

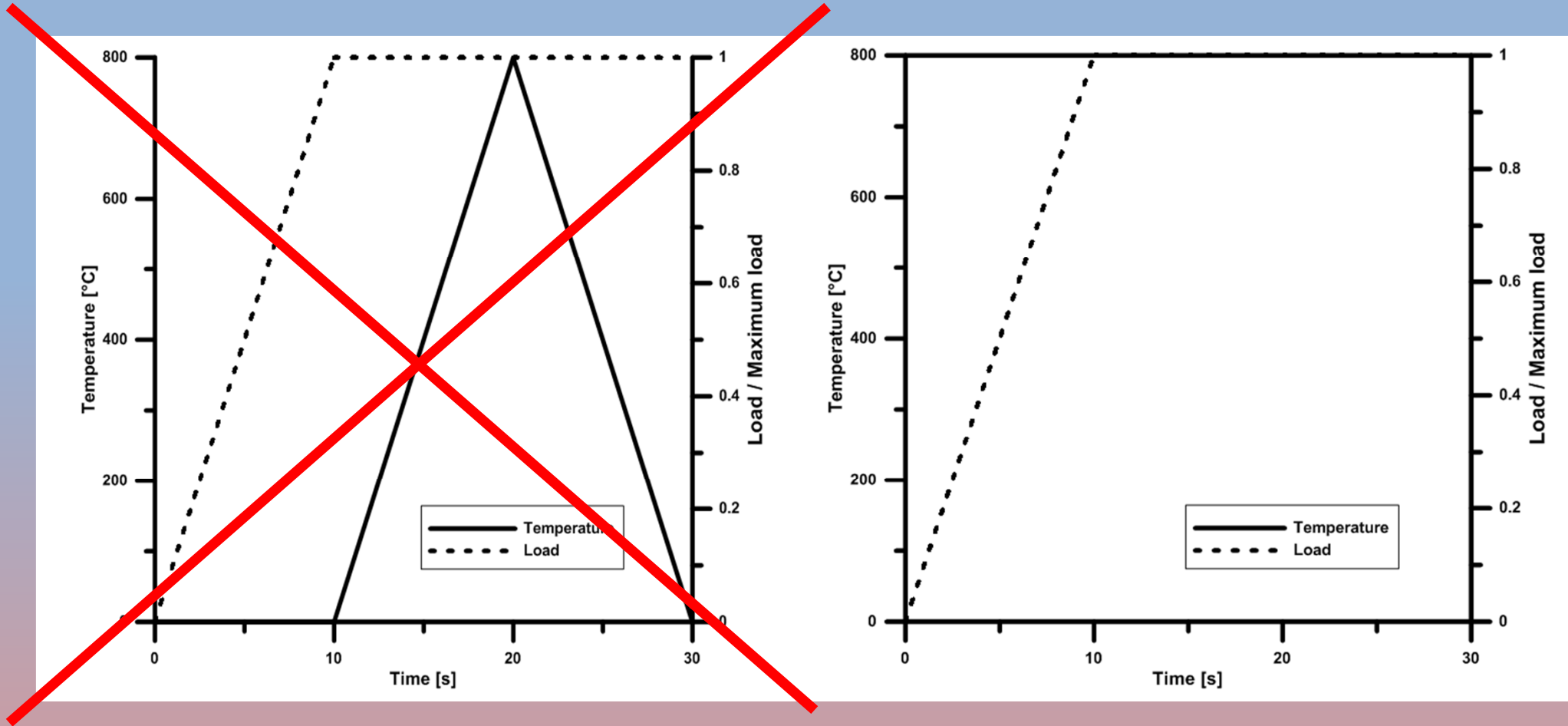
# Benchmarks for beams under elevated temperature

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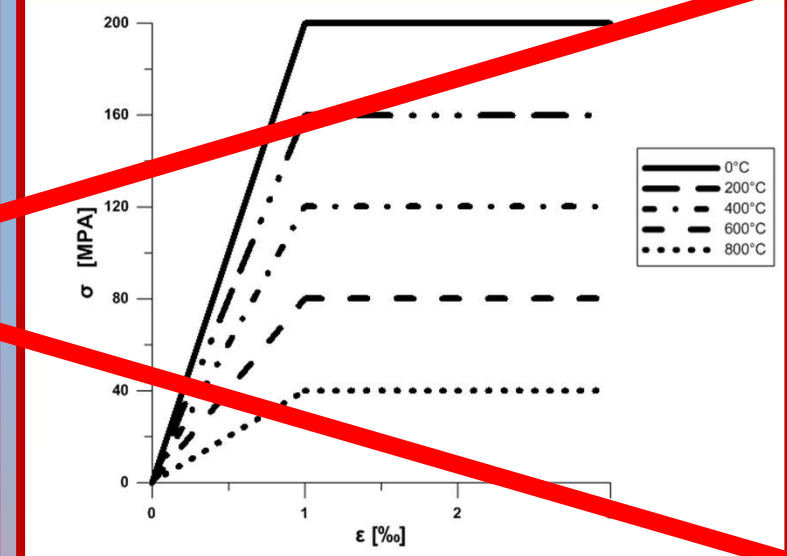
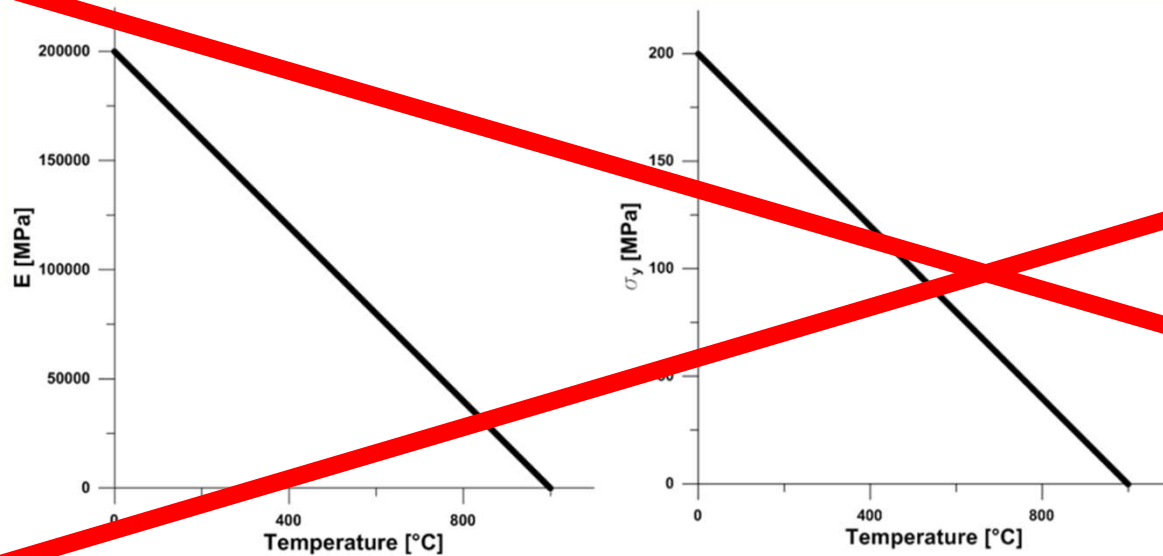
# Previous assumptions – temperature variaton



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# Previous assumptions – material properties

Yield stress and Young modulus are linearly temperature dependent



Change of material properties  
under elevated temperature

$$E=40\text{GPa}$$

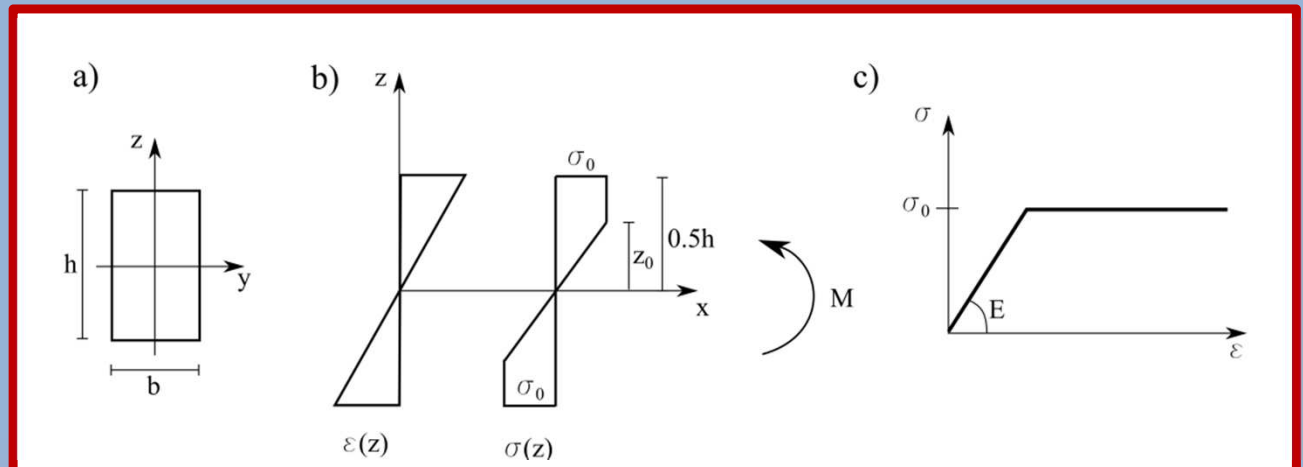
Stress-strain curves  
at chosen temperatures

$$f_y=40\text{MPa}$$

# Main assumptions of analytical solutions

## Assumptions:

- cross sections stay planar,
- the effect of shear is neglected,
- the approximate formula (second derivative) for the curvature can be applied to find beam deflection.

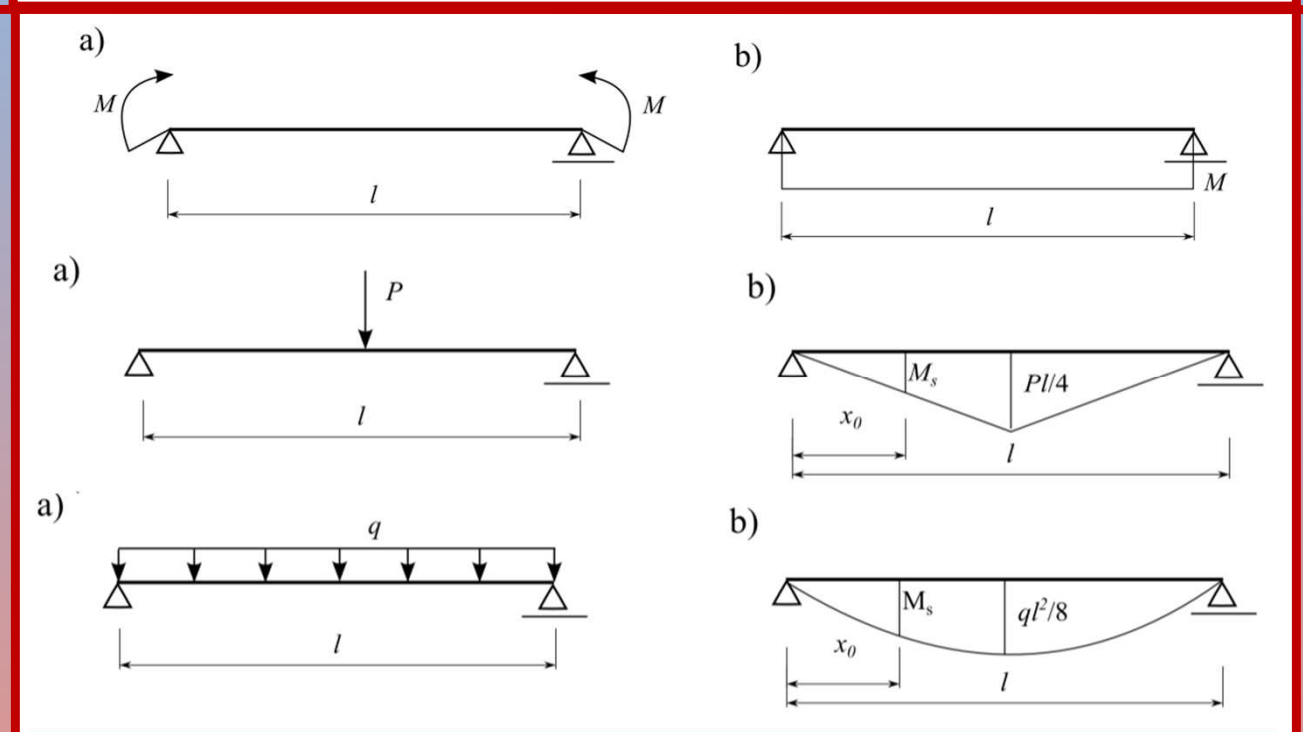


$$f = \frac{1}{8\sqrt{3}} \frac{\sigma_y l^2}{Eh} \left( \frac{1}{4} - \frac{1}{6} \mu \right)^{-1/2}$$

$$\mu = \frac{M}{\sigma_y \frac{bh^2}{6}}$$

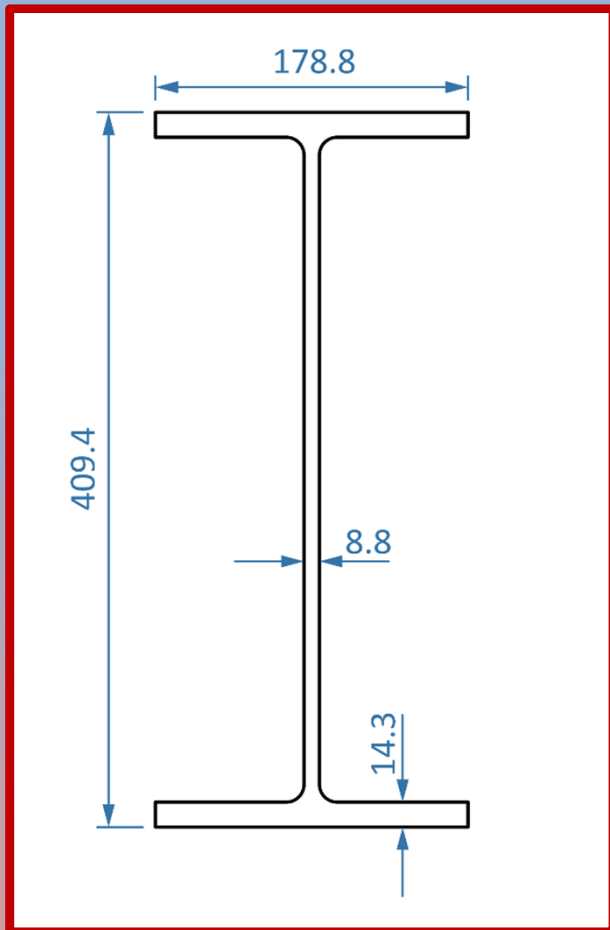
$$f = \frac{1}{6} \frac{\sigma_y l^2}{Eh} \frac{3\sqrt{3-2\psi} + \psi\sqrt{3-2\psi} - 5}{\psi^2}$$

$$\psi = \frac{P}{\frac{2}{3} \frac{\sigma_y bh^2}{l}}$$

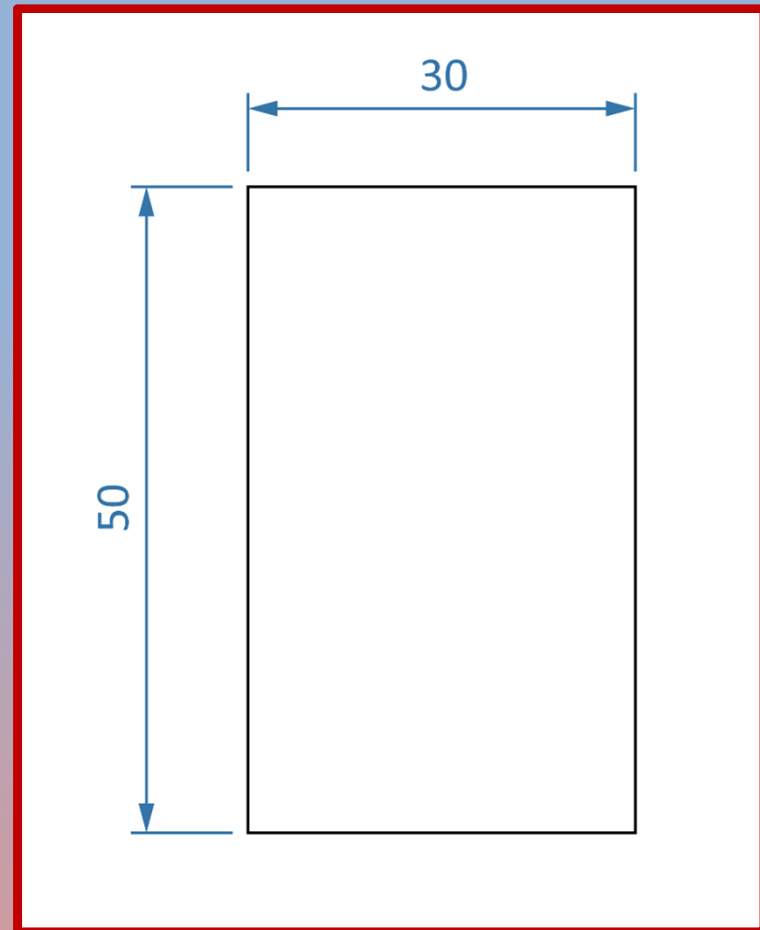


# Cross-sections

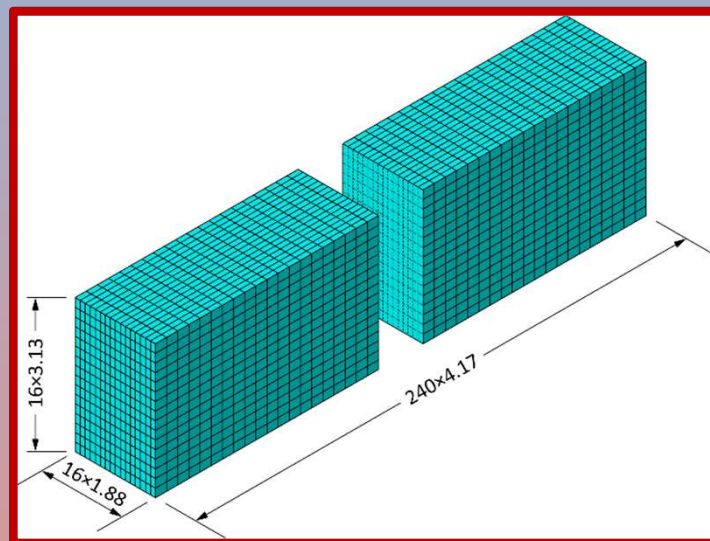
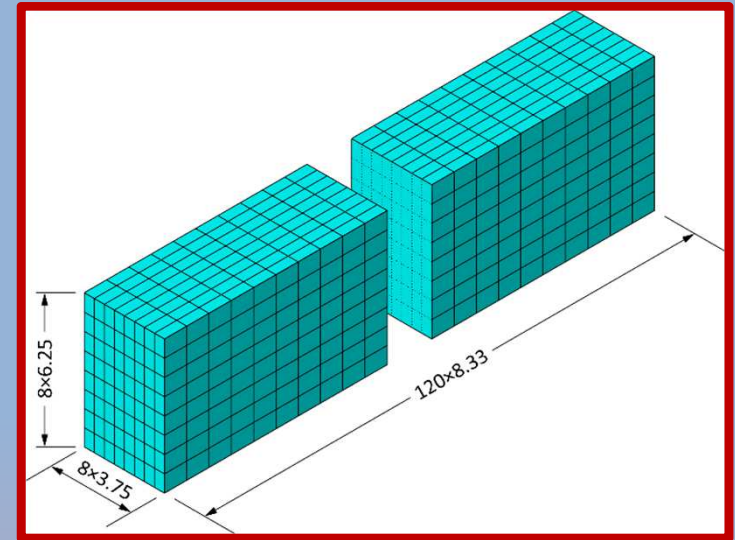
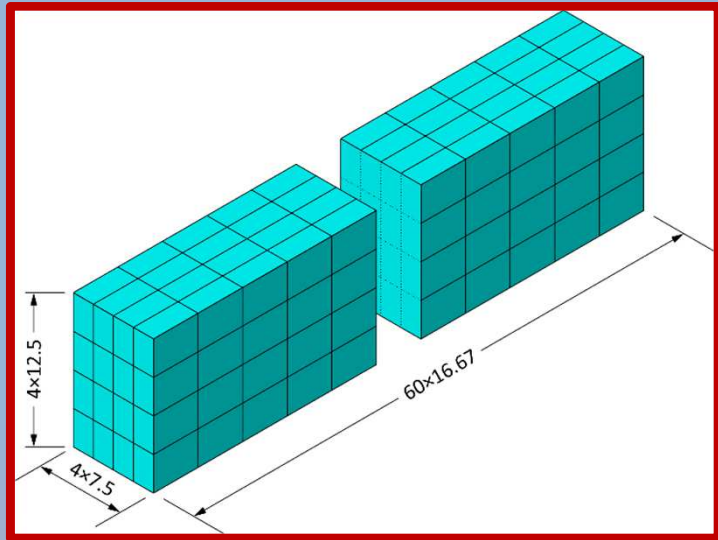
**UB 406×178×67 profile**



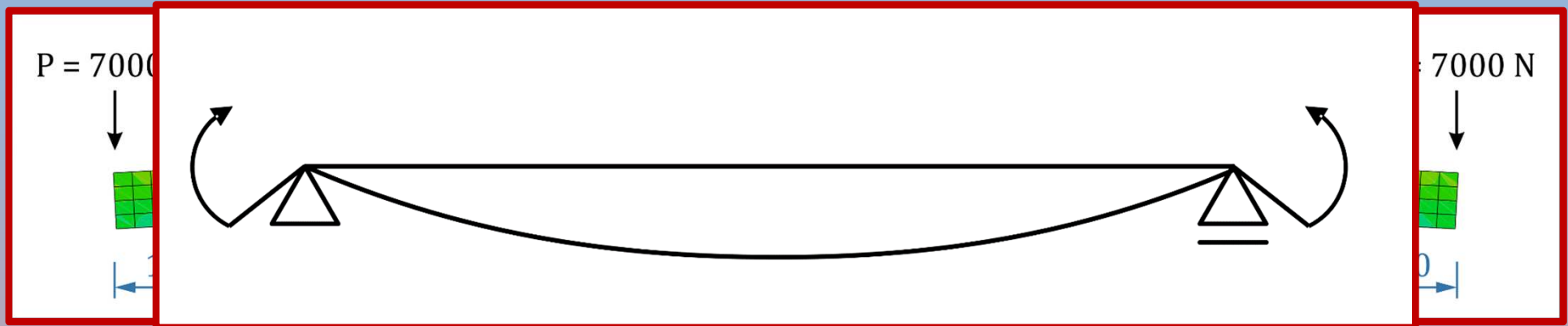
**Rectangular cross-section 30×50**



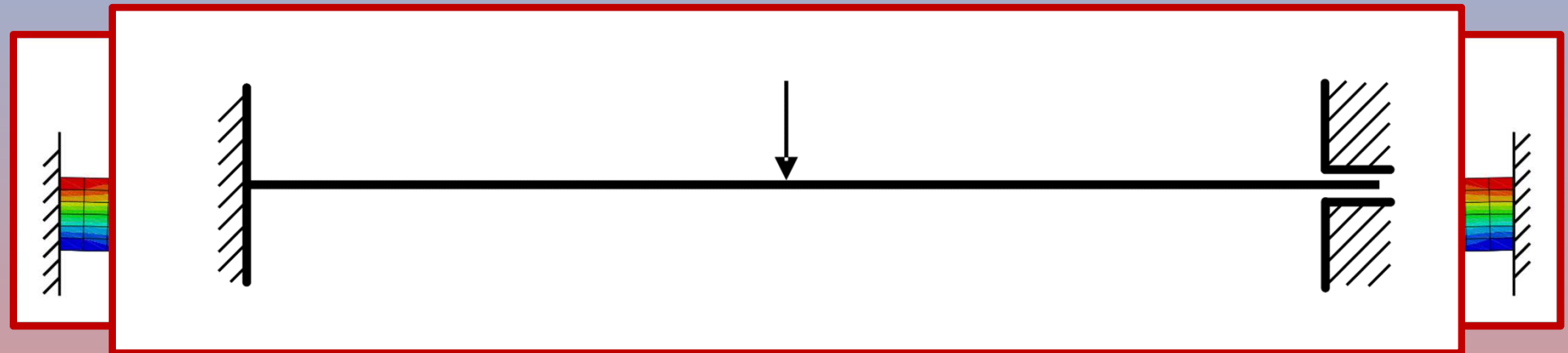
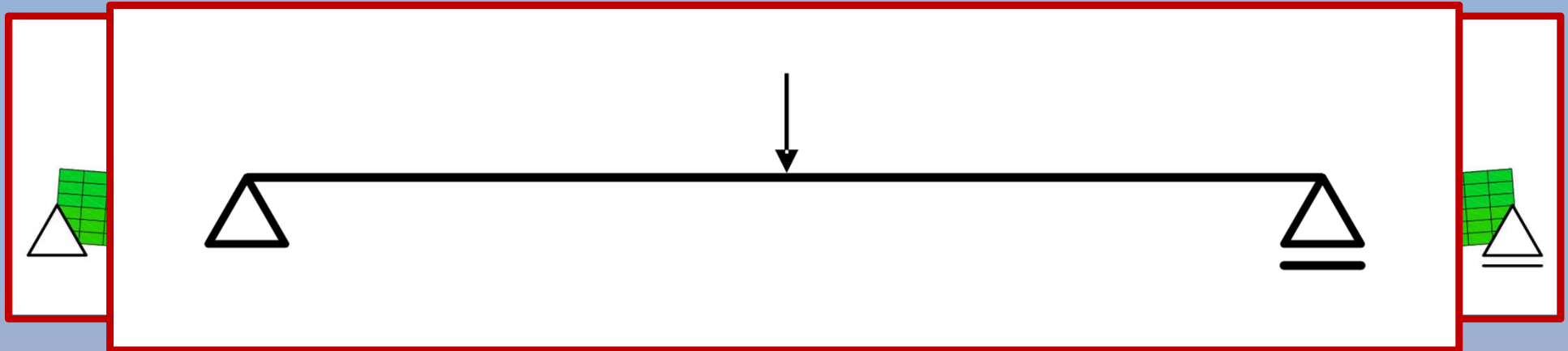
# Mesh densities



# Loading case 1 – pure bending

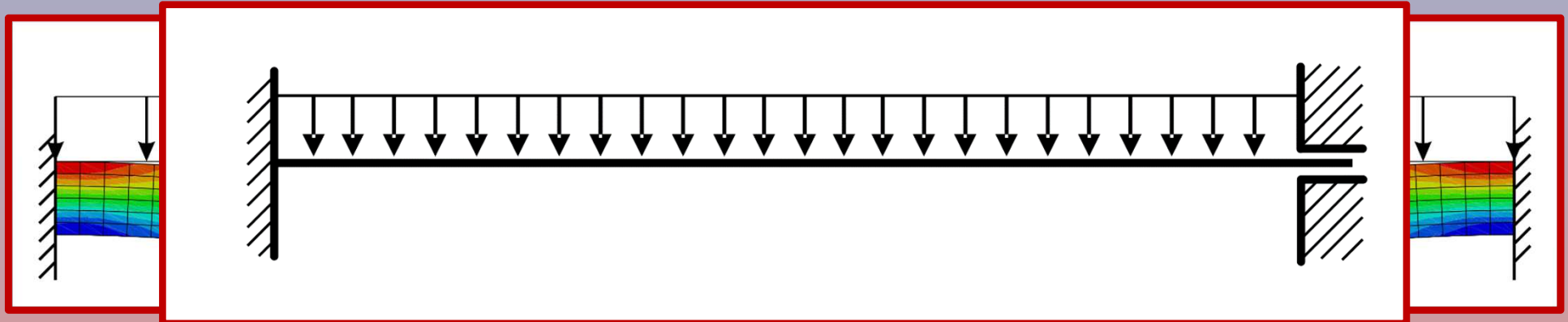
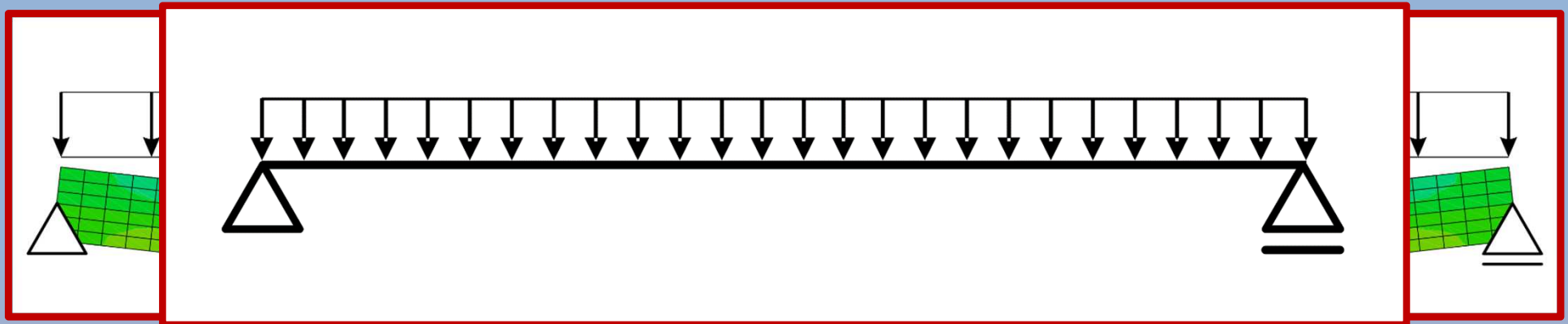


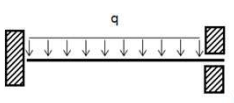
# Loading case 2 & 3 – point load





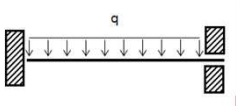
# Loading case 4 & 5 – distributed load



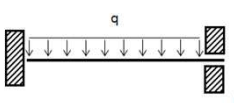


# Results – rectangular cross-section, k=0.80


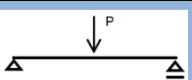
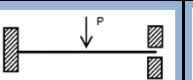
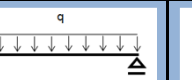
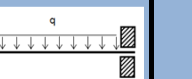
RECTANGULAR CROSS-SECTION (l×h×b) 1000×50×30mm, E=40GPa, $\sigma_y=40\text{MPa}$ , N=0 (no longitudinal constraints)								
J=312500mm <sup>4</sup> , M <sub>el</sub> = 500000Nmm, M <sub>u</sub> =750000Nmm								
			#	1	2	3	4	5
			BC					
			$M=M_{el}+0.8\cdot(M_u-M_{el})$	M=700000 Nmm	P=2800 N	P=5600 N	q=5.6 N/mm	q=8.4 N/mm
Analytical			f <sub>el</sub> [mm]	7.0000	4.6667	2.3333	5.8333	1.7500
			f [mm]	$\frac{Ml^2}{8EJ}$	$\frac{Pl^3}{48EJ}$	$\frac{Pl^3}{192EJ}$	$\frac{5ql^4}{384EJ}$	$\frac{ql^4}{384EJ}$
LS-DYNA	Solid	Mesh 3 (16.65× 12.5× 7.5)	f <sub>el</sub> [mm]	6.8753	4.5870	2.3270	5.7232	1.7425
			f [mm]	9.6829	4.8339	2.4212	6.5639	1.7631
		Mesh 2 (8.3× 6.3× 3.7)	f <sub>el</sub> [mm]	6.9970	4.6697	2.3722	5.8112	1.7768
			f [mm]	11.0130	5.0932	2.5523	7.0554	1.8193
		Mesh 1 (4.2× 3.2× 1.9)	f <sub>el</sub> [mm]	7.0303	4.6923	2.3848	5.8563	1.7866
			f [mm]	11.8070	5.1690	2.5905	7.3645	1.8354
	Shell	Mesh 3 (16.65 ×7.5)	f <sub>el</sub> [mm]	6.9959	4.6944	2.3823	5.8587	1.7850
			f [mm]	10.6410	5.1699	2.5705	7.3691	1.8272
		Mesh 2 (8.3×3. 7)	f <sub>el</sub> [mm]	6.9960	4.6955	2.3862	5.8608	1.7883
			f [mm]	10.6410	5.1758	2.5947	7.3755	1.8377
		Mesh 1 (4.2×1. 9)	f <sub>el</sub> [mm]	6.9962	4.6958	2.3874	5.8613	1.7894
			f [mm]	10.6420	5.1780	2.6024	7.3776	1.8431

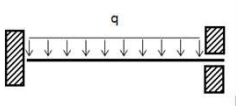


		VULCAN									
ABAQUS	Solid	Mesh 3 (33.3× 33.3× 5.5× 3.3)	$f_{el}$ [mm]	7.0841	4.7610	2.4352	5.9416	1.8264			
			f [mm]	10.9827	5.2611	2.6955	7.5532	1.9033			
		Mesh 2 (33.3× 2.5× 1.5)	$f_{el}$ [mm]	7.0143	4.7143	2.4119	5.8833	1.8090			
			f [mm]	11.0547	5.2081	2.6591	7.4779	1.8805			
		Mesh 1 (33.3× 2.0× 1.2)	$f_{el}$ [mm]	7.0080	4.7101	2.4098	5.8781	1.8074			
			f [mm]	11.1824	5.2000	2.6559	7.4825	1.8783			
	Solid	Mesh 3 (16.65× 12.5× 7.5)	$f_{el}$ [mm]	7.4579	4.8383	2.4625	6.0337	1.8398			
			f [mm]	14.3010	5.3121	2.6736	7.5906	1.8879			
		Mesh 2 (8.3× 6.3× 3.7)	$f_{el}$ [mm]	7.1071	4.7820	2.4365	5.9638	1.8195			
			f [mm]	11.5924	5.3089	2.6659	7.6962	1.8690			
		Mesh 1 (4.2× 3.2× 1.9)	$f_{el}$ [mm]	7.0245	4.7279	2.4104	5.8963	1.7996			
			f [mm]	11.0694	5.2339	2.6330	7.4909	1.8527			
	Shell	Mesh 3 (16.65 ×7.5)	$f_{el}$ [mm]	6.9960	4.7006	2.3973	5.8649	1.7969			
			f [mm]	10.6423	5.1738	2.5901	7.3749	1.8398			
		Mesh 2 (8.3× 3.7)	$f_{el}$ [mm]	6.9961	4.7017	2.3994	5.8669	1.7984			
			f [mm]	10.6416	5.1790	2.6051	7.3802	1.8476			
		Mesh 1 (4.2× 1.9)	$f_{el}$ [mm]	6.9961	4.7019	2.4000	5.8674	1.7988			
			f [mm]	10.6416	5.1799	2.6095	7.3820	1.8502			
	Beam	Mesh 3 (16.7)	$f_{el}$ [mm]	6.9968	4.7016	2.4040	5.8664	1.8031			
			f [mm]	8.9900	5.2169	2.6609	7.0034	1.8866			
		Mesh 2 (8.3)	$f_{el}$ [mm]	6.9968	4.7019	2.4045	5.8676	1.8034			
			f [mm]	8.9900	5.2184	2.6623	7.0056	1.8872			
		Mesh 1 (4.2)	$f_{el}$ [mm]	6.9968	4.7019	2.4046	5.8679	1.8035			
			f [mm]	8.9900	5.2186	2.6630	7.0062	1.8874			

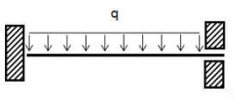


# Results – rectangular cross-section, k=0.95

RECTANGULAR CROSS-SECTION (l×h×b) 1000×50×30mm, E=40GPa, $\sigma_y=40$ MPa, N=0 (no longitudinal constraints)								
J=312500mm <sup>4</sup> , M <sub>el</sub> = 500000Nmm, M <sub>u</sub> =750000Nmm								
			#	1	2	3	4	5
			BC					
			<b>M=M<sub>el</sub>+0.95(M<sub>u</sub>-M<sub>el</sub>)</b>	M=737500 Nmm	P=2950 N	P=5900 N	q=5.9 N/mm	q=8.85 N/mm
Analytical			f <sub>el</sub> [mm]	7.3750	4.9167	2.4583	6.1458	1.8437
			$f$ [mm]	$\frac{Ml^2}{8EJ}$	$\frac{Pl^3}{48EJ}$	$\frac{Pl^3}{192EJ}$	$\frac{5ql^4}{384EJ}$	$\frac{ql^4}{384EJ}$
LS-DYNA	Solid	Mesh 3 (16.65× 12.5× 7.5)	f <sub>el</sub> [mm]	7.2467	4.8329	2.4517	6.0298	1.8359
			f [mm]	12.8190	5.4056	2.6726	7.8911	1.8782
		Mesh 2 (8.3× 6.3× 3.7)	f <sub>el</sub> [mm]	7.3748	4.9205	2.8765	6.1124	1.8720
			f [mm]	23.4980	5.8757	2.5523	9.1407	1.9493
		Mesh 1 (4.2× 3.2× 1.9)	f <sub>el</sub> [mm]	7.4087	4.9442	2.5129	6.1699	1.8823
			f [mm]	23.3990	6.0624	2.9454	10.2480	1.9716
	Shell	Mesh 3 (16.65 ×7.5)	f <sub>el</sub> [mm]	7.3711	4.9460	2.5100	6.1752	1.8806
			f [mm]	22.3510	6.0048	2.8973	11.0570	1.9598
		Mesh 2 (8.3×3. 7)	f <sub>el</sub> [mm]	7.3706	4.9477	2.5141	6.1747	1.8841
			f [mm]	22.3180	6.1508	2.9485	11.1080	1.9825
		Mesh 1 (4.2×1. 9)	f <sub>el</sub> [mm]	7.3700	4.9480	2.5157	6.1752	1.8852
			f [mm]	22.3530	6.1627	2.9780	11.1240	1.9888




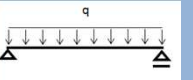
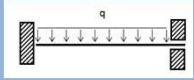


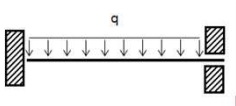
		VULCAN									
ABAQUS	Solid	Mesh 3 (33.3× 33.3× 5.5× 3.3)	$f_{el}$ [mm]	7.4633	5.0160	2.5657	6.2598	1.9243			
			f [mm]	20.7379	6.3268	3.2396	11.0557	2.0593			
		Mesh 2 (33.3× 2.5× 1.5)	$f_{el}$ [mm]	7.3897	4.9668	2.5411	6.1984	1.9059			
			f [mm]	20.1379	6.1889	3.1211	10.8031	2.0413			
		Mesh 1 (33.3× 2.0× 1.2)	$f_{el}$ [mm]	7.3830	4.9624	2.5389	6.1928	1.9042			
			f [mm]	20.3720	6.1732	3.1243	10.7609	2.0376			
	Solid	Mesh 3 (16.65× 12.5× 7.5)	$f_{el}$ [mm]	7.8570	5.0975	2.5944	6.3569	1.9384			
			f [mm]	18.1445	6.6285	3.0713	12.0794	2.0319			
		Mesh 2 (8.3× 6.3× 3.7)	$f_{el}$ [mm]	7.4875	5.0381	2.5670	6.2832	1.9170			
			f [mm]	25.3703	6.3609	3.0458	11.9572	2.0126			
		Mesh 1 (4.2× 3.2× 1.9)	$f_{el}$ [mm]	7.4005	4.9811	2.5395	6.2121	1.8960			
			f [mm]	21.3933	6.1998	3.0122	10.8401	1.9913			
	Shell	Mesh 3 (16.65 ×7.5)	$f_{el}$ [mm]	7.3704	4.9524	2.5258	6.1790	1.8931			
			f [mm]	22.3227	6.0595	2.9243	11.0846	1.9769			
		Mesh 2 (8.3× 3.7)	$f_{el}$ [mm]	7.3705	4.9535	2.5279	6.1811	1.8948			
			f [mm]	22.3196	6.1197	2.9549	11.1126	1.9911			
		Mesh 1 (4.2× 1.9)	$f_{el}$ [mm]	7.3705	4.9538	2.5285	6.1816	1.8952			
			f [mm]	22.3194	6.1339	2.9644	11.1207	1.9940			
	Beam	Mesh 3 (16.7)	$f_{el}$ [mm]	7.3713	4.9535	2.5328	6.1806	1.8996			
			f [mm]	9.7376	5.6347	2.8729	7.5919	2.0096			
		Mesh 2 (8.3)	$f_{el}$ [mm]	7.3713	4.9537	2.5333	6.1818	1.9000			
			f [mm]	9.7376	5.6357	2.8741	7.5943	2.0102			
		Mesh 1 (4.2)	$f_{el}$ [mm]	7.3713	4.9538	2.5334	6.1822	1.9001			
			f [mm]	9.7376	5.6360	2.8746	7.5949	2.0108			



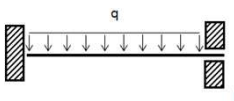
# Results – I-beam, k=0.80

UB 406×178×67 cross-section, l=8000mm, E=40GPa,  $\sigma_v=40$ MPa, N=0 (no longitudinal constraints)  
 $J=2.4015 \times 10^8 \text{ mm}^4$ ,  $M_{el}=4.6927 \times 10^7 \text{ Nmm}$ ,  $M_f=4.8915 \times 10^7 \text{ Nmm}$ ,  $M_u=5.3169 \times 10^7 \text{ Nmm}$

			#	1	2	3	4	5
			BC					
			$M=M_{el}+0.8 \cdot (M_u-M_{el})$	$M=5.1921 \times 10^7 \text{ Nmm}$	$P=25960.3 \text{ N}$	$P=51920.6 \text{ N}$	$q=6.49008 \text{ N/mm}$	$q=9.73511 \text{ N/mm}$
Analytical			$f_{el} \text{ [mm]}$	7.0000	4.6667	2.3333	5.8333	1.7500
			$f \text{ [mm]}$	11.1803	5.1569	2.5785	7.4301	1.8216
LS-DYNA	Shell	Mesh 3 (133× 100× 45)	$f_{el} \text{ [mm]}$	42.5210	29.3000	16.0730	36.9340	12.2290
			$f \text{ [mm]}$	61.6370	29.8120	16.1580	43.6440	12.2550
		Mesh 2 (66×50 ×23)	$f_{el} \text{ [mm]}$	42.5210	29.3100	16.1050	36.9490	12.2540
			$f \text{ [mm]}$	61.2590	29.9480	16.3230	44.1600	12.3320
		Mesh 1 (33×25 ×11)	$f_{el} \text{ [mm]}$	42.5210	29.3140	16.1190	36.9520	12.2620
			$f \text{ [mm]}$	61.4380	30.0060	16.4400	44.1820	12.3920
VULCAN		Mesh 3 (133.3× 34×15)	$f_{el} \text{ [mm]}$	43.3154	29.3631	15.3998	36.5825	11.5499
			$f \text{ [mm]}$	77.6325	30.8757	16.1075	45.2064	11.7563
		Mesh 2 (133.3× 20×9)	$f_{el} \text{ [mm]}$	43.2670	29.3294	15.3829	36.5404	11.5373
			$f \text{ [mm]}$	77.1231	30.7664	16.1002	44.9249	11.7369
		Mesh 1 (133.3× 17× 17.5)	$f_{el} \text{ [mm]}$	43.2595	29.3244	15.3804	36.5342	11.5354
			$f \text{ [mm]}$	76.8991	30.7611	16.0814	44.8710	11.7310


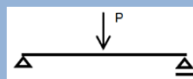
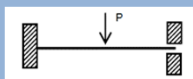
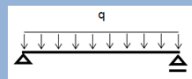
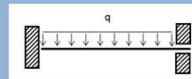


ABAQUS		Shell	Mesh 3 (133.3× 100× 45)	$f_{el}$ [mm]	43.0331	29.646	16.2621	37.4070	12.3754	
				$f$ [mm]	72.4345	30.2998	16.4472	48.4766	12.4048	
ABAQUS	Shell	Mesh 2 (66×50 ×23)	$f_{el}$ [mm]	42.6592	29.4112	16.1583	37.1111	12.2921		
			$f$ [mm]	62.3708	30.2167	16.4490	45.0364	12.3911		
		Mesh 1 (33×25 ×11)	$f_{el}$ [mm]	42.5674	29.3562	16.1344	37.0413	12.2720		
			$f$ [mm]	62.7691	30.1374	16.5224	44.6249	12.4236		
		ABAQUS	Beam	Mesh 3 (133.3)	$f_{el}$ [mm]	43.2761	29.7669	16.2490	36.9713	12.1868
					$f$ [mm]	69.8233	31.4506	17.0838	45.1008	12.3395
Mesh 2 (66.6)	$f_{el}$ [mm]			43.2761	29.7640	16.2432	36.9745	12.1825		
	$f$ [mm]			69.8232	31.4515	17.0858	45.1594	12.4153		
Mesh 1 (33.3)	$f_{el}$ [mm]			43.2761	29.7633	16.2418	36.9753	12.1814		
	$f$ [mm]			69.8232	31.4518	17.0863	45.1674	12.4150		

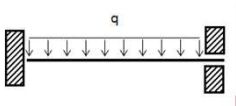


# Results – I-beam, k=0.95

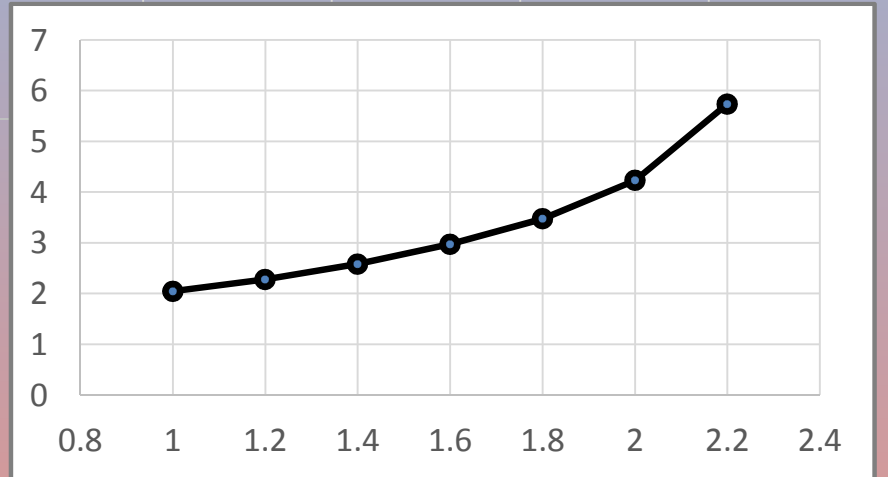
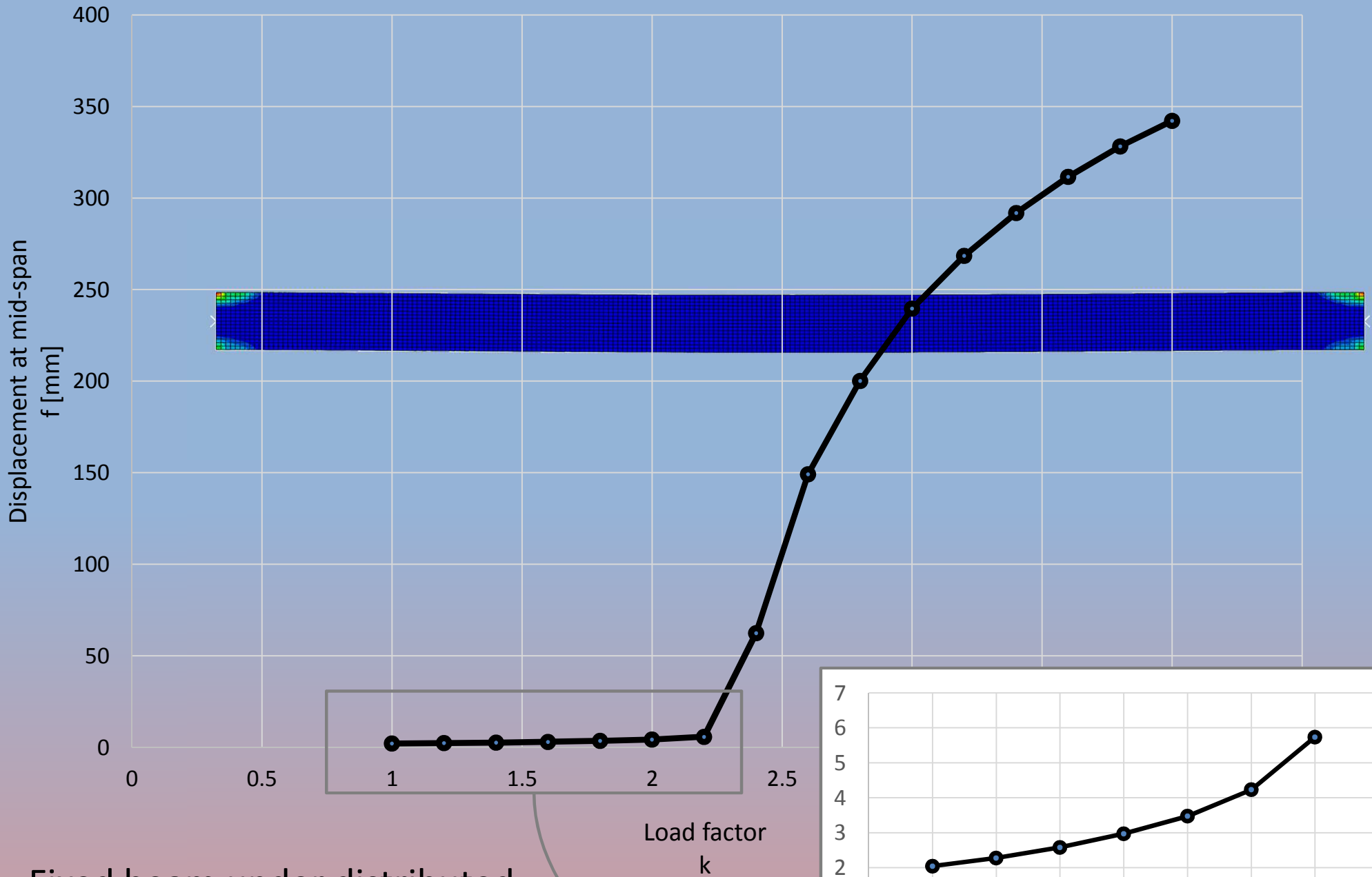
UB 406×178×67 cross-section, l=8000mm, E=40GPa,  $\sigma_y=40\text{MPa}$ , N=0 (no longitudinal constraints)  
 $J=2.4015 \times 10^8 \text{ mm}^4$ ,  $M_{el}=4.6927 \times 10^7 \text{ Nmm}$ ,  $M_f=4.8915 \times 10^7 \text{ Nmm}$ ,  $M_u=5.3169 \times 10^7 \text{ Nmm}$

		#	1	2	3	4	5	
		BC						
		$M=M_{el}+0.95(M_u-M_{el})$	$M=5.28569 \times 10^7 \text{ Nmm}$	$P=26428.5 \text{ N}$	$P=52856.9 \text{ N}$	$q=6.6071 \text{ N/mm}$	$q=9.910668 \text{ N/mm}$	
Analytical		$f_{el} \text{ [mm]}$	44.0203	29.3469	14.6733	36.683	11.0050	
		$f \text{ [mm]}$	155.111	31.5555	16.7100	57.0720	11.3768	
LS-DYNA	Shell	Mesh 3 (133× 100× 45)	$f_{el} \text{ [mm]}$	43.3010	29.8310	16.3670	37.6000	12.4520
			$f \text{ [mm]}$	87.9010	31.1390	16.6170	52.4430	12.4990
		Mesh 2 (66×50 ×23)	$f_{el} \text{ [mm]}$	43.3010	29.8410	16.4000	37.6150	12.4780
			$f \text{ [mm]}$	89.6360	31.2260	16.8950	55.0960	12.6280
		Mesh 1 (33×25 ×11)	$f_{el} \text{ [mm]}$	43.3010	29.8450	16.4140	37.6190	12.4860
			$f \text{ [mm]}$	88.2980	31.3030	17.1040	55.1600	12.7250
VULCAN		Mesh 3 (133.3× 34×15)	$f_{el} \text{ [mm]}$	44.0960	29.8926	15.6775	37.2420	11.7582
			$f \text{ [mm]}$	133.3375	33.6005	17.2481	60.5958	12.1158
		Mesh 2 (133.3× 20×9)	$f_{el} \text{ [mm]}$	43.6569	29.8583	15.5800	37.1991	11.7453
			$f \text{ [mm]}$	135.9685	33.2939	17.2415	--	12.0991
		Mesh 1 (133.3× 17× 17.5)	$f_{el} \text{ [mm]}$	44.0392	29.8532	15.6578	37.1928	11.7434
			$f \text{ [mm]}$	--	33.2742	17.2419	--	12.0944





ABAQUS				$f_{el}$ [mm]						
Shell	Mesh 3 (133.3× 100× 45)			43.8114	30.1812	16.5554	38.0912	12.6014		
				f [mm]	126.1600	31.4241	16.9272	63.0917	12.6622	
		Mesh 2 (66×50 ×23)			$f_{el}$ [mm]	43.4307	29.9422	16.4497	37.7899	12.5165
					f [mm]	97.2991	31.4816	17.1062	59.3565	12.7171
		Mesh 1 (33×25 ×11)			$f_{el}$ [mm]	43.3370	29.8862	16.4253	37.7188	12.4961
					f [mm]	92.6288	31.5062	17.2266	57.2381	12.7714
	Beam	Mesh 3 (133.3)			$f_{el}$ [mm]	44.0561	30.3037	16.5420	37.6379	12.4066
					f [mm]	78.0258	32.9218	17.8441	49.1606	12.7333
		Mesh 2 (66.6)			$f_{el}$ [mm]	44.0561	30.3008	16.5362	37.6411	12.4022
					f [mm]	78.0258	33.0015	17.8461	49.2409	12.7383
		Mesh 1 (33.3)			$f_{el}$ [mm]	44.0561	30.3001	16.5347	37.6419	12.4011
					f [mm]	78.0258	33.0020	17.8861	49.2501	12.7691



Fixed beam under distributed loading

$$M_{max} = M_{el} + k \cdot (M_u - M_{el})$$

# Fixed beam under distributed loading

$$M_{max} = M_{el} + k \cdot (M_u - M_{el})$$

