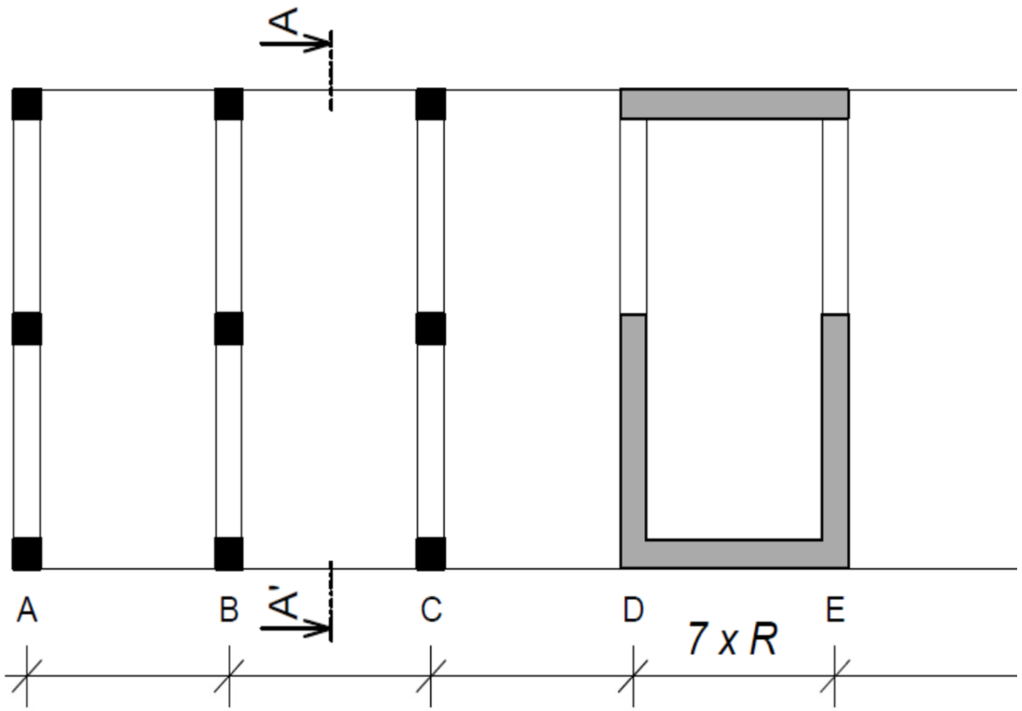
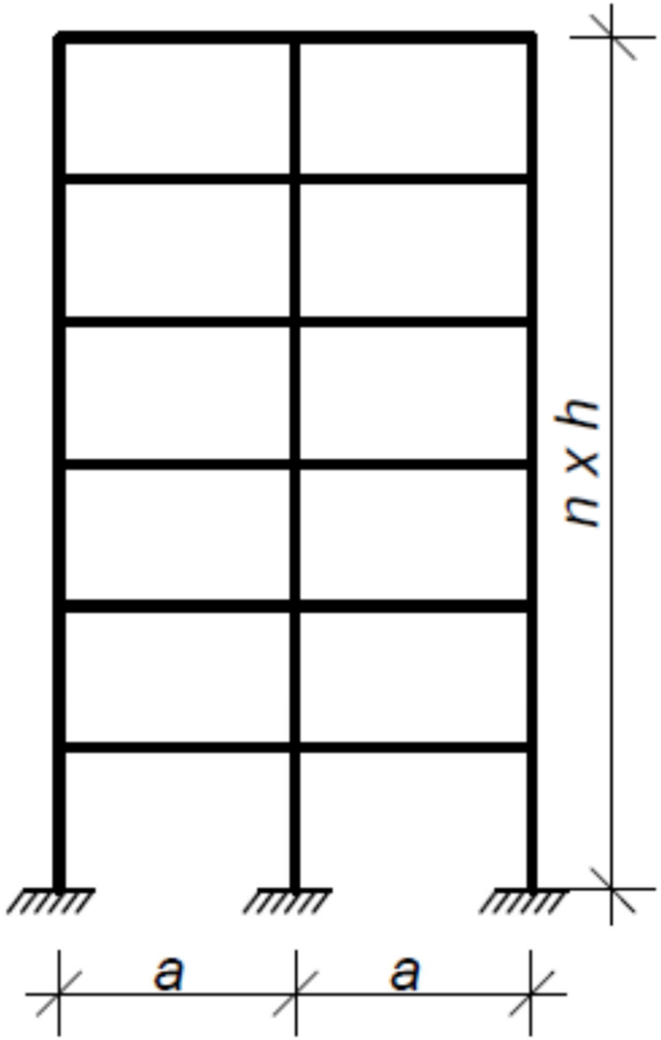
1st task:
Frame structure



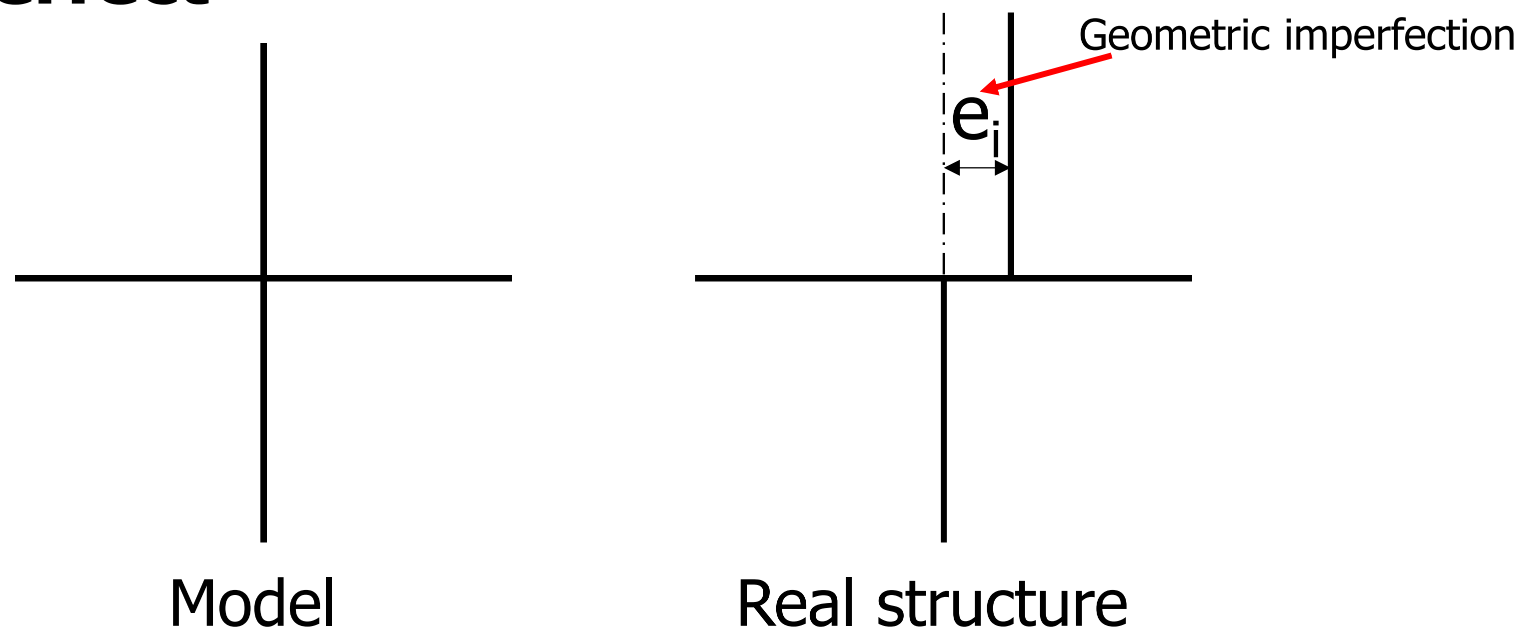


Design of reinforcement

- Design the reinforcement of the middle column in the 1st floor
- Geometric imperfections of column
- Slenderness of the column
- Reinforcement of the column

Geometric imperfections

- We calculated moments on ideal model of frame structure, but real structures are **not perfect**



- Geometric imperfections cause **additional bending moments**

Geometric imperfections

- Geometric imperfection

$$e_i = \theta_0 \cdot \alpha_h \cdot \alpha_m \cdot \frac{l_0}{2}$$

Basic value of imperfection, 1/200

Reduction factor for height

Reduction factor for number of members

Effective length of the column, in our case $l_0 = 0{,}8h$

$$\alpha_h = \frac{2}{\sqrt{h}}$$

$$\frac{2}{3} \le \alpha_h \le 1{,}0$$

$$\alpha_m = \sqrt{0{,}5 \cdot \left(1 + \frac{1}{m}\right)}$$

Clear length of the column

Number of columns in one frame, m = 3

- Additional moment due to geom. imperfection

$$M_{\mathrm{imp}} = N_{\mathrm{Ed}} e_{\mathrm{i}}$$

Normal force in given cross-section

Geometric imperfections

- Calculate bending moments with the influence of geometric imperfections $M_{Ed,I}$ in the head and foot of the column for both combinations
- Use table to calculate the values

	M [kNm]	Head of the column	Foot of the column
	$\|M_{impl}\|$	8,0	8,3
COMB1	M_{Ed}	-17,0	15,0
	$M_{Ed,I}$	-25,0	23,3
COMB2	M_{Ed}	-12,0	23,0
	$M_{Ed,I}$	-20,0	31,3

- We will use these values to check the load-bearing capacity (in the interaction diagram)

Slenderness of the column

- Slenderness of the column

$$\lambda = \frac{l_0}{i}$$

$$i = \sqrt{\frac{I}{A_c}}$$

Radius of gyration (i)

Cross-sectional area of the column (A_c)

$$I = \frac{1}{12} b_{col} h_{col}^3$$

Moment of inertia (I)

Dimensions of the **cross-section** of the column (b_{col}, h_{col})

- Limiting slenderness

$$\lambda_{lim} = \frac{20ABC}{\sqrt{n}} \leq 75$$

Effect of creep, A = 0.7

Effect of reinforcement ratio, B = 1.1

Effect of bending moments (C)

You can´t consider λ_{lim} to be more than 75

Relative normal force

$$n = \frac{N_{Ed}}{A_c f_{cd}}$$

Slenderness of the column

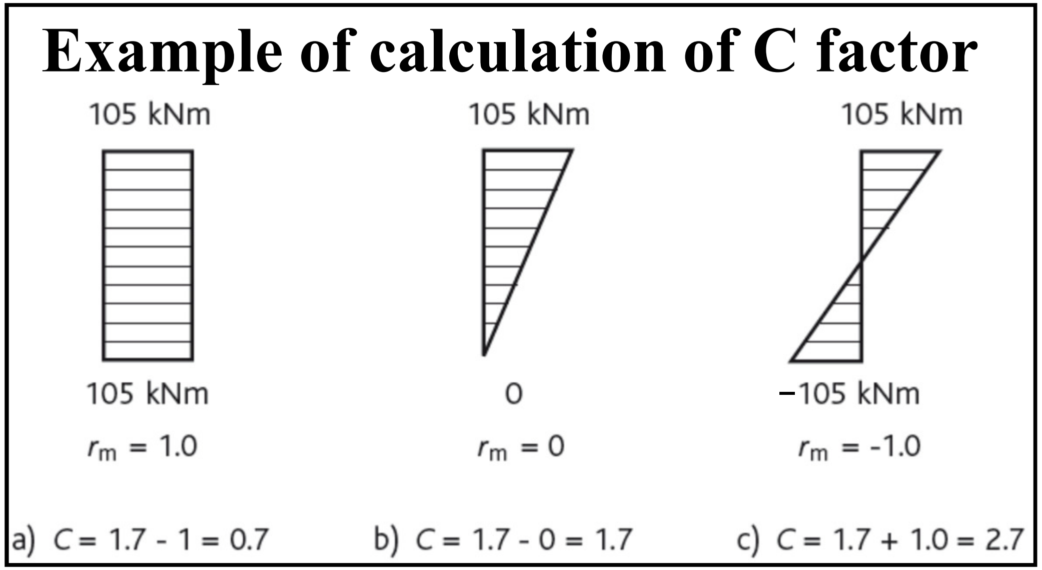
- Effect of bending moments

$$C = 1,7 - r_m$$

$$r_m = \frac{M_{01}}{M_{02}}$$

M_{01} and M_{02} are bending moments in the head and foot of the column,
$|M_{02}| > |M_{01}|$

Compare absolute values, but assign values WITH the sign into the equation for C



- Calculate λ_{lim} for all combinations and use the worst case (lowest λ_{lim})

Slenderness of the column

- BUT: If the bending moments are caused predominantly by the imperfections, we should always take C = 0.7
- This is the case of our COMB1 (there are NO moments from internal forces, just the moments caused by imperfections) => Take **C = 0.7**

- If $\lambda \leq \lambda_{lim}$, the column is robust
- If $\lambda > \lambda_{lim}$, the column is slender

Design of reinforcement

- *1st method*: Estimation with the presumption of uniformly distributed compression over the whole cross-section

Stress in reinforcement

$$A_{s,req,1} = \frac{N_{Ed} - 0,8 A_c f_{cd}}{\sigma_s}$$

$$\sigma_s = 400 \text{ MPa if } f_{yd} \geq 400 \text{ MPa}$$

$$\sigma_s = f_{yd} \text{ if } f_{yd} < 400 \text{ MPa}$$

- If the equation gives $A_{s,req,1} < 0$, minimum reinforcement 4ø 12 mm should be designed

Design of reinforcement

- *2nd method*: Chart for design of symmetrical reinforcement

$$\mu = \frac{M_{\mathrm{Ed,I}}}{b_{\mathrm{col}} h_{\mathrm{col}}^2 f_{\mathrm{cd}}} \qquad \nu = \frac{N_{\mathrm{Ed}}}{b_{\mathrm{col}} h_{\mathrm{col}} f_{\mathrm{cd}}} \qquad \xrightarrow{\text{chart}} \qquad \omega$$

Relative bending moment (μ)

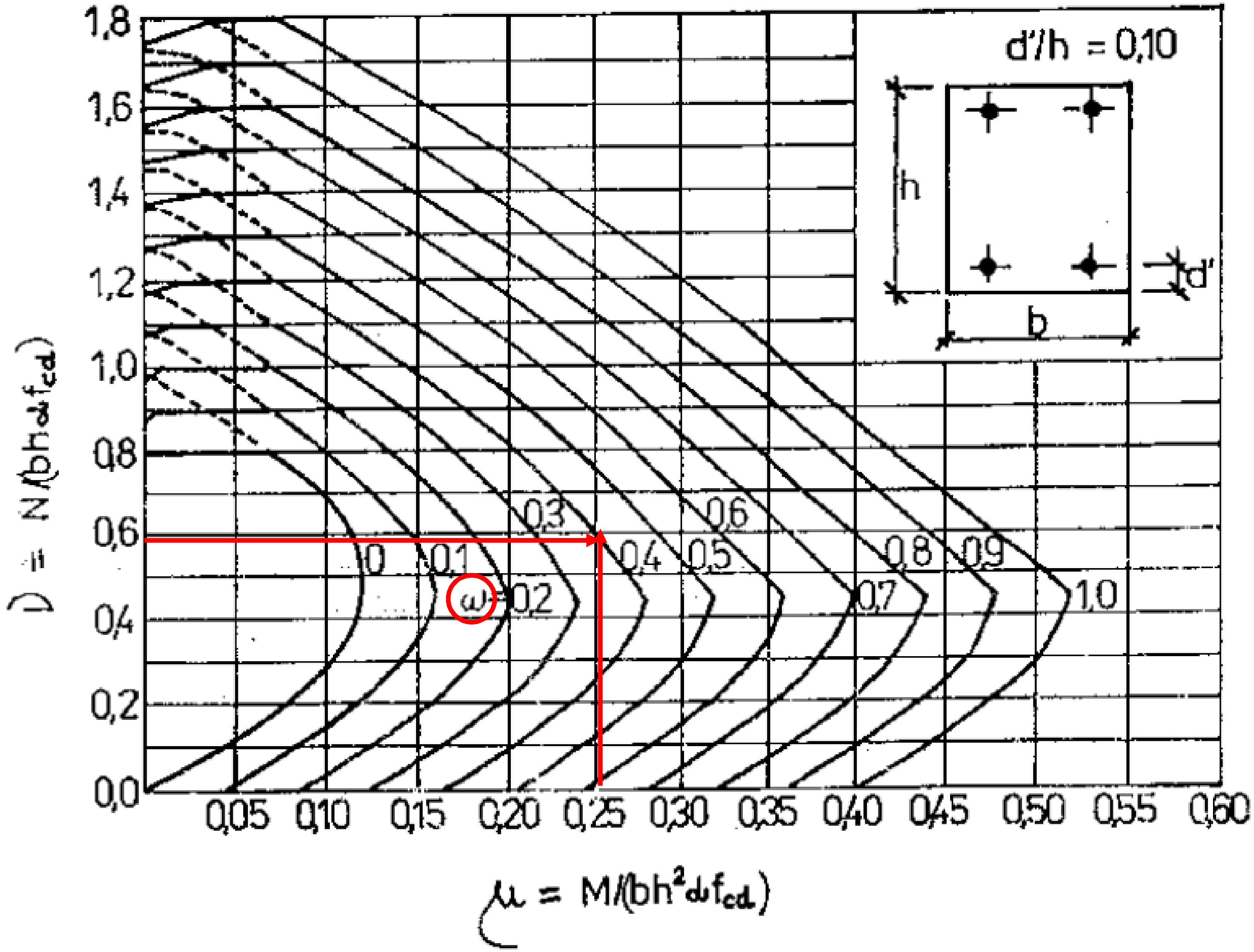
Relative normal force (ν)

Relative exploitation of concrete part of the cross-section (ω)

$$\rightarrow A_{\mathrm{s,req,2}} = \frac{\omega A_{\mathrm{c}} f_{\mathrm{cd}}}{f_{\mathrm{yd}}}$$

- Try both combinations and use higher value of $A_{s,req}$

Design of reinforcement



Design of reinforcement

- Final design:

$$A_{s,req} = \max\left(A_{s,req,1}; A_{s,req,2}\right) \rightarrow A_{s,prov} \geq A_{s,req}$$

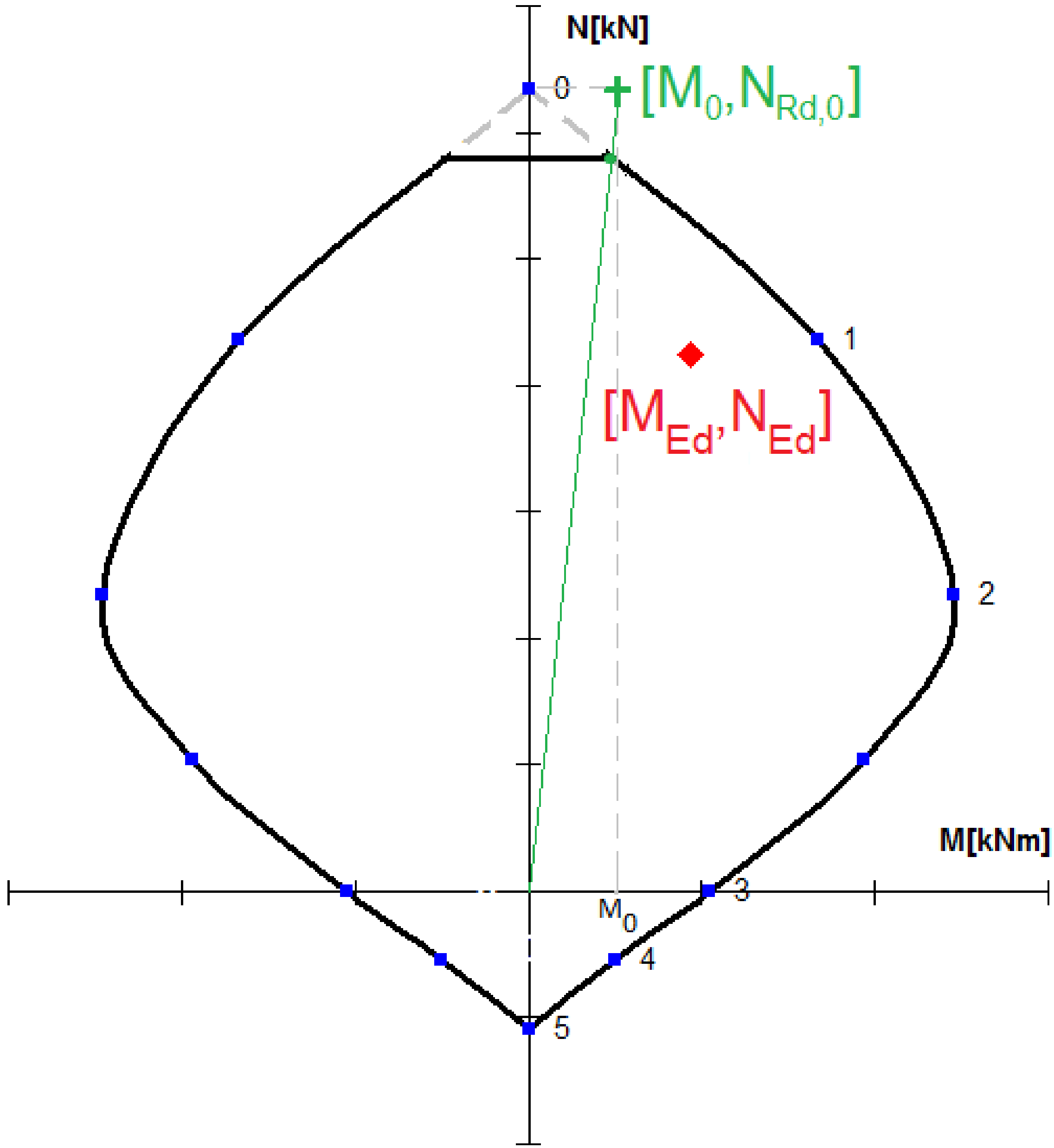
For example Design: 4x Ø16 ($A_{s,prov}$ = 804 mm^2)

- Check detailing rules

$$A_{s,prov} \geq A_{s,min} = \max\left(0.1\frac{N_{Ed}}{f_{yd}}; 0.002A_c\right)$$

$$A_{s,prov} \leq A_{s,max} = 0,04A_c$$

Check of column – interaction diagram

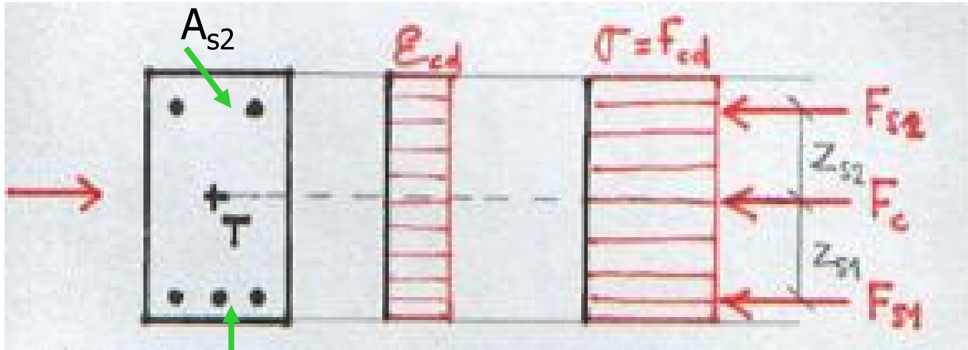


Check of column – interaction diagram

- Calculate main points of interaction diagram
- Connect them by lines (simplification)
- Calculate minimum bending moment M_0
- Restrict axial resistance
- If your column is **slender**, **increase bending moments** by approximately 30 % (simplification)
- If COMB1 and COMB2 lay inside the curve, column is checked
- If not, we will adjust the design (DO NOT recalculate the ID)
- **See the example on my website**

ID – point 0

- Maximum normal force resistance (pure compression)



$$N_{Rd,0} = F_c + F_{s1} + F_{s2} = b_{col} h_{col} f_{cd} + A_{s1}\sigma_s + A_{s2}\sigma_s$$

$$M_{Rd,0} = F_{s2} z_{s2} - F_{s1} z_{s1} = \left(A_{s2} z_{s2} - A_{s1} z_{s1}\right)\sigma_s$$

See design of reinforcement

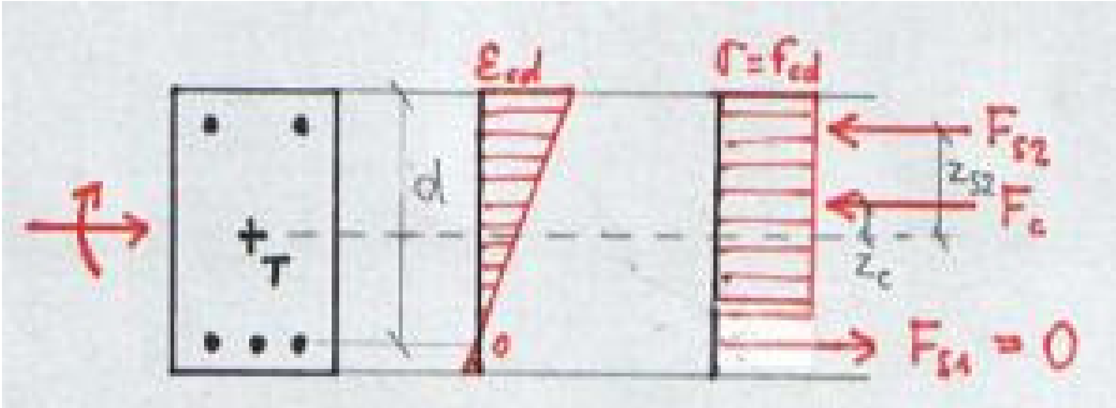
FOR ALL POINTS OF ID:

- To calculate normal force capacity – sum the internal forces
- To calculate bending moment capacity – sum the moments of these forces

- In our case, for all points $A_{s1} = A_{s2}$ and $z_{s1} = z_{s2}$ because we have symmetrical reinforcement

ID – point 1

- Whole cross-section is compressed (strain in tensile reinforcement is 0)



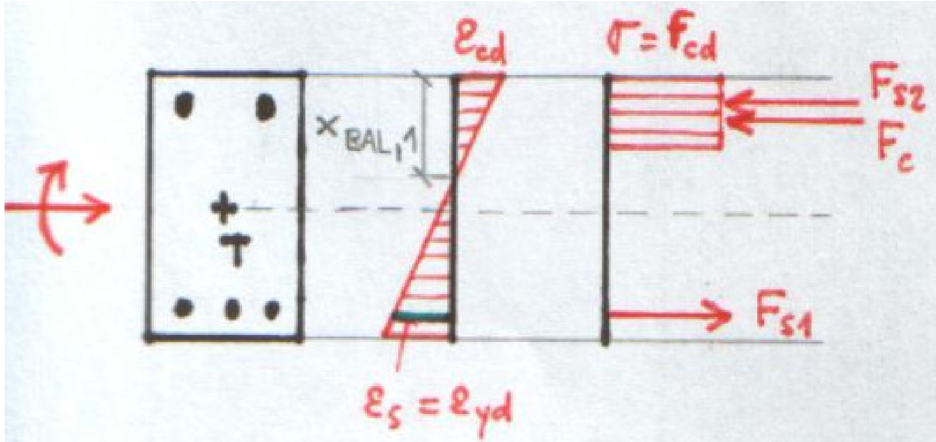
$$N_{\mathrm{Rd},1} = F_{\mathrm{c}} + F_{\mathrm{c2}} = 0.8 b_{\mathrm{col}} d f_{\mathrm{cd}} + A_{\mathrm{s2}} f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd},1} = F_{\mathrm{c}} z_{\mathrm{c}} + F_{\mathrm{s2}} z_{\mathrm{s2}} = 0.8 b_{\mathrm{col}} d f_{\mathrm{cd}} \left(\frac{h}{2} - 0.4d \right) + A_{\mathrm{s2}} f_{\mathrm{yd}} z_{\mathrm{s2}}$$

Factor expressing the difference between real and idealized stress distribution, see 3rd HW

ID – point 2

- Maximum bending moment resistance (stress in tensile reinforcement σ_{s1} = f_{yd}; that means x = $x_{bal,1}$)



$$N_{Rd,2} = F_c + F_{s2} - F_{s1} = 0.8 b_{col} x_{bal,1} f_{cd} + A_{s2}\sigma_{s2} - A_{s1} f_{yd}$$

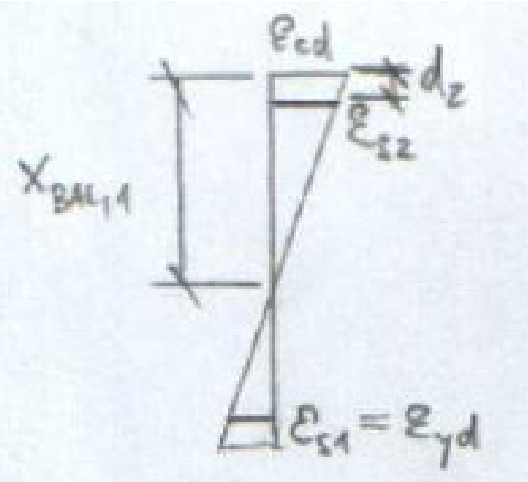
$$M_{Rd,2} = F_c z_c + F_{s2} z_{s2} + F_{s1} z_{s1} = 0.8 b_{col} x_{bal,1} f_{cd}\left(\frac{h}{2} - 0.4 x_{bal,1}\right) + A_{s2}\sigma_{s2} z_{s2} + A_{s1} f_{yd} z_{s1}$$

$$x_{bal,1} = \xi_{bal,1} d = \frac{700}{700 + f_{yd}} d$$

?

ID – point 2

- How to find σ_{s2} (stress in compressed reinforcement) ?



Distance from surface of the column to the centroid of compressed reinforcement

$$\varepsilon_{s2} = \varepsilon_{cd}\left(1 - \frac{d_2}{x_{bal,1}}\right)$$

Limit strain of concrete, ε_{cd} = 0.0035

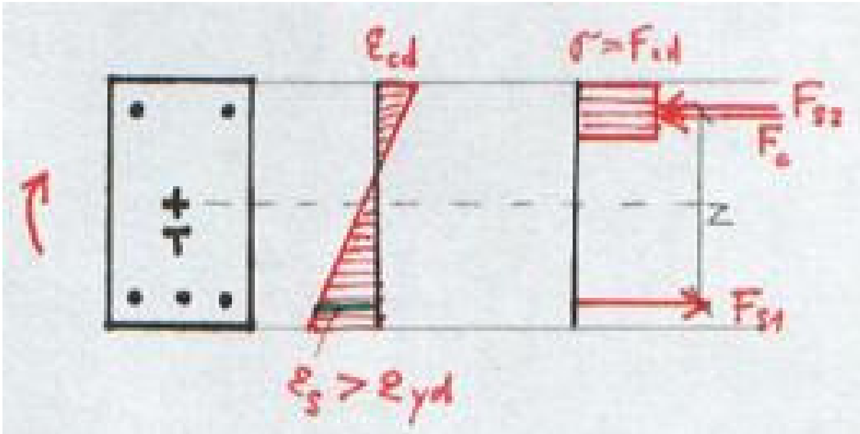
$$\text{If } \varepsilon_{s2} \geq \varepsilon_{yd} = \frac{f_{yd}}{E_s} \rightarrow \sigma_{s2} = f_{yd}$$

$$\text{else } \sigma_{s2} = \varepsilon_{s2} E_s$$

Elastic modulus of steel, 210000 MPa

ID – point 3

- Pure bending



$$N_{Rd,3} = F_c + F_{s2} - F_{s1} = 0$$

$$M_{Rd,3} = F_c z_c + F_{s2} z_{s2} + F_{s1} z_{s1} = 0.8 b_{col} x f_{cd} \left(\frac{h}{2} - 0.4x \right) + A_{s2} \sigma_{s2} z_{s2} + A_{s1} f_{yd} z_{s1}$$

We have two unknowns: height of compressed part of concrete cross section (x) and stress in compressed reinforcement (σ_{s2})

ID – point 3

- To find the value of σ_{s2}, we can derive quadratic equation:

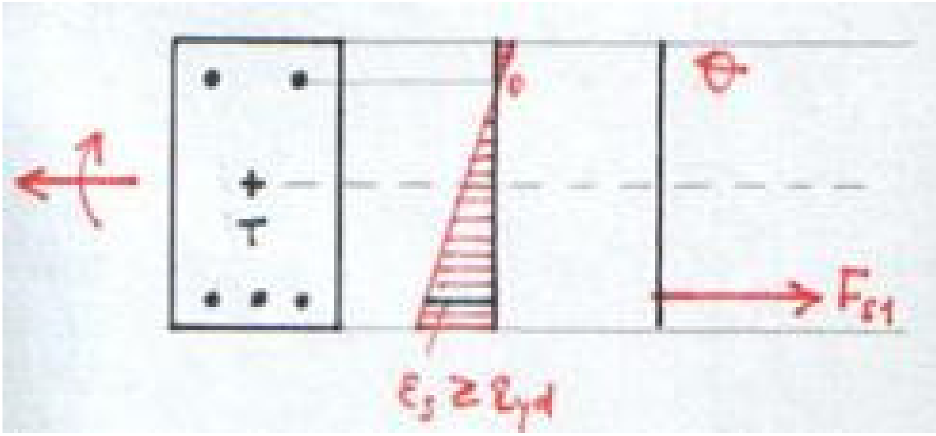
$$\sigma_{s2}^2 A_{s2} - \sigma_{s2}\left(A_{s1} f_{yd} + A_{s2}\varepsilon_{cd} E_s\right) + \varepsilon_{cd} E_s\left(A_{s1} f_{yd} - 0.8 b_{col} f_{cd} d_2\right) = 0$$

- By solving this equation, we will receive 2 roots
- Only one of them will „make sense" – we will use that one to calculate x:

$$x = \frac{A_{s1} f_{yd} - A_{s2}\sigma_{s2}}{0.8 b_{col} f_{cd}}$$

ID – point 4

- Whole cross-section is in tension (strain in compressed reinforcement is 0)

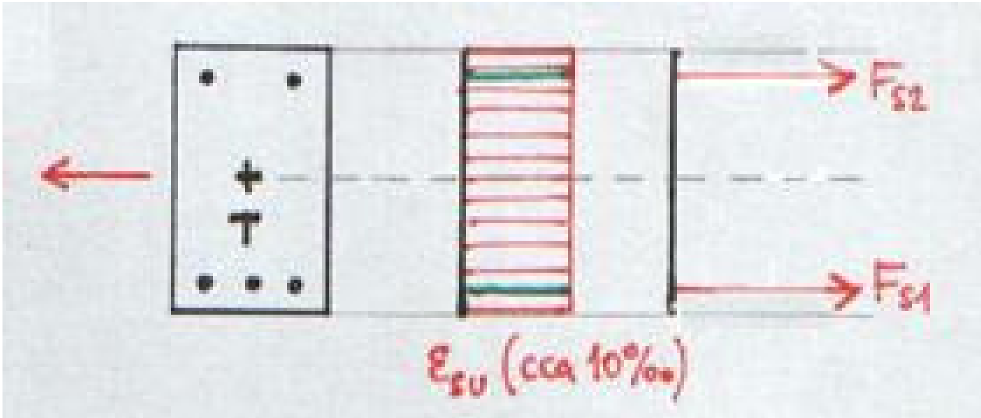


$$N_{\mathrm{Rd},4} = F_{\mathrm{s1}} = A_{\mathrm{s1}} f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd},4} = F_{\mathrm{s1}} z_{\mathrm{s1}} = A_{\mathrm{s1}} f_{\mathrm{yd}} z_{\mathrm{s1}}$$

ID – point 5

- Pure tension



$$N_{\mathrm{Rd},5} = F_{\mathrm{s1}} + F_{\mathrm{s2}} = \left(A_{\mathrm{s1}} + A_{\mathrm{s2}}\right) f_{\mathrm{yd}}$$

$$M_{\mathrm{Rd},5} = F_{\mathrm{s1}} z_{\mathrm{s1}} - F_{\mathrm{s2}} z_{\mathrm{s2}} = \left(A_{\mathrm{s1}} z_{\mathrm{s1}} - A_{\mathrm{s2}} z_{\mathrm{s2}}\right) f_{\mathrm{yd}}$$

ID – moment M_0

- Always consider minimum eccentricity

$$e_0 = \max\left(\frac{h_{\text{col}}}{30}; 20\ \text{mm}\right)$$

- Minimum bending moment

$$M_0 = N_{\text{Rd},0} e_0$$

- Restriction of ID – pure compression can never occur, minimum bending moment always has to be taken into account