Post-tensioned prestressed concrete bridge - assignment

Design a post-tensioned prestressed concrete bridge of a three-span arrangement. The construction is prestressed at the age of 7 days and put into operation at the age of 100 days. The durability is expected to be 100 years. The structure is loaded by the dead load $g_0 + g_{add}$ and live load $q$.

Individually assigned input parameters are:

- $L$ [m] length of the middle span
- $b$ [m] cross-sectional width
- $a$ [-] ratio of outer span to middle span
- $h$ [m] cross-sectional height
- $g_{add,k}$ [kN/m²] additional dead load except own weight – characteristic value
- $q_k$ [kN/m²] live load – characteristic value

Prestressing reinforcement: 7-wire tendons, diameter 15 mm;

$$f_{pk} = 1770 \text{ MPa}; f_{p0.1k} = 1560 \text{ MPa}$$

Tasks:

- cross-section geometry characteristics (area A; position of centre of gravity - cg; moment of inertia I)
- internal forces (N, V, M), extreme values of internal forces caused by live load
- number of tendons
- losses of prestress
- rigorous SLS assessment in decisive cross-sections (stress limit)
- ULS assessment in one of the decisive cross-section
- structural drawing, prestressing reinforcement drawing
Following model homework uses model input parameters. In your homework, use your own parameters obtained in the first lesson.

1. **Dimensions, cross-section geometry, material characteristics**
Define the geometry of the beam based on your own input parameters. The positions of the decisive cross-sections are marked as ⑤, ⑩ and ⑮.

![Diagram of beam with cross-sections](image)

Calculate following cross-sectional characteristics:

- **Area** \( A = 1.738 \, m^2 \)
- **Distance between cg and bottom surface** \( e_b = 0.782 \, m \)
- **Distance between cg and top surface** \( e_t = 0.518 \, m \)
- **Moment of inertia in vertical axis** \( I_y = 0.276 \, m^4 \)

Define the properties of used materials:

- **Concrete**: 30/37, \( f_{cd} = \frac{f_{ck}}{1.5} = 20 \, MPa \)
- **Prestressing steel**: 7-wire tendons, diameter 15 mm, area of one tendon \( A_{p1} = 150 \, mm^2 \)
  \( f_{pk} = 1770 \, MPa \) (ultimate stress)
  \( f_{p0.1k} = 1560 \, MPa \) (conventional yield stress)
2. Loading

<table>
<thead>
<tr>
<th></th>
<th>( g_t ) [kN.m(^{-1})]</th>
<th>( \gamma_f ) [-]</th>
<th>( g_d ) [kN.m(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of structure ( g_0 )</td>
<td>43.45</td>
<td>1.35</td>
<td>58.66</td>
</tr>
<tr>
<td>Other dead load ( g - g_0 )</td>
<td>7.56</td>
<td>1.35</td>
<td>10.21</td>
</tr>
<tr>
<td>Total</td>
<td>51.01</td>
<td>-</td>
<td>68.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( q_t ) [kN.m(^{-1})]</th>
<th>( \gamma_f ) [-]</th>
<th>( q_d ) [kN.m(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live load ( q )</td>
<td>16.5</td>
<td>1.5</td>
<td>24.75</td>
</tr>
<tr>
<td>Total</td>
<td>16.5</td>
<td>-</td>
<td>24.75</td>
</tr>
</tbody>
</table>

Loading cases listed below must be considered:

LC1 Own weight
LC2 Additional dead load
LC3 Live load I
LC4 Live load II
LC5 Live load III
LC6 Live load IV

3. Internal forces

Use any software for structural analysis to calculate internal forces on the structure (N, V, M). Calculate the internal forces in combinations listed in the table below.

Following table summarizes obtained internal forces in the decisive cross-sections (5, 10, 15) caused by loading cases and their combinations.

<table>
<thead>
<tr>
<th>Force, cross-section</th>
<th>( g_0 )</th>
<th>( g_{add,k} )</th>
<th>( g )</th>
<th>( q_{min} )</th>
<th>( q_{max} )</th>
<th>( g + q_{min} )</th>
<th>( g + q_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 5</td>
<td>617.74</td>
<td>109.6</td>
<td>727.34</td>
<td>-264.91</td>
<td>504.11</td>
<td>462.43</td>
<td>1231.45</td>
</tr>
<tr>
<td>M 10</td>
<td>-2773.09</td>
<td>-491.98</td>
<td>-3265.07</td>
<td>-1159.89</td>
<td>86.11</td>
<td>-4424.96</td>
<td>-3178.96</td>
</tr>
<tr>
<td>M 15</td>
<td>2020.78</td>
<td>358.52</td>
<td>2379.3</td>
<td>-190.74</td>
<td>973.21</td>
<td>2188.56</td>
<td>3352.51</td>
</tr>
<tr>
<td>Q 10, L</td>
<td>-537.57</td>
<td>-95.37</td>
<td>-632.94</td>
<td>-212.94</td>
<td>-</td>
<td>-845.88</td>
<td>-</td>
</tr>
<tr>
<td>Q 10, P</td>
<td>639.18</td>
<td>113.4</td>
<td>752.58</td>
<td>-</td>
<td>259.6</td>
<td>-</td>
<td>1012.18</td>
</tr>
</tbody>
</table>

3.1 Result combination

Prepare a result combination of maximal decisive values.
$M_{5,\text{max}} = 1231.45 \text{ kNm}$

$M_{10,\text{min}} = -4424.96 \text{ kNm}$

$M_{15,\text{max}} = 3352.51 \text{ kNm}$

(Notice: The position of the cross-section 5 is not in the centre of the outer span – it is placed at the extreme value of bending moment.)

### 3.2 Estimated secondary prestressing moment

As the structure is statically indefinite, secondary prestressing moments are created (moment diagram in the picture below). Firstly, its maximal value is estimated.

\[ \Delta M_p = (10 - 15\%) \cdot M_{10,\text{min}} \]

Chosen estimation: 10%

\[ \Delta M_p = 0.1 \cdot M_{10,\text{min}} = 0.1 \cdot 4424.96 \text{ kNm} = 442.5 \text{ kNm} \]

### 3.3 Moment values for prestress design

To obtain decisive moment values, both primary and secondary prestressing moment must be considered.

\[ M_5 = M_{5,\text{max}} + \Delta M_p \frac{5.4}{18} = 1231.45 \text{ kNm} + 442.5 \frac{5.4}{18} \text{ kNm} = 1364.05 \text{ kNm} \]

\[ M_{10} = M_{10,\text{min}} + \Delta M_p = -4424.96 \text{ kNm} + 442.5 \text{ kNm} = -3982.46 \text{ kNm} \]

\[ M_{15} = M_{15,\text{max}} + \Delta M_p = 3352.51 \text{ kNm} + 442.5 \text{ kNm} = 3795.01 \text{ kNm} \]
4. Tendon design (position, prestressing force)

4.1 Eccentricity of tendon in cross-section 10 and 15

The tendon will be placed at the position of maximal possible eccentricity \( e_{10} \) in cross-section 10. Maximal possible eccentricity \( e_5 \) or \( e_{15} \) will be also used in cross-section 5 or 15, depending on whichever of these two has a higher value of bending moment. In this model homework, \( M_{15} > M_5 \), so the maximal eccentricity is used for cross-section 15.

The maximal possible eccentricity is then given by the thickness of the cover layer \( c \) of concrete. For this model homework, cover layer is given 100 mm. Do not forget to consider the diameter of the tendon duct, which is preliminarily designed to be 100 mm.

Eccentricity of the tendon (cross-section 10):

\[
e_{P,10} = e_t - c - \frac{\phi_d}{2} = 0.518 - 0.1 - \frac{0.1}{2} = 0.368 \text{ m}
\]

Eccentricity of the tendon (cross-section 15):

\[
e_{P,15} = e_b - c - \frac{\phi_d}{2} = 0.782 - 0.1 - \frac{0.1}{2} = 0.632 \text{ m}
\]

(The eccentricity of tendon in cross-section 5 will be determined subsequently.)

4.2 Prestressing force design

The aim of prestressing is to reduce the tension in concrete: THE REQUIRED STRESS AT THE TENSIONED SIDE IS ZERO (= the sum of stress created by load and prestressing is zero.) – this requirement is a base for prestress design.

\[
\sigma_t^f + \sigma_t^p = 0 \text{ [MPa]} \quad \text{... for the cross-section 10 at the top of the beam}
\]

\[
\sigma_b^f + \sigma_b^p = 0 \text{ [MPa]} \quad \text{... for the cross-section 15 at the bottom of the beam}
\]

Cross-section 10:

\[
\sigma_t^f + \sigma_t^p = 0
\]

where \( \sigma_t^f \) is a stress at the top caused by loading

\[
\sigma_t^p \text{ is a stress at the top caused by prestressing}
\]

\[
\sigma_t^f = \frac{M_{10}}{I_y} \cdot e_t = \frac{3982.46}{0.276} \cdot 0.518 = 7.47 \text{ [MPa] (tension)}
\]
\[
\sigma_t^p = -\frac{N_{P,\infty}^{10}}{A} - \frac{N_{P,\infty}^{10} \sigma_{p,10}}{I_y} \cdot e_t
\]

Combining equations above, necessary prestressing force at the time of 100 years:

\[
N_{P,\infty}^{10} = \frac{\sigma_t^f}{1 - \sigma_{p,10} e_t} \cdot \frac{1}{I_y} [kN]
\]

\[
N_{P,\infty}^{10} = \frac{7.470 \times 10^6}{1.738 - 0.276} = 5900.28 [kN]
\]

**Cross-section 15**: (If \( M_{15} < M_5 \), continue with cross-section 5 instead of 15)

\[
\sigma_b^f + \sigma_b^p = 0
\]

where \( \sigma_b^f \) is a stress at the bottom caused by loading

\( \sigma_b^p \) is a stress at the bottom caused by prestressing

\[
\sigma_b^f = \frac{M_{15}}{I_y} \cdot e_b = \frac{3795.01}{0.276} \cdot 0.782 = 10.75 \text{ [MPa]} \text{ (tension)}
\]

\[
\sigma_b^p = -\frac{N_{P,\infty}^{15}}{A} - \frac{N_{P,\infty}^{15} \sigma_{p,15}}{I_y} \cdot e_b
\]

Combining equations above, necessary prestressing force at the time of 100 years:

\[
N_{P,\infty}^{15} = \frac{\sigma_b^f}{1 - \sigma_{p,15} e_t} \cdot \frac{1}{I_y} [kN]
\]

\[
N_{P,\infty}^{15} = \frac{10.750 \times 10^6}{1.738 - 0.276} = 4543.46 [kN]
\]

**The decisive value of prestressing force:**

There can be only one value of prestressing force in the tendon – it is necessary to choose the highest value for the design:

\[N_{P,\infty} = \max[N_{P,\infty}^{10}; N_{P,\infty}^{15}] = \max [5900.28; 4543.46] = 5900.28 [kN]\]

Due to losses of prestress, the force changes during the life of the structure. The prestressing force calculated above is designed for the age of 100 years of the construction.

**4.3 Eccentricity of tendon in cross-section 5**

To design the eccentricity in the cross-section 5, the ratio of primary moments will be used as a proportionality coefficient (This applies only for case \( M_{15} > M_5 \), otherwise the process is reverse):

\[
\frac{M_5}{M_{15}} = \frac{1231.45}{3352.51} = 0.37
\]

\[
e_5 = e_{15} \cdot \frac{M_5}{M_{15}} = 0.632 \cdot 0.37 = 0.234 \text{ m}
\]

(The position of the tendon peak in cross-section 5 is at the place of maximal bending moment, not in the mid-span).
Notice: If the extreme value of bending moment is higher in cross-section 5 than in cross-section 15, the prestressing force \( N_{P,\infty} \) should be determined based on the equation \( \sigma_f + \sigma_b = 0 \) [MPa] in cross-section 5 (not 15). Then the decisive value of prestressing force should be determined as: \( N_{P,\infty} = \max \{ N_{P,\infty}^5; N_{P,\infty}^{10} \} \).

4.4 Geometry of tendon position

Create points in the distance of 400 mm from the eccentricity of cross-section 10.

Split the interval between this point and the mid-span into halves.

Draw lines between created points.

Parabolic shape of tendon is approximated using circular arcs. The lines drawn in the previous step are tangent to circular arcs of the tendon.

Draw the upper (concave) circular arc from the cross-section 10 to the right side. The arc starts at the peak in the cross-section 10 and its length is up to the point placed 400 mm from the breaking point on the askew tangent. (In AutoCAD, use the circle described by two tangents and radius.)

The ending point of the arc drawn in previous step is also the starting point of the lower (convex) arc. It is also the contact point of the lower arc with its askew tangent. Draw the lower arc using its tangents. Fill the distance between the end of the lower arch and the mid-span with a straight line.

To draw the tendon shape to the left from the cross-section 10, use analogical procedure as in previous steps. The lowest point of the lower arch must be placed in the position of maximal bending moment \( M_5 \). Notice that the first 1500 mm of the tendon must be straight.
5. Preliminary prestressing design

Number of tendons:

Calculate the total cross-sectional area of all tendons $A_p$ necessary to carry the load:

$$N_{p,\infty} = \sigma_{p,\infty} \cdot A_p$$

$$A_p = \frac{N_{p,\infty}}{\sigma_{p,\infty}} [mm^2]$$

$$A_p = \frac{5900.28}{1053 \cdot 10^6} = 5603 [mm^2]$$

where $\sigma_{p,\infty} = \sigma_{p,max} \cdot (1 - 25\%) = 1404 \cdot 0.75 = 1053 [MPa]$.

The loss of prestress is estimated to be 25% at the end of the service life of the structure (100 years of age), the precise value will be determined in further calculation.

$$\sigma_{p,max} = \min[0.8 \cdot f_{pk}; 0.9 \cdot f_{p,0.1,k}] =$$

$$= \min[0.8 \cdot 1770; 0.9 \cdot 1560] =$$

$$= \min[1416; 1404] = 1404 [MPa]$$

Calculate the number of tendons into which the total cross-sectional area will be divided:

$$n = \frac{A_p}{A_{p,1}} = \frac{5603}{150} = 37.4 \rightarrow 38 \text{ tendons}$$

Two ducts of 19 tendons will be used. (The usual amount of tendons in a duct is 12, 15, 19 – choose from these three options.)
6. **Losses of prestress**

In previous step, the losses of prestress in the end of the service life were estimated to be 25% of the initial prestress. In further text, the losses of prestress are calculated precisely.

### 6.1 Immediate losses of prestress

#### 6.1.1 Loss to friction

There is a friction between the tendon and the tendon duct in the curved parts. Also, there is friction in the straight tendon parts caused by presence of the duct supports. The friction forces along the length of the tendon are in opposite direction to the prestressing force in the tendon, decreasing it. This is the cause of the loss of prestress.

\[
\Delta \sigma_{P,1} = -\sigma_{P,max} \cdot \left(1 - e^{-\mu(\alpha+k \cdot l)}\right)
\]

where \( \mu \)… friction coefficient in curved part

\( \alpha \)… central angle of circular arc in particular interval

\( k \)… friction coefficient in straight part (related to 1m of length)

\( l \)… length of particular interval

\( \mu = 0.19 \) (for metal duct)

\( k = 0.01 \ m^{-1} \)

**Angles:** (cross-section 5) \( \alpha = \alpha_1 = 0.06527 \ [rad] \)

(cross-section 10) \( \alpha = \alpha_1 + \alpha_2 + \alpha_3 = 0.06527 + 0.10097 + 0.10097 = 0.26721 \ [rad] \)

(cross-section 15) \( \alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0.06527 + 0.10097 + 0.10097 + 0.11835 + 0.11835 = 0.50391 \ [rad] \)

**Lengths:** (cross-section 5) \( l = l_1 + l_2 = 1.503 + 4.653 = 6.156 \ [m] \)

(cross-section 10) \( l = l_1 + l_2 + l_3 + l_4 + l_5 = 1.503 + 4.653 + 0.369 + 11.453 + 0.799 = 18.777 \ [m] \)

(cross-section 15) \( l = l_1 + l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 = 1.503 + 4.653 + 0.369 + 11.453 + 0.799 + 0.799 + 13.887 + 0.349 = 33.812 \ [m] \)

**Loss of prestress due to friction:**

\[
\Delta \sigma_{P,1,5} = -1404 \cdot \left(1 - e^{-0.19(0.06527+0.01\cdot6.156)}\right) = -33.43 \ [MPa]
\]

\[
\Delta \sigma_{P,1,10} = -1404 \cdot \left(1 - e^{-0.19(0.26721+0.01\cdot18.777)}\right) = -116.27 \ [MPa]
\]

\[
\Delta \sigma_{P,1,15} = -1404 \cdot \left(1 - e^{-0.19(0.50391+0.01\cdot33.812)}\right) = -207.57 \ [MPa]
\]

#### 6.1.2 Anchorage set loss

At the moment of anchoring, the anchor wedge slips into the anchor head, causing length reduction of the tendon, which leads to the loss of prestress. The slip is decreased by the friction – that means that the effect of slip decreases with the length of the tendon.

Initial slip: \( w = 0.005 \ m \)
Hooke’s law applies:

\[ \Delta \sigma = E_p \Delta \varepsilon = E_p \frac{w}{L} \]
\[ w = \frac{\Delta \sigma L}{E_p} = \frac{A_w}{E_p} \]
\[ A_w = w \cdot E_p = 0.005 \cdot 195 \, 000 = 975 \, [MPa] \]

where \( A_w \) is the area under the stress-length curve

changes of stress related to 1m of length in straight tendon:
\[ p_p = \sigma_{p,0,\text{max}} \cdot \left( 1 - e^{-\mu (\Delta \alpha + k \cdot l)} \right) = 1404 \cdot \left( 1 - e^{-0.19(0+0.01 \cdot 1)} \right) = 2.665 \, [MPa] \]

changes of stress related to 1m of length in curved part no. 1:
\[ p_1 = \sigma_{p,0,\text{max}} \cdot \left( 1 - e^{-\mu (\Delta \alpha_1 + k \cdot l)} \right) = 1404 \cdot \left( 1 - e^{-0.19(0.01403+0.01 \cdot 1)} \right) = 6.395 \, [MPa] \]
where: \( \Delta \alpha_1 = \frac{a_1}{l_1} = \frac{0.06527}{4.653} = 0.01403 \, [rad \cdot m^{-1}] \)

changes of stress related to 1m of length in curved part no. 2:
\[ p_2 = \sigma_{p,0,\text{max}} \cdot \left( 1 - e^{-\mu (\Delta \alpha_2 + k \cdot l)} \right) = 1404 \cdot \left( 1 - e^{-0.19(0.00882+0.01 \cdot 1)} \right) = 5.011 \, [MPa] \]
where: \( \Delta \alpha_2 = \frac{a_2}{l_1} = \frac{0.10097}{11.453} = 0.00882 \, [rad \cdot m^{-1}] \)

The effect of anchorage set loss is assumed to be eliminated in the curved part no. 2 of the tendon.

![Diagram of tendon stress-length curve](image)

Area under the stress-length curve:
(Notice: there is a straight part between curve no. 1 and curve no. 2 – make sure you omit that straight part, if you did not designed it in previous steps)

\[ A_w = 2 \cdot \left( \frac{1}{2} \cdot x_{P,1} \cdot p_p \cdot x_{P,1} + x_{P,1} \cdot (x_1 \cdot p_1 + x_{P,2} \cdot p_p + x_2 \cdot p_2) \right) + \text{straight part} \]
\[ + 2 \cdot \left( \frac{1}{2} \cdot x_1 \cdot p_1 \cdot x_1 + x_1 \cdot (x_{P,2} \cdot p_p + x_2 \cdot p_2) \right) + \text{curve no. 1} \]
\[
A_w = 2 \left( \frac{1}{2} \cdot 1.5 \cdot 2.655 \cdot 1.5 + 1.5 \cdot (4.65 \cdot 6.395 + 0.369 \cdot 2.665 + x_2 \cdot 5.011) \right) + \\
+2 \cdot \frac{1}{2} \cdot 4.65 \cdot 6.395 \cdot 4.65 + 4.65 \cdot (0.369 \cdot 2.665 + x_2 \cdot 5.011) + \\
+2 \cdot \frac{1}{2} \cdot 0.369 \cdot 0.369 \cdot 2.665 + 0.369 \cdot (x_2 \cdot 5.011) + \\
+2 \cdot \frac{1}{2} \cdot x_2 \cdot x_2 \cdot 5.011
\]

\[
A_w = 5.011x_2^2 + 65.333x_2 + 245.936 = 975
\]

The solution of \(x_2\) (only one reasonable):

\[
x_2 = 7.192 \, [m]
\]

Length of the tendon where the anchorage set loss is effective \(x_w\):

\[
x_w = x_{p,1} + x_1 + x_{p,2} + x_2 = 1.5 + 4.65 + 0.369 + 7.192 = 13.711 \, [m]
\]

The result shows that the effect of anchorage set loss will affect only the cross-section 5.

**Total anchorage set loss:**

\[
\Delta \sigma_{p,w} = -2 \cdot \left( x_{p,1} \cdot p + x_1 \cdot p_1 + x_{p,2} \cdot p + x_2 \cdot p_2 \right)
\]

\[
= -2(1.5 \cdot 2.665 + 4.65 \cdot 6.395 + 0.369 \cdot 2.665 + 7.192 \cdot 5.011)
\]

\[
= -141.51 \, [MPa]
\]

**Anchorage set loss in cross-section 5:**

\[
\Delta \sigma_{p,2,5} = - \left( 141.51 - 2 \cdot (x_{p,1} \cdot p + x_1 \cdot p_1) \right)
\]

\[
= -(141.51 - 2 \cdot (1.5 \cdot 2.665 + 4.65 \cdot 6.395)) = -74.04 \, [MPa]
\]

**Stresses in tendons after immediate losses – summary:**

\[
\bar{\sigma}_{p0} = \sigma_{p,0,max} + \Delta \sigma_{p,1} + \Delta \sigma_{p,2}
\]

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>(\sigma_{p,0,max})</th>
<th>(\Delta \sigma_{p,1})</th>
<th>(\Delta \sigma_{p,2})</th>
<th>(\bar{\sigma}_{p0})</th>
<th>(\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1404</td>
<td>-33.43</td>
<td>-74.04</td>
<td>1296.53</td>
<td>0.733</td>
</tr>
<tr>
<td>10</td>
<td>1404</td>
<td>-116.27</td>
<td>0</td>
<td>1287.73</td>
<td>0.728</td>
</tr>
<tr>
<td>15</td>
<td>1404</td>
<td>-207.57</td>
<td>0</td>
<td>1196.43</td>
<td>0.676</td>
</tr>
</tbody>
</table>

(Notice: \(\mu\) in this table does not stand for the friction coefficient)

\[
\mu = \frac{\bar{\sigma}_{p0}}{f_{pk}}, \text{ where } f_{pk} = 1770 \, MPa
\]
6.2 Long-term losses of prestress

6.2.1 Loss due to relaxation of prestressing reinforcement

Metal reinforcement bars have “the ability to relax” when elongated. This means, that the stress in the tendon decreases over time even though their elongation was not reduced.

\[ \Delta \sigma_{p,r} = -\bar{\sigma}_{p0} \cdot 0.66 \cdot 2.5 \cdot e^{0.1\mu} \cdot \left( \frac{t}{1000} \right)^{0.75(1-\mu)} \cdot 10^{-5} \]

where \( t \) is the time after prestressing [hours]

Loss of prestress at the time of 100 days:

**Cross-section 5**

\[ \Delta \sigma_{p,r,5}^{100} = -1296.53 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.733} \cdot \left( \frac{2400}{1000} \right)^{0.75(1-0.733)} \cdot 10^{-5} \]

\[ = -20.10 \text{ [MPa]} \]

**Cross-section 10**

\[ \Delta \sigma_{p,r,10}^{100} = -1287.73 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.728} \cdot \left( \frac{2400}{1000} \right)^{0.75(1-0.728)} \cdot 10^{-5} \]

\[ = -19.14 \text{ [MPa]} \]

**Cross-section 15**

\[ \Delta \sigma_{p,r,15}^{100} = -1196.43 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.676} \cdot \left( \frac{2400}{1000} \right)^{0.75(1-0.676)} \cdot 10^{-5} \]

\[ = -11.46 \text{ [MPa]} \]

Loss of prestress at the end of the service life (100 years):

Total relaxation \( \Delta \sigma_{pr}^{capacity} \) for \( t = 500\,000 \) h (approx. 57 years) – it is assumed that after this time the relaxation has no effect:

**Cross-section 5**

\[ \Delta \sigma_{p,r,5}^{\infty} = -1296.53 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.733} \cdot \left( \frac{500\,000}{1000} \right)^{0.75(1-0.733)} \cdot 10^{-5} \]

\[ = -58.56 \text{ [MPa]} \]

**Cross-section 10**

\[ \Delta \sigma_{p,r,10}^{\infty} = -1287.73 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.728} \cdot \left( \frac{500\,000}{1000} \right)^{0.75(1-0.728)} \cdot 10^{-5} \]

\[ = -56.89 \text{ [MPa]} \]

**Cross-section 15**

\[ \Delta \sigma_{p,r,15}^{\infty} = -1196.43 \cdot 0.66 \cdot 2.5 \cdot e^{0.10.676} \cdot \left( \frac{500\,000}{1000} \right)^{0.75(1-0.676)} \cdot 10^{-5} \]

\[ = -41.96 \text{ [MPa]} \]
6.2.2 Loss due to creep

Concrete as a material is slightly viscous. As a result, its deformation increases even though the loading conditions are constant. As the concrete creep, there is loss of prestress in the prestressing reinforcement.

**Force in the tendon after prestressing:**

\[
N_{P,0}^5 = \sigma_{P,0}^5 \cdot n \cdot A_p = 1296.53 \cdot 38 \cdot 150 = 7390.22 \text{ [kN]}
\]
\[
N_{P,0}^{10} = \sigma_{P,0}^{10} \cdot n \cdot A_p = 1287.73 \cdot 38 \cdot 150 = 7340.06 \text{ [kN]}
\]
\[
N_{P,0}^{15} = \sigma_{P,0}^{15} \cdot n \cdot A_p = 1196.43 \cdot 38 \cdot 150 = 6819.65 \text{ [kN]}
\]

**Bending moments caused by dead load and secondary prestressing moment:**

\[
M_g^5 = 727.34 + 132.75 = 860.09 \text{ [kNm]}
\]
\[
M_g^{10} = -3265.07 + 442.50 = -2822.57 \text{ [kNm]}
\]
\[
M_g^{15} = 2379.30 + 442.50 = 2821.8 \text{ [kNm]}
\]

**Elastic stress in concrete at the place of cg of the tendon:**

\[
\sigma_{cp}^{(g+p)} = -\frac{N_{p,0}}{A_i} \frac{N_{P,0} \cdot e_{p,i}}{I_{y,i}} \cdot e_{p,i} + \frac{M_g}{I_{y,i}} \cdot e_{p,i}
\]

\[
\sigma_{cp,5}^{(g+p)} = \frac{7.390}{1.757} - \frac{7.390 \cdot (0.25 - (0.782 - 0.78))}{0.277} + \frac{0.860}{0.277} = -0.50 \text{ [MPa]}
\]

\[
\sigma_{cp,10}^{(g+p)} = -\frac{7.340}{1.757} \frac{7.340 \cdot (0.368 - (0.786 - 0.782))}{0.278} = -3.98 \text{ [MPa]}
\]

\[
\sigma_{cp,15}^{(g+p)} = -\frac{6.820}{1.757} \frac{6.820 \cdot (0.5 - (0.782 - 0.777))}{0.280} = -4.86 \text{ [MPa]}
\]

**Loss of prestress due to creep:**

\[
\Delta \sigma_{pc} = -\frac{E_p}{E_c(\tau)} \cdot \sigma_{cp}^{g+p} \cdot \varphi(t; \tau)
\]

where \( \tau \)...time of prestressing \( \tau = 7 \text{ day} \)

\( t \)...time in the life of the beam \( t_1 = 100 \text{ days, } t_2 = 100 \text{ years} \)

The values of the Young’s modulus at the time of 7 days \( (E_c(\tau)) \) and the creep coefficient of the concrete \( (\varphi(t; \tau)) \) are usually determined using software. For the purpose of this homework, the values are given defaultly:
\[ E_c(7) = 21 \, 700 \, MPa \]
\[ \varphi(100 \, days) = 0.8 \]
\[ \varphi(100 \, years) = 2.8 \]

**Loss of prestress due to creep at the time of 100 days:**

\[
\Delta \sigma_{P,4,5}^{100} = - \frac{195000}{21748} \cdot 5.08 \cdot 0.83 = -37.81 \, [MPa]
\]
\[
\Delta \sigma_{P,4,10}^{100} = - \frac{195000}{21748} \cdot 3.98 \cdot 0.83 = -29.62 \, [MPa]
\]
\[
\Delta \sigma_{P,4,15}^{100} = - \frac{195000}{21748} \cdot 4.86 \cdot 0.83 = -36.17 \, [MPa]
\]

**Loss of prestress due to creep at the time of 100 years:**

\[
\Delta \sigma_{P,4,5}^{\infty} = - \frac{195000}{21748} \cdot 5.08 \cdot 2.81 = -127.99 \, [MPa]
\]
\[
\Delta \sigma_{P,4,10}^{\infty} = - \frac{195000}{21748} \cdot 3.98 \cdot 2.81 = -100.28 \, [MPa]
\]
\[
\Delta \sigma_{P,4,15}^{\infty} = - \frac{195000}{21748} \cdot 4.86 \cdot 2.81 = -122.45 \, [MPa]
\]

### 6.2.3 Loss due to shrinkage of concrete

Concrete shrinks over time, as the water leaves the matrix. As a result of shrinkage, there is a loss of prestress in the tendons.

\[
\Delta \sigma_{PS}(t) = -E_p \cdot (\varepsilon_c^S(t) - \varepsilon_c^S(\tau))
\]

The shrinkage coefficient is usually determined using software. For the purpose of this homework, the values are given defaultly:

\[
\varepsilon_c^S(\tau = 7\, days) = 8.22 \cdot 10^{-6}
\]
\[
\varepsilon_c^S(t = 100\, days) = 56.33 \cdot 10^{-6}
\]
\[
\varepsilon_c^S(t = 100\, years) = 439.51 \cdot 10^{-6}
\]

**Loss of prestress due to shrinkage at the time of 100 days:**

\[
\Delta \sigma_{P,5}^{100} = -E_p \cdot (\varepsilon_c^S(100) - \varepsilon_c^S(7)) = -195000 \cdot (56.33 \cdot 10^{-6} - 8.22 \cdot 10^{-6})
\]
\[
= -9.38 \, [kNm]
\]

**Loss of prestress due to shrinkage at the time of 100 years:**

\[
\Delta \sigma_{P,5}^{\infty} = -E_p \cdot (\varepsilon_c^S(36500) - \varepsilon_c^S(7)) = -195000 \cdot (439.51 \cdot 10^{-6} - 8.22 \cdot 10^{-6})
\]
\[
= -84.10 \, [kNm]
\]

Stresses in tendons after long-term losses – summary:

\[
\overline{\sigma_p} = \overline{\sigma_{p0}} + \Delta \sigma_{P,3} + \Delta \sigma_{P,4} + \Delta \sigma_{P,5}
\]
Summary of stresses in cross-sections 5, 10, 15 at different times of life:

<table>
<thead>
<tr>
<th></th>
<th>Stresses and forces</th>
<th>Cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Input stress $\sigma_{P,0,max}$ [Mpa]</td>
<td>1404.00</td>
<td>1404.00</td>
</tr>
<tr>
<td>Friction $\Delta \sigma_{P,1}$ [Mpa]</td>
<td>-33.43</td>
<td>-116.27</td>
</tr>
<tr>
<td>Anchorage set $\Delta \sigma_{P,2}$ [Mpa]</td>
<td>-74.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Stress after immediate losses $\sigma_{P,0}$ [Mpa]</td>
<td>1296.53</td>
<td>1287.73</td>
</tr>
<tr>
<td>Force after immediate losses $N_{P,0}$ [kN]</td>
<td>7390.22</td>
<td>7340.06</td>
</tr>
<tr>
<td>Relaxation $\Delta \sigma_{P,3}$ [Mpa]</td>
<td>-20.10</td>
<td>-19.14</td>
</tr>
<tr>
<td>Creep $\Delta \sigma_{P,4}$ [Mpa]</td>
<td>-37.81</td>
<td>-29.62</td>
</tr>
<tr>
<td>Stress at the time of 100 days $\sigma_{P,100}$ [Mpa]</td>
<td>1229.24</td>
<td>1229.59</td>
</tr>
<tr>
<td>Force at the time of 100 days $N_{P,100}$ [kN]</td>
<td>7006.67</td>
<td>7008.66</td>
</tr>
<tr>
<td>Relaxation $\Delta \sigma_{P,3}$ [Mpa]</td>
<td>-58.56</td>
<td>-56.89</td>
</tr>
<tr>
<td>Creep $\Delta \sigma_{P,4}$ [Mpa]</td>
<td>-127.99</td>
<td>-100.28</td>
</tr>
<tr>
<td>Shrinkage $\Delta \sigma_{P,5}$ [Mpa]</td>
<td>-84.10</td>
<td>-84.10</td>
</tr>
<tr>
<td>Stress at the end of service life $\sigma_{P,\infty}$ [Mpa]</td>
<td>1025.88</td>
<td>1046.46</td>
</tr>
<tr>
<td>Force at the end of service life $N_{P,\infty}$ [kN]</td>
<td>5847.52</td>
<td>5964.82</td>
</tr>
</tbody>
</table>