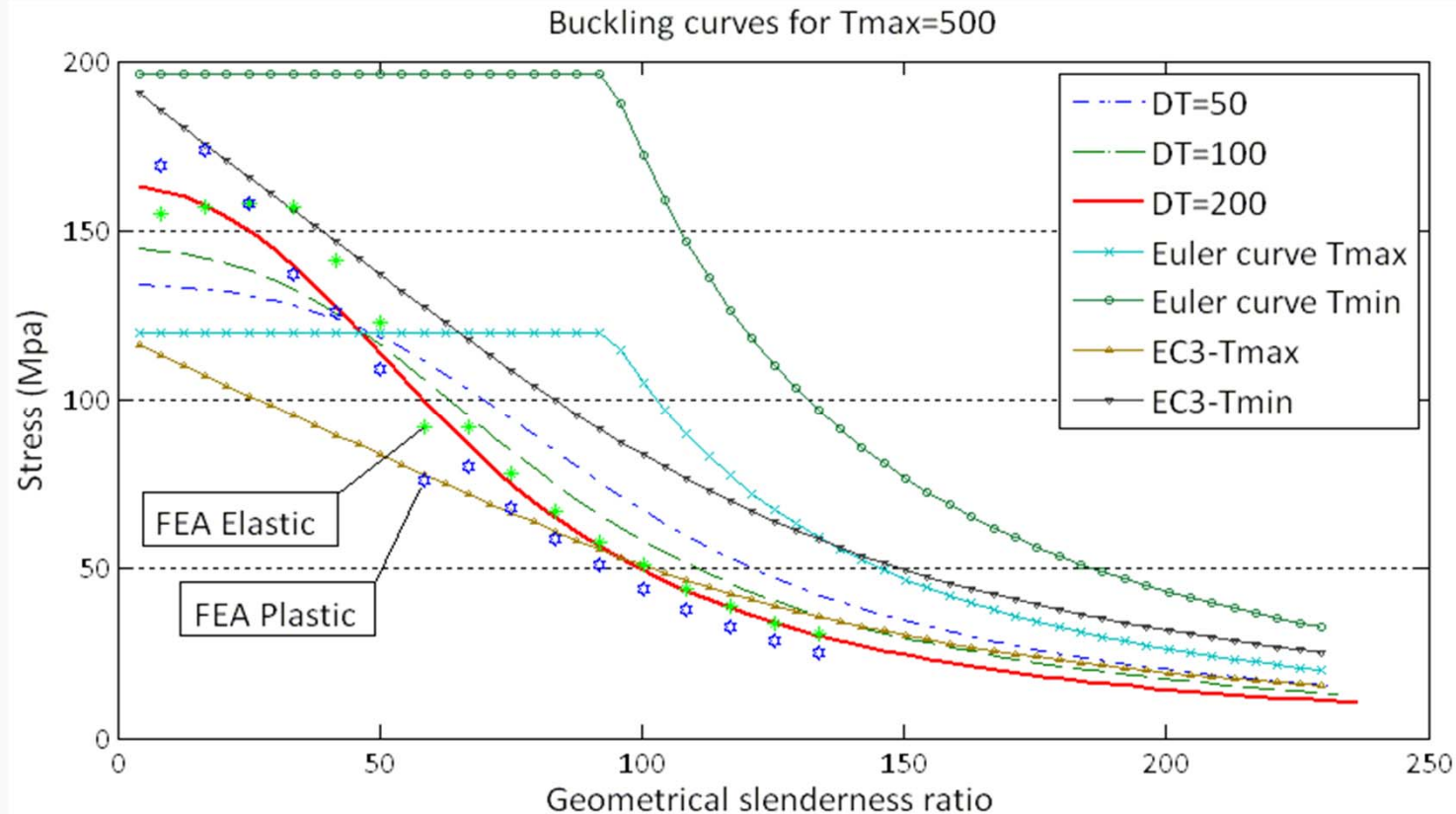


*Deflection:*  $w(x) = U_{bow} - U_{ecc} + e \Rightarrow$

$$w(x) = \frac{a_{\Delta\theta}}{1 - \frac{P}{P_{cr,eq}}} \sin\left(\frac{\pi x}{l}\right) - \frac{M_0}{2EI_{eq}} x(l-x) + e$$



# [ Conclusions ]



Good agreement between the analytical solution and the finite element analysis

The equation will be studied for various types of steel cross-sections and thermal cases

*...thank you*